On a fundamental Relationship between Newton’s Laws, Relativity and Quantum Theory

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Abstract

The motion of particles in the Newton/Mach universe is studied, especially their interaction with its collective properties, namely the universal existence and propagation of waves. It turns out that the motion of individual masses or local mass distributions is not only described correctly in relativistic terms on this basis, but that there is also a fundamental connection to quantum theory. A quantitative relationship between Planck's constant and fundamental parameters of the universe is derived. It does not appear to be a natural constant. The quantum property of nature seems to follow from Newton's laws, or at least correspond to them in a deeper way than previously known. Gravity and the quantum character of nature appear to be fundamentally linked within the Newton/Mach theory.

Key words: Newton's laws and the universe, Mach's principle, gravitational waves, cosmology, fundamental physics, Sine-Gordon equation, Klein-Gordon equation, Planck units, Quantum Gravity

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1. Introduction

In an earlier work [1)] it was shown that even in Newton's universe, time and space cannot be thought of separately from one another if Mach's principle is taken into account, i.e. that with every movement of a mass according to Newton's law of inertia the influence of all other masses of the universe has to be considered. For the motion of a mass, the exact results of the special theory of relativity are achieved (locally) solely from Newton’s laws and the consideration of Mach’s principle. There is a (local) maximum velocity and the equations of motion become (locally) Lorentz invariant, space and time are connected to form a unit in Minkowski's sense. The size of this maximum velocity is affected by gravity and the totality of the masses of the universe.

One finds the following relationships for the energy and momentum of a particle with rest mass m_0:

$$E = \frac{m_0}{\sqrt{1-\frac{v^2}{b^2}}}\, b^2 \,. \tag{1.1}$$

$$p = mv = \frac{m_0}{\sqrt{1-\frac{v^2}{b^2}}}\, v. \tag{1.2}$$

And with that also applies:

$$E^2 = p^2b^2 + m_0^2b^4 \,. \tag{1.3}$$

This is identical to the results of the special theory of relativity (SR). In contrast to Einstein's derivation, however, we did not postulate the constancy of the speed of light. Light and the speed of light are not included in the derivation at any point. According to (1.1) and (1.2) a particle obviously cannot move faster than the velocity v_{max} = b, just as according to the results of the SR. For the universe, we initially assumed the model of a sphere in three spatial dimensions with the radius R_0 and a time that is independent of this. Our consideration then shows that space and time are not independent of each other when considering Mach's principle.

For the size of b we found in [1]:

$$b = \sqrt{2\pi\, G\, \rho\, (\, {R_o}^2 - \tfrac{1}{3} r^2\,)}\ . \tag{1.4}$$

G is the gravitational constant, ρ the density of the universe and r the distance of the particle with rest mass m_0 from the center of the universe. For a particle close to this center we get:

$$b_0 = \sqrt{2\pi\, G\, \rho\, {R_o}^2}\ . \tag{1.5}$$

The maximum speed of a particle v_{max} can be determined experimentally [2] and proves to be identical to the (vacuum) speed of light c near the earth. However, the measurement of v_{max} does not depend on light or the speed of light. Like G, the density of the universe can also be measured and is on large scales [3]: $\rho = 5 \times 10^{-30}$ g/cm^3. With this one can calculate the radius of the universe from (1.4) or (1.5). If the measurement of v_{max} is made near the center of the universe, i.e. r = 0, the result is

$$R_0 = \sqrt{\frac{b_0^2}{2\pi G \rho}} = 2{,}07\ 10^{28}\ \text{cm}. \tag{1.6}$$

This agrees very well with today's value of the expansion of the universe, determined in a different way [3].
This result and the fact that the results of the SR derived by Einstein are identical in form to the formulas (1.1) to (1.3), which were derived without reference to the speed of light, leads to the conclusion that the quantity b (or b_0) must be identical to the speed of light c, because we obviously get results of the same shape for the same natural phenomenon in two completely different ways, which only match exactly if b = c. And of course this must be demanded, because the mathematically precise description of one and the same natural phenomenon must of course not depend on the way in which it was derived. However, since b according to (1.4) is partly determined from location-dependent parameters of the universe, we conclude that the speed of light is also location-dependent and cannot be a natural constant.
For further considerations in this work, it is important to point out and emphasize that the derivation of the results represented by (1.1) to (1.6) has shown that the motion of the individual masses of today's universe are irrelevant for this derivation play and the motion can therefore be neglected: From the point of view of a test mass, whose motion is characterised by (1.1) to (1.3), the universe consists of "fixed stars".
The test mass can be considered as a mass point, as it is also considered in the special theory of relativity. However, experience now shows that the image of a "point mass" does not correctly describe all motions of masses, but that the quantum-theoretical description becomes necessary for areas below certain thresholds (which are characterized, for example, by the Ehrenfest theorem). In quantum theory there are no point masses, only mass distributions. In the following, we now want to pursue the question of whether the movements of the masses of the universe must not also be taken into account in our theory

presented in [1] and whether this consideration perhaps necessarily leads to the description of mass distributions instead of point masses and thus perhaps to quantum theory.

2. Collective Properties of the Universe

In the paper [4] we were able to show that certain collective motions, namely waves, are possible in the Newton/Mach universe and that wave fields therefore exist. This also applies if no test mass is available. It turned out that the universe has a fundamental similarity to solid bodies: Both are many-particle systems whose building blocks can be understood as point masses that exert forces on one another. Of fundamental importance for the results is that one can apply Newtonian point mechanics. Because of Newton's third axiom (actio = reactio), the positions of the real masses and their distances from one another are not important for the collective properties. Every fictitious system in which the real masses are replaced by any number of fictitious point masses proves to be completely equivalent to the real system, as long as they each have the same center of mass as the real masses. In Newtonian point mechanics, even the number of particles involved in the movement remains undetermined (cf. the very detailed description in [5] or in [6]).

With regard to these basic properties, the system of masses in the universe does not differ in any way from a solid state system. A difference only becomes apparent when we consider the form of the forces interacting between the particles: in solids these are (approximately) assumed to be linear spring forces. This is of course different in the universe: Newton's gravitational force acts everywhere between the masses. This has the consequence (and is an essential result of work [4], that in contrast to solid bodies there can be no (gravitationally caused) longitudinal waves in the universe. In solids there are longitudinal and transverse waves and, accordingly, two speeds of sound. In contrast, there is only one speed of sound in the universe.

The simplest form of propagation of these transverse gravitational waves is a plane wave. For their propagation speed c_{tr} we find in work [4] (near the center of the universe):

$$c_{tr} = b_0 = \sqrt{2\pi\, G\, \rho\, R_o^{\,2}}\ . \qquad (2.1)$$

The speed of propagation of transverse gravitational waves is therefore identical to the maximum speed for a mass in the universe b_0, and this turns out to be (cf. [4]) identical to the (vacuum) speed of light c. This is a most amazing and unexpected result, and leads us to a connection which, to the author's knowledge, has never been discovered or described by anyone: the universe exhibits some collective properties like a solid. The “speed of sound” of the “solid universe” is identical to the (vacuum) speed of light. The waves propagating at these velocities are transversal.

Both the solid-state lattice dynamics and our theory presented in [4] are based on the idea of a very large but finite three-dimensional space lattice, whereby the "lattice points" do not have to be identical to the real masses. As long as only very small deflections of the mass points from their regular "rest positions" are considered, one speaks of movements in an "ideal crystal" in solid state physics. However, it only describes a small part of the physical phenomena to be observed, but not, for example, plastic deformations or the movement of "dislocations". Such movement patterns can be associated with any, not just small, deflections of the point masses from their rest positions, i.e. with any long migrations, i.e. movements of point masses through the crystal. These phenomena are of course dependent on the crystal environment in which they take place, so they are related to the properties of the "ideal crystal". They take place "on his background" and must include him in their description. It is therefore very obvious to ask whether something like a "plastic deformation" or a "displacement" is produced by an external test mass that is introduced into the "ideal crystal" of the undisturbed universe. Their movement would then have to be described "on the background" of the universe surrounding them as an "ideal crystal". It is precisely this question that we want to pursue in the following chapters. We start in chap. 3 with a brief description of the theory of dislocations in crystals, insofar as it is essential for our question, and we always limit ourselves to the mathematically simplest cases, because we are only looking at the fundamental relationships here.

3. Brief Description of the Theory of Dislocations in Crystals

As mentioned above, Newtonian point mechanics does not assume real atomism. Instead of the physical particle locations, the geometric centers of mass can be considered and described. For this reason it is possible to describe a system of atoms or molecules connected with springs, i.e. a crystal, by a continuum theory (cf. e.g. [7][8]). This applies not only to the "ideal crystal" mentioned above, but also to crystals that contain "disorders", e.g. vacancies, interstitial atoms or dislocations.

As an example of such a perturbation, we want to consider a dislocation that forms the half-plane $-\infty < x < +\infty$ and $y > 0$ at location $z = 0$ (e.g. as an interstitial plane). In this case, the line $y=0$, $z=0$ (i.e. the x-coordinate) is the "displacement line". Let us allow that the lattice building blocks can move perpendicularly to the x-coordinate by the amount q in the z-direction, where q is expressly no longer limited to a small distance from the equilibrium position. Then one can set up the following equation for q depending on x and t (preliminary work by Dehlinger [9], then Frenkel and Kontorova [10], Seeger [11, 12] and finally Rubinstein [13], who gave it the name Sine-Gordon equation):

$$\frac{\partial^2}{\partial x^2} q(x,t) - \frac{1}{v_s{}^2}\frac{\partial^2}{\partial t^2} q(x,t) - \frac{D}{\sigma}\sin\left(\frac{2\pi}{a} q(x,t)\right) = 0, \qquad (3.1)$$

where a stands for the lattice constant on the dislocation line, σ for an "elastic modulus" and D for a "spring constant" of the periodic force (both related to the dislocation line). Both σ and D are to be calculated from the constants of the lattice (cf. [12]). Eshelby has shown [14] that the signal velocity v_s, which also occurs in this equation, is identical to the transversal sound velocity c_{tr} of the crystal. It is the same in all directions in the crystal's rest frame if it is isotropic.

Eq. (3.1) and related equations have been extensively studied both mathematically and in terms of their physical meaning in very different areas. We want to limit ourselves here to very few aspects and only to the one-dimensional case. What is remarkable about this equation in our context is that it describes a distribution function q(x,t), i.e. a field, in which the directions of q and x are perpendicular to each other, i.e. q propagates transversely to x .

Eq. (3.1) can be integrated directly and has the solution [12][15][16] for $v < v_s$

$$q(x,t) = \frac{2a}{\pi} \operatorname{arctg} \exp\left(\frac{\pi(x-vt)}{L_0\sqrt{1-(\frac{v}{v_s})^2}}\right). \qquad (3.2)$$

This solution is called a "kink". Therein we have used the abbreviation

$$L_0 = \sqrt{\frac{a\sigma}{2\pi D}}\,. \qquad (3.3a)$$

This quantity describes the so-called "dislocation length". It is determined by how the surrounding lattice acts on the dislocation, so it only contains parameters of the crystal.
It can be seen that (3.1) is "Lorentz-invariant", but not using the speed of light c, but the speed of sound v_s. Eq. (3.2) describes the movement of a "kink" along the x-axis. It has been extensively described and discussed [16)18)] that the internal geometry of a kink line defines natural units of “length” and “time” in the crystal continuum.
One can think of L_0 and L' as "gauges" seen by an observer who is on the kink line.
There is a connection between L' and L_0

$$L' = L_0 \sqrt{1 - \frac{v^2}{{v_s}^2}} \,. \tag{3.4a}$$

This is equivalent to the Lorentz contraction "in the vacuum".
Correspondingly, the following can also be derived for time scales:

$$T' = \frac{T_0}{\sqrt{1 - \frac{v^2}{{v_s}^2}}} \,, \tag{3.4b}$$

i.e. the time dilation as in the SR, with

$$T_0 = \sqrt{\frac{4\pi a \boldsymbol{\rho_0}}{2D}} \,. \tag{3.3b}$$

$\boldsymbol{\rho_0}$ is the mass density related to the unit length of the dislocation line, which in turn is determined exclusively by parameters of the surrounding crystal lattice:

$$\boldsymbol{\rho_0} = \frac{\sigma}{{v_s}^2} \,. \tag{3.5}$$

A more detailed analysis has shown [5)] that kink lines can be understood as reference systems for events. If an event in S is determined by (x,t), then the same event in S' is determined by (x',t'). If S and S' move uniformly against each other and differ in the speed difference v, then the following applies:

$$x' = \frac{x - vt}{\sqrt{1 - (\frac{v}{v_s})^2}} \tag{3.6a}$$

and

$$t' = \frac{t - \frac{vx}{v_s^2}}{\sqrt{1 - (\frac{v}{v_s})^2}} \,. \tag{3.6b}$$

Events in kink systems moving uniformly relative to each other obviously behave in exactly the same way as events in systems moving uniformly relative to each other according to the SR, i.e. they are Lorentz-invariant.
But that's not all. The analogy between the movement of a kink line and a particle according to the SR goes further. It could be shown (by calculating an energy-momentum tensor using a Lagrangian formalism) [5)] that a mass and a velocity can be assigned to the field described by Eq. (3.2), and one finds:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{{v_s}^2}}} \tag{3.7a}$$

and

$$E = \frac{m_0 {v_s}^2}{\sqrt{1 - \frac{v^2}{{v_s}^2}}} \,. \tag{3.7b}$$

The rest mass m_0 is given by

$$m_0 = \rho_0 \frac{2}{\pi} \frac{a^2}{L_0}, \quad (3.8a)$$

or with (3.5):

$$m_0 = \frac{2}{\pi} \frac{a}{L_0} \frac{(\sigma a)}{c^2}. \quad (3.8b)$$

With regard to our further consideration, it should be particularly emphasized that this mass m_0 is defined exclusively by parameters of the dislocation line or the crystal surrounding it. It exists only when the crystal exists and has no existence without it. The product (σa) represents the energy of the line stress σ on the a/L_0 section of the dislocation line.

A dislocation line in a crystal, which is described by (3.2), obviously behaves like a (point) particle in the theory of relativity, but with the transversal speed of sound v_s of the crystal instead of the speed of light c of the vacuum. On the other hand, the particle is described in a similar way to quantum theory, because q(x, t) is a field, and the mass is accordingly "smeared" over a certain volume. This can best be seen by looking at the so-called "localized" solution or "breather solution" [12] of the Sine-Gordon equation:

$$q_l = \frac{2a}{\pi} \arctan \frac{\sin \frac{\pi v_s t}{L_0 \sqrt{2}}}{\cosh \frac{\pi x}{L_0 \sqrt{2}}}. \quad (3.9)$$

It describes a localized oscillating line.

In view of the consideration that we will carry out in the next sections, we want to use the above Eq. (3.1) reformulate a bit:

First we divide (3.1) by a/2π and define Q = q(x, t)/(a/2π). With this we can write the Sine-Gordon equation (3.1) with the normalized function Q(x, t) in the following form:

$$\frac{\partial^2}{\partial x^2} Q(x,t) - \frac{1}{v_s^2} \frac{\partial^2}{\partial t^2} Q(x,t) - \frac{1}{L_0^2} \sin(Q(x,t)) = 0. \quad (3.1a)$$

At this point we want to emphasize again that all previous results were developed exclusively on the basis of Newton's laws. Neither the theory of relativity nor the quantum theory entered the derivation in any way. As we have seen, Eq. (3.1a) describes the movement of a dislocation line to which a mass m or m_0 can be assigned (cf. (3.7a) and (3.8 a,b)). This mass is determined solely from the internal parameters of the crystal. It disappears when there is no crystal. Therefore, in the following, we always want to use the designation m* for this type of “effective” mass. In contrast, Newtonian point mechanics in absolute space (ignoring Mach's principle) and the SR describe the motion of a point mass that exists independently of the masses of the universe. The masses of the universe play no role in deriving the SR and have no bearing on its motion. However, as we were able to show with our work [1], this influence is present. Taking it into account (Mach's principle) then leads solely on the basis of Newton's mechanics to a maximum speed with which mass points can move, and this is identical to the speed of light. The influence of the (stationary) masses of the universe on an (additional) moving mass point is expressed in the postulate of the constancy of the speed of light in the SR.

4. What are Particles within the Universe?

In sections 1 and 2 above, we have given our earlier (in [4]) result that the universe has a (transverse) “speed of sound” and that this is identical to the vacuum speed of light. Both our results regarding the universe and the results given for crystals in Section 3 are based exclusively on Newton's point mechanics. In this comparison, the crystal and the universe differ only in the type of interaction forces between the point masses: in the universe it is the gravitational forces, in the crystal spring forces. If we can transfer the conditions in a crystal (with the transversal speed of sound v_s) to the universe (with the transversal speed of sound $v_s = b_0$), then we see from the consideration in Section 3 that a dislocation in the "crystal lattice universe“ then would move similar to how it would have to move in a vacuum according to the laws of the SR. However, the motion of a dislocation is described by the motion of a mass distribution. In contrast to this, the motion of a mass is described in the SR and our derivation in [1)] by the motion of a mass point.
If, contrary to our consideration in [1)], we allow that when an external test mass is introduced into the universe, its masses can also move (the universe is therefore no longer regarded as "rigid", the "fixed stars" are allowed to move (to an extremely small extent)), then the test mass will cause a “dislocation” in its environment. The extent of this is determined by the restoring forces of the surrounding masses involved.
Based on the conditions in crystals presented in Section 3, we can therefore assume that an equation very similar to Eq. (3.1) or (3.1a) also applies to the movement of such dislocations in the universe. According to our results in [4)] there is only one transversal “speed of sound” $v_s = b_0 = c$ in the universe. How the last term in Eq. (3.1) changes if instead of the spring forces in the crystal the gravitational forces between the masses are to be considered, we will investigate further below.
As mentioned above, the Sine-Gordon equation does not describe the movement of a point mass, but rather the movement of a distribution function or a field q(x, t). The solution (3.2) for the movement of a kink describes a uniform movement in the x-direction. This circumstance immediately brings to mind the Klein-Gordon equation, which describes the uniform movement of a mass in quantum mechanics. The former was derived solely on the basis of Newton's laws, the latter is a quantum theoretical equation. A physically and mathematically justifiable connection between the Sine-Gordon and Klein-Gordon equations would therefore mean a linking of the classical Newtonian theory with the quantum theory. The choice of the name "Sine-Gordon equation" already indicates the intuitive idea of such a link. It was deliberately chosen by Rubinstein [13)] based on the mathematically related and sound-similar "Klein-Gordon equation". In the next section we want to explore this linking idea in more detail.

5. Are there Properties of the Universe like Lattice Defects in Crystals?

As described in Section 1, a mass m_0 at rest in the universe is assigned an energy:

$$E_0 = m_0\, b_0^2\,. \tag{5.1}$$

Assuming that the masses of the universe do not move (i.e. are “fixed stars”) and are distributed over a limited volume of space, the following results near the center of the mass distribution (cf. Section 1):

$$b_0 = \sqrt{2\pi\, G\, \rho\, R_o^{\,2}} = c. \tag{1.5}$$

For the following consideration we now want to allow that the masses of the university lattice can move. The introduction of an external mass m_0 into the previously “undisturbed” lattice of the mass points of the universe then causes a distortion of the university lattice surrounding it. The mass m_0 brought in from the outside (and resting again after being brought in) must apply the corresponding distortion energy ΔE_n (s_{ng}). The index n designates the mass m_n of the university lattice that is influenced by the mass m_0. s_{ng} denotes their deflection from their equilibrium position in an undisturbed initial situation before the introduction of m_0. Eq. (5.1) must therefore be supplemented by this distortion energy:

$$E_0 = m_0\, b_0^2 + \Delta E_n\, (s_{ng}). \tag{5.1a}$$

We can also transform this into

$$E_0 = m_0\, b_0^2 + \frac{\Delta En\, (s_{ng})}{b_0^{\;2}}\, b_0^2 = \left(m_0 + \frac{\Delta En\, (s_{ng})}{b_0^{\;2}}\right) b_0^2. \tag{5.1b}$$

The second term in the bracket of (5.1b) can thus be regarded as an additional ("effective") mass:

$$m_0^* = \frac{\Delta En\, (s_{ng})}{b_0^{\;2}}\ . \tag{5.1c}$$

In the following we will calculate this effective mass approximately. We will then examine the effect of this mass on the surrounding world grid. We base ourselves on the calculation of the properties of dislocations in crystals, as was done, for example, by Seeger [12)] and then by many others. They all start from Newtonian point mechanics, but then often make the transition to continuum mechanics. The study of so-called "crowdions" (cf. [23], further references there) is probably the most similar to our question. This is an additional atom that is introduced into a densely packed, ideal crystal. The properties of the new overall system, consisting of this foreign atom and its (elastic!) environment, was determined by Kovalev et. al. [24)]. In their model, the crowdion is described as a linear chain of atoms embedded in the lattice environment. We follow this idea when we now describe the effect of introducing a point mass into the (otherwise thought of as ideal) grid of the universe. We restrict ourselves to one-dimensional solutions.
We first distribute the introduced mass m_0 to n mass elements along an imaginary line of length L' in the x-direction, to the right and left of the introduced mass m_0. The direction of x can of course be chosen arbitrarily. A mass element of this mass line then has the partial mass at a distance a

$$m_{0n} = \frac{a}{2L'}\, m_0 \tag{5.2}$$

According to Newton's point mechanics, we can carry out such a distribution as long as the center of mass of the distribution coincides with the center of mass of m_0.
Now let us imagine two linear chains of mass points m_{ni} of the universe, arranged parallel to each other in the z-direction and opposite to each other in the y-direction at a distance of a. They are an arbitrarily chosen part of the lattice of mass points that make up the universe. Let us now place the outer partial mass m_{0n} in the middle between these two linear chains, e.g. at z = 0. It then exerts a force F_{ni} on each of the mass points m_{ni} of the two chains, with s_i exerting a deflection of the m_{ni} from their original position caused by the force F_{ni}

$$F_{ni} = -\, G\, \frac{m_{0n} m_{ni}}{(\frac{a}{2} + s_{ni})^2}\ , \tag{5.3}$$

On the other hand, according to our consideration in [4)], the masses m_{ni} experience the restoring force from their neighboring masses $m_{n,i+1}$ when deflected by s_n in the y-direction (i.e. transverse to the direction of the chains).

$$F_{n,i\pm 1,y} = G \frac{{m_{ni}}^2}{a^2+(s_{n,i\pm1}-s_{n,i})^2} \frac{s_{n,i\pm1}-s_{n,i}}{\sqrt{a^2+(s_{n,i\pm1}-s_{n,i})^2}} . \tag{5.4}$$

If $s_{ni} \ll a$, this simplifies to

$$F_{n,i\pm 1,y} = G \frac{{m_{ni}}^2}{a^3} \left(s_{n,i\pm1} - s_{n,i}\right). \tag{5.5}$$

In order to present the core of the matter as simply as possible, we make the strong (but of course not necessary) simplification:

$$s_{n,i\pm1} \ll s_{n,i} \tag{5.6}$$

This approximation means that for each mass m_{ni} there is only one deflection $s_{ni}= s_n$ and only the index n is sufficient to identify the mass m_{ni}. From (5.5) with (5.6) we get:

$$F_{n,i+1,y} + F_{n,i-1,y} = - G \frac{{m_n}^2}{a^3} \left(2\, s_n\right). \tag{5.5a}$$

For $$s_n \ll a \tag{5.5b}$$

(5.3) also simplifies to:

$$F_{ni} = F_n = - 4\, G \frac{m_{0n} m_n}{a^2} \left(1 - 4 \frac{s_n}{a}\right) . \tag{5.7}$$

In equilibrium the following must apply:

$$F_{n,i+1,y} + F_{n,i-1,y} + F_{ni} = 0. \tag{5.8}$$

By inserting (5.5a) and (5.7) into (5.8) we then find for the equilibrium (index $_g$):

$$- \quad 2\, G \frac{{m_n}^2}{a^3}\, s_{ng} - \quad 4\, G \frac{m_{0n} m_n}{a^2} \left(1 - 4 \frac{s_{ng}}{a}\right) = 0,$$

or $$s_{ng} = \frac{2 m_{0n}}{(8 m_{0n} - m_n)}\, a. \tag{5.9}$$

With regard to a consideration that we will make below, we note that the denominator of (5.9) would have a plus sign if the force (5.3), i.e. the gravitational force, were repulsive. Now we further assume that

$$8\, m_{0n} \ll m_n . \tag{5.10}$$

Of course, we have to check whether this assumption is justified. With (5.10), (5.9) simplifies to:

$$s_{ng} = - \frac{2 m_{0n}}{m_n}\, a. \tag{5.11}$$

By introducing the additional mass m_0 or the partial masses m_{0n}, the original, "undisturbed" grid of the universe is somewhat distorted. A “residual stress” forms along the line with the distributed masses m_{0n}, just like it occurs in the crystal along a dislocation. We can also refer to this line as a "dislocation line" in the universe. The energy of the residual stress along this dislocation line results (based on the two linear chains with the lattice masses m_{ni} involved) as follows:

$$\Delta E_n\,(s_{ng}) \;=\; 2\int_0^{s_{ng}} (F_{n,i\pm1,y})\; ds_i\;, \tag{5.12}$$

or with (5.5a):

$$|\Delta E_n(s_{ng})| = 4\int_0^{s_{ng}} \left(G\,\frac{m_n{}^2}{a^3}\, s_i\right)\; ds_i$$

$$= 4\,G\,\frac{m_n{}^2}{a^3}\,\frac{1}{2}\,s_{ng}{}^2. \tag{5.13}$$

Let's insert (5.11) here:

$$|\Delta E_n(s_{ng})| = 2\,G\,\frac{m_n{}^2}{a^3}\,\left(\frac{2m_{0n}}{m_n}\,a\right)^2$$

or

$$|\Delta E_n(s_{ng})| = 8\,G\,\frac{m_{0n}{}^2}{a}\;. \tag{5.14}$$

This internal stress energy is "induced" by the "external" mass m_{0n}.
From this energy we can now calculate the effective rest mass m_{0n}* (cf. (5.1c))

$$m_{0n}{}^* \;=\; \frac{|\Delta En(s_{ng})|}{b_0{}^2}\;. \tag{5.15}$$

This mass obviously corresponds to the mass that can be assigned to a dislocation in the crystal (cf. Eq. (3.8)). It only exists when the "world grid" exists and has no meaning without it. We have shown in [1)] and [4)] that the quantity b_0 in the vicinity of the center of the universe corresponds to the speed of light c. With this and with (5.14) and (5.15) we finally find:

$$m_{0n}{}^* \;=\; 8\,G\,\frac{m_{0n}{}^2}{ac^2}\;. \tag{5.16}$$

What is interesting about this result is that the lattice masses of the universe m_{ni} do not appear explicitly. The effect of the universe on the masses m_{0n} is shown by the parameters of the universe a, G and c (m_{ni} is implicit in c).
We now consider the individual masses m_{0n}*, analogous to the conditions in a crystal, as elements of a "dislocation line" of length 2L' with the effective total mass

$$m_0{}^* \;=\; \frac{2L'}{a}\,m_{0n}{}^*\;. \tag{5.17}$$

As long as we assume the validity of Newton's point mechanics, nothing speaks against transferring the conditions in the crystal to the universe. Therefore we can examine the effect of this "dislocation line" on the surrounding "crystal", here the lattice of the universe. Because of Newton's point mechanics, it is again permissible to replace the actual distribution of masses by an equivalent distribution of fictitious masses, as long as the distribution of the centers of gravity is preserved. We have already examined this in [4)] and found that the grid of the universe can be described by a chain, i.e. a linear sequence of discs with the respective disc mass

$$m_s = \rho\pi\,R_0{}^2\,a. \tag{5.18}$$

These disk masses m_s correspond to the masses denoted by m_n above, i.e. it is

$$m_n = m_s. \tag{5.18a}$$

For the case of the homogeneous universe, whose density (in very large volume elements) is constant, the equation of motion for the disk masses reads (cf. [4)]):

$$m_s \ddot{s}_n = D_2 (s_{n+1} + s_{n-1} - 2s_n) \tag{5.19}$$

with
$$D_2 = 2\,G\frac{m_s^2}{a^3}. \tag{5.20}$$

It expresses the collective properties of a homogeneous universe. The deflections s_n are perpendicular to the chain line, so the chain oscillates transversally.
If we now place the "dislocation line", i.e. the stationary chain with the masses m_{0n}^* parallel to the linear chain with the masses m_s (which we are of course allowed to do), then the dislocation line with its chain links m_{on}^* exerts a (transversal) force F_n on each link in the chain with the masses m_s, and we get the equation of motion for the discrete mass points:

$$m_s \ddot{s}_n = D_2 (s_{n+1} + s_{n-1} - 2s_n) - F_n\,. \tag{5.21}$$

The change in dynamics is now expressed in this linear chain, which is caused by the introduction of a (resting) additive mass m_0 into the previously homogeneous universe. The mass elements with the index n must be in the interval 2L'/a (cf. (5.17)), i.e. within the "chain length".
We now mentally arrange the masses m_{0n}^* to the right and left of the chain elements m_s. For very small deflections s_n, the sum of the two forces acting from the left and right from m_{0n}^* to m_s is given by (cf. (5.7)):

$$F_n = 4x8xG\,\frac{m_s\frac{1}{2}(m_{0n}+m_{0n}^*)}{a^3}\,s_n. \tag{5.22}$$

The factor 1/2 in front of the bracket is due to the fact that we have distributed the mass elements m_{0n} and m_{0n}^* to the right and left of the chain with the mass elements m_s and the summation over n must of course result in the respective total mass m_0 or m_0^*.
Let us first consider only the portion of the mass that can be assigned to the induced residual stress, i.e. m_{0n}^* (cf. (5.16)), insert the corresponding portion of (5.22) into (5.21) and reduce m_s, like this surrendered

$$\ddot{s}_n = \frac{1}{m_s} D_2 (s_{n+1} + s_{n-1} - 2s_n) - 16\,G\,\frac{m_{0n}^*}{a^3}\,s_n. \tag{5.23}$$

If we assume that the distance a between the chain elements is small enough (see e.g. [5)]), then we can go from this description of a discrete chain to the continuum description.

With
$$s_n(t) \;\rightarrow\; q(x,t) \tag{5.24}$$

and using (5.2), (5.16), (5.18) and (5.20) we get

$$\frac{\partial^2}{\partial t^2} q = 2\,\frac{G}{a}\,\rho\pi\,R_0^2\,a\,\frac{\partial^2}{\partial x^2} q - 32\,G^2\,\frac{m_0^2}{a^2c^2L'^2}\,q \tag{5.25}$$

or with (see 1) and 4)):
$$b_0^2 = c^2 = 2\,G\,\rho\pi\,R_0^2 : \tag{5.26}$$

$$\frac{\partial^2}{\partial x^2} q - \frac{1}{c^2}\,\frac{\partial^2}{\partial t^2} q - \Gamma' q = 0. \tag{5.27}$$

We have abbreviated:

$$\Gamma' = 32\,G^2\,\frac{m_0^2}{a^2c^4L'^2}\,. \tag{5.28}$$

Now we define the dimensionless quantity

$$Q = \frac{q}{a} = Q(x,t), \tag{5.29}$$

then we can finally write (5.27) in the following form:

$$\frac{\partial^2}{\partial x^2}Q(x,t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}Q(x,t) - \Gamma' \, Q(x,t) = 0. \tag{5.30}$$

This equation apparently describes a "dislocation line" in the universe, whereby the displacement was caused by the introduction of an external mass m_0 into the previously undisturbed world grid. The entire derivation is based on classical Newtonian point mechanics based on the theory of plastic deformations in the crystal. Neither the quantum theory nor the theory of relativity nor Maxwell's electrodynamics have been incorporated at any point. In the derivation of (5.30) we restricted ourselves to immediately adjacent chains (cf. (5.3)). When deriving the Sine-Gordon equation for crystals, more neighbors are usually considered, which is then expressed in a Peierls potential. Then, instead of the linear term in (5.30), one obtains, for example, a sine function. For very small values of Q(x,t), as we use it here, the sine-function turns into a linear function.

It remains to examine the validity of the approximation (5.10) or to derive a condition for the maximum size of the introduced mass m_0. With (5.17) it follows from (5.10):

$$8\, m_{0n} = \frac{a}{2L'}\, 8\, m_0 \ll m_s \tag{5.31}$$

or, since the minimum size for L' is equal to a (and with the values for ρ and R_0 used in [1)]), the condition results:

$$m_0 \ll \frac{1}{4}\, \rho\pi\, R_0^2\, a = 0{,}25 \text{ x } 5 \text{ x}10^{-30}\, \frac{g}{cm^3} \text{ x } 3{,}14 \text{ x } 4\text{x}10^{56}\, cm^2 \text{ x } a$$

$$m_0 \ll 1{,}57 \text{ x } 10^{27}\, \frac{g}{cm} \text{ x } a. \tag{5.32}$$

We will see in the next section that there appears to be a smallest value for a, namely in the order of magnitude of the Planck length $l_p = 1.61 \text{ x } 10^{-33}$ cm (cf. (6.7). If one inserts this value for a, then our derivation above is valid only if m_0 stays below a maximum size:

$$m_0 \ll 2{,}54 \text{ x } 10^{-6} \text{ g}. \tag{5.33}$$

This is about a tenth of the Planck mass ($m_p = 2.176 \text{ x } 10^{-5}$ g). Given the strong approximations we have made above, quantitative results should of course be interpreted with great caution.

In (5.23) we only considered the induced mass fraction m_{0n}*. Because of the linearity of the equations of motion (in our approximation), the total motion for the mass m_0 results from the superposition of the solutions of Eq. (5.30) and the motion of the mass m_0 without the influence of the world environment induced by it, as described by (1.1) to (1.3).

6. A Possible Connection to Quantum Theory

We now want to leave the realm of Newton's laws and enter the world of quantum mechanics and write down the Klein-Gordon equation:

$$\frac{\partial^2}{\partial x^2}\,\Phi(x, t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\,\Phi(x, t) - \frac{m_0{}^2 c^2}{\hbar^2}\,\Phi(x, t) = 0. \qquad (6.1)$$

It describes a free (spinless) particle that moves in the x-direction. As is well known, it is obtained from the energy formula for relativistic particles (1.3) by replacing energy and momentum by the appropriate operators (and applying them to a wave function).
Obviously (5.30) and (6.1) are identical in form. Physically, they also describe the same process, namely the free motion of a particle in space, and they are both Lorentz invariant. However, they are derived in a completely different way.
Now Φ(x, t) in Eq. (6.1) is a wave function, and the question arises as to the meaning of the function Q(x, t) in Eq. (5.30). If one considers the form of the Breather solution (3.9), then it seems obvious to interpret the square of Q(x, t) as proportional to the probability w(x, t) that a "particle" on the dislocation line moves at the time t and location x. We could then write:

$$w(x, t) = \frac{1}{A^2}\,|\,Q(x, t)\,|^2. \qquad (6.2)$$

As in quantum theory, the normalization factor A^2 results from the condition that the particle must be somewhere at all times:

$$\int_{-\infty}^{+\infty} \frac{1}{A^2}\,|\,Q(x,t)\,|^2\,dx = 1. \qquad (6.3)$$

With this interpretation, Q and Φ seem to become consubstantial, and we can then put:

$$Q(x, t) = \Phi(x, t). \qquad (6.4)$$

Using this relation we now subtract the two equations (5.30) and (6.1) from each other and find:

$$32\,G^2\,\frac{m_0{}^2}{a^2 c^4 L'^2} = \frac{m_0{}^2 c^2}{\hbar^2} \qquad (6.5)$$

or

$$\hbar = \frac{c^3 a L'}{4\sqrt{2}G}. \qquad (6.6)$$

Since a can be chosen arbitrarily (Newton’s point mechanics!) and L' is also not yet fixed, we can use the abbreviation

$$l_p{}^2 = \frac{aL'}{4\sqrt{2}}. \qquad (6.7)$$

Using (6.6) we can finally write:

$$l_p = \sqrt{\frac{G\hbar}{b_0{}^3}} = \sqrt{\frac{G\hbar}{c^3}}. \qquad (6.8)$$

This is the well-known relation for the Planck length. However, it does not result from dimensional considerations, as Max Planck had done, but follows from the comparison between the Sine-Gordon and Klein-Gordon equations. One recognizes that this makes a fundamental difference when one brings (6.6) into the following form using the relation (5.26):

$$ħ = \frac{\sqrt{(2\pi\, G\, \rho\, R_0{}^2)^3}}{G}\, l_p{}^2 \,. \tag{6.9}$$

Here either the Planck length lp or the Planck constant ħ can be considered as a natural constant. The respective other quantity then also depends on the parameters of the universe G, ρ and R_0. One sees that ħ is non-zero only if l_p is finite and vice versa. Since we know from countless experiments that ħ exists and is finite (and known quantitatively), there must be a smallest value for l_p, which is determined by (6.8). Because of (6.7) L' is then also fixed, because for a = l_p we get

$$L' = 4\sqrt{2}\, l_p = 5{,}65\, l_p \,. \tag{6.7a}$$

We can still bring Eq. (6.9) into the following form:

$$ħ = 2\pi\, c\, \rho\, R_0{}^2 l_p{}^2. \tag{6.10}$$

We have thus found a simple and universally valid form (c is location-dependent!). Perhaps the form gains even greater simplicity or "beauty" if we introduce a universal area:

$$A_u = R_0\, l_p \tag{6.11}$$

$$ħ = 2\pi\, c\, \rho\, A_u{}^2. \tag{6.12}$$

All of our previous results (including those of previous papers [1] and [4]) are based solely on Newton’s laws before comparing (5.30) and (6.1). Therefore, we must interpret the fact that there is apparently a smallest distance at which two particles can approach each other in such a way that Newton's law for the gravitational force for very small distances between two gravitational masses is to be supplemented by a repulsive term, which works at very short distances. Such a force law can be formulated mathematically in many ways. In order to better discuss the consequences, we arbitrarily choose the following form:

$$F_g = G\, \frac{m_1 m_2}{r^2}\, \left(e^{\left(1 - \frac{r^2}{l_p{}^2}\right)} - 1\right). \tag{6.13}$$

For r = l_p the repulsive and the attractive forces cancel each other out, for $r \gg l_p$ F_g turns into the usual Newtonian attractive force, for $r < l_p$ the force causes an asymptotically increasing repulsion.
If we assume the validity of (6.13), then in our above consideration of the introduction of a foreign mass m_0 into the undisturbed lattice of the universal environment, we must not start from the attractive force (5.3), but we must start from the following repulsive force:

$$F_{ni} = -\, G\, \frac{m_{0n} m_{ni}}{(\frac{a}{2} + s_{ni})^2}\, \left(\exp\left(1 - \frac{(\frac{a}{2} + s_{ni})^2}{a^2}\right) - 1\right). \tag{6.14}$$

Otherwise, if we carry out the calculation exactly as above from (5.3) to (6.5), we get the relation instead of (6.5):

$$8 \text{x} 5 \text{x} e^{\frac{5}{4}}\, G^2\, \frac{m_0{}^2}{a^2 c^4 L'^2} \;=\; \frac{m_0{}^2 c^2}{ħ^2} \,. \tag{6.5a}$$

The factor in front of G^2 on the left side of (6.5) is now larger by f = 5/4 $e^{(5/4)}$ = 4.36 than in the case of the attractive gravitational interaction. In terms of quality, nothing changes in our above results. Only the length L' is slightly more than twice as large as according to (6.7a). Given the significant approximations we made in the above study, this does not affect our fundamental result. It is interesting that both values of L' are of the same order of magnitude (ca. 10 interatomic distances) as in the theory of point defects in anisotropic crystals, which can be described by a linear chain of atoms [24)].

If (6.13) (or a mathematically comparably formulated force) represents the fundamental force effect of gravitation, then two masses can never come arbitrarily close if we assume that their energy (sum of their kinetic and potential energy) can only ever be finite . There is also no singularity in their mathematical description, in contrast to the original Newtonian form without a repulsion term.

If equation (6.13) has a universal character, then l_p is the universal natural constant. In this case, the Planck constant would drop to the rank of a derived quantity. This would presumably be experimentally verifiable, because according to (6.9) or (6.12) ħ would depend, among other things, on the density of the universe and its radius. At an earlier point in time after the creation of the universe ("Big Bang"), these could have had significantly different values than today. Measured values that result from quantum theory (and are thus determined quantitatively by ħ) would then have to show different values for these earlier points in time than today.

Finally, we would like to point out some other facts:

a) As we know, the cosmological constant term in Einstein's field equation (for large curvature) also represents a repulsive effect of gravity, also on the Planck length scale. Here again there is obviously a similarity between the theory developed here (and in [1), 4)] and [22)]) and Einstein's theory of relativity.

b) Our derivation above leads to the picture of a linear chain with a certain length L', which is of the order of the Planck length. It expresses a connection between gravity in the universe and quantum theory. Should there be a connection to string theory here?

c) It has long been known [17), 20) 21)] that there are solutions to the Sine-Gordon equation in the crystal, which describe the formation or mutual annihilation of so-called kinks and anti-kinks. According to the properties of particle movements in the universe presented above, such solutions correspond to the creation and annihilation of particle pairs, as described by relativistic equations in quantum theory, which in turn are closely related to the Klein-Gordon equation. This seems to be further evidence for a close connection between the quantum mechanical world and the universe described by classical Newtonian point mechanics. Possibly, the universe made up of Newtonian point masses might behaves like the "vacuum" in quantum theory.

In view of the assumptions and approximations that we have made on the way up to here and also because of the previous limitation to a one-dimensional view, we do not want to draw any hasty conclusions here, but see the thoughts presented above as indications of possible connections. In order to convert the clues into evidence, in-depth investigations are of course necessary. And of course, it is also necessary to examine how the total motion of the mass m_0 will behave as a superposition of the "undisturbed solution" (without m_0*) and the solution of Eq. (5.30) (when does which part predominate? correspondence principle? Ehrenfest theorem?). But this is not the subject of the work presented here.

7. Summary

The investigation carried out here is based on the fact that some essential movements of particles in the universe are determined in the same way as in crystals. A relationship between Newton's laws and quantum theory can thus be derived. The universal validity of Planck's constant is possibly less important than that of Planck's length, and it then depends on the parameters of the universe, namely its density, its expansion and Newton's gravitational constant. The Newtonian gravitational force apparently has to be supplemented by a repulsion term at very small distances between the involved masses, which is determined by the Planck length.

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