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DETERMINISM in QUANTUM SLIT-EXPERIMENTS

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ABSTRACT

A mathematical model for the slit-experiments in the heart of quantum mechanics, is developed to gain insight in quantum theory.
The proposed system-theoretical approach of the mathematical model takes a different route compared to matrix mechanics in complex vector spaces: it is entirely based on commutative mathematics, eg. convolution, integral transformations and starts with spacetime functions with inherent energy based cause and effect relations of the statefunction Ψ in the complex Hilbert space.
The benefits of his approach are 1. invariance in time reversal, 2. deterministic result functions in the model in line with the outcome of slit-experiments, 3. separation of causality and fi. cross-correlations of attained states, 4. disappearance of (a posteriori added) probability of quantum states and 5. quantum a priori fixed states after causality interactions have ended, (even) when quanta are lightyears separated.
The model predicts the patterns in the experiments with mathematical functions of the energy distributions. The quantum mechanical description of physical reality of slit experiments thus may be considered complete in the sense of [10], and requires the thought experiment of reality still, which is not contradictory with the exact results.

At quantum slit-experiment energy level, the patterns found in double slit experiments actually are found to be an effect of energy (amplitude-*) modulation.* An *equivalent* double-slit pattern can be retrieved from an input modulated 1-slit experiment; two distributions in the (k-space) energy frequency domain appear mathematically *as if* produced by two slits, ánd the relation between k_0 and the physical slits is unambiguous. Due to the absence of multiple slits this excludes interference interpretations (of particles, waves). In principle it may be possible to experimentally verify the effect with a modulated input function of a one-slit experiment.

The system-theoretical method uses well known generic properties of quanta and evolves into determinism in quantum mechanics slit experiments, without a direct observation/measurement or direct description with variables of the individual quanta at the heart of the state-function Ψ.
The mathematics in the system theoretical approach handles *beables* [9,10] by treatment of momentum p in system theoretical I/O relations of the transformed functions and allows the proposed description by the avoidance of a direct addressing of the individual quanta through

variables. The followed method yields exact, non-probabilistic results.

INTRODUCTION

Results of quantum slit experiments are usually explained with wave-theory aiming for diffraction and interference [e.g. 6] with constructive and destructive interference of waves in analogy with waves in fluids and gases. At energy levels in quantum slit-experiments, no direct interference between photons has ever been observed or predicted, and when reduced to one photon/electron experiments without possible interaction and with in time separated detections, repeated experiments also yield the pattern, indicating that interaction of particles does not play a significant role. Wave theory leads to interpretations to explain the observed patterns with interference, in violation with conservation of quantum energy and experiment practices, while in case of a source delivering electrons, similar dark-light distribution patterns are being found, and interpretations of particle-wave duality [5] are being put forward.

The purpose of models of nature, and consequently of physics, is to provide insight into what happens in reality, i.e. with quanta which although invisible, are as messenger an ubiquitous part of our reality and leave tracks, by which we may (indeed) indirectly observe them.
The result of this study is straightforward and indicates that ‘Herr Gott nicht wuerfelt’ with the consequence that determinism is ruling our world. This impacts many interpretions that seemingly use an inverse way to construct ‘reality’ from a framework of interpreted explanations.
The process of finding determinism avoids directly tracing/addressing quanta in mathematical treatment by using a system theoretical approach of input-output relations.
Quanta stay invis-ables on individual basis in the heart of the state function Ψ, cannot be observed or measured and can’t individually be traced by virtual/mathematical descriptions directly without violation of the Heisenberg relation. This invisibility appears to be the paradox of the found determinism, leading to the proposal of a system-theoretical approach using I/O relations instead of matrix mechanics and direct variables. The approach excludes all non-commutative operators that affect time reversal invariance (or symmetry).

This doesn`t make research in determinism any easier, although quanta actually are being manifest indirectly. The proposed descriptions in a different virtual reality of mathematics, avoiding treatment as *observables,* may be a glimpse of light at the end of this tunnel.

QUANTUM EXPERIMENTS

1. The mathematics

This paper takes a system-theoretical approach of the quantum-mechanical description of the slit experiments instead of the usual matrix mechanics.
Why? One of the reasons certainly is that it is not possible to directly address quanta with variables and simply start calculating. Many attempts were made, but eg. only one of the

properties p or r of a photon/quantum can be known *exactly* (Heisenberg) while both are required in calculations when trying to predict exact behaviour. In general, when the information i.e. (minimum required) energy to arrive at exact values is approaching the energy of the particle under study, it *has to* change properties when its energy is significantly changed, usually indicated as a ‘collaps’ of the describing function. This is the principal reason to describe behaviour from a different domain where one determines the characteristics of the frequencies (of occurrence) of energy or quanta per location r, with exact frequencies and amplitudes but without knowledge *which* individual quantum arrives at a certain exact location i.e. mathematically, the quanta are not ‘tagged’ by variables for calculations.
The introduction of the system theoretical approach ‘links’ the causal interactions by input/output relations in the domain of the integral transformations of eg. LaPlace, Fourier, Hilbert etc. which can be chosen to suit the particular properties of the study (phase-, stability-, intensity-, amplitude-, etc.).
Matrix mechanics yields eigenvalues and necessitated introduction of probabilities as ‘a posteriori’ addition (Born) of the mathematical result. Not an exact solution but a very clever breakthrough to proceed and to advance with the acceptance of the theory at the time. Nonetheless as it seems also one of the reasons by which probability became poured in concrete in the theory and interpretations became paramount due to lacking mathematical progress.

The complex Hilbert space and the principle of superposition stay fully intact in this proposal for all of the states and superpositions attained in the experiment.
The system approach (developed in the 60’s – 70’s) uses input/output relations starting from linear spacetime (r, t) functions developed in accordance with the entities of reality (beables) and the physical setup of the experiment.
To calculate the energy distributions, the Fourier transformation is suitable and follows with spatial frequency $\xi = 1/\lambda$, and wavenumber $k = 2\pi\xi$.

2. The one slit experiment model

The experiment model consists of a source i as input for the system s (manipulation in the experiment), with result g of the output, in which all functions are spacetime-functions: e.g. the momentum of a photon is $h.\nu/c$, in which h and value of c are contants. The momentum value is constant for certain λ (monochromatic source), and the functions of source and system introduce a local causality relation.

In case of separate i(r) and s(r), g(r) may be calculated by the convolution i(r) * s(r)

$$g(r) = i(r) * s(r) = \int_{-\infty}^{+\infty} i(\rho).\, s(r-\rho)d\rho \qquad (1)$$

The result of the convolution is considered to be the weighted average outcome of the interactive effect between i(r) and s(r) over certain (limited) r. The convolution is commutative i.e. i(r) * s(r) = s(r) * i(r).
A local causal relation is thus introduced between the functions: g(r) obviously is a direct effect of

i(r) and s(r) ; g(r) must be absolutely integrable on the interval $-\infty < r < \infty$ to apply integral transformations.

Since we know the approximate results of the experiments, focus is on i and s.
In the slit-experiment the system experiment setup is well known and a description of the process with the quanta is to be found *because* of the system interaction (manipulation) i.e. with the energy on quantum level in the experiment. Therefore, for the application in quantum mechanics, the Fourier transform is suitable also when multiple slits are to be considered. The Fourier transformation - abbreviated <=F=> - is unambiguous in both directions, commutative and suitable for our purpose.
The Fourier transformed functions [8, 11] in the k-space frequency domain then are

g(r), i(r), s(r) <=F=> G(k), I(k), S(k) and

$$G(k) = I(k) \cdot S(k) \qquad (2)$$

The following step deviates from the usual approach: instead of turning to vector space matrix mechanics and linear algebra, systems-theory [11, 12, 14] with its roots in commutative mathematics is applied to derive the spatial location-frequency i.e. in the (k, r) space domain to arrive at k-space frequency functions in the detection plane.

2.1 System functions in the (k, r) -space

The start is the input i for the system i.e. the source of quanta with a spatial momentum distribution that is uniform and all frequencies (m^{-1}) have the same amplitude. This source is considered ideal for the experiments, as all spatial locations at a distance r from the source at $t = t_0$ have the same energy amplitude.
Because of the patterns found in the experiments, therefore the deviation from the ideal uniform distribution of quanta in space may provide a model that describes the actual manipulation in the experiment.

The source and the slit manipulate the quanta in momentum p in the interaction and also the uncertainty in momentum Δp is reduced to p-components in the direction of the z-axis of the slit. When quanta are being captured in the slit, the assumption is made that the energy inside the slit remains unchanged i.e. there are no processes that require external energy and the boundaries of the slit do not absorb/emit energy in interaction with quanta.

The slit geometry reduces uncertainty Δr in spatial location r, however the Heisenberg relation [2] is to remain valid in the experiment with a lower boundary $\Delta r \Delta p \geq h/4\pi$.
The manipulation is in the internal energy states (degrees of freedom) of the photons.

The source is supplying the surface of the slit(s) with a uniform distribution in momentum[1] of N photons that may be repeated for a continuous input, to arrive at the result in detection (when

[1] Obviously, a non-ideal source would influence as well the frequency slit output (the frequency domain product) and thereby the (2D) graphics projected detection result.

required for clear detection). In principle the model thus is capable to fit a single photon experiment when provided by such a source repeatedly.
The energy distribution of an ideal source in terms of momentum is the normalised uniform distribution of quanta on the sphere around the point source. For quanta (photons) this represents the amount of quanta at a certain frequency i.e. the energy amplitude at the particular location.
The uniform distribution $D_p(k)$ determines the distribution of amplitudes of energy at the spatial frequencies and is by definition the Fourier transformed complex function of the ideal impuls function of momentum in the r-domain

$$D_p(k) < =F= > \delta_p(r) . \qquad (3)$$

Actually $\delta_p(r)$ is the generalised equivalent of a Dirac pulse of the momentum function p(r) with availability of *all* (!) momenta at $t = t_0$ and represents the ideal source[2]. In practice, this is quite demanding for a source, however repeating the pulse until all momenta are present is acceptable as the patterns may be built in time.
During the formation of the quantum pulse inside the slit, at the input of the slit ideally all spatial angles of the momentum vectors are captured with identical amplitude of all frequencies in the slit surface.
For a normalised amplitude, $D_p(k) = 1$ in the k-space domain i.e. all frequencies have the same normalized value '1' and in principle the entire spectrum of frequencies is covered in $D_p(k)$. The detection is a summation of all quanta in time in the frequencies, therefore the phase behaviour of quanta in the process (i.e. source and manipulation) does not play a significant role.
Typical for the point source is the decreasing density of the radiation as a function of r in free space. In contrast, in the confinement of the slit, the energy inside the slit (i.e. in each 'pulse' filling the slit) does not change, until the quanta start emanating from the slit. The values of momentum of the quanta do not change as internal energy state values are conserved.
To find the ideal system function $\delta_p(r)$ of the momentum function p(r), the system-theoretical approach is followed with the convolution property[2]

$$\delta_p(r) * p(r) = p(r) \qquad (4)$$

and the requirement that $D_p(k) = 1$.

The function normalising the momentum function p(r) is introduced as $p_n(r)$ with Fourier transform $P_n(k)$. $P_n(k)$ is the inverse Fourier transform function of P(k) to arrive at the uniform distribution $D_p(k)$[3] :

$$\delta_p(r) = p(r) * p_n(r) \text{ with } < =F= > \; D_p(k) = P(k) \,.\, P_n(k) = 1, => \text{ or}$$

$$P_n(k) = 1/P(k) \qquad (5)$$

2 In systems theory, the measured result r of the system s(r) with input $\delta_p(r)$ actually is s(r): $\delta_p(r)*s(r) = s(r)$. The description is by I/O relations.
3 It is not directly needed in this modelling to derive the function $p_n(r)$, only the Fourier transform 1/P(k) is required.

$\delta_p(r)$ represents the ideal momentum pulse: all quanta having momentum with components in the positive z-axis direction that are captured by the slit opening are present, propagate and start building the pulse.

In practice, 1. the source s, 2. distance d_s of the source to the slit , and 3. geometry of the slit, influence the result function g(r) to a large extent in the sense that the frequency content of g(r) is facing three low-pass momentum filters, shaping G(k) into mainly low frequency content around r = 0.

The photon energy is $E = h\,\nu = h\,c/\lambda$ of photons arriving at the surface O until the slit is filled with N quanta. When slit length is l, and the time t = l/c for the first components to emanate, the total energy E_p of each pulse inside the slit becomes

$$E_p = N.h.l/\lambda \quad \text{(J)} \qquad (6\ )$$

After the pulse leaves the slit, the process may be repeated to capture all of the momenta of the source until the source momentum distribution consists of all frequencies and the re-distribution is clearly observable at detection.

The model of the slit filled with quanta follows as a rectangular function s(r) of passing the emanating pulse through the slit output surface O , i.e. as the rectangular s(r) function of the pulse Π (r, t=l/c), building the pulse in surface layers of emanating quanta with their unique momentum created by the slit interaction.

Each pulse of constant energy emanating from the slit can be described by a convolution of the Dirac momentum pulse $\delta_p(\,r)$ and s(r), because i(r) * s(r) yield the result function g(r);

with $i(r) = \delta_p(\,r)$, then $g(\,r) = \delta_p(\,r) * s(\,r)$ with transformed function < =F= >

$$G(k) = D_p(k).S(k) = 1\ .S(k) = S(k) \qquad (7)$$

with S(k) < =F= > s(r)

The transformed result of g(r) <=F=> G(k) = S(k) is the frequency distribution function of the energy in case of the ideal source.
With the Fourier transform of a rectangular spatial pulse of quanta s(r), G(k) then is a sinc (k.r) with amplitude N.h.l/λ, and

$$G(k) = (N.h.l/\lambda).\ (\text{sinc}\ k.r) = (N.h.l/\lambda).\ (\sin k.r)/k.r \qquad (8)$$

with Planck's constant h, slitlength l, wavenumber $k = 2\pi\xi$, $\xi = 1/\lambda$ and λ the wavelength of the used source.

The result (8) shows that a *generic* 1 slit experiment model yields *sinc (k.r) function type patterns* at detection in the (x,y | z=0) plane, which in the experiment directly depend on

1. the momentum distribution and wavelength of the source and
2. the actual geometry used in the experiment: slit length l, slit in- and output surface O (capture, confinement) and source distance d_s[4] .

2.2. Experiment & practical system functions

The source at distance d_s from the slit supplies it with quanta covering all spatial angles with a momentum component in the positive direction of the z-axis, all other quanta are blocked i.e. the slit acts effectively as a *momentum filter* for the quanta. With $d_s = 0$, and the source in the centre of the input surface of the slit, all quanta with a momentum component in the positive z- direction are present and the frequencies/locations are symmetric in x and y. A uniform distribution at slit entrance is possible however depends on the used source's capability to emit all momenta and therefore in practice acts as a momentum filter as well.

Although the model is based upon an ideal source containing all momenta within +/- 90 degrees spatially with the positive z-axis, the distance of the source has a huge impact. The angle captured by the slit is governed by sin $\varphi_m/2$, where $\pm$arctan $\varphi_m/2$ is the angle caused by slit geometry and d_s. The angle φ_m is highly depending on the source distance d_s and geometry of the slit, and thus increasing d_s acts as an additional momentum filter by restricting φ_m in the higher values and therefore functions as a potential low-pass filter. The pulse then is built of momenta with a maximum in sin φ_m and thus substantially shapes the result function g(r) by restricting the frequency content in G(k), thereby creating a narrowed sinc shape around r = (x,y =0 | z=0). This may be modelled in i(r) by taking instead of the Dirac generalised δ_p(r) function, a rectangular filter function that restricts the higher frequency location components of the source.

The source, distance to the slit and slit-geometry therefore influence the pattern directly by sensitivity for higher momentum frequencies: when the momentum distribution is far off the ideal situation, creating little interaction with the boundaries inside the slit, a less broad pulse of small divergence and less frequency content emanates and vice versa, an ideal source δ_p(r) creates maximum interaction inside the slit and shows a broad diverging pulse of much higher frequency content.

3. Double-slit experiment

The result for two slits follows the one slit result. For this extension, one considers the one-slit result with an additional distribution of energy in the k-spatial frequency domain: the detection plane (x,y | z=0) of the experiments is a graphical representation of the quanta per location r i.e. an amplitude. The transformed function in the frequency domain represents the amplitude of energy in y of the frequencies locations in x, the spatial locations.

The double slit manipulation consists of *addition* of a identical second slit with therefore an *identical distribution* of quanta with shift in x. With the origin O in the middle between the slits, then

[4] Not integrated in the model as the purpose of the paper is to show (exact) predictability of the found patterns.

$\pm x_{slit} = n.\lambda = 2\pi/k_0$ on the x-axis, which is represented by a frequency shift in the Fourier transform G(k) of g(r) and the two distributions are

$$G(k + k_0) = G(k + 2\pi/x_{slit}) \quad \text{and} \quad G(k - k_0) = G(k - 2\pi/x_{slit}) \quad . \tag{9}$$

With spatial frequency $\xi = 1/\lambda$, wavenumber $k = 2\pi\xi = 2\pi/\lambda$ and $k_0 = 2\pi/ x_{slit}$.

Two slits thus produce 2 distributions $G(k - k_0)$ and $G(k + k_0)$, that seem to show a pattern with 'interference'. At levels of energy in the experiment, photon interference however has never been reported or theoretically predicted.

Two shifted distributions can be explained more clearly by calculating a modulated g(r) for *one* slit by the product

$$g(r) . \cos k_0.r = m(r) \tag{10}$$

with the Fourier transform M(k), resulting in the shifts in the k space domain by convolution:

$$m(r) <=F=> M(k) = ½ G(k + k_0) + ½ G(k - k_o) \tag{12}$$

which shows the 2 distributions due to modulation of the 1 slit g(r) in the amplitude of the frequencies.

The product in (10) yields in the k-domain the convolution $G(k)^* F<\cos k_0.r>$. For a limited r (i.e. possibly large but not unlimited, to be integrable $-\infty < r < \infty$), $f(r) = \Pi(r / \delta).\cos k_0.r$ and one finds the addition of the two (limited by $\Pi(r / \delta)$ filter function) expected sinc functions in the frequency k – domain.

The k_0 relation in the 2 distributions of M(k) is unambiguously related to the modulation $\cos k_0.r$; the two slit identical result and an identical k_0 relation between slits, therefore is related unambiguously to the modulating function $\cos k_0 r$ as well, obviously with double energy amplitude because of *two* slits:

from (9) and (12) one finds $G(k) = G(k + k_0) + G(k - k_0) = 2.M(k)$ (13)

The foregoing is known in Fourier transformation theory as the m*odulation theorem*, in which the modulated g(r) may be considered an envelope function with k_0 the wavenumber of the carrier frequency k[5].

From this theorem, one may conclude that one slit with modulated g(r) i.e. m(r), yields 2 k_0 shifted distributions, showing an identical re-distribution of quanta by the modulation effect as a two-slit experiment, creating an identical pattern, however by the absence of a second slit without possibility of interference.

This rules out interference caused by a separated physical second slit, and consequently this renders interpretations of a quantum partially being in two states at the same time [1] (by splitting

[5] From the (k, r) domain, by substitution into the (ω, t) pair domain for $\omega,t > 0$ (of information transmission and signal theory), likewise this property of the Fourier transformation has firm roots in radio technology deployments in which frequency sideband, envelope function and carrier wave frequency $f_0 = \omega_0 /2\pi$ have (had) a prominent role in communication and broadcasting.
The sideband contains all transformed information (obviously the main reason for radio transmission), which information may fully (or partially) be recovered by de-modulation. Thus the input information contained in the frequency domain is preserved to re-build the time functions (directly in e.g. broadcast, reception, de-modulation) but also any time when the data is conserved for this purpose.

up energy or other interpretations) and entering two slits leading to some kind of interference, unlikely.

The modulation effect paves the way for explanation of the experiment when performed with mass particles e.g. electrons and single particles with randomized momentum in repeated experiments, that are modulated equally into a pattern by manipulation of their momentum inside the slit and thus by yielding identical patterns in the final distribution when emanating, without (any) reverting to wave properties of mass particles and interference in general.
The model is suitable for single particle experiments with many repetitions, all other things equal including sources that emit (ideally) a randomized but full spectrum in momentum of the single particles.

In the practice of the photon experiments, due to slits of say 0.1 mm distance apart (different in experiments depending on used wavelengths) then k_0=0.05 mm, thus two distributions are present in reality with a tiny x-shift between distributions; due to this small value between slits, these are projected seemingly as one distribution with identical shape at the detection plane.

At detection of the resulting function, information in the frequency domain (sinc kr and k_0) can be preserved and may be used to reconstruct the momentum function g(r) as all mathematical operators (c.q. operations) are commutative; this means that the causality relations are invariant for time reversal [7].

The preservation of information in the modulation operation becomes more obvious in the (ω, t) pair frequency domain[5] of information transmission where all types of modulation including amplitude modulation as is the case here, are well known and widely deployed for the purpose of recovering the information e.g. of a radio broadcast locally elswhere. This example shows as well that causality not necessarily is ‘local’ and may exist *anywhere* in space-time.

Note that the arrow of time cannot be reversed in case of de- or con-structive interference, as information is lost and cannot be retrieved. This is as well the case in attemps where correlations exist between results of calculations or experiments, as the correlation operations are not commutative ($\Phi_{12} \neq \Phi_{21}$) . This in principle separates (Bell, 1964) correlation functions as well as cross vector products of matrix multiplications in vector spaces from causality eg. for invariance of time reversal in this system theoretical approach. A cross correlation operation is a ‘one way street’ by definition and is separated from causality even despite a statistical relation in datasets[6] anywhere in spacetime.

This result shows that the quanta in the frequencies of the re-distribution of energy as detected in double-slit experiments, therefore become manifest because of an *amplitude modulation* effect in the k-space frequencies, which is fundamentally different from interference[7].
This result is useful to abandon - on these quantum experiments energy level - wave

6 The existence of statistical relations is not excluded nor denied; the consequence mathematically is merely that from the ‘result’ function no reconstruction of the ‘cause’ function is possible as the original mathematical causality relation is reserved for commutative operators only.

7 Hence author’s preference for state-function instead of wave-function.

interpretations with interference and consequently, further interpretations thereof. Wave-particle dualism [5] and related interpretations[8] [6]) need not to be assumed nor are required to explain quantum slit experiments.

INTERPRETATION

A thought experiment of reality

In entering the slit a pulse of constant energy is created by the captured quanta i.e. the energy of the pulse does not change during the confinement in space, in contrast with the momentum functions which are heavily affected inside the slit. The interaction of the quanta with the slit is in internal energy (therefore re-active[9] or inter-re-active and requires time) and causes the re-distribution in momentum by creating a myriad of trajectories in the slit with equally different phases until finally emanating from the slit. When emanating layers of quanta of the pulse, the layers thus contain sets of quanta with a momentum distribution that differs in each layer, i.e. the sets of momentum in the layers are unique in composition due to inter-re-action of the quanta with the boundaries of the confinement: quanta with the largest momentum components in z-direction emanate first, the ones with the smallest components and most inter-re-action with the slit last, indicating that the phase of the emanating quanta does not play a significant role in the redistribution for pattern detection.

As result, the momentum distribution is re-arranged during the entire propagation of the pulse in the slit, until the layers reach the free space and momenta are ‘frozen’. During propagation, the momentum vectors of quanta are in transition of their internal energy state (almost continuously, depending on momentum distribution) i.e. are in a new superposition after each inter-re-action with the slit, and attained states are deterministic however not observable, while their vector *values* of energy h.ν stay preserved. The latter strongly indicates that observation/measurement will destroy quanta and convert their energy to results in eigen*values* with consequences both in virtual treatment mathematically (i.e. by ‘collapse’ of a state-function) and in reality by a full transition of energy when being observed/detected [10].

8 The result indicates that the de Broglie duality relation e.g. [5] in principle explains the energy equivalence formula, relating different properties of photon field-energy and matter energy of mass particles.

9 Inter-re-active and re-active is in the sense of local and temporal exchange of internal energy / ‘degrees of freedom’.

10 This provides support to change the way of mathematical treatment for quantum processes and gives ground to failing descriptions with e.g. hidden variables. When quanta are associated with variables, calculations and mathematical operators require e.g. momentum p and location r + related variables as exact values, thereby violating the Heisenberg relation. This leads to results in eigenvalues with passed on probability as encountered in the probability result functions from vector space matrix mechanics. For exact results, the quanta cannot be addressed individually nor be subjected to operators as for observables, and one has to rely on indirect descriptions.

CONCLUSIONS

This paper started by embedding cause and effect relations in the system model and continued with the introduction of system-theory, based upon commutative mathematics allowing reversibility in time, to derive the ideal system response in terms of k-space energy distribution. The practice of the experiments is treated by inclusion of the (low-pass) restrictions in the frequency content of the resulting functions (10).
It demonstrates that quantum slit experiments are deterministic in the resulting (and experimentally found) functions of patterns when accepting the deviation of the usual matrix mechanics in the vector space by the system-theoretical approach in commutative mathematics, including the in-/output relations and the modulation effect. This approach excludes interference and correlations in the description or treatment as they cannot support time reversal invariance.

The exact results may seem 'ímpossible' as one cannot observe the indiviual quanta exactly in the heart of the statefunction Ψ without destroying their state, and therefore remain 'hidden' from observation in reality's classical sense and as well in the virtual reality of mathematics. This led the author to the system theoretical approach[11] in the transformed domain instead of complex vector space mechanics following Schrödinger's approach; it is emphasized that the system is described with input-output relations based on *commutative* convolution & transformation (solely[12]) and in principle may not be unique for the described subject by system theory (i.e. physics, experiment). The latter however is often the case e.g. in equivalent behaviour and description of eg. mechanical and electrical systems (apart from the variables) with identical transformed system functions.

The functions in the results seem to rule out any form of quantum state probability in the quantum experiments, whereas all quantum state complex energy-vectors in the experiments are part of the Hilbert space and the state function Ψ has *no* restrictions in evolving into *any and all* of the states. This includes all possible linear combinations of state superpositions required to represent the quantum experiment (or any other manipulation or operation with quanta) - the result stays deterministic by applying mathematics in an indirect system approach thus excluding the direct description of individual quanta using exact values of any variables eg. related to r and p at the same time. This appears to support the expectation that all state functions Ψ_n in the complex orthonormal Hilbert space can have a deterministic result: with the consequence that exact solutions actually exist for *all* Ψ_n.

The determinism found in the model has substantial impact on current views and interpretations when extrapolation of the result in quantum mechanics can be verified into further aspects of the theory: from mathematical treatment to quantum computing (e.g. qubits) in subjects related to probability and interference.
When this verification can be established, probability including interpretations of quanta being "partly in each of two or more states" [1] apparently then disappear from stage, whereas the

11 Also named 'black-box' [11, 14]
12 Excluding the correlation function (-operator), being not commutative.

principle of quantum superposition of states in a complex Hilbert space holds, be it with the restriction of application of cross -vector and -correlation operators.

To date, mathematics including use of hidden variables and complex state-vector space matrix mechanics of linear algebra, are attempts requiring a direct description of quanta (and quantum-processes) leading to eigenvalues that do not exactly describe the actual states apparently due to a direct violation of the Heisenberg relation or implicit use of cross- operators. At the same time the introduced probability interpretation (Born) has been a blessing in proceeding with the heavily attacked [10] theory, rendering it one of the most successful theories in physics.

The proposed method gives an indirect description leading to exact results of what one actually expects physics to describe in almost a classical way; the difference is that it does not attempt to trace individual quanta or to reveal individual quantum behaviour in the heart of the state function (for which one must rely on the thought experiment[13]) due to the transformations into the different domain. We cannot calculate directly with or acquire information on the exact (p, r) of individual quanta, as W. Heisenberg already predicted.

[13] A.Einstein would have liked this.

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