A power sequence model as a bridge between fundamental natural constants and macroscopic liquid systems

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Abstract

A simple mathematical interaction function, with the limit value $w = e^{2\pi/e}$, derived as a basis for modelling emergent properties of nanoparticles, nanodrops and liquids, has been further developed. A correlation between physical systems of uniform particles and fundamental properties of natural numbers leads to a model, which acts as a bridge between properties of macroscopic systems and fundamental physical constants.
The four fundamental coupling constants, and the masses of fundamental elementary particles, including the Higgs and Z bosons, are derived by using this interaction function. A proposal for the mass of the electron neutrino is given. Finally, the nuclear binding interaction is modeled with the interaction function. Only the experimental values of the *Planck* constant h and the speed of light c are needed for the model.

1. Introduction

Recently an interaction function for liquid particles has been published [1,2]. For this interaction function a system of uniform particles with finite energies, as a basic assumption of quantum mechanics, has been correlated with the system of natural numbers and for x interacting particles. The function:

$$y = x^{q/x} \tag{1}$$

with the maximum values:

$y_{max} = 3^{q/3}$ for $x = 3$

and $Y_{max} = e^{q/e} > y_{max}$ for $x >> 1$

and

$(1+1/x)^x = e = 2.71828\ldots$

results.

Using the de Broglie relation for linear momentum, $p = h/\lambda$ of the particle, along with the Planck constant h and a wavelength λ, a second basic assumption of quantum mechanics has been applied. With a relative wavelength $\lambda = 2\pi = q$, the following power sequence is defined [2]:

$$w_{n,e} = \left(1 + \frac{2\pi}{n}\right)^{\frac{n}{e}}, \tag{2}$$

with the limit value:

$\lim_{n\to\infty} w_{n,e} = e^{2\pi/e} = 10.08909.. \approx \frac{2\pi}{w_1 - w_{1,e}} = 10.0898\ldots,$

and

$w_1 = (1+2\pi)^{1/2} = 2.6987\ldots$

and $w_{1,e} = (1+2\pi)^{1/e} = 2.076\ldots$

The power sequence in Equation (2) is denoted as the interaction function.

The functions y_{max} and Y_{max} in Equation (1) suggests two different kinds of interactions between physical particles.

As shown in [1], properties of liquids can be modelled with relations based on the interaction function in Equation (2). The interactions are all of electromagnetic nature.

Most properties of macroscopic systems are described with exponential functions. Positive exponents stand for high kinetic values, like for example diffusion or reaction constants. Negative exponents are responsible for strong attractive interactions in the matrix, like the activation energy of diffusion in polymers and of chemical reactions as two examples.

The maximum $y_{max} = 3^{q/3}$ for three interacting particles in Equation (1), refers to the first term, $w_1 = (1+2\pi)^{1/2} = 2.698738$ with n =1 in the power sequence:

$w_n = \left(1+\frac{2\pi}{n}\right)^{\frac{n}{(1+1/n)^n}}$ with the same limit w as in Equation (2) . The exponent $n/(1+1/n)^n$ approaches to a maximum at $e = 2.718$ with increasing n.

Subsequently $n/(1+1/n)^n = 2.59$ for n = 10 and 2.705 for n = 100.

The value w_1 represents the relative energy of one elementary particle. In conformity with Equation (1) a maximum of interaction results with a combination of three elementary particles and this leads to:

$y_{max} = w_1^3 = 19.6554 = 3^{q/3}$,

with a value $q = 8.1330397$

and $q/3 = 2.711\ldots \approx e$.

As shown later, the strong force is responsible for the interaction between three elementary particles, with $y_{max} = w_1^3$.

The three interacting particles have a rest mass, m_0 and in conformity with the Einstein Equation, $E = mc^2$, a rest energy $E_0 = m_0c^2$, with the speed of light c.

Now a relative rest energy is defined as coming from the three interacting particles w_1:

$E_1^3 = 1/[exp(w_1^3)\cdot w_1^3] = 1.48\cdot 10^{-10}$, The exponent in E_1^3 shows the strong attractive interaction resulting from the three particles.

In order to transform the relative energy value w_1 into a quantity expressed in IS units the limit value

$$\lim_{n\to\infty} w_n = \lim_{n\to\infty}\left(1+\frac{2\pi}{n}\right)^{\frac{n}{(1+1/n)^n}} = e^{2\pi/e}$$

is related in this case to the decimal system with the basic number 10. In this way $w_1 \cdot w/10$ represents the energy unit in J.

That means, the rest energy

$E_0 = E_1^3 \cdot w/10$ J = $1.49326 \cdot 10^{-10}$ J

and with $c = 2.99792458 \cdot 10^8$ ms^{-1}, the atomic mass unit $m_0 = u_0$ results:

$u_0 = E_0/c^2 = (1/[exp(w_1^3) \cdot w_1^3])w/10c^2 = 1.661475 \cdot 10^{-27}$ kg

where $1.661475 \cdot 10^{-27}$ kg $\approx 1.66054 \cdot 10^{-27}$ kg = u [3]. (3)

The interaction function is of fundamental importance for emergent properties in fluid phases and the first term w_1 forms a bridge between the fundamental atomic mass unit and macroscopic properties.

With the Avogadro Constant N_A and the atomic mass unit, the atomic mass for hydrogen, A_H is:

$A_H = 6.022 \cdot 10^{23} \cdot$mol^{-1} x $1.661 \cdot 10^{-27}$ kg = 1 g mol^{-1}.

The relative atomic mass for hydrogen is H = <u>1.</u>

This connection explains the importance of the relative mass values M of particles in macroscopic systems and of subparticles (atomic groups) in the molecules of the system.

As will be shown later, Equation (1) should be taken into account for all individual particles, independent of their nature, with the gravitational interaction as an example.

2. A general interaction relation

Now a further development of the above model is presented. Initially only a finite energy of the uniform particles in the physical system is assumed, with the relative value q = 1. Again, the system is correlated with the system of natural numbers.

The maximum value $e^{1/e}$ with q = 1 in Equation (1), resulted as a natural organization process with natural numbers in the direction of maximum interactions. This process can be presented formally as a result from the first two steps in the following algorithm:

$$\begin{aligned}
&y_1 = exp(q) \\
&y_2 = exp(q/y_1) \\
&y_3 = exp(q/y_2) \\
&\ldots\ldots\ldots\ldots\ldots \\
&y_i = exp(q/y_{i-1}).
\end{aligned} \tag{4}$$

With q =1, $y_1 = e$; $y_2 = e^{1/e}$.

A continuation of the interaction process, towards maximum values, is now assumed in the form of the algorithm (4).

The algorithm provides an oscillating sequence. Starting with q = 1, one monotone decreasing sequence y^{I}_{2n-1} and one monotone increasing sequence y^{II}_{2n} results, which asymptotically reach the same limit value (Figure 1),

$y^{I}_{\infty} = y^{II}_{\infty} = y_{\infty}(1)$ = 1.763222834351897…> $e^{1/e}$ = 1.44466…

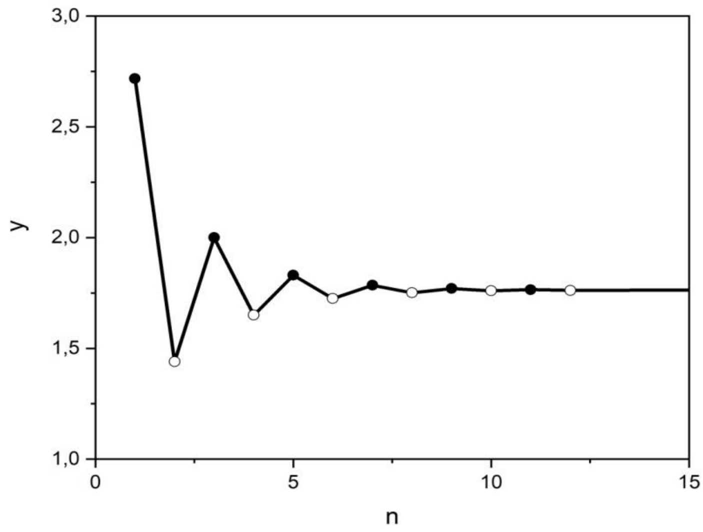

Figure 1. Graph (▬) of the oscillating sequence y_n in the algorithm (4) for q = 1: y^{I}_{2n-1} (●) and y^{II}_{2n} (○) with the same limit value $y^{I}_{\infty} = y^{II}_{\infty} = y_{\infty}(1)$ = 1.7632…

The algorithm (4) based on the exponential function is used as the backbone in this investigation, with $y_{\infty}(1)$ = 1.763222834351897…as the first oscillation limit point.

The limit value $y_{\infty}(1)$ obtained with q = 1 can now be used again as starting value $q_2 = y_{\infty}(1)$ in the algorithm (4) with the result of a new limit value $y_{\infty}(q_2)$, and so on. In this way a final limit value $y_{\infty}(e) = e$ results asymptotically.

The above results are a consequence of the before mentioned organization process with natural numbers based on their basic properties. The correlation with a physical system based only on the assumption of uniform particles with finite relative energies, q = 1, leads in this way to a maximum relative interaction density of energy:

$$y_{\infty}(e) = exp(e/y_{\infty}(e)) = e. \tag{5}$$

The value $y_\infty(e) = e$ is understood as the dimensionless limit value for an interaction constant. As will be shown later, this limit is the coupling constant for the strong force.

With the introduction of the second fundamental quantum mechanical assumption for the physical system [1], in form of the relative wavelength $\lambda = 2\pi$, a special result is obtained with algorithm (4), for $q = w > e$.
For an extension of the algorithm (4) into a region with $q > e$, the following invariant structure is postulated:

$$exp(e\cdot A\cdot B/y_\infty(q)) = e\cdot A\cdot B. \qquad and \quad B = (1/e\cdot A)\cdot e^{e\cdot A} \text{ for } y_\infty(q). \qquad (6)$$

The graph of the function $B = f(A)$ from Equation (6) is shown in Figure 2.

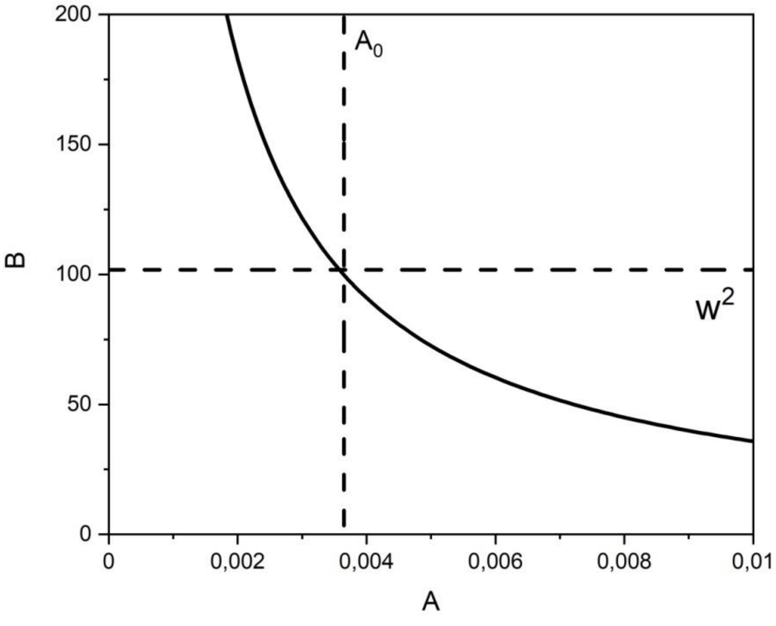

Figure 2. Graph of $B = (1/e\cdot A)\cdot e^{e\cdot A}$.

In contrast to $y_\infty(1)$, with q =1, two different limit values, $y^I_\infty(w)$ and $y^{II}_\infty(w)$ are obtained for $q=w$ with algorithm (4):

$y^{I}_{\infty}(w)$ = 23976.90006232883…> $y^{II}_{\infty}(w)$ = 1.000420872319414… and their ratio $Y_{\infty}(w) = y^{I}_{\infty}(w)/y^{II}_{\infty}(w)$ = 23966.81309411294…

The value $Y_{\infty}(w)$ is the second basic limit value in this investigation. Of special interest for the following relations is the ratio obtained from $y_{\infty}(1)$ and $Y_{\infty}(w)$:

$$Y_{\infty}(1,w) = exp\left(\frac{y_{\infty}(1)}{Y_{\infty}(w)}\right) = 1.000073572055.$$

Equation (6) is the fundamental relation for connections with natural constants.

3. Connection between the interaction function and fundamental natural constants

With $B = w^2$, the number $A = A_0/2$ with A_0 = 1/136.9807 results from Equation (6), Fig.2. This result with A_0 indicates a connection between w and the fine structure constant [3]:

$$\alpha = \frac{e_q^2}{4\pi\varepsilon_0} \cdot \frac{2\pi}{hc} = 1/137.035999084, \tag{7}$$

with the elementary charge e_q, the vacuum permittivity ε_0, the speed of light, c = $2.99792458\cdot10^{8}$ $m\cdot s^{-1}$, and the Planck constant h = $6.62607015\cdot10^{-34}$ Js [3].
α is a dimensionless fundamental constant of nature, derived in 1916 by Sommerfeld, representing the electromagnetic coupling constant.

The ratio

$$Y_0(A,B) = exp(A_0/w^2) = 1.00007172196$$

from the initial values A_0 and $B = w^2$ in Equation (6) is similar to the above derived ratio $Y_\infty(1,w) = 1.00007357206$.

With the limits $Y_\infty(w)$ and $Y_\infty(1,w)$ the following number b, similar to w^2, can be defined:

$$b = \left(\frac{Y_\infty(w)\cdot Y_\infty(1,w)}{(2\pi/e)}\right)^{1/2} = \pm\ 101.8306291\ldots \qquad (8)$$

With $b = 101.8306291$ and Equation (6) the following iteration, for representing a_n values related to b_n values, is used:

$$a_n = \left(\frac{2}{e\cdot b}\right)\cdot e^{\frac{e\cdot a_0}{2}}\cdot\frac{Y_\infty(1,w)}{Y_0(A,B)} \qquad \text{and} \qquad b_n = b\cdot\frac{Y_n(A,B)}{Y_\infty(1,w)} \qquad (9)$$

Starting with the initial values $a_0 = A_0 = 1/136.98$ and $b_0 = w^2$, the iteration procedure (9) leads to limit values a_∞ and b_∞:

$a_1 = 1/137.035466$, $b_1 = 101.8304378$; $a_2 = 1/137.036002$, $b_2 = 101.830435$;
$a_3 = 1/137.0360069$, $b_3 = 101.8304348$; $a_4 = 1/137.03600696$, $b_4 = 101.8304348$.

$a_\infty = 1/137.03600696$ and $b_\infty = 101.8304348$.

With the above iteration, the ratio $\alpha/a_\infty = 1.000000057$ results.

With $A = \alpha/2 = (1/137.035999084)/2$ and with Equation (6) a value $b_\alpha = 101.830429028$ results directly, with the ratio $b_\infty/b_\alpha = 1.000000057$. However, in this way the limit property of b_∞ as result from the algorithm (4) is not apparent, because it is related directly to α as an experimental value. Due to these small differences between experimental and calculated values, a_∞ is used in the following instead of α as the electromagnetic coupling constant.

With Equation (6) and iteration (9) the limit value a_∞ results as a modelled value for the electromagnetic coupling constant. From this modelled value, instead of α, in combination with the two experimental values for h and c, the value for $e_q^2/4\pi\varepsilon_0$ = $2.3070775 \cdot 10^{-28}$ Jm results with Equation (7), instead of the experimental value found to be $2.3070758 \cdot 10^{-28}$.

The results obtained until now demonstrate the importance of the asymptotical approach introduced with algorithm (4) for the corresponding maximal limit values of the different constants. That supports the assumption of a natural evolution process which occurs in multiparticle systems, based on fundamental properties of natural numbers.

The model developed in this investigation [1,2] has no similarity with the well known Standard Model for interaction between fundamental natural constants [5].

The following derivations of coupling constants and masses of elementary particles emphasize the strong connection between different kinds of interactions, following from the same initial assumption. This can be seen in the following example which demonstrates the strong connection between the electromagnetic coupling constant a_∞ and the limit value w of the interaction function (2). In the application of the interaction function presented in [1], with the vapor pressure of droplets as a special example, the interactions between the particles occurred in the form of strings. This behavior appeared as a characteristic property of electromagnetic interactions between the particles. The number $B = w^2$ in Equation (6), similar with the limit value b_∞, suggests a formal similarity between w and an electron as a multiparticle system as shown in Equation (1) for $n >> 1$. With such a formal correlation, an electron may appear as a system of interacting subparticles in the form of strings along a linear arrangement, with the possibility of place exchanges between the linear arrangements of the interactions in space of the electron at a certain time. Only after an interaction between a string and another particle placed in the string direction, the exact position of the electron becomes observable [4].

Although very speculative, this discussion generates an idea about the structure of the electron, as being similar to a liquid phase. The interactions between subparticles in the electron in form of strings, have as consequence the a priori unpredictable direction of the occurring interaction. This situation is fundamentally different from that in the nucleus, were the strong force is responsible for the interactions.

The electromagnetic coupling constant a_∞ results in this way as interaction of a pair of particles, corresponding to w^2, with

$A_0 = 1/136.98$, $b_0 = w^2$ and $Y_0(A,B) = \exp(A_0/w^2)$.

A formal similarity between w and an electron, as mentioned above, can be in conformity with Equation (1) as a triplet of interacting subparticles representing a real electron, as will be shown later.

The gravitational coupling constant

The limit value

$$\pm b_\infty\cdot(a_\infty\cdot e/2)\cdot e^{-a\infty\cdot e/2} = \pm 1 \tag{10}$$

resulting from Equation (6) is obtained using algorithm (4), along with the limit value $w > e$ in Equation (6). This result represents the two extremes obtained in the form of an extension process and a contraction process, as a consequence of interactions occurring in a physical system. As a total result, this leads to

$$+b_\infty\cdot(a_\infty\cdot e/2)\cdot e^{-a\infty\cdot e/2} + (-b_\infty)\cdot(a_\infty\cdot e/2)\cdot e^{-a\infty\cdot e/2} = 1 + e^{\pi i} = 0. \tag{11}$$

That means, despite the great changes in interaction energies during these processes, no additional building or destruction of parts of the total energy of the system occurs.

The small value of $e^{-b\infty} = 5.964912489 \cdot 10^{-45}$ indicates a connection with the dimensionless, gravitational coupling constant of nature, a_g.
The relation $a_g = Gm_e^2 2\pi/hc$ contains the same reference group $2\pi/hc$ like that in α.

$a_g = 1.75168850975 \cdot 10^{-45}$, using the constant of gravitation
$G = 6.67408(31) \cdot 10^{-11}$ m^{3}kg^{-1}s^{-2}
and the electron mass
$m_e = 9.1093837015 \cdot 10^{-31}$ kg = 0.5109989461(31) MeV [4]
represent real, experimentally measured constants, as well as the dimensionless ratio $a_g/\alpha = 2.4004438 \cdot 10^{-43}$ [4].

If the limit value w is considered to represent a particle, than in conformity with Equation (1), w^3 represents a particle (electron) having the highest interaction intensity and this leads to an interaction constant $(w/exp(w))^3$ for particle triplets.

By taking into account the limit value $w_b = (b_\infty)^{1/2} = 10.0911067$ from Equation (10), the following gravitational coupling constant is defined:

$$a_{g,\infty} = \left[w_b \left(\frac{w_b}{e^w} \right)^3 \right]^3 \frac{1}{e} \frac{1}{c^2} \cdot C_{ag} = 1.75169013 \cdot 10^{-45} \qquad (12)$$

With the correction factor C_{ag} :

$$C_{ag} = \frac{exp(b_\infty)}{exp\,(w^2)} \left(\frac{w_b}{w} \right)^9 \left(\frac{w_b}{w} \right)^3 \cdot \left(\frac{w_b}{lnY_\infty(w)} \right)^{1/2} = 1.04437119\ .$$

This gives the ratio $a_g/a_{g,\infty}$ = 1/1.00000092.

Neglecting the correction factor C_{ag} , a value

$a_{g,\infty} = \left[w_b \left(\frac{w_b}{e^w}\right)^3\right]^3 \frac{1}{e}\frac{1}{c^2} = 1.67727 \cdot 10^{-45}$ with the ratio $a_g/a_{g,\infty}$ = 1.0444 results.

The definition of $a_{g,\infty}$ in Equation (12) relates the interaction constant to the limit value w_b and the product $\left[w_b \left(\frac{w_b}{e^w}\right)^3\right]^3$ to the limit e from the domain $q \leq e$ and to c^2, in conformity with the Einstein relation, $E = mc^2$.

The limit values result as asymptotical limits from the algorithm (4) and this procedure has the necessity of considering corresponding correction factors as consequence, taken into account in C_{ag} . The correction numbers result from the algorithm (4) with asymptotically reached limits and are related to well defined numbers. No pure empirical numbers are used at all to reach these results.

This procedure for correcting factors is applied for all the following cases as well.

The dimensionless gravitational coupling constant (12) can be split into two factors, G_∞ and $m_{e,\infty}{}^2$:

$$G_\infty = \left(\frac{w_b}{e^w}\right)^3 \frac{e}{3} \cdot C_g = 6.67404 \cdot 10^{-11}\ \mathrm{Nm^2kg^{-2}} \tag{13}$$

with

$$C_g = \left(\frac{w_b}{w}\right)^3 \left(\frac{w_b}{lnY_\infty(w)}\right)^{1/6} = 1.00071008\ .$$

Where G/G_∞ = 1.000006. In this way the constant of gravitation results as a consequence of the extension of the algorithm into a region with $q > e$, with the corresponding limit b_∞.

A similar expression for electron mass can be derived:

$$m_{e,\infty}^2 = 3(w_b)^3 \left[\left(\frac{w_b}{e^w}\right)^3\right]^2 \frac{1}{e^2}\frac{1}{c^2} \cdot C_m/\left(\frac{2\pi}{hc}\right) \text{ kg}^2 = 8.29784557 \cdot 10^{-61} \qquad (14)$$

with a correction factor:

$$C_m = \frac{exp\ (b_\infty)}{exp\ (w^2)}\left(\frac{w_b}{w}\right)^9 \cdot \left(\frac{w_b}{lnY_\infty(w)}\right)^{1/3} = 1.0436301.$$

With $m_{e,\infty} = 9.1093837015 \cdot 10^{-31}$ kg, $m_e / m_{e,\infty} = 1.000015$.

As shown from the above relations, the mass of the electron results as a triple of the interaction value w. This behavior can be seen also from the 1/3 or 2/3 electrical charge of Quarks for the proton [5].

The electromagnetic coupling constant

The ratio between $a_g = Gm_e{}^2 2\pi/hc$ and $\alpha = (e_q{}^2/4\pi\varepsilon_0)2\pi/hc$ gives the dimensionless value:

$a_g/\alpha = 2.4004438 \cdot 10^{-43}$ [4].

This relation allows a definition of the electromagnetic coupling constant with the interaction function:

$$a_\infty = \frac{3b_\infty}{(e^w)^3}\frac{1}{c^2}\frac{2\pi}{hc}C_{e,\varepsilon} = 1/137.035954 \quad , \qquad (15)$$

$$C_\alpha = \left(\frac{exp(w^2)}{exp(b_\infty)}\right)^{4/3} \cdot \frac{b_\infty}{w^2} = 0.947575712913\ .$$

$\alpha/a_\infty = 1/1.0000003.$

The relation $\frac{3b_\infty}{(e^w)^3}\frac{1}{c^2}$ can be split into two factors, $e_{q,\infty}^2$ and $4\pi\varepsilon_0$:

$$e_{q,\infty}^2 = 3\left[\left(\frac{w_b}{e^w}\right)^3\right]^2 \frac{1}{e^2} \cdot \frac{1}{c^2} \cdot C_e = 2.566794 \cdot 10^{-38}. \qquad (16)$$

$$C_e = \left(\frac{exp(w_b)}{exp(w^2)}\right)^{7/6} \cdot \left(\frac{w_b}{w}\right) \cdot \left(Y_\infty(1,w)\right)^{-0.5} = 1.0487826$$

$e_{q,exp} = 1.602176634 \cdot 10^{-19}$, $\qquad e_{q,exp}/e_{q,\infty} = 1.00003$

$$4\pi\varepsilon_0 = w_b\left(\frac{w_b}{e^w}\right)^3\frac{1}{e^2}C_\varepsilon = 1.1125734\cdot 10^{-10}\ . \qquad (17)$$

$$C_\varepsilon = \frac{C_e}{C_\alpha} = 1.106806\ ,$$

$4\pi\varepsilon_{0,exp} = 1.11265\cdot10^{-10}$; $\frac{4\pi\varepsilon_{0,exp}}{4\pi\varepsilon_{0,\infty}} = 1.00007$.

From all above derived relations the importance of the initial Equation (1), with the maximum for three interacting particles is shown.

The strong and weak coupling constants

The coupling constant a_s for the strong force and the constant for the weak coupling a_w are not constant in reality [5]. Experimentally, the corresponding coupling constant for the strong force a_s - as determined from the force between two protons - is greater than one [5]. But a decrease of the strong force with decreasing distances between the interacting quark particles occurs. This "running" coupling constant is a result of the phenomenon known as asymptotic freedom. The weak coupling a_w, also behaves similarly but at a slower rate [5].

With the four dimensionless coupling constants a_g (gravimetrical), a_∞ (electromagnetic), a_w (electroweak), a_s (strong coupling) and Equation (6) the following relation results:

$$ln(a_g\cdot a_w/(a_\infty\cdot a_s/2))\cdot(a_\infty\cdot a_s/2)\cdot e^{-a_\infty\cdot a_s/2} = -1 = -b_\infty\cdot(a_\infty\cdot e/2)\cdot e^{-a_\infty\cdot e/2} = e^{\pi\cdot i} = cos(\pi)+i\cdot sin(\pi) \qquad (18)$$

The four fundamental constants of nature are connected to a fundamental mathematical relation between the transcendental numbers e and π. This relation is the result from the use of the limit value w of the interaction function (2) in the algorithm (4).

The structure of this relation and the limit value $y_\infty(e) = e$ from Equation (5) suggests the value $a_s = e$ for the strong coupling constant. With this value

$$a_w = exp(-b_\infty)\cdot a_\infty \cdot a_s / a_g \cdot 2 = 0.0337736 = 1/29.609, \qquad (19)$$

results for a weak coupling constant a_w.

The value a_w = 1/29.60895 is similar to the published value 1/29.5 [5]. The value for the weak mixing Weinberg angle $a_\infty/a_w = sin^2(\Theta)$ = 0.216067, calculated from Equation (10) is 3 % smaller in comparison with the CODATA [3] value 0.2229(30). With this value the weak coupling constant is a_w = 1/30.545 = 0.03274. These results demonstrate the variability of the a_w and a_s values. But from Equation (9) the following constant connection results:

$$a_s/a_w = 80.4856. \qquad (20)$$

The gravimetric coupling constant a_g and the electromagnetic coupling constant a_∞ appear only in connection with the Equation (6), beyond the limit $q = e$. But both domains, $q \leq e$ and $q \geq e$ must be considered for the strong coupling constant a_s and the weak coupling constant a_w. As a consequence this has a variability in the coupling constant values.

The weak coupling appears as a consequence from the extension of Equation (5) to Equation (6), with $q = w$ instead $q = e$ as a starting point in the algorithm (4). The small value $q_{v,0} = 2\cdot\pi/w$ = 0.62277 is inside of the range for the application of the algorithm (4) in conformity with Equation (5). This suggests the definition of two limit values for a_w, based on e from Equation (5) and w from Equation (6). With $e \cdot w$ = 27.425 the maximum value $a_{w,max} = 1/e \cdot w$ = 0.036463 is defined.

The dimensionless limit value for the interaction $y_\infty(e) = e^1$ from Equation (5) corresponds to b_∞ = 101.8304348 in Equation(6). If $-1/y_\infty(e) = -e^{-1}$ is related to -b_∞, then a minimum value

$a_{w,min} = -e^{-1} \cdot (-b_\infty)$ = 1/37.46132 is defined.

Similar to the above two limit values for $a_{w,}$ two limit values:

$a_{s,max} = a_{w,max} \cdot 80.4856 = 2.93475 > a_s = e$,

and

$a_{s,min} = a_{w,min} \cdot 80.4856 = 2.1485$

are obtained with Equation (20).

The coupling constant a_g contains the mass m_e of an electron and therefore a connection between m_e and w can be established. From the approximation $exp(w^2)/(e^w)^2 \cdot m_e \approx e^w \cdot w$, the following reference number M_e is defined:

$$M_e = \frac{e^w \cdot w_\infty}{Y_\infty \cdot (1,w)} \cdot \frac{1}{m_e} = 242813.98 \cdot \frac{1}{m_e} \qquad (21)$$

The numbers $w_\infty = ln(y_\infty^I(w))$ *and* $Y_\infty(1,w)$ take into account the asymptotical limits attained with the algorithm (4) at q =1 and $q = w$, respectively.

With Equation (21) the masses of other elementary particles can be related to the electron mass, as shown in the following examples.

The mass of the electron neutrino

The weak and electromagnetic interactions can be treated as different manifestations of a single electroweak force [5]. The Weinberg angle Θ shows the relation $a_\infty/a_w = sin^2(\Theta)$. For neutrinos no electromagnetic interactions like those for electrons occur. Instead of using w as starting point in the algorithm (4), a starting value

$$q_{\nu,0} = 2\cdot\pi/w = 0.62277 \; . \tag{22}$$

is used. With the corresponding values $e^{q\nu,0}$ and $q_{\nu,0,\infty} = 0.412315$, resulting with the algorithm (4), and with the weak coupling constant a_w, an expression for the neutrino mass, $m_{\nu,0}$ results:

$$m_{\nu,0} = \frac{e^{q_{\nu,0}} \cdot q_{\nu,0,\infty}}{M_e} \cdot a_W . \tag{23}$$

A term corresponding to $Y_\infty(1,w)$ in Equation (21) is not necessary in Equation (23), because $2\pi/w < e$ and only one limit value from the algorithm (4) results:

$$q_{\nu,0,\infty} = \ln(y_\infty(q_{\nu,0}))$$

With $a_w = 1/29.609$ from Equation (19), $m_{\nu,0} = 0.05436$ eV.

With different a_w values, different values for $m_{\nu,0}$ are obtained. But given the connection discussed below between the neutrino and the Z boson [5], only the minimum value $a_{w,min} = 1/37.46132 = 0.02669$ is selected and the mass $m_{\nu,0} = 0.0432$ eV is calculated with Equation (23).

Whereas w results as the limit value from electromagnetic interactions in conformity with Equation (2), $w_{1,e} = (1+2\pi)^{1/e} = 2.076$ can be understood as a limit value resulting from Equation (2) if no such interactions occur between the n particles. The three terms, w_1, $w_{1,e}$ *and* w_1-$w_{1,e} = 0.6227 \approx 2\pi/w$ are then used representing three different neutrino masses with Equation (2).

Starting with $q_{v,1} = w_{1,e}$ in algorithm (4), $q_{v,1,\infty} = 0.87$ is obtained and with a relation similar to Equation (23), $m_{v,1} = 0.388$ eV is obtained.

With $q_{v,2} = w_1$ in the algorithm (4), $q_{v,2,\infty} = 0.896$ and with a relation similar to Equation (23), $m_{v,2} = 0.745$ eV results for $a_{w,min}$.

It is interesting to note an additional connection between fundamental elementary particles and macroscopic systems. For one mol of neutrino particles, Equation (23), leads to an energy of 4168 Jmol^{-1}, with the minimum value for a_w and the Einstein relation $E = m \cdot c^2$. If the difference w_1-$w_{1,e}$ in entropy units, is related to the entropy 108.85 JK^{-1}mol^{-1} in the gas phase, then
$4168 \cdot (w_1 - w_{1,e})/108.85 = 23.85$ Jmol^{-1} results [1]. This value is 4 % smaller in comparison to the value 24.78 Jmol^{-1} obtained with the state equation $pV = RT$. A neutrino mass corresponding to the value from the state equation leads to 4331 Jmol^{-1} and 0.0449 eV instead of the minimum value 0.0432 eV in Equation (23). This case emphasizes the possible decrease of a_w as function of its source.

Experimentally an upper limit to the neutrino mass from a direct kinematic method by KATRIN is < 0.8 eV [6]. From combined cosmological observations and particle physics experiments an upper bound of the sum of neutrino masses, ≤ 0.26 eV and an approximation-independent upper bound for the lightest neutrino mass species, < 0.086 eV [7] are found.

The mass of the Higgs and Z boson

Whereas the electron mass m_e is related to the electromagnetic coupling constant a_∞, a mass m_1 can be derived from the connection between the term $w_1 = (1+2\cdot\pi)^{0.5} = 2.6987..$ in the power sequence (2) and the strong force a_s. Similar to the above use of two limit values $a_{w,}$ two limit values are obtained with Equation (20):

$a_{s,max} = a_{w,max}$·80.4856 = 2.93475 > $a_s = e$ > 2.1485 = $a_{s,min}$.

In conformity with Equation (1), a combination of three particles with relative values w_1 is considered to be representative of three quarks in a particle with a mass m_3. The interaction of three terms w_1 as w_1^3 can be considered as resulting from the strong force. With a similar relation like Equation (21) the following expression for m_3 is derived:

$$m_3 = \frac{e^{w1^3} \cdot w_{1,\infty}^3}{M_e} \cdot 3a_S \tag{24}$$

The factor 3 takes into account the interaction of the three particles (quarks). Corresponding to $Y_\infty(1,w)$ in Equation (21), the term $Y_\infty(1,w_1^3) = 1$.

With $a_{s,max}$ instead of a_s, the value m_H = 125.186 GeV results from Equation (24). It is the mass for the Higgs boson. The experimental value for this is m_H = 125.10 ± 0.14 GeV [8,9].

With $a_{s,min}$ a mass m_Z = 91.647 GeV results with Equation (24). This is similar with the mass of the Z boson having the experimental value m_Z = 91.19 GeV [5].

The mass of the proton

The term $w_1 = 2.699 < e$ from the interaction function is inside of the application range for algorithm (4) in conformity with Equation (5). That means, only a strong force coupling constant exists. This can be defined as $a_{s,p} = e^{-e}$ in comparison with a_s from Equation (18), as only one of 4 coupling constants, resulting with $e^{-b\infty}$. That means, e^{-e} stands for $e^{-b\infty}$, as a consequence of the extension into a region with $q > e$. With $a_{s,p}$ and an analogous relation to Equation (21) the following mass of the proton is obtained:

$$m_p = \frac{e^{w1^3} \cdot w_{1,\infty}^3}{M_e} \cdot \frac{1}{e^e} \cdot 1836.15073 \cdot m_e \tag{25}$$

The term $w_{1,\infty}^{3} > e$ is treated as a single particle.

The experimental value for the ratio m_p/m_e = 1836.1527 [3].

From this result the similarity with the atomic mass unit derived in Equation (3) is shown.

The value $1/e^e$ = 0.066 demonstrates a decrease of the strong force with decreasing distances between the interacting quark particles [5].

A hypothetical mass $\mathbf{m_x}$

From symmetrical reasons, in addition to the value *w* used in algorithm (4), a value $q = 1/w = w_x$ is defined and introduced in the algorithm (4), producing a limit value $w_{x,\infty}$ = 0.0905374. Whereas in conformity with Equation (21), $m_e = \frac{e^{w} \cdot w_{\infty}}{Y_{\infty}(1,w) \cdot M_e}$, there is a resulting hypothetical particle mass m_x analogous to Equation (23),

$$m_x = \frac{e^{w_x} \cdot w_{x,\infty}}{M_e} \cdot a_{\infty} = 3.00447 \cdot 10^{-9} \cdot m_e = 0.00153 \cdot \text{eV}. \qquad (26)$$

The discussed mass of Axion, supposed as a component of Dark Matter lies within such dimensions [10].

The numerical values obtained above with the simple model based on the interaction function (2), emphasizes the role of this function as a bridge between fundamental constants and emergent properties in liquids.

4. Modelling nuclear binding using the interaction function

The interaction function is not related *a priori* to particles with a special physical structure. Consequently the atomic nucleus represents another system with some

similarity in comparison with liquid droplets [11]. n similar particles (the nucleons) are interacting under the influence of strong forces [11,12].
In the following expression for the relative binding energy, E_n, between the n nucleons, referred to one nucleon, the term $(w_{n,e})^3$ plays the main role, where the exponent 3 takes into account the three quarks as separate interacting sub-particles in one nucleon. In addition, a negative term proportional to n is used and the n-1 interactions related to one nucleon give:

$$E_n = \left(w_{n,e}^3 \cdot \pi r^2 - n \cdot \pi(2r)^2\right)\frac{1}{\pi r^2} \cdot \frac{n-1}{n} = \left(w_{n,e}^3 - 4n\right) \cdot \frac{n-1}{n}\ . \tag{27}$$

The first term $w^3_{n,e}$ represents the mobility of one nucleon resulting from the interactions, in the form of its cross sectional area. But the movement of the nucleon must overcome the impedance produced by the relative total cross sectional area, $n\pi(2r)^2/\pi r^2 = 4n$ of the n nucleons related to the cross section area πr^2 of one nucleon.

The significant difference between the application of the interaction function (2) in this case and the examples with liquid properties presented in [1] lies in the involving all n nucleons in this treatment and not only the particles along a string with a linear arrangement. The place exchange of the n nucleons must overcome the resistance from the total cross sectional area of the nucleons. Instead of the electromagnetic interactions, like in the liquid, strong force interactions are involved in the nuclear "droplets".

Equation (27) shows a maximum value for $n = 60$ and approaches zero at $n = 238$. That means, no stable elements exist beyond uranium (Figure 3). A correlation between the dimensionless function E_n to the binding energy B/A in MeV for the atomic mass $n = A$, can be established in the following manner:

1) The elements Be, Co and Au are selected as references, because each of them has one isotope with 100 % abundance. Their binding energies are $B/9$ = 6.46253 MeV, $B/59$ = 8.76775 MeV and $B/197$ = 7.91554 MeV, respectively, obtained from the atomic masses in [13, section 1-14].
2) The corresponding E_n values from Equation (27) are E_9 = 139.039, E_{59} = 483.025 and E_{197} = 132.89.
3) With $a1 = (B/59-B/9)/(E_{59}-E_9)$ = 0.00670222 and from $E_{59}\cdot a1+b1 = B/59$, with $b1$ = 5.53066, the relation $B1/A$ for $A \leq 59$ in Equation (28) results. In the same way the relation $B2/A$ for $A \geq 59$ is obtained from $B/59$, $B/197$, E_{59} *and* E_{197}, respectively.
4) In combination with E_n from Equation (27) the curves B_1/A and B_2/A in Equation (28) represent the calculated B/A values between A = 9 and A = 250 (Figure 4).

$$B_1/A = 0.00670222\cdot E_n + 5.53066 \quad \text{(MeV)} \qquad \text{for } n = A \leq 59$$
$$B_2/A = 0.00243466\cdot E_n + 7.592 \quad \text{(MeV)} \qquad \text{for } n = A \geq 59. \qquad (28)$$

The following Table 1 below contains 26 elements having only one single isotope, or one isotope with > 95 % abundance. The experimental data are obtained from the atomic masses in [13].

Table 1.

Experimental binding energies B/A from [13, section 1-14] and $\Delta(B/A)$ between calculated values with Equation (28) and experimental values of 26 elements.

Element	B/A (MeV) a)	$\Delta(B/A)$ b)	Element	B/A (MeV) a)	$\Delta(B/A)$ b)
Be	6.46253	0.0	Y	8.714	-0.022
C	7.680	-0.821	Nb	8.664	0.0
F	7.779	-0.192	Rh	8.584	0.035

Na	8.111	-0.223	I	8.445	0.021
Al	8.331	-0.207	Cs	8.410	0.014
P	8.481	-0.172	Pr	8.354	0.011
Ar	8.595	-0.013	Tb	8.189	0.039
Sc	8.619	0.053	Ho	8.147	0.033
V	8.742	0.0	Tm	8.114	0.033
Mn	8.765	0.0	Au	7.91554	0.0
Fe	8.790	-0.028	Bi	7.848	-0.035
Co	8.76775	0.0	Th	7.615	0.0
As	8.701	0.044	U	7.570	-0.01

[a] [13]; [b] $\Delta(B(A) = (B/A)_{calc}$-$(B/A)_{exp}$ (MeV)

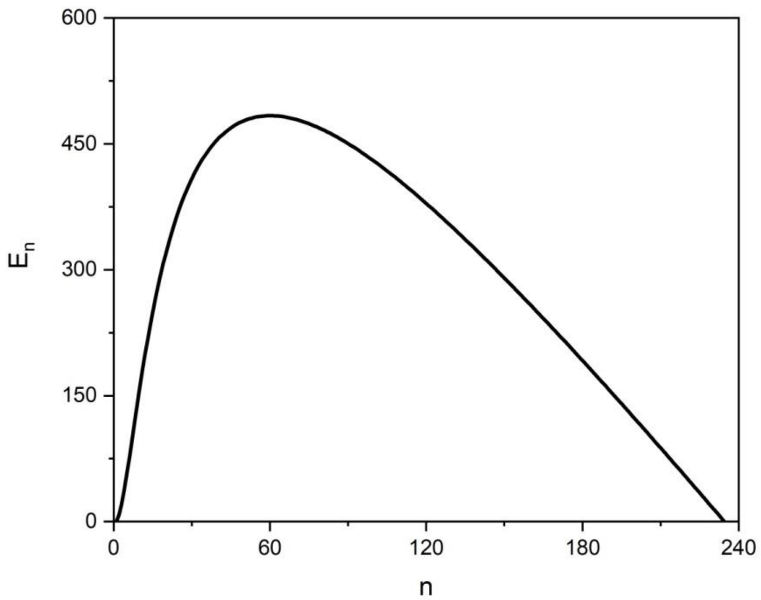

Figure 3. The interaction function E_n from Equation (27) versus the number n of nucleons.

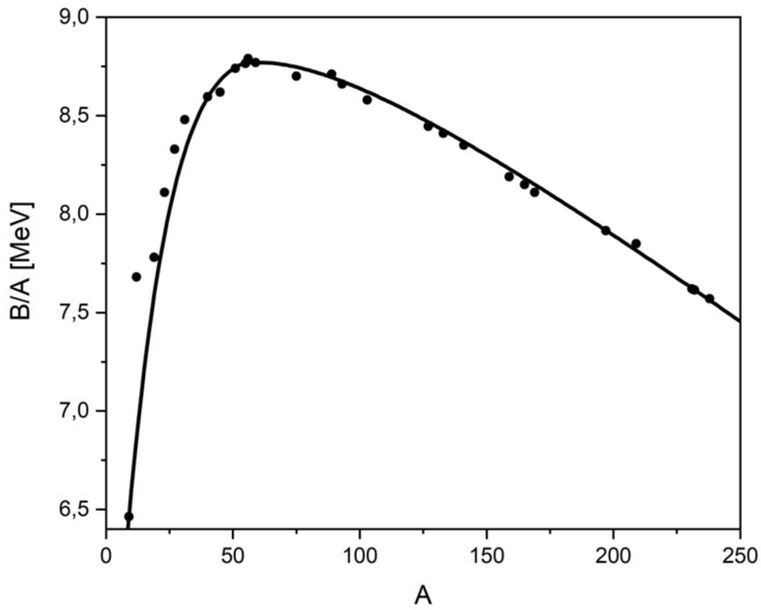

Figure 4. Binding energy/nucleon (MeV) as function of the nucleon number *A*. (●) experimental values; (▬) calculated values with Equation (28).

It is widely believed that ^{56}Fe is the most tightly bound atomic nuclide. The higher binding energy of ^{62}Ni in comparison with ^{56}Fe has been discussed recently [12]. The scatter of the binding energies for 10 isotopes of Fe, Co and Ni, obtained from their atomic masses published in [13, section 1-14], is compared with the corresponding values calculated with Equation (27) and Equation (28) and are shown in Table 2 and Fig.5.

The calculated values with Equation (27) have a maximum for A = 60. Even if ^{62}Ni, is used as reference instead of Co, the calculated B/A values with Equation (28) give a maximum value for A = 60, as shown in Fig.5.

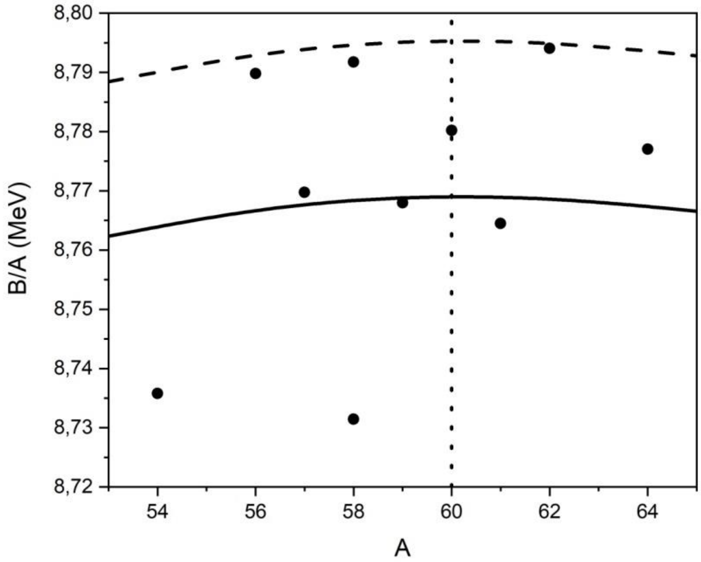

Figure 5. Binding energies *B/A* (●) obtained from atomic masses [13] for the 10 isotopes from Table 10 [13] and calculated (▬) with Equation (27). Calculated (- - -) values with *B/62* = 8.794 for ^{62}Ni as reference instead of ^{59}Co and Equation (27).

Table 2.

Experimental atomic masses *M* in mass units *u* and binding energies *B/A* for 10 isotopes from [13, section 1-14].

Isotope	*M* (u)	*B/A* (MeV)	Isotope	*M* (u)	*B/A* (MeV)
^{54}Fe	53.9396 1	8.736	^{58}Ni	57.9353	8.732
^{56}Fe	55.9349	8.790	^{60}Ni	59.9308	8.780
^{57}Fe	56.9354	8.770	^{61}Ni	60.9311	8.765
^{58}Fe	57.9333	8.792	^{62}Ni	61.9283	8.794
^{59}Co	58.9332	8.768	^{64}Ni	63.9280	8.777

An essential difference between Equation (27) for nuclear binding and the vapor pressures in liquid droplets [1] is the inclusion of all *n* nucleons in the interaction in Equation (27) and not only $i = n^{1/3}$ molecules, as in liquid droplets. This emphasizes the different interaction forces in the two systems, one as strong and the other as electromagnetic, respectively.

5. Conclusions

In addition to interactions between a few elementary particles, powerful principles of organization in the whole physical system being studied must be considered. The properties of natural numbers play a crucial role in this case. The fundamental properties of natural numbers reflect the asymptotical limits reached at equilibrium and these reflect the properties followed by physical systems. This situation is the fundamental position acting as a bridge between a few particles and macroscopic systems with a very large number of interacting particles. In this connection it is necessary to emphasize the practically unlimited number of small differences between specific systems, because one particle with a different structure can produce different properties in the whole system, but without fundamental changes in the overall organization.

This bridge, specific for liquid systems, has not been considered until about 1900, when the finite value of h and a little later the velocity of light c were introduced. With these two empirical constants of nature, most fundamental interactions between particles in physical systems were predictable with models. Neglecting limited physical values for energy of elementary particles, the Schrödinger Equation does not make sense. The algorithm (4) developed in this manuscript is the exact result of this bridge and demonstrates the impossibility of correctly modelling interactions based only on pair interactions, as the correct starting points for macroscopic parts of matter, as demonstrated in the fundamental laws of Newton and Einstein.

A physical system of uniform particles with finite energies is correlated with the system of natural numbers. With an algorithm based on the properties of natural numbers, a maximal limit value, $y(e) = e$ results asymptotically for a relative interacting energy density of the system.

In order to continue the application of the algorithm for initial values q > e, an invariant relation (6), with $B = (1/e\cdot A)\cdot e^{e\cdot A}$ for $B = y_\infty(q)$ has been postulated. In this way a strong correlation between the electromagnetic coupling constant α and the limit value *w* from the interaction function results. From the limit value of the algorithm with *w* as an initial value, a limit value $\pm b_\infty$ is obtained. The number $exp(-b_\infty)$ contains the four fundamental coupling constants. The values of the strong and weak coupling constants lie between two extremes, which allow the determination of the masses of the Higgs and Z bosons. In the same way the mass of the neutrino, as related to the Z boson, is derived.

The limit value *w* acts as bridge between the emergent properties of macroscopic systems and the fundamental constants of nature.

Of fundamental importance in the results obtained with this model is the difference of nature between the electromagnetic interaction contained in the coupling constant α and the strong coupling constant a_s. Related to α is the electron and the many interactions between atoms and molecules treated in [1]. Related to a_s is the proton and the interactions between nucleons.

A special relation has been derived for the nuclear binding energy of the elements. A maximum value results for n = 60 nucleons and a minimum for n = 238. Beyond the element U no stable elements can be formed.

Different forces interact in liquid droplets and between nucleons. In liquid droplets the interaction of electromagnetic forces occurs in the form of strings, whereas strong forces between nucleons produce a place exchange between all particles in the system.

For all kinds of particles resulting from different forces, the gravitational interactions between them, based on Equation (12) must be considered.

As a general conclusion, the strong interactions between the different kinds of particles and macroscopic liquid systems, based on the interaction function as bridge, should be emphasized.

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