Measurement of a Continuous Gravitational Wave near 2619.9 μHz

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Superconducting gravimeters respond to deformations of the test body Earth by gravitational waves. The frequencies of continuous gravitational waves are identified by means of selective integration of long-term data. The modified superhet method is a suitable method to determine the direction of the GW source. The measured frequency deviation of the phase modulation exceeds the upper limit allowed by the Doppler effect. This problem can be can be solved by assuming that the propagation velocity of GW is lower than the speed of light.

1 Introduction

Superconducting gravimeters register minute changes in gravity over a wide range of frequencies. Until now, they have only been used for earthquake research and are therefore directly connected to the ground. This kind of mounting transfers a high noise level, which could easily be reduced by improved mounting. Using the novel modified superhet evaluation method (MSH) and by comparing several years of data chains from different sensors, it has been possible to measure the CGW of the nearby binary star system HM Cancri with known orbital frequency [1].

With good S/N, few criteria are sufficient to identify the CGW in the noisy background. With increasing distance of the sources, the S/N of the signals decreases and the identification of CGW requires the help of a further criterion: The amplitude of a CGW must be constant during a period of several years. Thus the signatures of a CGW are fixed:

- During a period of several years, the amplitude of a CGW is constant.
- The signal is phase modulated with the frequency 1/365.256 days. During several years of observation, the spectrum consists of a broad bundle of separated spectral lines of low amplitude. The closer the source is to the ecliptic plane, the higher the modulation index and the number of spectral lines.
- The propagation speed of the CGW limits the highest value of the modulation index. Its measurement requires several years of observation.
- The average frequency of the signal increases slightly over time.

The MSH method of compensating the phase modulation has been described in detail in [1]. It also eliminates the frequency drift of the signal and causes a concentration of the total energy of the CGW on a single spectral line, improving the S/N. A spectrum shows the average value over the entire measurement period and contains no information about

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time-dependent changes in amplitude. Therefore, amplitude constancy over several years - an essential signature of any CGW - must be demonstrated by another method. Most methods require a very good S/N and are not very suitable for detecting CGW in noise. Only a method with excellent selectivity can solve this task.

2 Selective Integration

A phase sensitive integrator is an extremely narrow band filter with selectable frequency and a "memory" for the phase of already processed measured values.

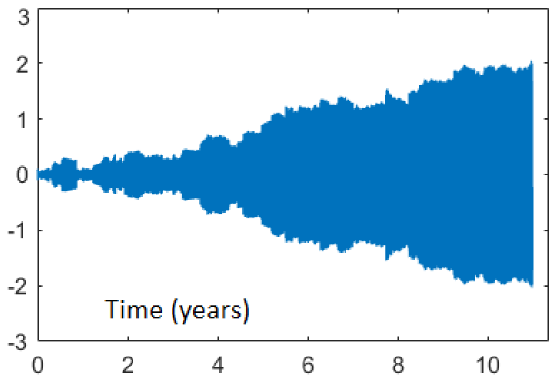

Figure 1): *Feeding a mixture of a weak signal of constant amplitude and a high noise component into a phase-sensitive integrator, the output signal increases approximately in proportion to time. The prerequisite is a vanishing frequency change during a long period of time.*

A gravimeter provides recordings as a sequence of single values z_n and z_{n+1} measured at the separation of the sampling time T_s. If one analyzes the oscillations of a fixed frequency f, the phase angle increases with each step by $\alpha = 2\pi f T_s$. Although each measured value z_n results from the sum of many single frequencies, the selective integration hardly reacts to noise and neighboring frequencies. The realization of the integrator chosen here is based on the addition theorem of trigonometric functions. The basis are the two CORDIC formulas

$$x_{n+1} = cos(\alpha)x_n + sin(\alpha)y_n + z_n \quad (1)$$

$$y_{n+1} = cos(\alpha)y_n - sin(\alpha)x_n \quad (2)$$

The choice of parameters determines the sequence of calculated values x_n and y_n:

- Without an injected signal ($z_n = 0$) and with initial values $x_1 = 0$ and $y_1 = 1$, the formulas (1) and (2) calculate a table of values for $x = sin(2\pi tf)$ and $y = cos(2\pi tf)$. The amplitudes are constant.
- If one sets $x_1 = y_1 = 0$ and feeds a monochromatic signal z_n of frequency f, the formulas calculate an oscillation of this frequency whose amplitude increases proportionally with time (see Fig 1).
- If the programmed and injected frequency differ or if the phase or amplitude of the injected signal z_n changes, the output signal of the integrator varies and the linear

increase of the envelope is lost. If the signal is in phase opposition, the output signal decreases proportionally to time.

- If only noise is fed in, the formulas (2) calculate a low bandwidth signal with a frequency near f whose amplitude varies irregularly.

3 A survey near 2619.9 μHz

So far, few binary star systems with precisely known orbital frequency in the range around 1 mHz are known. Since our galaxy probably hosts several such systems, the arbitrarily chosen frequency range between 2600 μHz and 2650 μHz was searched in small steps for anomalies (figure 2). This range contains no interfering natural resonances of the Earth and possibly only noise.

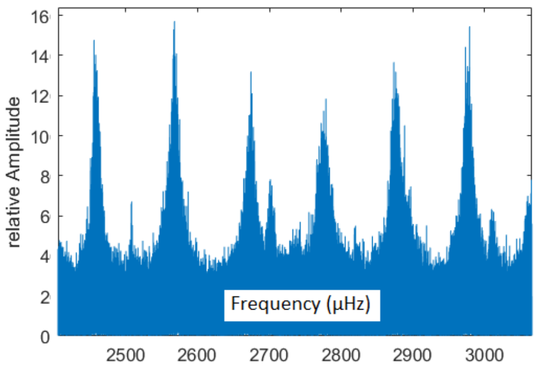

Figure 2): *Typical spectrum of gravitational data recorded by superconducting gravimeters. The broad maxima are natural resonances of the earth excited by earthquakes (damped oscillations). The background noise could be reduced by improved mounting of the gravimeters.*

Numerous experiments have shown that a phase-sensitive integrator is very well suited to detect CGW based on their constant amplitude, even if the S/N is very low. If one wants to analyze a candidate in more detail, the MSH method provides accurate information about the modulation index of the phase modulation and the frequency drift.

The spectra of especially many gravimeter recordings show a strong signal at 2619.95 ± 0.05 μHz, which could be a CGW. All further investigations are therefore limited to this narrow range, which apparently contains other candidates with good S/N that also show characteristics of a CGW. These are ignored below because they can be masked out due to the directionality of the MSH procedure.

4 Data basis of the investigations

Until now, the gravimeters have been used for earthquake research and are mounted directly on the ground. Therefore, they react to even the smallest ground movements that occur far away. Stronger earthquakes overload the sensors and cause data gaps of several minutes. These ground shaking events affect the quality of all records. Irregularly active disturbance sources such as earthquakes can be well suppressed by long-term integration.

Gravimeters respond not only to changes in the shape of the sample *earth* but also to the variable air mass above the instrument. Therefore, gravitational data (G in column 9) are stored together with parallel recorded air pressure values (P in column 10) at IGETS Potsdam [2]. The noise level can be minimized by a weighted sum of both records. The factor with which the air pressure is to be considered depends on the frequency and on the height of the gravimeter above sea level. The formula $a = G + 3 \cdot P$ usually delivers good results.

5 Methodology of measurement of a CGW

In the search for CGW, the goal is to discover signals that undoubtedly satisfy the signatures listed above. With the MSH method explained in [1] reasonable values can be found for the four free parameters in the ansatz

$$y = sin(2\pi t(f_{CGW} + t \cdot k_{Drift}) + \eta \cdot sin(2\pi t f_{year} + \varphi)) \tag{3}$$

The parameters mean:
f_{CGW} = estimated frequency of CGW; depends on the year
$k_{Drift} \approx 1$ nHz per year
$\eta \approx 8$ (Modulation index, from this follows the ecliptic latitude of the source)
$0 < \varphi < 2\pi$ (from this follows the ecliptic longitude of the source)
The parameter f_{year} = 31,688 nHz is the constant orbital frequency of the earth.

The parameters must not only respect interpretable boundaries, they must also not vary too much when changing the data basis. The data were recorded by 17 gravimeters, which are characterized by a low noise level and were in operation almost without gaps in the period 1997 to 2020: BF1+2 (Germany), ST (France), BH (Germany), CO (Austria), DJ (Benin), MC (Italy), MO1+2 (Germany), PE (Czech Republic), SU1+2 (South Africa), OS (Sweden), WE (Germany), YS (Spain), CB (Australia), MB (Belgium).

The readings from each source are linked to form a long chain of data that begins on January 1 of each year and covers a period of about ten years. The data reduction of each chain is done step by step:

1. A narrow frequency range around 2619.95 μHz is reduced to the intermediate frequency 2 ± 1.6 μHz. This corresponds to a superhet of common design. A too small bandwidth distorts the phase modulated signal and causes erroneous results. The modulation index determines the necessary Carson bandwidth, which can be up to 1 μHz at the frequency investigated here.

2. A phase modulated auxiliary oscillator reduces the intermediate frequency to 1 μHz. The frequency drift of this oscillator is freely selectable. This stage is the core element of a modified superhet MSH.

3. One iterates the phase φ, the modulation index η and the drift of the auxiliary oscillator until the amplitude of a spectral line near 1 μHz reaches a maximum. Then

all the energy of the CGW flows into the central spectral line and the amplitudes of the accompanying spectral lines of the phase modulated signal decrease. The height of the remaining spectral line is no reliable criterion to distinguish a CGW from noise.

4. By selective integration (see figure 1) one checks whether the amplitude of the signal is approximately constant during the entire period of about ten years. This test is necessary to distinguish CGW signals from noise.

5. Small frequency deviations ($\Delta f < 5$ nHz) are corrected. Larger deviations mean the signal may not be coming from the CGW we are looking for and require a new iteration with modified parameters.

6. If we obtain a reproducible result with sufficient S/N, the parameters of the phase-modulated auxiliary oscillator match the characteristics of the CGW. These are tabulated with indication of the start date.

6 Results

The formula (3) was applied to all data chains, each covering ten years. In the frequency range 2619.9 μHz to 2620 μHz there are several CGWs, that can be separated using different values for φ and η. The following mean values characterize the strongest signal (Figure 3): $\varphi = 5.262 \pm 0.035$, $\eta = 11.347 \pm 0.311$, and $k_{drift} = (153.9 \pm 14.6) \times 10^{-18}\ s^{-2}$. The standard deviation was calculated using the jackknife method.

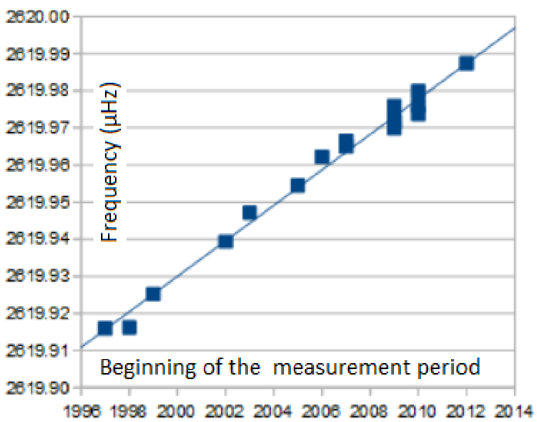

Figure 3): *The start frequencies of all data chains as a function of the start date. Each chain covers a period of ten years. The last data points extend to the year 2020. The slope of the plotted straight line is 4.8 nHz per year.*

After converting the phase shift φ and the modulation index η (results of the MSH procedure) into ecliptic coordinates, one may check with electromagnetic waves whether there is an object in the calculated direction which could generate the CGW. For the ecliptic longitude, the conversion [3] is simple: a near-Earth observer measures maximum blueshift on the 60th day of the year and maximum redshift on the 242nd day. So the RA of the source could be about 16h30'.

The calculation of the ecliptic latitude leads to a problem that is also observed in other frequency ranges. The earth moves with high velocity in the wave field of the CGW, therefore the Doppler effect causes a seasonal frequency shift. With a sufficiently long observation period, one does not measure a variable frequency, but a phase-modulated signal. Its parameter is the ratio of the maximum frequency deviation Δf (from the mean frequency) to the modulation frequency $f_{Mod} = 1/365$ days, called modulation index η.

Assuming that GW and electromagnetic waves propagate at the same speed, the maximum possible frequency shift Δf is calculated according to the formula (4).

$$\Delta f \leq f_{CGW} \cdot \left(\sqrt{\frac{c + v_{Earth}}{c - v_{Earth}}} - 1 \right) \approx 262 \; nHz \tag{4}$$

This maximum value applies when the source of the CGW is close to the ecliptic plane; As the ecliptic latitude increases, Δf decreases. The modulation index η should never become larger than

$$\eta = \frac{\Delta f}{f_{Mod}} = \frac{262 \; nHz}{31.688 \; nHz} = 8.27 \tag{5}$$

However, the actual measured values for η are usually higher and the discrepancy increases with decreasing frequency of CGW. In the study of HM Canri [1] ($f_{CGW} =$ 6220 μHz), the excess was still within the measurement accuracy and may be explained by poor S/N and measurement inaccuracy. Now, at $f_{CGW} = 2620$ μHz, the actually measured value $\eta = 11.347$ is 37% above the maximum value allowed by the theory of relativity.

How can this deviation be explained? The Doppler formula (4) allows one way out: The speed of GW is smaller than the speed of light. Evaluations already done, not yet published, show that the propagation speed of gravitational waves obviously depends on the frequency and decreases with decreasing frequency. That's a different story.

References

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