Towards Settling Doubly Special Relativity*

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Abstract

Seven months ago (viXra:2201.0082) I put forward a method to resolve the contradiction between the existence of Planck units (length, time, mass) and Special Relativity (length contraction, time dilation, relativistic mass). In this note, as a timestamp, I present the final solution, but postpone the derivation and implications to a detailed future treatment.

Keywords— length contraction, Planck length, time dilation, Planck time, Doubly Special Relativity, Deformed Special Relativity

It is known that the existence of Planck length and time is in contradiction with fundamental results of special relativity, i.e. *Lorentz-FitzGerald contraction* (length contraction) and time dilation[1, 2, 3, 4, 5]. In a previous attempt[6] I approached the problem from a blindly-formal perspective, but Nature can be more subtle than what formal reasoning can achieve. Although that attempt of mine is not satisfactory to myself, I (and for that matter nobody) have not still come up with a completely satisfactory solution, leaving [6] the only work that at least gets *some* transformations.

About seven months ago [7] I adopted a fresh perspective which I am optimist will settle the issue:
There is *but one* simple straightforward way to resolve this contradiction if we are to avoid adding an extra dimension: we must modify length contraction, time dilation and 'relativistic mass' in the following manner

$$\begin{cases} L' = L\gamma(v)f\big(L,v\big), \text{ s.t. } f(l_P,v) = 1/\gamma, \\ T' = T\gamma(v)g\big(T,v\big), \text{ s.t. } g(t_P,v) = 1/\gamma, \\ M' = M\gamma(v)h\big(M,v\big), \text{ s.t. } g(M_P,v) = 1/\gamma, \end{cases} \tag{1}$$

for unknown sufficiently-smooth functions $\zeta, \xi : \mathbb{R} \to \mathbb{R}$. As we expect to recover special relativity for lengths $L \gg l_P$, time durations $T \gg t_P$ and masses $M \gg M_P$, we must impose the condition

$$\begin{cases} \lim_{L\to\infty} f(L,v) = 1, \\ \lim_{T\to\infty} g(T,v) = 1, \\ \lim_{M\to\infty} h(M,v) = 1. \end{cases} \tag{2}$$

I shall now make a further assumption that

$$\begin{cases} f(L,v) = \phi(\frac{v}{L}), \\ g(T,v) = \Gamma(\frac{v}{L}), \\ h(M,v) = \eta(\frac{v}{L}). \end{cases} \tag{3}$$

Justification of this assumption will be done in the promised full treatment in future.

The final solution is

$$\boxed{\begin{aligned} f(L,v) = \phi(\frac{v}{L}) &= \sqrt{1-(\frac{vt_P}{L})^2} \\ g(T,v) = \Gamma(\frac{v}{L}) &= \sqrt{1-(\frac{v}{Ta_P})^2} \\ h(M,v) = \eta(\frac{v}{L}) &= \sqrt{1-(\frac{vM_P}{Mc})^2} \end{aligned}} \tag{4}$$

The functions presented satisfy (2) as required.

As a result,

$$\boxed{E = mc^2\gamma\sqrt{1-(M_P/c)^2(\frac{v}{m})^2}} \tag{5}$$

Naturally the energy of a particle with Planck mass would be Planck energy.

References

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