Some High-precision Relation Equations of Physical Constants

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Abstract

In this paper, we find some relation equations of physical constants. Their calculated results are coincidentally consistent with the measured values in the laboratory or recommended values for 2018 CODATA. They are: the anomalous magnetic moment of the electron, the anomalous magnetic moment of the muon, the muon mass, the fine structure constant, the formula for the universal gravitational constant, etc.

Introduction

The relations in this paper have no derivation process, they are just some calculation results with coincidence relationship. We go directly to the results.

1. The formula for the universal gravitational constant:

$$G = \frac{ke^2}{g_n} \frac{\lambda_p^7}{a_0^7} \frac{m_n}{m_p^3} \frac{1}{9\alpha\pi^9} \tag{1}$$

where:

G is the Newtonian constant of gravitation.

k is the Coulomb constant.

e is the elementary charge.

λ_p is the Compton wavelength of the proton.

m_n is the mass of the neutron.

m_p is the mass of the proton.

g_n is the spin g-factor of the neutron.

a_0 is the Bohr radius.

α is the fine structure constant.

π is the pi.

The result of equation (1) is $G = 6.67434039314 \times 10^{-11} m^3/kg \cdot s^2$.

The recommended value for 2018 CODATA is $G = 6.67430 \times 10^{-11} m^3/kg \cdot s^2$.

2. The anomalous magnetic moment of the electron a_e:

$$a_e = \frac{\alpha}{2\pi} - \frac{\alpha^2}{\pi^3} - \frac{9\pi}{2}\alpha^4 - \left(\frac{\pi}{3}\right)^4 \alpha^5 \tag{2}$$

The result of equation (2) is $a_e = 0.001159652181247$.

According to the 2018 CODATA recommendations, the anomalous magnetic moment of the electron is:

$$a_e = \frac{g_e - 2}{2} = 0.00115965218128. \tag{3}$$

g_e is the spin g-factor of the electron.

3. The anomalous magnetic moment of the muon a_μ:

$$a_\mu = \frac{\alpha}{2\pi} + \frac{\alpha^2}{4\pi} + \frac{2}{3}\alpha^3 + 5\alpha^4 \tag{4}$$

The result of equation (4) is $a_\mu = 0.001165920582$.

The latest laboratory measurement of the muon anomalous magnetic moment [1] is $a_\mu = 0.00116592061$.

A change to equation (4) gives:

$$a_\mu = \frac{\alpha}{2\pi} + \frac{\alpha^2}{4\pi} + \frac{g_e}{3}\alpha^3 + 5\alpha^4 \tag{5}$$

The result of equation (5) is $a_\mu = 0.001165920883$.

According to the 2018 CODATA recommendations, the muon anomalous magnetic moment is:

$$a_\mu = \frac{g_\mu - 2}{2} = 0.0011659209 \tag{6}$$

g_μ is the spin g-factor of the muon. If we replace g_e with g_μ in equation (5). then, we can get that the formula of g_μ is:

$$\frac{g_\mu}{2} = \frac{1 + \frac{\alpha}{2\pi} + \frac{\alpha^2}{4\pi} + 5\alpha^4}{1 - \frac{2}{3}\alpha^3} \tag{7}$$

The result of equation (7) is $g_\mu = 2 \times 1.001165920884$.

4. The mass of the muon m_μ:

$$\frac{m_e}{m_\mu} = 2\pi\alpha^2 \left[\frac{\pi\alpha m_p^2}{4\left(m_n - m_p\right)^2}\right]^{\frac{1}{3}} \left(1 + \frac{m_e}{m_p}\frac{4}{g_n\sqrt{g_\mu}}\right) \tag{8}$$

m_e is the mass of the electron.

The result of equation (8) is $m_\mu/m_e = 206.768283004$.

The 2018 CODATA recommendation is $m_\mu/m_e = 206.768283$.

5. The spin g-factor of the proton g_p:

$$g_p = \frac{m_p}{m_n}\left(1 + \frac{m_e}{m_n - m_p}\right)\frac{g_e g_\mu}{1 + \sqrt{2g_e}\alpha^3/\pi} \tag{9}$$

In equation (9), we take the value of g_μ as the latest laboratory measurement, which is $g_\mu = 2 \times 1.00116592061$, instead of $g_\mu = 2 \times 1.0011659209$.

So, we get that the result of equation (9) is $g_p = 5.5856946892547$.

The recommended value for 2018 CODATA is $g_p = 5.58569468926$.

6. The fine structure constant α:

$$\begin{cases} \dfrac{\pi^4 + \pi^3 + \pi^2}{A} = \alpha \\ \left(\dfrac{\pi}{A-1}\right)^3 \alpha^2 - \dfrac{\pi}{A-1} = \dfrac{m_p}{m_e} + 3g_{el}^4 \dfrac{m_e}{m_p} \end{cases} \tag{10}$$

g_{el} is the orbital g-factor of the electron; A is a dimensionless parameter.

The calculation result of equations (10) is $\alpha = 137.03599908396$.

The recommended value for 2018 CODATA is $\alpha = 137.035999084$.

If equation (10) does not have that $3g_{el}^4 m_e/m_p$, the result of its calculation is $\alpha = 137.035998773$.

7. The equation for the spin g-factor of the electron and muon:

$$g_e \times \left[1 + \frac{\pi\alpha(1 + 4\alpha/\pi^2)}{2m_p/m_e}\right] = g_\mu \tag{11}$$

The result of equation (11) is $g_\mu = 2 \times 1.001165920662$.

The latest laboratory measurement is $g_\mu = 2 \times 1.00116592061$.

8. The spin g-factor relationship between electron, muon, proton and neutron：

$$\frac{\pi\alpha g_e^3 g_n^3 g_p}{\left(g_e+g_p\right)^2}-\frac{m_e}{8m_p}\frac{m_n^2}{m_p^2}\frac{\sqrt{\alpha^3}g_e g_\mu^2}{\sqrt{1+a_e/\pi^2}}=1 \tag{12}$$

a_e is the electron anomalous magnetic moment from equation (3).

In equation (12), we take the value of g_n as $g_n = 3.82608545$, if you take the value of g_n as $g_n = 3.82608546$, then you will get a different result than ours.

The calculation result on the left side of equation (12) is $1.000\ 000\ 000\ 000\ 0601$.

$$\frac{\pi\alpha g_e^3 g_n^3 g_p}{\left(g_e+g_p\right)^2}=1.00000034160615 \tag{13}$$

The last

The values of physical constants in this paper are taken from 2018 CODATA, except for those specifically accounted for. Since we need to perform high-precision calculations, there are requirements for the value of each physical constant, because any slight change may cause a large error in the calculation results.

References

[1] arXiv:2104.03281