Ultimate Acceleration in Quantum Mechanics to Obtain Spin

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Abstract: In analogy with the ultimate speed c, there is an ultimate acceleration β, nobody's acceleration can exceed this limit β, for electrons and quarks, β=2.327421e+29(m/s^2). Because this ultimate acceleration is a large number, any effect connecting to β will become easy to test, including quantum gravity tests. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, the spin concept can be derived out from the matter wave. This paper also carefully explains how the matter wave to display its spin effect in Stern-Gerlach experiments. It is completely a new aspect to quantum mechanics for the relativistic matter wave to contain spin.

1. Introduction

This year is 100th anniversary of the initiative of de Broglie's matter wave. In 1922, the Louis de Broglie considered blackbody radiation as a gas of light quanta [1], he tried to reconcile the concept of light quanta with the phenomena of interference and diffraction. In 1923 and 1924, the concept that matter behaves like a wave was proposed by Louis de Broglie [2,3]. It is also referred to as the de Broglie hypothesis, matter waves are referred to as de Broglie waves.

Using ultimate acceleration, this paper shows that matter wave has spin. In analogy with the ultimate speed c, there is an ultimate acceleration β, nobody's acceleration can exceed this limit β.

At the first, de Broglie matter wave is generalized in terms of the ultimate acceleration, and applied to the solar system to explain quantum gravity effects. Consider a particle, its relativistic matter wave is given by the path integral

$$\psi = \exp\left(\frac{i\beta}{c^3}\int_0^x (u_1 dx_1 + u_2 dx_2 + u_3 dx_3 + u_4 dx_4)\right) \quad . \tag{1}$$

where u is the 4-velocity of the particle, β is the ultimate acceleration determined by experiments. Because this ultimate acceleration is a large number, any effect connecting to β will become easy to test, including quantum gravity tests. This paper show that the generalized matter wave can quantize the solar system correctly.

Next, electronic matter wave is re-examined using the ultimate acceleration, consequently, the spin concept can be derived out from the matter wave. This paper also carefully explains how the matter wave to display its spin effect in Stern-Gerlach experiments. It is completely a new aspect to quantum mechanics for the relativistic matter wave to contain spin.

2. Extracting ultimate acceleration from the solar system

In the orbital model as shown in Fig.1(a), the orbital circumference is n multiple of the wavelength of the relativistic matter wave, according to Eq. (1), consider a planet, we have

$$\left.\begin{aligned} \frac{\beta}{c^3}\oint_L v_l dl = 2\pi n \\ v_l = \sqrt{\frac{GM}{r}} \end{aligned}\right\} \Rightarrow \sqrt{r} = \frac{c^3}{\beta\sqrt{GM}} n; \quad n = 0,1,2,\ldots \quad . \tag{2}$$

This orbital quantization rule only achieves a half success in the solar system, as shown in Fig.1(b), the Sun, Mercury, Venus, Earth and Mars satisfy the quantization equation; while other outer planets fail. But, since we only study quantum gravity effects among the Sun, Mercury, Venus, Earth and Mars, so this orbital quantization rule is good enough as a foundational quantum theory. In Fig.1(b), the blue straight line expresses a linear regression relation among the quantized orbits, so it gives β=2.956391e+10 (m/s^2) by fitting the line. The quantum numbers n=3,4,5,... were assigned to the solar planets, the sun was assigned a quantum number n=0 because the sun is in the **central state**.

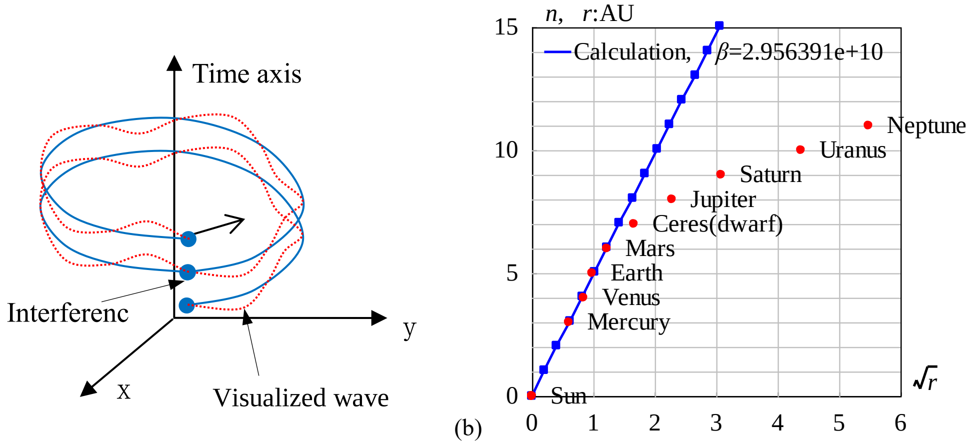

Fig.1 (a)The head of the relativistic matter wave may overlap with its tail. (b) The inner planets are quantized.

The relativistic matter wave can be applied to determine the solar density and radius. In a central state, if the coherent length of the relativistic matter wave is long enough, its head may overlap with its tail when the particle moves in a closed orbit, as shown in Fig.1(a). Consider a point on the solar equatorial plane, the overlapped wave is given by

$$\begin{aligned} &\psi = \psi(r)T(t) \\ &\psi(r) = 1 + e^{i\delta} + e^{i2\delta} + \ldots + e^{i(N-1)\delta} = \frac{1-\exp(iN\delta)}{1-\exp(i\delta)} \quad . \\ &\delta(r) = \frac{\beta}{c^3}\oint_L (v_l)dl = \frac{2\pi\beta\omega r^2}{c^3} \end{aligned} \tag{3}$$

where N is the overlapping number which is determined by the coherent length of the relativistic matter wave, δ is the phase difference after one orbital motion, ω is the angular speed of the solar self-rotation. The above equation is a multi-slit interference formula in optics, for a larger N it is called as the Fabry-Perot interference formula.

The relativistic matter wave function ψ needs a further explanation. In quantum mechanics, $|\psi|^2$ equals to the probability of finding an electron due to Max Burn's explanation; in astrophysics, $|\psi|^2$ equals to the probability of finding a nucleon (proton or neutron) *averagely on an astronomic scale*, we have

$$|\psi|^2 \propto \text{nucleon-density} \propto \rho \quad . \tag{4}$$

It follows from the multi-slit interference formula that the overlapping number N is estimated by

$$N^2 = \frac{|\psi(0)_{multi-wavelet}|^2}{|\psi(0)_{one-wavelet}|^2} = \frac{\rho_{core}}{\rho_{surface\_gas}} \quad . \tag{5}$$

The solar core has a mean density of 1408 (kg/m^3), the surface of the sun is comprised of convective zone with a mean density of 2e-3 (kg/m^3) [18]. In this paper, the sun's radius is chosen at a location where density is 4e-3 (kg/m^3), thus the solar overlapping number N is calculated to be N=593. Since the mass density ρ has spherical symmetry, then the ψ has the spherical symmetry.

Sun's angular speed at its equator is known as ω=2π/(25.05x24x3600) (s^{-1}). Its mass 1.9891e+30 (kg), well-known radius 6.95e+8 (m), mean density 1408 (kg/m^3), the constant β=2.956391e+10 (m/s^2). According to the N=593, the matter distribution of the $|\psi|^2$ is calculated in Fig.2(a), it agrees well with the general description of star's interior. The radius of the sun is determined as r=7e+8 (m) with a relative error of 0.72% in Fig.2, which indicates that the sun radius strongly depends on the sun's self-rotation.

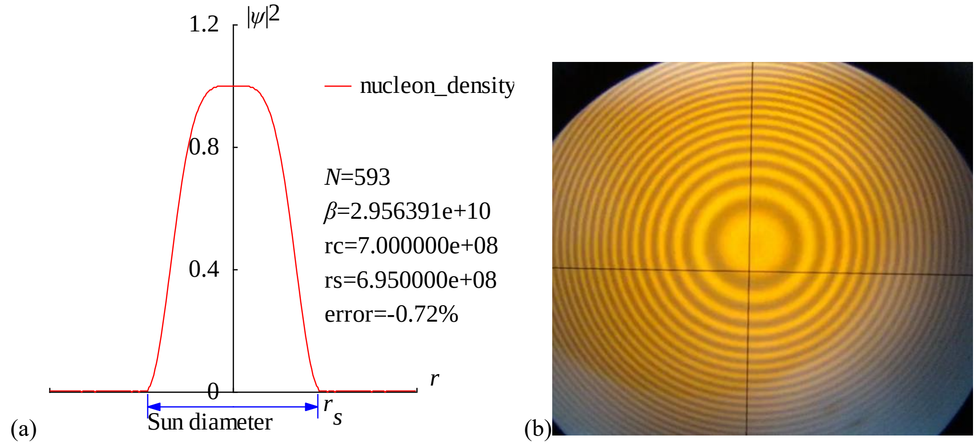

Fig.2 (a)The nucleon distribution $|\psi|^2$ in the Sun is calculated in the radius direction. (b) As contrast, sodium Fabry-Perot interference (δ=const.).

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<Clet2020 Script>//C source code [17]
int i,j,k,m,n,N,nP[10];
double beta,H,B,M,r,r_unit,x,y,z,delta,D[1000],S[1000], a,b,rs,rc,omega,atm_height; char str[100];
main(){k=150;rs=6.95e8;rc=0;x=25.05;omega=2*PI/(x*24*3600);n=0; a=1408/0.004; N=sqrt(a);
beta=2.956391e10;H=SPEEDC*SPEEDC*SPEEDC/beta;M=1.9891E30; atm_height=2e6; r_unit=1E7;
for(i=-k;i<k;i+=1) {r=abs(i)*r_unit;
if(r<rs+atm_height) delta=2*PI*omega*r*r/H; else delta=2*PI*sqrt(GRAVITYC*M*r)/H;//around the star
x=1;y=0; for(j=1;j<N;j+=1) { z=delta*j; x+=cos(z);y+=sin(z);} z=x*x+y*y; z=z/(N*N);
S[n]=i;S[n+1]=z; if(i>0 && rc==0 && z<0.0001) rc=r; n+=2;}
SetAxis(X_AXIS,-k,0,k,"#ifr; ; ; ;");SetAxis(Y_AXIS,0,0,1.2,"#if|ψ|#su2#t;0;0.4;0.8;1.2;");
DrawFrame(FRAME_SCALE,1,0xafffaf); z=100*(rs-rc)/rs;
SetPen(1,0xff0000);Polyline(k+k,S,k/2,1," nucleon_density"); SetPen(1,0x0000ff);
r=rs/r_unit;y=-0.05;D[0]=-r;D[1]=y;D[2]=r;D[3]=y; Draw("ARROW,3,2,XY,10,100,10,10,",D);
Format(str,"#ifN#t=%d#n#ifβ#t=%e#nrc=%e#nrs=%e#nerror=%.2f%",N,beta,rc,rs,z);
TextHang(k/2,0.7,0,str);TextHang(r+5,y/2,0,"#ifr#sds#t");TextHang(-r,y+y,0,"Sun diameter");
}#v07=?>A
```

3. Extracting ultimate acceleration from the earth

The moon is assigned a quantum number of n=2 because some quasi-satellite's perigees have reached a depth almost at n=1 orbit, as shown in Fig.3. Here, the ultimate acceleration β=1.377075e+14(m/s^2) is determined uniquely by the line between the earth and moon by Eq. (2).

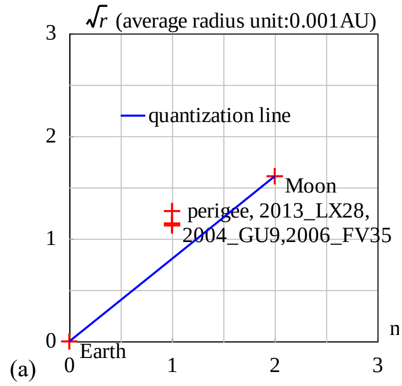

Fig.3 Orbital quantization for the moon.

The earth has a mean density of 5530 (kg/m^3), its surface is covered with air and vapor with a density of 1.29 (kg/m^3). The earth's radius is chosen at the sea level, it follows Eq.(5) that the earth's overlapping number N is calculated to be N=65.

The earth's angular speed is known as ω=2π/(24x3600) (s^{-1}), its mass 5.97237e+24 (kg), the well-known radius is 6.378e+6 (m), the earth's constant β=1.377075e+14 (m/s^2). The matter distribution $|\psi|^2$ in radius direction is calculated by Eq.(3), as shown in Fig.4(a). The radius of the earth is determined as r=6.4328e+6 (m) with a relative error of 0.86%, it agrees well with common knowledge. The secondary peaks over the atmosphere up to 2000 km altitude are calculated in Fig.4(b) which agree well with the space debris observations [19].

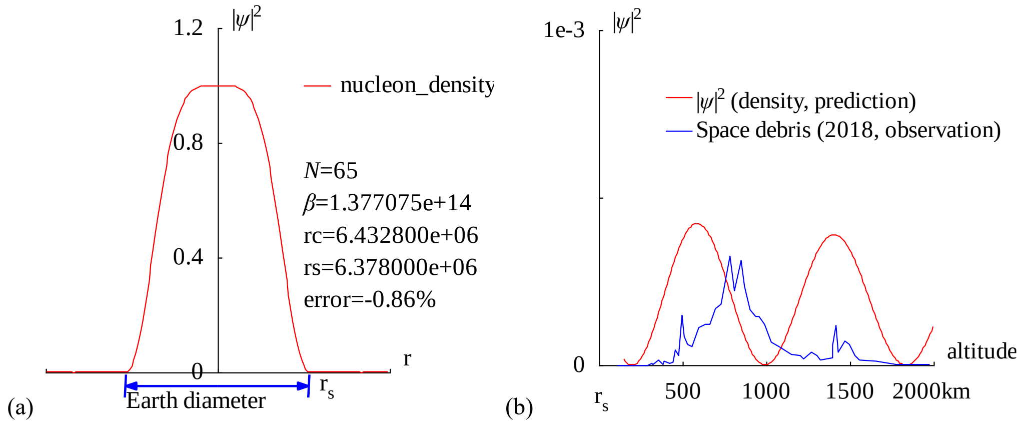

Fig.4 (a) The radius of the Earth is calculated out r=6.4328e+6 (m) with a relative error of 0.86% by the interference of its relativistic matter wave; (b) The space debris distribution up to 2000 km altitude.

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<Clet2020 Script>//C source code [17]
int i,j,k,m,n,N,nP[10]; double H,B,M,v_r,r,AU,r_unit,x,y,z,delta,D[10],S[1000];
double rs,rc,rot,a,b,atm_height,beta; char str[100];
main(){k=80;rs=6.378e6;rc=0;atm_height=1.5e5;n=0; N=65;
beta=1.377075e+14;H=SPEEDC*SPEEDC*SPEEDC/beta;
M=5.97237e24;AU=1.496E11;r_unit=1e-6*AU; rot=2*PI/(24*60*60);//angular speed of the Earth
for(i=-k;i<k;i+=1) {r=abs(i)*r_unit;
if(r<rs+atm_height) v_r=rot*r*r; else v_r=sqrt(GRAVITYC*M*r);//around the Earth
delta=2*PI* v_r/H; y=SumJob("SLIT_ADD,@N,@delta",D); y=y/(N*N);
if(y>1) y=1; S[n]=i;S[n+1]=y; if(i>0 && rc==0 && y<0.001) rc=r;    n+=2;}
```

```
SetAxis(X_AXIS,-k,0,k,"r; ; ; ;");SetAxis(Y_AXIS,0,0,1.2,"#if|ψ|#su2#t;0;0.4;0.8;1.2;");
DrawFrame(FRAME_SCALE,1,0xafffaf); x=50;z=100*(rs-rc)/rs;
SetPen(1,0xff0000);Polyline(k+k,S,k/2,1," nucleon_density");
r=rs/r_unit;y=-0.05;D[0]=-r;D[1]=y;D[2]=r;D[3]=y;
SetPen(2,0x0000ff); Draw("ARROW,3,2,XY,10,100,10,10,",D);
Format(str,"#ifN#t=%d#n#ifβ#t=%e#nrc=%e#nrs=%e#nerror=%.2f%",N,beta,rc,rs,z);
TextHang(k/2,0.7,0,str);TextHang(r+5,y/2,0,"r#sds#t");TextHang(-r,y+y,0,"Earth diameter");
}#v07=?>A#t
<Clet2020 Script>//C source code [9]
int i,j,k,m,n,N,nP[10]; double H,B,M,v_r,r,AU,r_unit,x,y,z,delta,D[10],S[10000];
double rs,rc,rot,a,b,atm_height,p,T,R1,R2,R3; char str[100]; int
Debris[96]={110,0,237,0,287,0,317,2,320,1,357,5,380,1,387,4,420,2,440,3,454,14,474,9,497,45,507,26,527,19,557,17,597,34,63
4,37,664,37,697,51,727,55,781,98,808,67,851,94,871,71,901,50,938,44,958,44,991,37,1028,21,1078,17,1148,10,1202,9,1225,6,
1268,12,1302,9,1325,5,1395,7,1395,18,1415,36,1429,12,1469,22,1499,19,1529,9,1559,5,1656,4,1779,1,1976,1,};
main(){k=80;rs=6.378e6;rc=0;atm_height=1.5e5;n=0; N=65;
H=1.956611e11;M=5.97237e24;AU=1.496E11;r_unit=1e4;
rot=2*PI/(24*60*60);//angular speed of the Earth
b=PI/(2*PI*rot*rs*rs/H); R1=rs/r_unit;R2=(rs+atm_height)/r_unit;R3=(rs+2e6)/r_unit;
for(i=R2;i<R3;i+=1) {r=abs(i)*r_unit; delta=2*PI*sqrt(GRAVITYC*M*r)/H;
y=SumJob("SLIT_ADD,@N,@delta",D); y=1e3*y/(N*N);// visualization scale:1000
if(y>1) y=1; S[n]=i;S[n+1]=y;n+=2;}
SetAxis(X_AXIS,R1,R1,R3,"altitude; r#sds#t;500;1000;1500;2000km ;");
SetAxis(Y_AXIS,0,0,1,"#if|ψ|#su2#t;0; ;1e-3;");DrawFrame(FRAME_SCALE,1,0xafffaf); x=R1+(R3-R1)/5;
SetPen(1,0xff0000);Polyline(n/2,S,x,0.8,"#if|ψ|#su2#t (density, prediction)");
for(i=0;i<48;i+=1) {S[i+i]=R1+(R3-R1)*Debris[i+i]/2000; S[i+i+1]=Debris[i+i+1]/300;}
SetPen(1,0x0000ff);Polyline(48,S,x,0.7,"Space debris (2018, observation) "); }#v07=?>A#t
```

4. Extracting electronic ultimate acceleration with quantum theory

In the relativity, the speed of light c is an ultimate speed, nobody's speed can exceed this limit c. The relativistic velocity u of a particle in the coordinate system ($x_1,x_2,x_3,x_4=ict$) satisfies

$$u_1^2+u_2^2+u_3^2+u_4^2=-c^2 \quad . \tag{6}$$

No matter what particles (electrons, molecules, neutrons, quarks), their 4-vector velocities all have the same magnitude: $|u|=ic$. All particles gain **equality** because of the same magnitude of the 4-velocity u. The acceleration a of a particle is given by

$$a_1^2+a_2^2+a_3^2=a^2; \quad (a_4=0; \quad \because x_4=ict) \tag{7}$$

Assume that particles have an ultimate acceleration β as limit, no particle can exceed this acceleration limit β. Subtracting the both sides of the above equation by β^2, we have

$$a_1^2+a_2^2+a_3^2-\beta^2=a^2-\beta^2; \qquad a_4=0 \tag{8}$$

It can be rewritten as

$$[{a_1}^2+{a_2}^2+{a_3}^2+0+(i\beta)^2]\frac{1}{1-a^2/\beta^2}=-\beta^2 \tag{9}$$

Now, the particle subjects to an acceleration whose five components are specified by

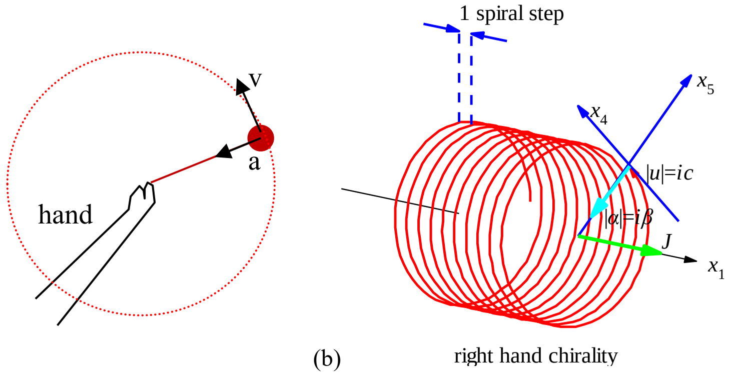
$$\begin{aligned} &\alpha_1=\frac{a_1}{\sqrt{1-a^2/\beta^2}}; \quad \alpha_2=\frac{a_2}{\sqrt{1-a^2/\beta^2}}; \\ &\alpha_3=\frac{a_3}{\sqrt{1-a^2/\beta^2}}; \quad \alpha_4=0; \quad \alpha_5=\frac{i\beta}{\sqrt{1-a^2/\beta^2}}; \end{aligned} \tag{10}$$

where α_5 is the newly defined acceleration in five dimensional space-time ($x_1,x_2,x_3,x_4=ict,x_5$). Thus, we have

$$\alpha_1^2+\alpha_2^2+\alpha_3^2+\alpha_4^2+\alpha_5^2=-\beta^2; \quad \alpha_4=0 \tag{11}$$

It means that the magnitude of the newly defined acceleration α for every particle takes the same value: $|\alpha|=i\beta$ (constant imaginary number), all particle accelerations gain **equality** for the sake of the same magnitude.

How to resolve the velocity u and acceleration α into x, y, and z components? In realistic world, a hand can rotate a ball moving around a circular path at constant speed v with constant centripetal acceleration a, as shown in Fig.5(a).



(a) (b)

Fig.5 (a) A hand rotates a ball moving around a circular path at constant speed v with constant centripetal acceleration a. (b) The particle moves along the x_1 axis with the constant speed $|u|=ic$ in the u direction and constant centripetal force in the x_5 axis at the radius iR (imaginary number).

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<Clet2020 Script>// Clet is a C compiler [17]
double D[100],S[2000];int i,j,R,X,N;
int main(){R=50;X=50;N=600;D[0]=-50;D[1]=0;D[2]=0;D[3]=X;D[4]=0;D[5]=0;D[6]=-50;D[7]=R;D[8]=0;
D[9]=600;D[10]=10;D[11]=R;D[12]=0;D[13]=3645;
Lattice(SPIRAL,D,S);SetViewAngle(0,80,-50);DrawFrame(FRAME_NULL,1,0xffffff);
Draw("LINE,0,2,XYZ,0","-150,0,0,-50,0,0");Draw("ARROW,0,2,XYZ,10","50,0,0,150,0,0");
SetPen(2,0xff0000);Plot("POLYLINE,0,600,XYZ",S[9]);i=9+3*N-6;Draw("ARROW,0,2,XYZ,10",S[i]);
TextHang(S[i],S[i+1],S[i+2],"   #if|u|=ic#t");TextHang(150,0,0,"   #ifx#sd1#t");SetPen(2,0x005fff);
Draw("LINE,1,2,XYZ,8","-50,0,50,-50,0,100");Draw("LINE,1,2,XYZ,8","-40,0,50,-40,0,100");
Draw("ARROW,0,2,XYZ,10","-80,0,100,-50,0,100");Draw("ARROW,0,2,XYZ,10","-10,0,100,-40,0,100");
TextHang(-50,0,110,"1 spiral step");i=9+3*N;S[i]=50;S[i+1]=10;S[i+2]=10;
Draw("ARROW,0,2,XYZ,10","50,0,0,50,80,80");TextHang(50,80,80," #ifx#sd5#t");
Draw("ARROW,0,2,XYZ,10","50,72,0,50,0,72");TextHang(50,0,72,"   #ifx#sd4#t");
SetPen(3,0x00ffff);Draw("ARROW,0,2,XYZ,15",S[i-3]);TextHang(S[i],S[i+1],S[i+2],"   #if|α|=iβ#t");
SetPen(3,0x00ff00);Draw("ARROW,0,2,XYZ,15","50,0,0,120,0,0");TextHang(110,5,5," #ifJ#t");
TextHang(-60,0,-80," right hand chirality");}#v07=?>A
```

In analogy with the ball in a circular path, consider a particle in one dimensional motion along the x_1 axis at the speed v, in the Fig.5(b) it moves with the constant speed $|u|=ic$ almost along the x_4 axis and slightly along the x_1 axis, and the constant centripetal acceleration $|\alpha|=i\beta$ in the x_5 axis at the constant radius iR (imaginary number); the coordinate system (x_1,x_4=ict,x_5=iR) establishes a cylinder coordinate system in which this particle moves spirally at the speed v along the x_1 axis. According to usual centripetal acceleration formula $a=v^2/r$, the acceleration in the x_4-x_5 plane is given by

$$a=\frac{v^2}{r} \quad \Rightarrow \quad i\beta=\frac{|u|^2}{iR}=-\frac{c^2}{iR}=i\frac{c^2}{R} \quad . \tag{12}$$

Therefore, the track of the particle in the cylinder coordinate system (x_1,x_4=ict,x_5=iR) forms a shape, called as **acceleration-roll**. The faster the particle moves along the x_1 axis, the longer the spiral step is.

As like a steel spring with elastic wave, the track in the acceleration-roll in Fig.5(b) can be described by a wave function whose phase changes 2π for one spiral step. Apparently, this wave is the de Broglie's matter wave for electrons, protons or quarks, etc.

Proof: The wave function phase changes 2π for one spiral circumference $2\pi(iR)$, then a small displacement of the particle on the spiral track is $|u|d\tau=icd\tau$ in the 4-vector u direction, thus this wave phase along the spiral track is evaluated by

$$phase=\int_0^\tau \frac{2\pi}{2\pi(iR)}icd\tau=\int_0^\tau \frac{c}{R}d\tau \quad . \tag{13}$$

Substituting the radius R into it, the wave function ψ is given by

$$\psi=\exp(-i\cdot phase)=\exp(-i\int_0^\tau \frac{c}{R}d\tau)=\exp(-i\frac{\beta}{c}\int_0^\tau d\tau) \quad . \tag{14}$$

In the theory of relativity, we known that the integral along $d\tau$ needs to transform into realistic line integral, that is

$$\begin{aligned} d\tau &=-c^2\frac{d\tau}{-c^2}=(u_1^2+u_2^2+u_3^2+u_4^2)\frac{d\tau}{-c^2} \\ &=(u_1dx_1+u_2dx_2+u_3dx_3+u_4dx_4)\frac{1}{-c^2} \end{aligned} \quad . \tag{15}$$

Therefore, the wave function ψ is evaluated by

$$\begin{aligned} \psi &=\exp(-i\frac{\beta}{c}\int_0^\tau d\tau) \\ &=\exp(i\frac{\beta}{c^3}\int_0^x(u_1dx_1+u_2dx_2+u_3dx_3+u_4dx_4)) \end{aligned} \quad . \tag{16}$$

This wave function may have different explanations, depending on the particle under investigation. If the β is replaced by the Planck constant for electrons, the wave function is given by

$$\begin{aligned} &assume: \quad \beta=\frac{mc^3}{\hbar} \\ &\psi=\exp(\frac{i}{\hbar}\int_0^x(mu_1dx_1+mu_2dx_2+mu_3dx_3+mu_4dx_4)) \end{aligned} \quad . \tag{17}$$

where $mu_4dx_4=-Edt$, it strongly suggests that the wave function is just the de Broglie's matter wave [1,2,3].

In Fig.5(b), the acceleration-roll of the particle moves with two distinctions: **right-hand roll** (You curl the four fingers of your right hand along the spiral direction, the extended thumb, which is at a right angle to the fingers, points in the direction of x_1) and **left-hand roll** (You curl the four fingers of your left-hand along the spiral direction, the extended thumb, which is at a right angle to the fingers, points in the direction of x_1). The direction of the angular momentum J would be slightly different from the x_1 due to spiral precession. It is easy to calculate the ultimate acceleration β, the radius R and the angular momentum J in the plane x_4-x_5 for a spiraling electron as

$$\begin{aligned} &\beta=\frac{c^3m}{\hbar}=2.327421\text{e+29 (M/s}^2\text{)}; \quad R=\frac{c^2}{\beta}=3.861593\text{e-13 (M)} \\ &J=\pm m|u|iR=\mp\hbar \end{aligned} \quad . \tag{18}$$

```
<Clet2020 Script>// Clet is a C compiler [17]
double beta,R,J,m,D[10];char str[200];
int main(){m=ME;beta=SPEEDC*SPEEDC*SPEEDC*m/PLANCKBAR;
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```
R=SPEEDC*SPEEDC/beta;J=PLANCKBAR;Format(str,"beta=%e, #nR=%e, #nJ=%e",beta,R,J);
TextAt(50,50,str);ClipJob(APPEND,str);}#v07=#t
```

Tip: actually, ones cannot get to see the acceleration-roll of a particle in the relativistic space-time ($x_1,x_2,x_3,x_4=ict$) ; only get to see it in the cylinder coordinate system ($x_1,x_4=ict,x_5=iR$).

5. The first clue to spin and uncertainty from matter wave

Dimension is defined as the number of independent parameters in a mathematical space. In the field of physics, dimension is defined as the number of independent space-time coordinates. 0D is an infinitesimal point with no length. 1D is an infinite line, only length. 2D is a plane, which is composed of length and width. 3D is 2D plus height component, has volume.

In this section we at first discuss how to measure dimension by wave. In Fig.6(a), one puts earphone into ear, one gets 1D wave in the ear tunnel.

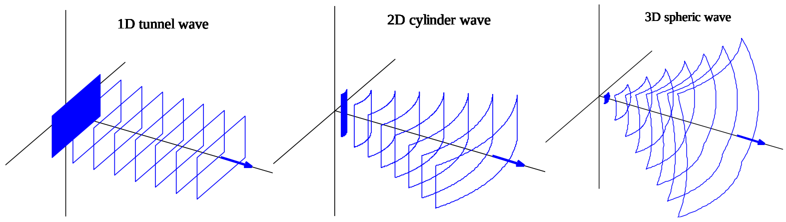
$$1D: \quad y = A\sin(kr - \omega t) = \frac{A}{r^0}\sin(kr - \omega t)\,. \tag{19}$$

where r is the distance between the wave emitter and the receiver. In Fig.2(b), one touches a guitar spring, one gets 2D cylinder wave.

$$2D: \quad y = \frac{A}{r^{1/2}}\sin(kr - \omega t)\,. \tag{20}$$

In Fig.2(c), one turns on a music speaker, one gets 3D spherical wave.

$$3D: \quad y = \frac{A}{r}\sin(kr - \omega t)\,. \tag{21}$$



(a) 1D tunnel wave (b) 2D cylinder wave (c) 3D spherical wave

Fig.6 The wave behavior in various dimensional spaces.

```
<Clet2020 Script>// Clet is a C compiler [17]
int i,j,k,type,nP[10]; double D[20],S[1000];
int main(){SetViewAngle("temp0,theta60,phi-30");SetAxis(X_AXIS,0,0,200,"X;0;200;");
DrawFrame(FRAME_LINE,1,0xafffaf); type=2;SetPen(1,0x00ff);
for(i=10;i<160;i+=20){D[0]=i;D[1]=0;D[2]=0;D[3]=i+5;D[4]=0;D[5]=0;D[6]=i;D[7]=10;D[8]=0;
if(type==0) {D[9]=4;D[10]=40;D[11]=20;D[12]=i;TextHang(50,0,100,"1D tunnel wave");k=CARD;}
else if(type==1) {D[9]=200;D[10]=i/2;D[11]=20;D[12]=i;TextHang(50,0,100,"2D cylinder wave");k=50;}
else {D[9]=200;D[10]=i/2;D[11]=i/2;D[12]=i;TextHang(50,0,100,"3D spheric wave");k=40;}
Lattice(k,D,S);nP[0]=POLYGON;nP[1]=0;nP[2]=200;nP[3]=XYZ;
if(i==10) nP[1]=3; if(type==0) nP[2]=4; Plot(nP,S[9]);}
j=30;D[3]=D[0]+j*S[0];D[4]=D[1]+j*S[1];D[5]=D[2]+j*S[2];
SetPen(3,0x00ff);Draw("ARROW,0,2,XYZ,10",D);}
#v07=?>A
```

In general, we can write a wave in the form

$$y = \frac{A}{r^w}\sin(kr - \omega t)\,. \tag{22}$$

It is easy to get the dimension of the space in where the wave lives, the dimension is D=2w+1. Nevertheless, wave can be used to measure the dimension of space, just by determining the parameter w.

Waves all contain a core oscillation (wavelength invariance, i.e. k is constant)

$$\frac{d^2y}{dr^2}+k^2y=0, \tag{23}$$

Substituting y into the core oscillation, we obtain the radial wave equation

$$\frac{d^2y}{dr^2}+\frac{2w}{r}\frac{dy}{dr}+(k^2+\frac{w(w-1)}{r^2})y=0. \tag{24}$$

This equation expresses the wave behavior modulated by the spatial dimension parameter w. For 1D wave w=0, it is trivial, but for 2D wave w=1/2, it reduces to the Bessel equation in a cylinder coordinate system (r,φ)

$$\frac{d^2y}{dr^2}+\frac{1}{r}\frac{dy}{dr}+(k^2-\frac{1}{4r^2})y=0 \quad \text{(2D wave)}$$

comparing to the Schrodinger's equation: (25)

$$\frac{d^2R(r)}{dr^2}+\frac{2}{r}\frac{dR(r)}{dr}+[k^2-\frac{l(l+1)}{r^2}]R(r)=0$$

In quantum mechanics, y is an electronic wave function, comparing to the Schrodinger radial wave equation in textbooks [9,10,11], we find that the $-1/4r^2$ term represents the electronic spin effect. However, here according to the above radial Bessel equation we can simply conclude: sound wave, electromagnetic wave, or any wave can have spin effect in 2D space! Let us use ***k*** denote the wave-vector, then the above 2D wave equation tells us

$$k\equiv\frac{2\pi}{\lambda};\quad k_r^{\ 2}=k^2-k_\varphi^{\ 2};\quad k_\varphi=\pm\frac{1}{2r}. \tag{26}$$

The k_φ causes the 2D wave-vector ***k*** to spin little by little as illustrated Fig.7. The positive and negative k_φ corresponds to spin up and spin down respectively; as r goes to the infinity, the spin effect vanishes off.

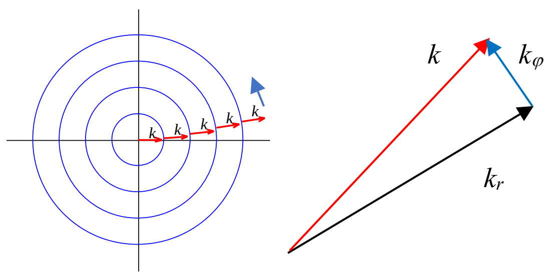


Fig.7 2D wave-vector ***k*** spins little by little in the cylinder coordinates (r,φ).

```
<Clet2020 Script>// Clet is a C compiler [17]
int i,j,k;double r,x,y,a,D[100];
int main(){DrawFrame(FRAME_LINE,1,0xafffaf); SetPen(1,0x0000ff);
for(i=0;i<90;i+=20) {D[0]=-i;D[1]=-i;D[2]=i;D[3]=i; Draw("ELLIPSE,0,2,XY,0",D);}
for(i=0;i<90;i+=20) {a=0.2*(i-i*i/200)*PI/180;r=i;D[0]=r*cos(a);D[1]=r*sin(a);
r+=18;D[2]=r*cos(a);D[3]=r*sin(a); SetPen(2,0xff0000);
Draw("ARROW,0,2,XY,8",D);TextHang(D[2]-10,D[3]+5,0,"#ifk");}
}#v07=?>A
```

If the 2D wave is the de Broglie matter wave for a particle beam, in a cylinder coordinate (r,φ), then the matter wave has a spin angular momentum given by

$$k_r=\frac{p_r}{\hbar};\quad k_\varphi=\frac{p_\varphi}{\hbar}=\frac{J_\varphi}{r\hbar};\quad J_\varphi=\pm\frac{1}{2}\hbar. \tag{27}$$

According to the angular momentum formula in general physics, it is recognized that the particle total momentum p is a constant given by

$$\left(\frac{p}{\hbar}\right)^2=\left(\frac{p_r}{\hbar}\right)^2+\left(\frac{p_\varphi}{\hbar}\right)^2 .$$
$$k^2=k_r{}^2+k_\varphi{}^2=const. \tag{28}$$

Since the particle total wave vector k is a constant, the wave-vector k_r must vary as r changes. The wave-vector in the radial direction would change as the wave attenuates.

For 3D wave in a spherical coordinate system (r, θ, φ), the modulated wave reduces to

$$\frac{d^2y}{dr^2}+\frac{2}{r}\frac{dy}{dr}+(k^2+0)y=0 \quad \text{(3D wave)} \cdot \tag{29}$$

It gives us an impression there is not spin effect, actually the impression is not true. Because spin effect belongs to 2D rotation in a plane, it is required for us to transform the 3D spherical wave into a 2D platform for observation, in the plane x-y as shown in Fig.8, we re-arrange the terms to fit the 2D cylinder Bessel equation as

$$\frac{d^2y}{dr^2}+\frac{1}{r}\frac{dy}{dr}+(k^2+\frac{1}{ry}\frac{dy}{dr})y=0 \quad . \tag{30}$$

(2D plateform for 3D wave, radial equation)

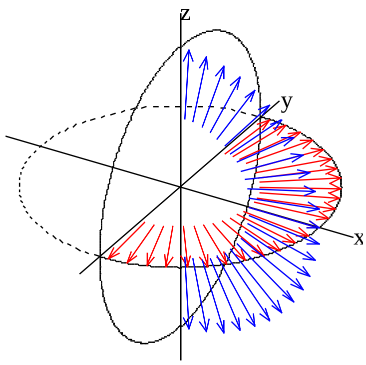


Fig.8 In the plane x-y as a 2D platform for observing the 3D spherical wave.

```
<Clet2020 Script>// Clet is a C compiler [17]
int i,j,k; double r,rot,x,y,z,dP[10],D[1000];
int main(){k=100;SetViewAngle("temp0,theta60,phi-30");
DrawFrame(FRAME_LINE,1,0xafffaf);Overlook("2,1,80", D);
r=80;k=12;rot=0.04*PI;SetPen(1,0xff0000);
for(i=-k;i<=k;i+=1) {x=r*cos(rot*i);y=r*sin(rot*i);z=0; D[0]=x/2;D[1]=y/2;D[2]=z/2;D[3]=x;D[4]=y;D[5]=z;
Draw("ARROW,0,2,XYZ,10",D);} SetPen(1,0x0000ff);//k=5;
for(i=-k;i<=k;i+=1) {y=0;x=r*cos(rot*i);z=r*sin(rot*i);D[0]=x/2;D[1]=y/2;D[2]=z/2;D[3]=x;D[4]=y;D[5]=z;
Draw("ARROW,0,2,XYZ,10",D);}r+=20;
TextHang(r,0,0,"x");TextHang(0,r,0,"y");TextHang(0,0,r,"z");
}#v07=?>A
```

Substituting expected 2D wave $y=(1/r^{1/2})\exp(ikr-i\omega t)$ into the newly separated term, we could get the wave equation in the radial direction

$$\frac{d^2y}{dr^2}+\frac{1}{r}\frac{dy}{dr}+(k^2-\frac{1}{2r^2}+\frac{ik}{r})y=0 \quad .$$
$$k_r{}^2=k^2-\frac{1}{2r^2}+\frac{ik}{r}=(k+\frac{i}{2r})^2-\frac{1}{4r^2} \tag{31}$$

The observed 2D wave in the 2D platform for the spherical wave is approximately given by

$$y \simeq \frac{A}{\sqrt{r}} \exp(ir[\sqrt{k_r^2 + \frac{1}{4r^2}} - \frac{i}{2r}] - i\omega t) + O(\frac{1}{r^3}) \tag{32}$$
$$k + \frac{i}{2r} = \sqrt{k_r^2 + k_\varphi^2}; \quad k_\varphi = \pm \frac{1}{2r}$$

It contains an imaginary number $i/2r$ term which approximately represents the attenuation of the wave. The k_φ causes the 2D wave-vector to spin little by little as shown Fig.7, this is a kind of visualization of the spin effect. The positive and negative k_φ corresponds to spin up and spin down respectively; as r goes to the infinity, the spin effect vanishes off.

If the 3D wave is the de Broglie matter wave for a particle beam, in a cylinder coordinate (r,φ) as a 2D platform for observation, then the matter wave has a spin angular momentum given by

$$k_r = \frac{p_r}{\hbar}; \quad k_\varphi = \frac{p_\varphi}{\hbar} = \frac{J_\varphi}{r\hbar}; \quad J_\varphi = \pm \frac{1}{2}\hbar \cdot \tag{33}$$

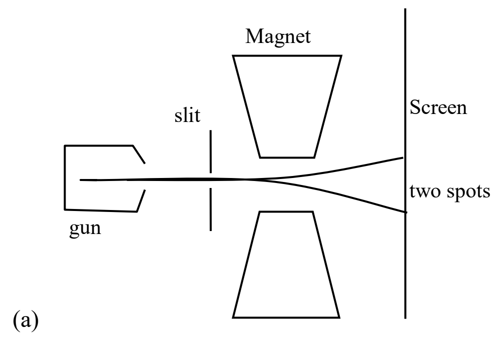
According to the angular momentum formula in general physics, it is recognized that the particle total momentum p holds

$$\left(\frac{p}{\hbar}\right)^2 = \left(\frac{p_r}{\hbar}\right)^2 + \left(\frac{p_\varphi}{\hbar}\right)^2, \quad (k + \frac{i}{2r})^2 = k_r^2 + k_\varphi^2 \cdot \tag{34}$$

Actually, spin visualization belongs to 2D world.

6. Spin in Stern-Gerlach experiment

In Stern-Gerlach experiments, as shown in Fig.9(a), silver (Ag) atoms are heated in an oven, the oven has a small hole through which some silver atoms escape. The silver vapor busts out the oven and go through a slit as the collimator and is then subjected to an inhomogeneous magnetic field. In this experiment, the single valent election of silver atom moves on its Bohr orbit, as shown in Fig.9(b).



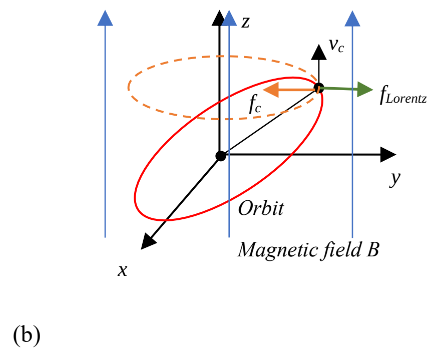


Fig.9 The Stern-Gerlach experiment.

It concerns with three issues:

(1)The Lorentz force can transfer energy but fails to flip the orbital ring in external magnetic field

The electronic orbit is subject to the Lorentz force which cannot flip the orbital ring, because the electronic velocity v_c in the x-y plane gives rise to a centripetal force f_c which can balance off the Lorentz force $f_{Lorentz}$, as illustrated in Fig.9(b), so that the orbital ring would not flip its normal direction, unlike a small magnetic needle in the external magnetic field. In Fig.9(b), we have the balance

$$f_c = m\frac{(v_c + \Delta v_c)^2}{r_c} - m\frac{v_c^{\ 2}}{r_c} \quad \Leftrightarrow \quad f_{Lorentz} = eBv_c \ . \tag{35}$$

If the orbital ring rotates its orientation by other external force, a work should be done by the Lorentz force against the force f_c so that an energy is transferred between the atom and the external force.

(2)Spin up and spin down

In the section 4 we have mentioned, the acceleration-roll of particle moves with two distinctions: **right-hand roll** (You curl the four fingers of your right hand along the spiral direction, the extended thumb, which is at a right angle to the fingers, points in the direction of x_1) and **left-hand roll** (You curl the four fingers of your left-hand along the spiral direction, the extended thumb, which is at a right angle to the fingers, points in the direction of x_1). Here we directly declare, if an electron moves with right-hand roll, then it is in spin-up state; if an electron moves with left-hand roll, then it is in spin-down state.

The spin-up and spin-down have two distinct behaviors in a magnetic field. As shown in Fig.10, the spin-up electron in a hydrogen atom with right-hand roll on its orbital ring meets with an external magnetic line B, it will eat the magnetic line into its acceleration-roll, as illustrated in Fig.10(a), finally it involves the magnetic line in its orbital ring, as illustrated in Fig.10(b). As contrast, the spin-down electron in a hydrogen atom with left-hand roll on its orbital ring meets with an external magnetic line B, it *seems to eat* the magnetic line into its acceleration-roll, as illustrated in Fig.11(a), but finally it spits the magnetic line out of its orbital ring due to geometry of the roll, as illustrated in Fig.11(b).

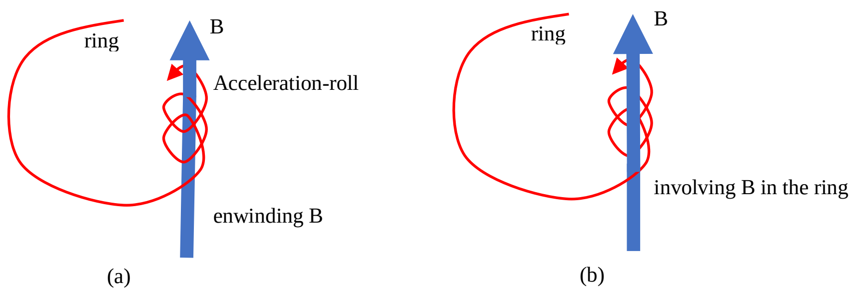


Fig.10 The spin-up electron in a hydrogen atom with right-hand roll on its orbital ring meets with an external magnetic line B, it will involve the magnetic line in its orbital ring.

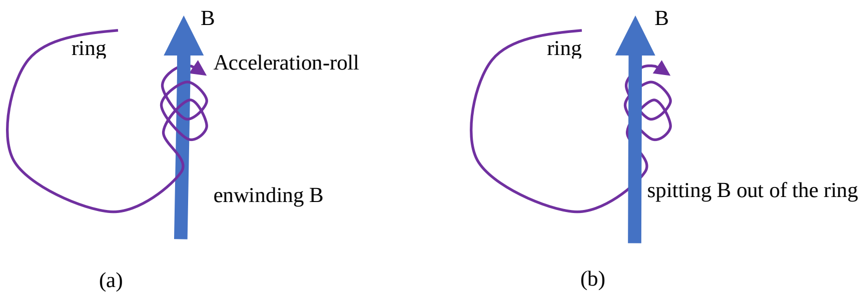


Fig.11 The spin-down electron in a hydrogen atom with left-hand roll on its orbital ring meets with an external magnetic line B, it will spit the magnetic line out of its orbital ring.

Consequently, if the spin-up electron does not satisfy orbital quantization condition, its acceleration-roll will open its rolling-mouth and make a tempt to devour the magnetic lines in a Stern-Gerlach experiment, as illustrated in Fig.12, it flips its orbital ring like a small magnetic needle, until its orbit satisfies a quantization condition and shut up its rolling-mouth, stop to eat the magnetic line and stop flipping. Finally, the magnetic moment of the orbital ring is μ_z=-μ_{Bohr}.

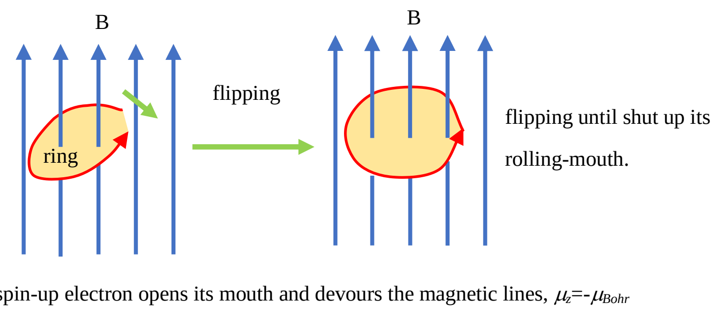


Fig.12 The spin-up electron flips its orbital ring like a small magnetic needle, until its orbit satisfies a quantization condition and shut up its rolling-mouth, and stop flipping.

As contrast, if the spin-down electron does not satisfy orbital quantization condition, its acceleration-roll will open its rolling-mouth and make a tempt to dodge the magnetic lines in a Stern-Gerlach experiment, as illustrated in Fig.13, it flips its orbital ring, until its orbit satisfies a quantization condition and shut up its rolling-mouth, stop to dodge the magnetic line and stop flipping. Finally, the magnetic moment of the orbital ring is μ_z=μ_{Bohr}.

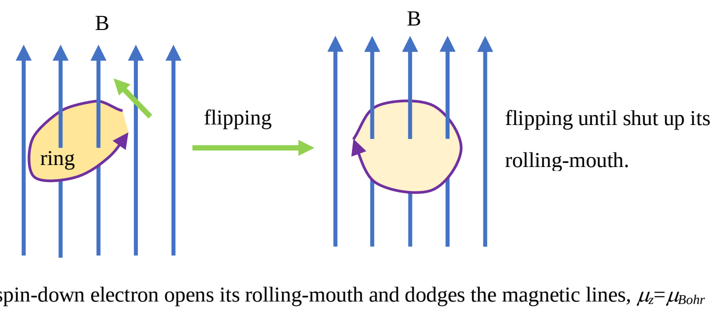
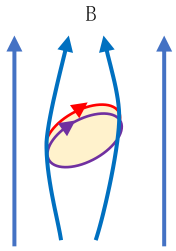


Fig.13 The spin-down electron flips its orbital, until its orbit satisfies a quantization condition and shut up its rolling-mouth, and stop flipping.

As the result, in a Stern-Gerlach experiment, the spin-down electron rotates its orbital magnetic moment to the most-low energy state like a small magnetic needle yielding to the external magnetic field. While, the spin-up electron rotates its orbital magnetic moment to the most-high energy state which does not yield to the external magnetic field, this is the result of the interference of the matter wave. Thus, the 50% spin-down electrons have a positive Bohr magnetic moment; while the 50% spin-up electrons have a negative Bohr magnetic moment, this is confirmed by the two spots on the screen in Stern-Gerlach experiments.

(3)a pair of spin-up and spin-down in helium atom

A helium atom consists of a nucleus and a pair of electrons, in spin-up and spin-down respectively. The two electrons chase each other on the common orbit, why the helium goes along without magnetic moment? In an external magnetic field B, the spin-up electron opens its rolling-mouth and make a tempt to devour the magnetic lines, while the spin-down electron opens its rolling-mouth and make a tempt to dodge the magnetic line. Thus, this pair of electrons would clip up the external magnetic lines and excludes off the magnetic field out of the atom. As shown in Fig.14, in other words, the helium atom loses its atomic magneticity in this way, appearing the Meisner effect like superconductor ring. In the helium case, Lorentz force didn't work to the atom due to no atomic magnetic moment. The helium atom does not flip its atom orbital ring in the external magnetic field.



The helium atom does not flip its atom orbital plane in the external magnetic field.

Fig.14 The spin-up electron devours the magnetic lines, while the spin-down dodges the magnetic lines. Thus, this pair of electrons would clip up the external magnetic lines and excludes off the magnetic field out of the atom.

7. Conclusions

This year is 100th anniversary of the initiative of de Broglie's matter wave, it is a good time for rediscovering the matter wave. In analogy with the ultimate speed c, there is an ultimate acceleration β, nobody's acceleration can exceed this limit β, for electrons and quarks, β=2.327421e+29(m/s^2). Because this ultimate acceleration is a large number, any effect connecting to β will become easy to test, including quantum gravity tests. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, the spin concept can be derived out from the matter wave. This paper also carefully explains how the matter wave to display its spin effect in Stern-Gerlach experiments. It is completely a new aspect to quantum mechanics for the relativistic matter wave to contain spin.

References

[1] de Broglie, L., CRAS,175(1922):811-813, translated in 2012 by H. C. Shen in Selected works of de Broglie.
[2] de Broglie, Waves and quanta, Nature, 112, 2815(1923): 540.
[3] de Broglie, Recherches sur la théorie des Quanta, translated in 2004 by A. F. Kracklauer as De Broglie, Louis, On the Theory of Quanta. 1925.
[4] A. Einstein, B. Podolsky, N. Rosen, Can quantum-mechanical description of physical reality be considered complete, Phys. Rev., 1935, 47, 777-780,
[5] D.Bohm, A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables. I, Phys. Rev., 1952, 85, 166-179,
[6] D.Bohm, A Suggested Interpretation of the Quantum Theory in Terms of Hidden Variables. II, Phys. Rev., 1952, 85, 180-193,
[7] R. G. Chambers, Shift of an electron interference pattern by enclosed magnetic flux[J], Phys. Rev. Lett., 1960, 5:3-5.,
[8] A.Tonomura , N.Osakabe , T.Matsuda , T.Kawasaki , J.Endo , S.Yano , H.Yamada , Evidence for Aharonov-Bohm Effect with Magnetic Field Completely Shielded from Electron wave[J], Phys. Rev. Lett., 1986, 56, 8:792-795.,
[8] N.Osakabe , T. Matsuda, T.Kawasaki , J.Endo , A.Tonomura , Experimental confirmation of Aharonov-Bohm effect using a toroidal magnetic field confined by a superconductor[J], Phys. Rev. , 1986, A 34, 2:815-819.,
[9] L.I. Schiff, quantum mechanics, McGraw-Hill Book Company, 1968, third ed.,
[10] E.G. Harris , Introduction to Modern Theoretical Physics, John Wiley & Sons, 1975, Vol.1 and 2, https://doi.org/10.1007/BF00207145
[11] D.J.Griffiths, Introduction to quantum mechanics, Pearson Education,Inc., 2005,
[12] F. Halzen, A.D.Martin, Quarks and Lepton: An Introductory Course in Modern Particle Physics, Johm Wiley & Sons,Inc., 1984,
[13] Huaiyang Cui, Relativistic Matter Wave and Its Explanation to Superconductivity: Based on the Equality Principle, Modern Physics, 10,3(2020)35-52. https://doi.org/10.12677/MP.2020.103005

[14] Huaiyang Cui, Relativistic Matter Wave and Quantum Computer, Amazon Kindle ebook, 2021.
[15] Huaiyang Cui, Evidence of Planck-Constant-Like Constant in Five Planetary Systems and Its Significances, viXra:2204.0133, 2022.
[16] Huaiyang Cui, Approach to enhance quantum gravity effects by ultimate acceleration, viXra:2205.0053, 2022.
[17] Clet Lab, Clet: a free C compiler, download at https://drive.google.com/file/d/1OjKqANcgZ-9V5 6rgcoMtOu9w4rP49sgN/view?usp=sharing
[18] NASA. https://solarscience.msfc.nasa.gov/interior.shtml
[19] Orbital Debris Program Office, (2018) History of On-Orbit Satellite Fragmentations. 15th Edition, National Aeronautics and Space Administration, Washington DC.