A statistical test of gravitational wave events

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Abstract

Here I show some statistics of all the 93 gravitational wave (GW) events observed by LIGO in 3 phases during the last 6 years, with 3, 8 and 82 GW events for each phase. The detection sensitivity in O3 phase was increased by 40% than that in O2 phase. The co-working ratio of the two LIGO observatories was 0.42 (O2 phase) and 0.60 (O3 phase), respectively. The product of sensitive volume and time (VT) was thus increased by a factor of $1.4^3 \times (0.60/0.42) \approx 4$. Statistical analyses of all the 93 GW events suggest that the observations so far do not meet an intuitive expectation, say, with higher detection sensitivity and longer observation time, we should observe more GW events.

A Chinese version of this paper is included in the appendix.

Key Words: gravitational wave; gravitational wave detection; statistical test.

Since the observation of the GW150914 gravitational wave event[1], the Laser Interferometric Gravitational Wave Observatory (LIGO)[2,3] has not only validated the general relativity[4] proposed by Einstein 100 years ago, thus opening a new era of for gravitational wave (GW) multi-messenger astronomy observations[5,6], but also won the 2017 Nobel Prize in Physics[7–9]. Here I give some statistical analyses of all the observed GW events, using the public information published online by the LIGO scientific team (LIGO Scientific Collaboration, LSC)[10–12].

LIGO Scientific Collaboration (LSC) has gone through three phases of observation, O1, O2 and O3 (O3a & O3b). Their analysis in 2016 showed[13] that the product of sensitive volume and time (VT) of O2 was between 7 and 20 (with respect to a reference value, VT0), and O3 had a VT between 30 and 70. LSC has found 93 certified gravitational wave events (GW events), with 3, 8 and 82 GW events for each phase[14,15]. At first glance, it seems to meet nicely the intuitive expectation that we should observe more gravitational wave events with higher detection sensitivity and longer observation time. However, my analysis suggests the contrary.

As shown in Fig.1(a), as the accumulated days of observation increase, there comes a knee point beyond which the rate of the detection of GW events increases dramatically. Two indicators measure the detection sensitivity of the LIGO, the angular averaged range for dual neutron star fusion events (Fig.1(b)) and the amplitude density of detectable gravitational wave (Fig .1(c)). Between the O2 and O3 phases (around day 400), the detection sensitivity was increased by 40% or more, thus leading a triple increase of the detection volume. However, the rate of the detection of gravitational events remained almost constant, or even slightly decreased, far away from the expected triple increase. Fig.1(d) shows the daily average working ratio of each LIGO/Virgo observatories during the O2 and O3 phases. As shown in Fig.1(b-d), the knee point in Fig.1(a) appeared around the time when Virgo was put into work (about a month before O2 phase ended), although its detection sensitivity was much worse than the two LIGO observatories.

Observation of a GW event demands at least two observatories to work in the "observation mode" simultaneously. For sake of simplicity, Figure 2(a) shows the ratio of two LIGO observatories operating simultaneously (co-working ratio): in the 24 hours of a day (UTC time), the chance of both observatories work properly at the same time. The co-working ratio varies at each moment of the day, as shown in Fig.2(a), probably due to the fact that trouble-shooting is more likely to be made at daytime, while detection is more likely to be made at night. The averaged co-working ratio of a day was 0.42 (O2 phase) and 0.60 (O3 phase), respectively.

The detection rate of the GW events may be expected to be proportional to the co-working

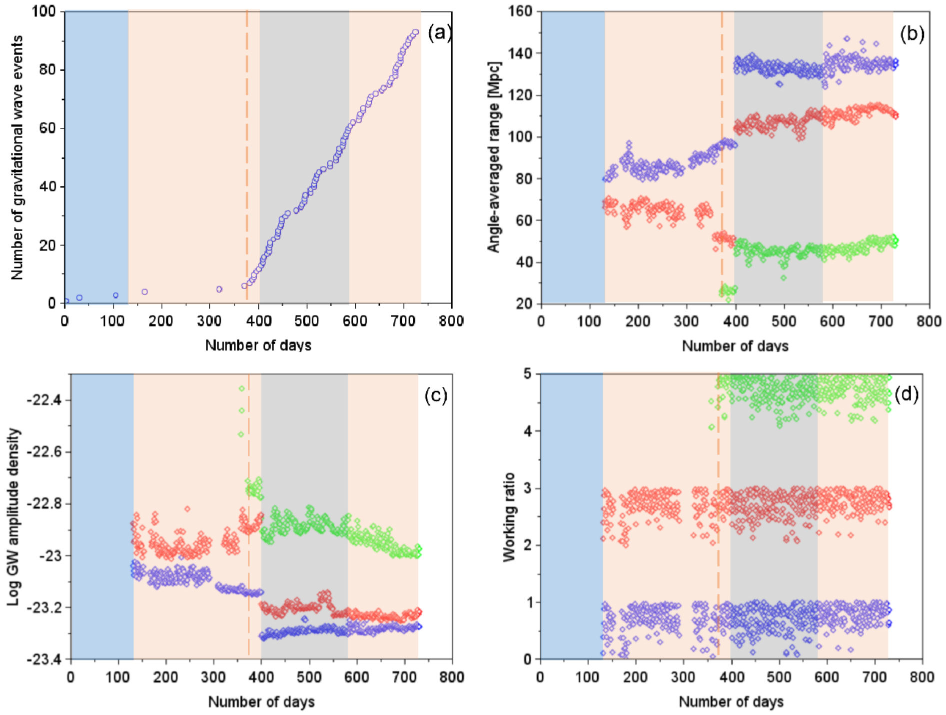

FIG. 1: Over the past 6 years, LIGO went through three observation phases and found 93 gravitational wave events. Colorful zones represent different observation phases. (a) Number of gravitational wave events. (b) Angle-averaged range of gravitational wave events due to binary neutron stars; (c) Amplitude density of detectable gravitational waves (data at 80 Hz). (d) Averaged daily working ratio for three stations (For sake of clarity, base-lines are raised by 2 (red) and 4(green), respectively.). All of the horizontal coordinates are cumulative detection days.The red dash line indicates the turning point (a GW evnet was observed on Jul. 29th, and Virgo officially announced cooperation with LIGO on Aug. 1st). And in (b-d), Blue: LIGO Livingston; Red: LIGO Hanford; Green: Virgo.

ratio, however, it is not the case, as shown in Fig.2(b). Probability distribution 1 (PD1, red symbols) is the reduced probability, via dividing the co-working ratio of one day in the O3 phase into 12 equal periods (2 hours each), probability distribution 2 (PD2, green symbol) is an equal probability distribution, and 82 GW events in the O3 phase give the observed rate (blue symbol). The cross variance of the observed rate with respect to PD1 (PD2) is 0.0076 (0.0044). Thus, the rate of the detection of GW events is closer to the equal probability distribution, instead of what one may expect from the co-working ratio. The cross variance is defined as the following, $\sum_{i=1}^{k}\left(p_i^o - p_i^t\right)^2$, where i represents the k types of stochastic event that may occur ($k = 12$ here),

p_i^0 is the actual occurrence rate of the i-th type of stochastic events, and p_i^t is the corresponding theoretical probability (here are two kinds of theoretical probabilities, corresponding to PD1 and PD2, respectively).

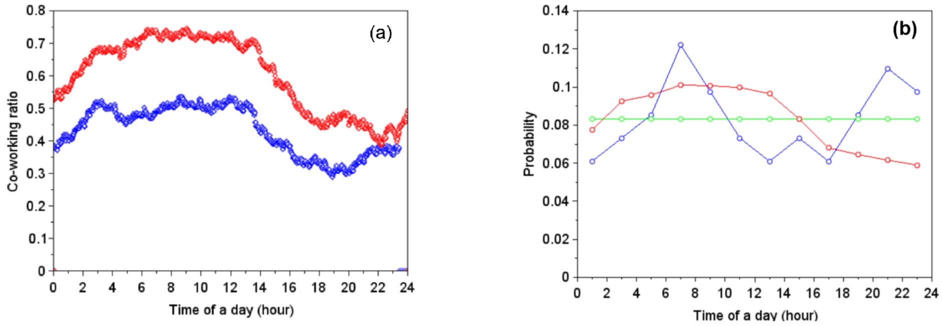

FIG. 2: Expected and actual results of the occurrence rate of gravitational wave events observed by LIGO. (a) The co-working ratio of the two LIGO stations operating properly at the same time. Blue: O2 phase; Red: O3 phase. (b)Expected and actual results of the occurrence rate of gravitational wave events. Red (probability distribution 1): the expected occurrence rate according to the co-working ratio of the two LIGO stations. Blue: the actual occurrence rate of gravitational wave events. Green (probability distribution 2): an equal probability distribution. Horizontal axis is UTC time.

This is in contrast to numerical simulations. Among 10,000 simulations generated with probability PD2, about 19% have the results whose cross variance with respect to PD1 is smaller than that with respect to PD2. On the other hand, among 10,000 simulations generated with probability 1, about 82% have the results whose cross variance with respect to PD1 is smaller than that with respect to probability 2. In other words, for the simulation results with one kind of probability, its cross variance with respect to this probability has a significant chance ($\sim 4:1$) to be smaller than the cross variance with respect to the other probability.

It is also worth noting that, in the last month of the O2 (Jul. 26th - Aug. 25th), there appeared 6 GW events, 5 of which are in August (9th - 23rd). LSC thought[14,16] that it is not unusual to see 5 GW events in August. Taking into account the different efficiency of detection and observation time in O1 and O2, these two periods can be divided into non-overlapping 10 months. The probability of observing at least 5 GW events in a month is 5.3%. However, the real question is, how likely it is to see 6 GW events (one in Jul. 29th, five in August) in a month while the total number is no more than 11 in ten months? With the same assumptions and data from LSC, the

	0.030	0.033	0.037	0.040	0.043	0.047	0.050
11	15	16	21	16	13	11	15
12	22	34	42	33	29	26	22
13	33	46	62	66	60	57	53
14	47	64	93	108	108	105	98
15	63	89	134	151	156	165	170

TABLE I: Results of computer simulations. There is a probability of about 0.2% to observe 11 GW events in 10 with 6 and more times occurring within 30 consecutive days. The first row is the odds per day to observe at least one gravitational wave event, the first column is the number of gravitational wave events within 10 months, and the corresponding position in the table gives the number of 6 and more GW events over 30 consecutive days (for 10,000 runs of simulation).

probability is 1% or less. Numerical simulation (Table 1.) gives a probability of 0.2%. This may not be ascribed to luck.

Note that the GW events discussed here are those with probability of astrophysical origin (P_{astro}) greater than 0.5. Recently, LSC announced some marginal events[17], and some groups used independent algorithms to find new GW events[18]. The "August anomaly" discussed here may be less unusual. However, it is not yet suitable for discussion.

Apart from the occurrence time of the GW events, all the data presented here are extracted from the pictures published online by LSC. I think the inaccuracy of the retrieved data would not affect our analysis. More accurate and convenient data and possibly more detection results in the future will further test the preliminary statistical analysis given here. Obviously, such a statistical test can be applied to more data analysis, say, the location of GW events on the celestial sphere, their distance to the earth, the mass and spin of the corresponding massive bodies (black holes or neutron stars), and so on. However, the public pictures are not suitable for further analysis.

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[1] Abbott B P, et al. (LIGO Scientific Collaboration and Virgo Collaboration), 2016, Phys. Rev. Lett. 116, 061102

[2] Aasi J, et al. (LIGO Scientific Collaboration), 2105, Classical Quantum Gravity 32, 074001

[3] Abbott B P, et al. (LIGO Scientific Collaboration and Virgo Collaboration), 2016, Phys. Rev. Lett. 116, 131103 (2016).

[4] Einstein A, 1916, Sitzungsber. K. Preuss. Akad. Wiss. 1, 688

[5] Abbott B P, et al. (LIGO Scientific Collaboration and Virgo Collaboration), 2017,Phys. Rev. Lett. 119, 161101

[6] Abbott B P, et al. (LIGO Scientific Collaboration and Virgo Collaboration), 2017, The Astrophysical Journal Letters, 848, L12

[7] Weiss R, 2018, Rev.Mod.Phys., 90, 040501

[8] Barish B C, 2018, Rev.Mod.Phys., 90, 040502

[9] Thorne K S, 2018, Rev.Mod.Phys., 90, 040503

[10] The Gravitational Wave Open Science Center, https://www.gw-openscience.org/about/

[11] Gravitational-Wave Observatory Status, https://www.gw-openscience.org/detector_status/

[12] The Gravitational-Wave Candidate Event Database (GraceDB) https://gracedb.ligo.org/

[13] Abbott B P, et al. (LIGO Scientific Collaboration and Virgo Collaboration), 2019, The Astrophysical Journal Letters, 833:L1, 1

[14] Abbott B P, et al. (LIGO Scientific Collaboration and Virgo Collaboration), 2019, Phys. Rev. X, 9, 031040

[15] The LIGO Scientific Collaboration, The Virgo Collaboration, The KAGRA Scientific Collaboration, The population of merging compact binaries inferred using gravitational waves through GWTC-3,arXiv 2111.03634

[16] W. M. Farr and J. R. Gair, LIGO Scientific Collaboration and Virgo collaboration Technical Report No. LIGO-T1800529, https://dcc.ligo.org/LIGO-T1800529/public.

[17] https://www.gw-openscience.org/eventapi/html/allevents/

[18] Nitz A H, et al., 4-OGC: Catalog of gravitational waves from compact-binary mergers, arXiv:2112.06878

Appendix: Chinese version of this paper

引力波事件的统计分析

摘要： 利用LIGO科学团队（LIGO Scientific Collaboration, LSC）在网上公开的数据，对过去6年里发现的93次引力波事件进行了一些统计分析。在第二个观测期里，在Virgo正式投入使用、而LIGO的探测灵敏度并没有显著变化的时候，探测到引力波事件的速率突然增大了许多；在第三个观测期期开始后，尽管每个观测站的探测灵敏度都大幅提升，探测到引力波事件的速率却几乎维持不变，远没有达到预期的三倍增长。此外，在第三个观测期里，82次引力波事件的出现频率更接近于等概率分布，而不符合同时正常工作比的预期。由此可见，目前公布的引力波观测结果并不符合一种朴素的预期，即探测灵敏度越高，有效观测时间越长，观测到的引力波事件就越多。这个结果将会让更多的人关注LIGO引力波事件的探测和分析。

关键词：引力波；引力波探测；统计分析　PACS：04.30.-w, 04.80.Nn, 02.50.-r

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自从首次观测到引力波事件GW150914以来[1]，激光干涉引力波观测站（LIGO）[2,3]不仅验证了爱因斯坦在100年前提出的广义相对论[4]，开辟了引力波多信使天文学观测的新时代[5,6]，还获得了2017年诺贝尔物理学奖[7–9]。本文利用LIGO科学团队（LIGO Scientific Collaboration, LSC）发布在网上的公开信息[10–12]，对目前已经观测到的引力波事件做了一些统计分析。对引力波这种没有明显发生规律的事件来说，一种朴素的预期是：探测灵敏度越高，有效观测时间越长，观测到的引力波事件就越多。LIGO团队在2016年的分析表明[13]，O2的敏感体积和时间的乘积（VT）介于7至20之间，而O3的VT介于30至70之间（二者都是相对于一个参考值VT0而言），有显著的增加。然而，统计分析的结果表明，目前公布的引力波观测结果并不符合这个预期。

在过去的6年里，LIGO进行了三期观测，发现了93次引力波事件[10–12]，如图1(a)所示。第一个观测期有3次（O1，2015.09.12-2016.01.19，共130天）；第二个观测期有8次（O2，2016.11.30-2017.08.25，共269 天）；第三个观测期有82次（O3a，2019.04.01-2019.09.30，共183天，观测到47次引力波事件；O3b，2019.11.01-2020.03.27，共148天，35次引力波事件）[14,15]。乍一看，图1(a)的结果符合预期：LIGO的探测灵敏度逐步增强，探测体积随之增大，相同时间内探测到引力波事件的数目也就增加。然而，更仔细地观察就会看到，引力波事件的出现频率在红色虚线处有一个拐点（7月29日观测到一次引力波，8月1日Virgo宣布正式投入合作），虚线两侧的出现频率有巨大差别，而在不同观测期之间（颜色区的边界处），却没有显著的变化。

有两个指标可以衡量LIGO的探测灵敏度：针对双中子星并合事件的角度平均的探

测范围（图1(b)），能够探测的引力波振幅密度（图1(c)，这里是80Hz处的数据）。在O2和O3之间（测量天数400附近），探测灵敏度有40%甚至更大的增强，探测体积增大为此前的大约3倍（$1.4^3 \approx 2.7$）。图1(b-d)仍然用虚线标出了图1(a)拐点出现的时刻，那时候发生的重大事件是Virgo投入正常使用，尽管它的探测灵敏度比LIGO的两个观测站差得很远。与此同时，LIGO利文斯顿站的观测灵敏度略有上升，LIGO汉福德站却略有下降。图1(d)给出了每个观测站在O2和O3时期的每日平均工作比（working ratio），O3时期的平均工作比（~ 0.60）也要高于O2时期（~ 0.42 ）。

总结图1的结果：从Virgo正式投入使用以后（在O2结束之前大约1个月），LIGO在探测灵敏度没有显著变化的情况下，探测到引力波事件的速率突然增大了许多；而在O3开始以后，尽管每个观测站的探测灵敏度都大幅提升，探测到引力波事件的速率却几乎维持不变，甚至略有下降，远没有达到预期的三倍增长。

观测引力波事件，至少需要两个观测站正常工作。因为Virgo的探测灵敏度远低于LIGO的两个观测站，为了简单起见，图2(a)只给出了在O2和O3时期，LIGO的两个观测站同时正常工作的比值（co-working ratio）：在24小时以内（以90秒为计时单位），两者同时正常工作的几率。这里采用的是协调世界时（UTC时间），与各个探测器所在的时区无关。在O2和O3时期，两个LIGO观测站同时正常工作的几率分别是0.42和0.60，差别来自于技术的进步；这个几率在每天的各个时刻也不一样，主要原因也许是人类活动等因素（调试和检测更经常在白天进行，而正常探测更多是在夜晚）。

两个观测站正常工作的几率并没有明显体现在引力波事件的探测频率上。更仔细地分析数据后发现，在图1里虚线标识的时间附近，LIGO的两个观测站每天同时正常工作的比值从不到0.4跳到大约0.6。这是一个有趣的观察：同时正常工作比增加了50%，对应探测事件的频率增加了5倍甚至更多；探测灵敏度预期把探测事件的频率增大到3倍，却并没有什么体现。

根据同时正常工作比，还可以预期，引力波事件的出现频率应该有大致类似的分布，图2(b)的结果却并非如此。几率1（红色符号）是把O3时期里的一天内的同时正常工作比划分为12个等时段（每段2小时）后得到的约化几率，几率2（绿色符号）是等概率几率，而O3时期的62个引力波事件给出了引力波事件的出现频率（蓝色符号）。引力波的出现频率与几率1（几率2）的交叉方差（cross variance）是0.0076 （0.0044），也就是说，它更接近于等概率分布（也就是几率2）而不是符合由两个LIGO观测站的同时正常工作比所预期的概率分布（几率1）。

交叉方差的定义是$\sum_{i=1}^{k}\left(p_i^o - p_i^t\right)^2$，其中，$i$表示$k$种可能的随机事件的出现几率（在这里，$k = 12$），$p_i^0$是第$i$种随机事件实际出现的几率，而$p_i^t$是相应的理论几率（这里有两

种理论几率，分别对应于几率1和几率2）。以两种概率分别进行10000次仿真实验（每次有82个随机事件按照该几率出现在一天以内），得到的结果与两种概率的交叉方差表明：以几率1为概率分布进行的仿真实验，在大约82%的实验中，仿真结果相对于几率1的交叉方差小于它相对于几率2（等概率分布）的交叉方差；以几率2为概率分布进行的仿真实验，只有大约19%的仿真结果，相对于几率1的交叉方差小于它相对于几率2 的交叉方差。简单地说，以某个概率进行随机仿真的结果，与这种概率的交叉方差有显著的几率（～4 : 1）小于它和另一种概率的交叉方差。

总结图2和仿真实验的结果：在O3观测期里，引力波事件的出现频率更接近于等概率分布，而不符合同时正常工作比的预期。

还需要注意的是，在O2观测期间的最后一个月里（8月25日结束），观测到的引力波事件的数目显著增多。7月29日观测到1次，8月观测到5次（而且是发生在15天内，8月9-23日）。Virgo在8月1日宣布正式投入与LIGO 的合作，但此前几天已经投入正常使用；与此类似的是，O1正式宣布开始观测是2015年9月18日，而第一次引力波事件GW150914发现于9月14日。LIGO团队认为[14,16]，在一个月里观测到5次引力波时间并不奇特。考虑到两个观测时期（O1和O2）的观测探测效率的差别以及有效的观测时间，这两个时期的等效观测时间可以划分为互不重叠的10个月，由于引力波事件的观测是泊松过程，每个月观测事件的平均数是1.1次（把11次引力波事件分配在10个月里），在某个月里观测到至少5次引力波事件的几率是5.3%。

但是，5.3%并非在某个特定月份能够观测到至少5次引力波事件的几率。真正需要回答的问题是：为什么在Virgo宣布正式合作以后不到一个月的时间里，发现了5次引力波事件？按照LIGO团队的假设和计算方法[14,16]，这个事情的几率是0.5%。如果认为Virgo加入合作并没有什么特殊性，那么需要回答的问题是：从7月29日（这一天的特殊性在于，当天观测到一次引力波事件）到O2观测期间结束，为什么观测到了5次引力波事件？换个方式说，在某一个非特定的30天里（这里是从7月26日到8月25日），为什么发现了6次引力波事件？同样按照LIGO 团队的假设和计算方法，这个几率是1%。计算机模拟的结果表明（表1），假设探测效率和引力波事件发生几率没有显著变化，在10个月的观测期间，观测到总数为11次的引力波事件，而且有6次（含）以上的事件发生在30 天以内，这样的几率是大约0.2%。发生这种几率为1%（甚至更低）的事件，恐怕不能完全归结于运气。

需要说明的是，这里讨论的引力波事件是检测信号源自太空的几率（probability of astrophysical origin，P_{astro}）大于0.5的事件。最近，LSC公布了一些边缘性事件（marginal event，弱化了引力波事件的判据，O1观测期3个，O2观测期11个，使得O1和O2观测到的事件总数由原来的11次增加为25次）[17]，有些团队采用独立的算法从LSC公布的数据中发

现新的引力波事件[18]。如果考虑这些事件，本文讨论的“8月异常”可能就没有那么异常了。但是现在还不适合进行这种讨论。

除了引力波事件的发生时间以外，本文选用的数据都是从LIGO团队公布的图片中提取出来的，也许会有一些误差，但是并不影响这里采用的分析方法，以及得到的结论。更准确、更方便的数据以及今后可能的更多的探测结果，将会进一步检验这里给出的初步的统计分析。这里给出的统计检验显然可以应用于更多的数据分析，例如引力波事件在天球上出现的位置，到地球的距离，相应的致密天体（黑洞或者中子星）的质量和自旋，等等。但是，目前公开的图片还不适合分析。

引力波事件都发生天文距离（几千万乃至几十亿光年），彼此之间没有关联。对于它们被观测到的几率来说，一种朴素的预期是：探测灵敏度越高，有效观测时间越长，观测到的引力波事件就越多。然而，从上述统计分析来看，目前公布的引力波观测结果并不符合这个预期。本文可能会让更多的人关注LIGO引力波事件的探测和分析。

致谢：我感谢几位匿名审稿人的有益建议（尽管最后还是拒稿了）。感谢LIGO科学团队在网上提供的公开数据。感谢沈庭扬和施婷婷的讨论和帮助。

图1. 在过去6年里，LIGO进行了三期探测，总共发现了93次引力波事件。不同的颜色区表示了不同的观测时期。(a)引力波事件的个数。(b) 针对双中子星引力波事件的角度平均的探测范围；(c)能够探测的引力波振幅密度（80Hz处的数据）。(d)每个观测站的每日平均工作比（为了清楚起见，基线分别上移了2（红色）和4（绿色））。横坐标都是累计的探测天数。红色虚线标出了拐点出现的位置（7月29日观测到一次引力波，8月1日Virgo宣布正式投入合作）。在(b-d)里，蓝色：LIGO利文斯顿；红色：LIGO汉福德；绿色：Virgo。

图2. 由LIGO的工作状态对引力波事件出现频率的预期和实际结果。(a) LIGO两个观测站同时正常工作的比值。蓝色：O2时期；红色：O3时期。(b)预期的引力波事件的出现频率和实际结果。红色（几率1）：根据LIGO的同时正常工作比所预期的频率分布。蓝色：引力波事件出现的实际频率。绿色（几率2）：等概率分布。横坐标是协调世界时（UTC时间）。

表1. 计算机模拟的结果表明：在10个月内观测到11次引力波事件、且有6次（含）以上的事件发生在30天以内的几率是大约0.2%。第一行是每天发生引力波事件的几率（第一行），第一列是10个月内发生引力波事件的次数，而表格里相应的位置给出了连续30天里有6次（含）以上引力波事件出现的次数（总模拟次数是10000次）。