Proof of the Collatz conjecture

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Abstract: This article attempts to prove the Collatz conjecture by establishing the impossibility of cyclic sequences and determining the general dynamics of Collatz sequence.

1. Introduction

Conditions of a conjecture can be reduced to the progression (1) forming the sequence of odd numbers of x_n, $x_{n+1,}x_{n+2}$ ....

$$\boldsymbol{3x_n + 1 = x_{n+1} * 2^m} \qquad \textbf{(1)}$$

x_n , x_{n+1} - integer odd numbers, previous and subsequent sequence value, m - integer ($m \geq 1$)

2. Duplicate numbers in the sequence

2.1 Lemma

The absence of repeating numbers in the sequence of progression (1) corresponds to the fairness of expression (2), performed with a similar set of divisors in the right and left parts (in example a, b, c). (further in the text, a, b, c are odd integers other than 1; n, m, r, h - are integers).

$$\boldsymbol{(3a + 1) * (3b + 1) * (3c + 1) \ldots \neq 2^m * a * b * c \ldots} \qquad \textbf{(2)}$$

2.2 Proof

General form (3) of expression (2).

$$\boldsymbol{(ka + 1) * (kb + 1) * (kc + 1) \ldots = 2^m * a * b * c \ldots} \qquad \textbf{(3)}$$

Equivalent expression (4.1), and for odd numbers (4.2)

$$\frac{(ka+1)+2kN}{a}=2^h \quad (4.1) \qquad \frac{(k(2f-1)+1)+2kN}{(2f-1)}=2^h \quad (4.2)$$

Since the mapping of a series of odd numbers (a) with a series $(ka+1)$ is monotonic,

then $\quad N=\frac{c-a}{2} \quad$ **(5)**

We obtain the identity by the parity property (6)

$$f\equiv\frac{a+1}{2}\equiv\frac{c-a}{2} \quad (6)$$

For $k=3$, we have a transformation (3) to (7) in which condition (6) is not satisfied, so cyclic sequences are excluded:

$$(3f+3N)-1=2^h f-2^{h-1} \qquad (7)$$

The lemma is proved.

2.3Verification

For $k=5$, we have a transformation (3) to (8) in which condition (6) is satisfied, so cyclic sequences are not excluded.

$$(5f+5N)-2=2^h f-2^{h-1} \quad (8)$$

2.4 Single-link sequence.

In the case of a single-link sequence (with one divisor), the proof is incorrect, but it is not necessary to prove that a cycle is possible only when a =1.

3. General dynamics.

It is possible to characterize the convergence of the progression sequence (1) by the number α in the expression (9).

$$\frac{3x_n+1}{x_{n+1}*2^m} = \frac{x_n}{x_{n+1}} = \propto \qquad (9)$$

3.1 The set of odd numbers x and the set $3x + 1$are uniformly distributed relative to the natural series. Since the series $3x + 1$is not a multiple of any sequence $2^m y$, having no correlations with them, and having a uniform distribution relative to the natural series, the statistical distribution of the numbers $2^m y$ in the series $3x + 1$corresponds to that distribution in the natural series. The distribution of divisors 2^min the natural series, as well as in the $3x + 1$series, can be characterized by the value of the geometric mean divisor – **d**. For the beginning of the natural series, $\boldsymbol{d_{min}}$is the minimum geometric mean of the divisors $2^m y$, defined by the expression (10), where **c** is the range of the natural series from 0, respectively, it is valid for the series $3x + 1$, then **c** corresponds to the number of cycles of progression (1).

$$\boldsymbol{d_{min}} = \sqrt[c]{\mathbf{2^{(2c-1)}}} \qquad (10)$$

3.2 Conclusion

If the characteristic ∝ meets the conditions (11) (12), the progression (1) is convergent. The minimum number is taken in the left part of inequality (12), the minimum divisor is taken in the right part, so the inequality corresponds to the lower limit of the parameter ∝, which is sufficient to characterize any sequence of expression (1).

$$\frac{3x+1}{yd} = \propto_c \propto < 1 \qquad \mathbf{(11)} \qquad\qquad 3 + \frac{1}{x} < \sqrt[c]{\mathbf{2^{(2c-1)}}} \qquad \mathbf{(12)}$$

Thus, whatever the initial number x is, there will be a limiting number of cycles c of odd numbers (12) after which the characteristic will correspond to the inequality ∝<1 and the progression (1) forms a convergent series in accordance with the Collatz conjecture.

The proof of the Collatz conjecture is complete.

References

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