Not allowed odd maxima of cyclic Collatz sequences.

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Abstraction: The preprint provides a calculation of the not allowed odd maxima of cyclic Collatz sequences

1. *Formulation of the lemma on cyclic sequences:*

If the numbers $(ka+1) = b2^q, (kb+1) = c2^t, (kc+1) = a2^u$ *form a Collatz conjecture cycle, then the expression (1) holds*

$$(ka+1) * (kb+1) * (kc+1) \ldots = 2^m * a * b * c \ldots \quad (1)$$

In the text: a, b, c - are odd integers; k, m, n, h, f, N, q, t, u – are integers

2.

Expression (1) in another form (2)

$$\frac{(ka+1)(kb+1)(kc+1)}{abc} = 2^m \quad (2)$$

If the numbers $b, c, (ka+1), (kb+1)$ *form a Collatz conjecture corresponding to expression (3) , then the only condition for the formation of a cycle is expression (4)*

$$\frac{(ka+1)(kb+1)}{bc} = 2^n \quad (3)$$

$$\frac{(kc+1)}{a} = 2^h \quad (4)$$

Thus, the lemma can be expressed by condition (4).

Odd numbers can be represented in the form (5), then the numbers $(ka+1)$ *have the form (6)*

$$a = 2f - 1 \quad (5)$$

$$(ka+1) = 2kf - k + 1 \quad (6)$$

The number $(kc+1)$ *can be represented in the form (7)*

$$(ka+1) - (kc+1) = 2k\Delta f$$

$$\boldsymbol{N} = \Delta f = \frac{\boldsymbol{a+1}}{\boldsymbol{2}} - \frac{\boldsymbol{c+1}}{\boldsymbol{2}} = \frac{\boldsymbol{a-c}}{\boldsymbol{2}}$$

$$(kc+1) = (ka+1) + 2kN \quad (7)$$

Then the condition lemma can be represented by (8.1) and (8.2)

$$\frac{(\boldsymbol{ka}+\mathbf{1})+\mathbf{2}\boldsymbol{kN}}{\boldsymbol{a}} = \mathbf{2}^{\boldsymbol{h}} \quad \textbf{(8.1)} \quad \frac{(\boldsymbol{k}(\mathbf{2}\boldsymbol{f}-\mathbf{1})+\mathbf{1})+\mathbf{2}\boldsymbol{kN}}{(\mathbf{2}\boldsymbol{f}-\mathbf{1})} = \mathbf{2}^{\boldsymbol{h}} \quad \textbf{(8.2)}$$

3. *Expression (8.2) is converted to (9)*

$$\frac{(k(2f_a-1)+1)+2kN}{(2f_a-1)} = 2^h, \quad kf_a - \frac{(k-1)}{2} + kN = 2^{h-1}(2f_a-1) \quad (9)$$

Accordingly,

If $k = 3$ *, then the expression (9) has the form (10):*

$$(\mathbf{3}\boldsymbol{f} + \mathbf{3}\boldsymbol{N}) - \mathbf{1} = \mathbf{2}^{\boldsymbol{h}}\boldsymbol{f} - \mathbf{2}^{\boldsymbol{h}-\mathbf{1}} \quad (\mathbf{10})$$

If $k = 5$ *, then the expression (9) has the form (11).*

$$(\mathbf{5}\boldsymbol{f} + \mathbf{5}\boldsymbol{N}) - \mathbf{2} = \mathbf{2}^{\boldsymbol{h}}\boldsymbol{f} - \mathbf{2}^{\boldsymbol{h}-\mathbf{1}} \quad \textbf{(11)}$$

4. *With a certain parity ratio between the numbers f and N, it is possible to exclude cycles.*

$$f_a = \frac{a+1}{2}, \qquad N = \frac{c-a}{2}, \qquad \frac{a+1}{2} \equiv \frac{c-a}{2}$$

$$\boldsymbol{f_a} \equiv \boldsymbol{N} \quad \textbf{(9)}$$

$$2f = c - (2f-1)$$

$$4f - 1 = c$$

Thus, the numbers (c) of the form $4x-1$ *- (3, 7, 11, 15, 19...)they correspond to condition (9), and cannot be the maximum of cycles for the sequence Collatz conjecture (k = 3), but are the only maxima of possible cycles for sequencesuwith k = 5. The remaining numbers (c) of the form* $4x+1$ *have the inverse property with respect to the maxima of cycles.*