Mass Spectrum of the N and Delta Baryons

D.G. Grossman
March 20, 2022

Abstract

All subatomic particle masses can be expressed as multiples of hypersphere surface volumes times ‘h’ – Planck’s constant’s coefficient. How one determines which hypersphere surface volume is a multiple of a given subatomic particle’s mass is from knowledge of the particle’s quark content. The details of how to do this are explained in the introduction. In the case of N and Δ baryons, the hypersphere surface volume needed is that of a 6-sphere (S6 for short). The purpose of this paper is to *show* that the masses of all N and Δ baryons can be expressed as simply defined multiples of S6h, by matching experimental N and Δ masses with their theoretical values in the mass spectrums generated by S6h that are presented in the paper.

Contents

1. Introduction

Baryons are subatomic particles composed of three quarks. The three quarks in N and Δ baryons are combinations of ‘up’ and ‘down’ quarks. According to Quark Theory, the three quarks inside baryons orbit one another in 3d space under the influence of a central force called the Strong Force. The mathematics for predicting baryon masses using the Quark Model is very complicated and uncertain since two key elements necessary for making accurate predictions are missing, namely the exact masses of the quarks, and an exact mathematical expression for the Strong Force. In this paper, quarks are treated as hypersphere surface volumes of mass of various dimensions, instead of point particles. The highest mathematics involved are the formulae for the hypersphere surface volumess.

It is hoped that the study of hypersphere surface volume factoring of particle masses will *result* in a comprehensive theory of particle structure, including discovering the complex mathematics that is likely associated with physics in higher dimensions.

2. How to Calculate the Factoring Unit from the Quark Content

To find the factoring unit of a subatomic particle, its quark content must be translated to the corresponding hypersphere surface volume formulae, then multiplied together along with 'h' - Planck's constant's coefficient. The quark content of the N baryon is either 'uud' or 'udd' depending on the charge of N. The quark content of the Δ baryon is either 'uuu', 'uud', 'udd', or 'ddd' depending on the charge of Δ. The mass of the 'u' quark corresponds to 2-sphere surface volume (call it S2) and the mass of the 'd' quark corresponds to 3-sphere surface volume (call it S3). So, to get the *factoring unit* of the 'udd' baryon (in units of MeV/c^2) multiply the corresponding quark content surface volume formulae together along with 'h' - the coefficient of Planck's constant. All four combinations of 'u' and 'd' taken three at a time result in basically the same factoring unit. (This may not strictly be true, especially for udd vs ddd, but is used as a working hypothesis.) They differ only in their coefficients and in their powers of 'r', which is irrelevent because the value of 'r' is set equal to one when the value of the factoring unit is calculated.

Factoring unit of **udd** = S2 S3 S3 h

Factoring unit of **udd** = $(2 \pi r)\ (4 \pi r^2)\ (4 \pi r^2)\ h$

Factoring unit of **udd** = $32 \pi^3 r^5\ h$

The surface volume of a 6-sphere is: S6 = $\pi^3 r^5$. As you can see, the factoring unit of **udd** is 32 times bigger, but except for that constant of multiplication, they are the same. Throughout this paper S6h will be used instead of S2S3S3h for factoring N and Δ masses, because it is more concise (there is less to type) and it is a smaller factoring unit, both of which are advantageous.

To get the value of the factoring unit that is used in this paper, set r=1, then multiply S6 by the coefficient of Planck’s constant (h = 6.62607015).

S6 h = $\pi^3\ 1^5\ h$

S6 h = 205.4497644 MeV/c^2

This is the *factoring unit* that was used to construct the mass spectrums in this paper.

3. How to Read the Mass Spectrums

Col 1: **n** The first column of the mass spectrum holds '**n**', the *multiplier* of **S6h**.

Col 2: **nS6h** The second column shows the *theoretical mass*, which resuls from multiplying '**n**' with **S6h**.

Col 3: **ExpMass** The third column holds 'ExpMass', the *experimental mass*. It is placed next to the theoretical mass in the mass spectrum that most closely matches it.

Col 4: **Error** The fourth column holds the *error of the experimental mass*. It is plus or minus.

Col 5: **dm** This is the difference between the experimental and theoretical masses.

Col 6: **dm /Error** This is **dm** divided by the **Error,** and the result is shown as a percentage. It shows the difference between theoretical and experimental mass as a percentage of error size.

4. Low Resolution Mass Spectrum of the N and Δ Baryons

Resolution: (.50)S6h = 102.724 MeV/c^2

n	n S6h	ExpMass	Error	Baryon
6	1232.698	**1232.9**	1.2	Δ(1232)
6.5	1335.423			
7	1438.148	**1439**	19	N(1440)
7.5	1540.873			
8	1643.598	**1643**	6/3	Δ(1700)
8.5	1746.322	**1748**	~	Δ(1750)
9	1849.047	**1848**	9/19	Δ(1930)
9.5	1951.772	**1951**		N(1875)
10	2054.497			
10.5	2157.222	**2157**	3/14	N(2250)
11	2259.947	**2260**	60	Δ(2400)
11.5	2362.672			
12	2465.397	**2468**	50	Δ(2400)
12.5	2568.122	**2570**	19/10	N(2570)
13	2670.846			
13.5	2773.571			
14	2876.296			
14.5	2979.021			
15	3081.746	**3100**	~	N(~3000)
15.5	3184.471	**3200**	200	Δ(~3000)
16	3287.196	**3300**	200	Δ(~3000)
16.5	3389.921			
17	3492.645	**3500**	200	N(~3000)
17.5	3595.370			
18	3698.095	**3700**	200	Δ(~3000)
18.5	3800.820	**3800**	200	N(~3000)
19	3903.545			
19.5	4006.270			
20	4108.995	**4100**	300	Δ(~3000)

Note: The placement of the experimental masses in the mass spectrum above do not necessarily reflect the best factoring for each experimental mass shown, given its error size. Some do, but some don’t.

5. Mass Spectrum of the Δ(1232) Baryon's Experimental Mass Data

n	6+n S6h / 2048	ExpMass	Error	dm	dm/Error
-28	1229.8897				
-27	1229.9900	**1230**	2	.0100	0.5%
-26	1230.0903				
-25	1230.1907				
-24	1230.2910				
-23	1230.3913				
-22	1230.4916	**1230.5**	0.2	.0084	4.2%
-21	1230.5919				
-20	1230.6922				
-19	1230.7926	**1230.8**	0.4	.0074	1.9%
-18	1230.8929	**1230.9**	0.3	.0074	2.5%
-17	1230.9932				
-16	1231.0935	**1231.1**	0.2	.0065	3.3%
-15	1231.1938				
-14	1231.2941	**1231.3**	0.6	.0059	1.0%
-13	1231.3945				
-12	1231.4948				
-11	1231.5951				
-10	1231.6954				
-9	1231.7957				
-8	1231.8960	**1231.88**	0.29	.0160	5.5%
-7	1231.9964	**1232**	3	.0036	0.1%
-6	1232.0967				
-5	1232.1970				
-4	1232.2973				
-3	1232.3976				
-2	1232.4980				
-1	1232.5983				
0	**1232.6986**				
1	1232.7989				
2	1232.8992	**1232.9**	1.2	.0008	0.1%
3	1232.9995	**1233**	2	.0005	0.0%
4	1233.0999	**1233.1**	0.3	.0001	0.2%
5	1233.2002				
6	1233.3005				
7	1233.4008	**1233.4**	0.4	.0008	0.2%
8	1233.5011				
9	1233.6014	**1233.6**	0.5	.0014	0.3%
10	1233.7018				
11	1233.8021	**1233.8**	0.2	.0021	0.3%
12	1233.9024				
13	1234.0027				
14	1234.1030				
15	1234.2033				
16	1234.3037				
16.5	1234.3538	**1234.35**	0.75	.0050	0.5%
17	1234.4040				
18	1234.5043				
19	1234.6046				
20	1234.7049				
21	1234.8052				
22	1234.9056	**1234.9**	1.4	.0056	0.4%
23	1235.0059				
24	1235.1062				
25	1235.2065				
26	1235.3068				

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

5.1 Commentary on D(1232)'s Mass Spectrum

Spectrum Resolution and ExpMass's Error Size

The correct resolution size of a mass spectrum depends on the average error size of the data being displayed. The data associated with the Δ(1232) baryon is of high accuracy, so the mass spectrum showing that data must also be of high accuracy, or, resolution. Of the 16 data points plotted in this mass spectrum, 11 have errors of 0.75 or less (three have errors of 0.20). So, in order to resolve masses that may be that close together, a high resolution mass spectrum is needed. The resolution of this mass spectrum is set to **(1/ 2048) S6h =** 0.100317268 MeV/c2, which is just small enough to resolve all of Δ(1232)'s experimental masses. If the step size of this mass spectrum was any bigger (if its resolution was any lower) four pairs of data points would *not* be resolved. These are: (1230.8, 1230.9), (1231.88, 1232), (1232.9, 1233), and (1233, 1233.1), because the masses of each pair are about 0.10 MeV/c^2 apart. Since these experimental masses – the ones that are closest together - appear next to one another in this mass spectrum, it means that (1/ 2048) S6h is the correct resolution size for this set of data.

Distribution of the Data in the Spectrum

Exactly half the data points lie above **6.00 S6h** = 1232.6986 and half lie below it. Why none are found exactly at **6.00 S6h** is a mystery.

6. Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
6.00000	1232.699	**1232.9**	1.2	0.201	16.8%	Δ(1232)
6.03125	1239.119					
6.06250	1245.539					
6.09375	1251.960					
6.12500	1258.380					
6.15625	1264.800					
6.18750	1271.220					
6.21875	1277.641					
6.25000	1284.061					
6.28125	1290.481					
6.31250	1296.902					
6.34375	1303.321					
6.37500	1309.742					
6.40625	1316.163					
6.43750	1322.583					
6.46875	1329.003					
6.50000	1335.423					
6.53125	1341.844					
6.56250	1348.264					
6.59375	1354.684	**1355**		0.316		N(1440)
6.62500	1361.105					
6.65625	1367.525					
6(170/256)	1369.130	**1369**	3	0.130	4.3%	N(1440)
6(171/256)	1369.933	**1370**	4	0.067	1.7%	N(1440)
6.68750	1373.945					
6(177/256)	1374.748	**1375**	30	0.252	0.8%	N(1440)
6.71875	1380.366					
6(191/256)	1385.983	**1386**		0.017		N(1440)
6.75000	1386.786					
6.78125	1393.206					
6.81250	1399.627					
6.84375	1406.047					
6(221/256)	1410.060	**1410**	12	0.060	0.5%	N(1440)
6(447/512)	1412.065	**1412**	2	0.065	3.3%	N(1440)
6.87500	1412.467					
6(230/256)	1417.282	**1417**	4	0.282	7.1%	N(1440)
6.90625	1418.887					
6.93750	1425.308					
6.96875	1431.728					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
7.00000	1438.148	**1439**	19	0.852	4.5%	N(1440)
7.03125	1444.569					
7.06250	1450.989					
7.09375	1457.409	**1457**		0.409		N(1440)
7.12500	1463.830					
7.15625	1470.250					
7.18750	1476.670					
7.21875	1483.090					
7.25000	1489.511					
7.28125	1495.931	**1496**		0.069		N(1535)
7(76/256)	1499.141	**1499**		0.141		N(1535)
7(77/256)	1499.944	**1500**	4	0.056	1.4%	N(1535)
7(78/256)	1500.746	**1501**	4	0.254	6.4%	N(1535)
7.31250	1502.351	**1502**		0.351		N(1535)
7(86/256)	1507.167	**1507**	3	0.167	5.6%	n(1520)
7.34375	1508.772	**1509**	4/2	0.228	5.7%	N(1535)
7(92/256)	1511.982	**1512.0**	1.5	0.018	1.2%	N(1520)
7.37500	1515.192	**1515**	15	0.192	1.3%	N(1440)
7(97/256)	1515.995	**1516**	2	0.005	0.3%	N(1520)
7(98/256)	1516.797	**1517**	3	0.203	6.7%	N(1520)
7.40625	1521.612	**1522**	11	0.388	3.5%	N(1535)
7.43750	1528.033	**1528**	6	0.033	0.6%	N(1535)
7.46875	1534.453					
7.50000	1540.873					
7.53125	1547.294					
7(143/256)	1552.911	**1553**	8	0.089	1.1%	N(1535)
7.56250	1553.714					
7.59375	1560.134					
7.62500	1566.554					
7.65625	1572.975					
7.68750	1579.395					
7.71875	1585.815					
7.75000	1592.236					
7.78125	1598.656	**1599**		0.344		Δ(1600)
7.81250	1605.076					
7.84375	1611.497					
7.87500	1617.917					
7.90625	1624.337					
7.93750	1630.758					
7(245/256)	1634.770	**1634.7**	1.1	0.070	6.4%	N(1650)
7.96875	1637.178					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
8.00000	1643.598	**1643**	6/3	0.598	10.0%	Δ(1700)
8.03125	1650.018	**1650**		0.018		N(1650)
8.06250	1656.439					
8.09375	1662.859	**1663**	4	0.141	3.5%	N(1675)
8(29/256)	1666.872	**1667**	5	0.128	2.6%	N(1680)
8.12500	1669.279	**1669**	2	0.279	14.0%	N(1675)
8(38/256)	1674.095	**1674**	2/1	0.095	4.8%	N(1675)
8.15625	1675.700	**1676**	6	0.300	5.0%	N(1680)
8(43/256)	1678.107	**1678**	5	0.107	2.1%	N(1680)
8.18750	1682.120					
8.21875	1688.540	**1689**	12	0.460	3.8%	N(1650)
8.25000	1694.961	**1695.0**	1.3	0.039	3.0%	Δ(1700)
8.28125	1701.381					
8.31250	1707.801					
8.34375	1714.221	**1714**		0.221		Δ(1750)
8(95/256)	1719.839	**1720**	5	0.161	3.2%	N(1720)
8.37500	1720.642					
8.40625	1727.062					
8.43750	1733.482					
8.46875	1739.903					
8.50000	1746.323					
8(135/256)	1751.941	**1752**	3	0.059	2.0%	N(1710)
8.53125	1752.743					
8.56250	1759.164					
8.59375	1765.584					
8.62500	1772.004					
8.65625	1778.425					
8.68750	1784.845					
8.71875	1791.265					
8.75000	1797.685	**1797**	26	0.685	2.6%	N(1895)
8.78125	1804.106					
8(201/256)	1804.908	**1805**	10	0.092	0.9%	Δ(1905)
8.81250	1810.526					
8.84375	1816.946					
8.87500	1823.367	**1824**		0.633		N(1875)
8.90625	1829.787	**1830**	40	0.213	0.5%	Δ(1905)
8.93750	1836.207	**1836**		0.207		Δ(1930)
8.96875	1842.628					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
9.00000	1849.048	**1848**		1.048		Δ(1930)
9.03125	1855.468	**1856**		0.532		N(1900)
9(14/256)	1860.283	**1860**	35	0.283	0.8	N(1880)
9.06250	1861.888					
9.09375	1868.309					
9.12500	1874.729	**1875**	11	0.271	2.5%	N(1880)
9.15625	1881.149					
9.18750	1887.570	**1888**		0.430		Δ(1910)
9.21875	1893.990					
9.25000	1900.410	**1900**	8	0.410	5.1%	N(1900)
9(71/256)	1906.028	**1906**	17	0.028	0.2%	N(1895)
9.28125	1906.831	**1907**	10	0.169	1.7%	N(1895)
9.31250	1913.251	**1913**		0.251		N(1990)
9.34375	1919.671					
9(90/256)	1921.276	**1921.3**	0.2	0.024	12.0%	Δ(1950)
9.37500	1926.092					
9.40625	1932.512					
9(106/256)	1934.117	**1934**	10	0.117	1.2%	N(1875)
9.43750	1938.932					
9.46875	1945.352	**1945**	35	0.352	1.0%	N(1900)
9(127/256)	1950.970	**1951**	27	0.030	0.1%	N(1875)
9.50000	1951.773					
9.53125	1958.193					
9.56250	1964.613					
9(147/256)	1967.021	**1967**	20	0.021	0.1%	N(1880)
9.59375	1971.034					
9.62500	1977.454					
9.65625	1983.874					
9.68750	1990.295	**1990**	45	0.295	0.7%	N(1990)
9.71875	1996.715					
9.75000	2003.135	**2003**	18	0.135	0.8%	N(1875)
9.78125	2009.556	**2010**		0.444		N(2060)
9.81250	2015.976					
9.84375	2022.396					
9.87500	2028.816					
9.90625	2035.237					
9.93750	2041.657	**2042**		0.343		N(2190)
9.96875	2048.077	**2048**	65	0.077	0.1%	N(1875)
9(253/256)	2052.090	**2052**	6/3	0.090	1.5%	N(2100)

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
10.00000	2054.498					
10.03125	2060.918					
10.06250	2067.338					
10(17/256)	2068.141	**2068**	3	0.141	4.7%	N(2040)
10.09375	2073.759	**2074**		0.241		N(2190)
10(33/256)	2080.981	**2081**	20	0.019	0.1%	N(2120)
10.12500	2080.179					
10.15625	2086.599					
10.18750	2093.019					
10.21875	2099.440	**2100**	50	0.560	1.1%	N(2190)
10.25000	2105.860	**2107**		1.140		N(2190)
10.28125	2112.280					
10.31250	2118.701	**2119**	11/1	0.299	2.7%	N(2060)
10.34375	2125.121	**2125**	61	0.121	0.2%	N(2190)
10.37500	2131.541					
10.40625	2137.962					
10.43750	2144.382	**2144**	30	0.382	1.3%	N(2060)
10(119/256)	2150.000	**2150**	35	0.000	0.0%	N(2220)
10.46875	2150.802					
10(122/256)	2152.407	**2152.4**	1.4	0.007	0.5%	N(2190)
10.50000	2157.223	**2157**	3/14	0.223	7.4%	N(2250)
10.53125	2163.643					
10.56250	2170.063					
10.59375	2176.483	**2176**	40	0.483	1.2%	Δ(2200)
10.62500	2182.904					
10.65625	2189.324					
10(175/256)	2194.942	**2195**	45	0.058	0.1%	N(2250)
10.68750	2195.744					
10.71875	2202.165					
10.75000	2208.585					
10.78125	2215.005	**2215**	16	0.005	0.0%	Δ(2200)
10.81250	2221.426					
10(209/256)	2222.228	**2222**	15	0.228	1.5%	N(2190)
10.84375	2227.846	**2228**	30	0.154	0.5%	N(2060)
10.87500	2234.266					
10.90625	2240.686					
10.93750	2247.107					
10.96875	2253.527					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
11.00000	2259.947	**2260**	60	0.053	0.1%	Δ(2400)
11.03125	2266.368					
11.06250	2272.788					
11(25/256)	2280.011	**2280**	40	0.011	0.0%	N(2250)
11.09375	2279.208					
11.12500	2285.629					
11.15625	2292.049					
11.18750	2298.469					
11(52/256)	2301.679	**2302**	6	0.321	5.4%	N(2250)
11.21875	2304.890	**2305**	26	0.110	0.4%	Δ(2350)
11.25000	2311.310	**2311**	16	0.311	1.9%	N(1990)
11.28125	2317.730					
11.31250	2324.150					
11.34375	2330.571					
11.37500	2336.991					
11.40625	2343.411					
11.43750	2349.832	**2350**	100	0.168	0.2%	Δ(2390)
11.46875	2356.252					
11.50000	2362.672					
11.53125	2369.093					
11(137/256)	2369.895	**2370**	80	0.105	0.1%	Δ(2300)
11.56250	2375.513					
11.59375	2381.933					
11.62500	2388.354					
11.65625	2394.774					
11.68750	2401.194					
11.71875	2407.614					
11.75000	2414.035					
11.78125	2420.455					
11.81250	2426.875	**2427**		0.125		Δ(2350)
11.84375	2433.296					
11.87500	2439.716					
11.90625	2446.136					
11.93750	2452.557					
11.96875	2458.977					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
12.00000	2465.397					
12(3/256)	2467.804	**2468**	50	0.196	0.4%	Δ(2400)
12.03125	2471.817					
12.06250	2478.238					
12.09375	2484.658					
12.12500	2491.078					
12.15625	2497.499					
12.18750	2503.919					
12.21875	2510.339					
12.25000	2516.760					
12.28125	2523.180					
12.31250	2529.600	**2529**		0.600		Δ(2420)
12.34375	2536.021					
12.37500	2542.441					
12.40625	2548.861					
12.43750	2555.281					
12.46875	2561.702					
12.50000	2568.122					
12(130/256)	2569.727	**2570**	19/10	0.273	1.4%	N(2570)
12.53125	2574.542					
12(139/256)	2576.949	**2577**	50	0.051	0.1%	N(2600)
12.56250	2580.963					
12.59375	2587.383					
12.62500	2593.803					
12.65625	2600.224					
12.68750	2606.644					
12(183/256)	2612.261	**2612**	45	0.261	0.6%	N(2700)
12.71875	2613.064					
12.75000	2619.484					
12.78125	2625.905					
12.81250	2632.325					
12(209/256)	2633.127	**2633**	29	0.127	0.4%	Δ(2420)
12.84375	2638.745					
12.87500	2645.166					
12.90625	2651.586					
12.93750	2658.006					
12.96875	2664.427					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
13.00000	2670.847					
13.03125	2677.267					
13.06250	2683.688					
13.09375	2690.108					
13.12500	2696.528					
13.15625	2702.948					
13.18750	2709.369					
13.21875	2715.789					
13.25000	2722.209					
13.28125	2728.630					
13.31250	2735.050					
13.34375	2741.470					
13.37500	2747.891	**2750**		2.109		Δ(2750)
13.40625	2754.311					
13.43750	2760.731					
13.46875	2767.152					
13.50000	2773.572					
13.53125	2779.992					
13.56250	2786.412					
13.59375	2792.833					
13.60000	2794.116	**2794**	80	0.116	0.1%	Δ(2750)
13.62500	2799.253					
13.65625	2805.673					
13.68750	2812.094					
13.71875	2818.514					
13.75000	2824.934					
13.78125	2831.355					
13.81250	2837.775					
13.84375	2844.195					
13.87500	2850.615	**2850**	150	0.615	0.4%	Δ(2850)
13.90625	2857.036					
13.93750	2863.456					
13.96875	2869.876					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
14.00000	2876.297					
14.03125	2882.717					
14.06250	2889.137					
14.09375	2895.558					
14.12500	2901.978					
14.15625	2908.398					
14.18750	2914.819					
14.21875	2921.239					
14.25000	2927.659					
14.28125	2934.079					
14.31250	2940.500					
14.34375	2946.920					
14.37500	2953.340	**2950**		3.340		Δ(2950)
14.40625	2959.761					
14.43750	2966.181					
14.46875	2972.601					
14.50000	2979.022					
14.53125	2985.442					
14.56250	2991.862					
14.59375	2998.282					
14.62500	3004.703					
14.65625	3011.123					
14.68750	3017.543					
14.71875	3023.964					
14.75000	3030.384	**3030**		0.384		N(3030)
14.78125	3036.804					
14.81250	3043.225					
14.84375	3049.645					
14.87500	3056.065					
14.90625	3062.486					
14.93750	3068.906					
14.96875	3075.326					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
15.00000	3081.746					
15.03125	3088.167					
15.06250	3094.587					
15.09375	3101.007					
15.12500	3107.428					
15.15625	3113.848					
15.18750	3120.268					
15.21875	3126.689					
15.25000	3133.109					
15.28125	3139.529					
15.31250	3145.950					
15.34375	3152.370					
15.37500	3158.790					
15.40625	3165.210					
15.43750	3171.631					
15.46875	3178.051					
15.50000	3184.471					
15.53125	3190.892					
15.56250	3197.312					
15.59375	3203.732					
15.62500	3210.153					
15.65625	3216.573					
15.68750	3222.993					
15.71875	3229.413	**3230**		0.587		Δ(3230)
15.75000	3235.834					
15.78125	3242.254					
15.81250	3248.674					
15.84375	3255.095					
15.87500	3261.515					
15.90625	3267.935					
15.93750	3274.356					
15.96875	3280.776					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
16.00000	3287.196					
16.03125	3293.617					
16.06250	3300.037	**3300**	200	0.037	0.0%	Δ(3000)
16.09375	3306.457					
16.12500	3312.877					
16.15625	3319.298					
16.18750	3325.718					
16.21875	3332.138					
16.25000	3338.559					
16.28125	3344.979					
16.31250	3351.399					
16.34375	3357.820					
16.37500	3364.240					
16.40625	3370.660					
16.43750	3377.081					
16.46875	3383.501					
16.50000	3389.921					
16.53125	3396.341					
16.56250	3402.762					
16.59375	3409.182					
16.62500	3415.602					
16.65625	3422.023					
16.68750	3428.443					
16.71875	3434.863					
16.75000	3441.284					
16.78125	3447.704					
16.81250	3454.124					
16.84375	3460.544					
16.87500	3466.965					
16.90625	3473.385					
16.93750	3479.805					
16.96875	3486.226					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
17.00000	3492.646					
17.03125	3499.066	**3500**	200	0.934	0.5%	N(3000)
17.06250	3505.487					
17.09375	3511.907					
17.12500	3518.327					
17.15625	3524.748					
17.18750	3531.168					
17.21875	3537.588					
17.25000	3544.008					
17.28125	3550.429					
17.31250	3556.849					
17.34375	3563.269					
17.37500	3569.690					
17.40625	3576.110					
17.43750	3582.530					
17.46875	3588.951					
17.50000	3595.371					
17.53125	3601.791					
17.56250	3608.211					
17.59375	3614.632					
17.62500	3621.052					
17.65625	3627.472					
17.68750	3633.893					
17.71875	3640.313					
17.75000	3646.733					
17.78125	3653.154					
17.81250	3659.574					
17.84375	3665.994					
17.87500	3672.415					
17.90625	3678.835					
17.93750	3685.255					
17.96875	3691.675					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
18.00000	3698.096	**3700**		1.904		Δ(3000)
18.03125	3704.516					
18.06250	3710.936					
18.09375	3717.357					
18.12500	3723.777					
18.15625	3730.197					
18.18750	3736.618					
18.21875	3743.038					
18.25000	3749.458					
18.28125	3755.879	**3755**		0.879		N(3755)
18.31250	3762.299					
18.34375	3768.719					
18.37500	3775.139					
18.40625	3781.560					
18.43750	3787.980					
18.46875	3794.400					
18.50000	3800.821	**3800**	200	0.821	0.4%	N(3000)
18.53125	3807.241					
18.56250	3813.661					
18.59375	3820.082					
18.62500	3826.502					
18.65625	3832.922					
18.68750	3839.342					
18.71875	3845.763					
18.75000	3852.183					
18.78125	3858.603					
18.81250	3865.024					
18.84375	3871.444					
18.87500	3877.864					
18.90625	3884.285					
18.93750	3890.705					
18.96875	3897.125					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

Mass Spectrum of the N and Δ Baryons

Resolution = (**1/32**)S6h = 6.420305137 MeV/c^2

n	n S6h	ExpMass	Error	dm	dm/Error	Baryon
19.00000	3903.546					
19.03125	3909.966					
19.06250	3916.386					
19.09375	3922.806					
19.12500	3929.227					
19.15625	3935.647					
19.18750	3942.067					
19.21875	3948.488					
19.25000	3954.908					
19.28125	3961.328					
19.31250	3967.749					
19.34375	3974.169					
19.37500	3980.589					
19.40625	3987.009					
19.43750	3993.430					
19.46875	3999.850					
19.50000	4006.270					
19.53125	4012.691					
19.56250	4019.111					
19.59375	4025.531					
19.62500	4031.952					
19.65625	4038.372					
19.68750	4044.792					
19.71875	4051.213					
19.75000	4057.633					
19.78125	4064.053					
19.81250	4070.473					
19.84375	4076.894					
19.87500	4083.314					
19.90625	4089.734					
19.93750	4096.155					
19.96875	4102.575	**4100**	200	2.575	1.3%	N(3000)
20.00000	4108.995					

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

6.1 Commentary on the N and Δ Baryons Mass Spectrum from 1.2 to 4.1 Gev

As stated peviously, the correct resolution size of a mass spectrum depends on the average error size of the data being displayed. The average error size of Δ(1232)'s experimental mass data is less than one. The error size for the rest of the experimental mass data for the N and Δ baryons is much greater. It probably averages about +/-15 MeV/c^2. The good news is that that is probably not the true error size, meaning that it probably does not reflect the true accuracy of the experimentalists measurements. The true error size is probably 1/20th of +/-15, or about +/-0.75. How do I know this? Because experimentalists routinely give error sizes of 10, 15, or higher to measurements of masses that they measure to be only 1.0 MeV/c^2 apart. How could they distinguish between two masses that are only 1.0 MeV/c^2 apart if their experimental resolution was only +/-15 MeV/c^2? Another way to tell that experimenters mesurements are better than stated is because many experimental masses fall very close to their probable theoretical values. Most experimental masses match their probable theoretical values to within 5% of their Error size (that's what dm/Error calculates). That means most error sizes are too big, or, too conservative, by about 20 times. Some are too big by much more. A few are about right, but the vast majority are too big.

So, since the *true* average errror size is probably about +/-0.75 MeV/c^2, the best resolution size for the N and Δ mass spectrum is (**1/256**)**S6h** = 0.802538142 MeV/c^2. The range of the mass spectrum has to be from 1.2 to 4.1 GeV, or, from **6 S6h** to **20 S6h,** to cover all N and Δ data**.** The number of lines needed to cover that range, at that resolution would be 14(256) = 3584 lines. At 50 lines per page, that spectrum would be 72 pages long. A mass spectrum of that length would be sparsely populated, since there are only about 400 data points that could be plotted. A mass spectrum with a resolution of (**1/32**)**S6h** = 6.420305137 MeV/c^2, covering the same range would be only 14 pages long, but because of the coarser resolution, it would match far fewer experimental masses. As a compromise, the mass spectrum decided upon for this paper is the (1/32)S6h resolution 14 page one, but with theoretical mass additions at a resolution of **(1/256)S6h** where needed to match experimental masses that do not match any values at the (**1/32**)**S6h** level of resolution. To easily differentiate between the two levels of resolution, each **n** next to a theoretical mass entry that is a multiple of (**1/32**)**S6h** will be shown with a decimal expansion. Each **n** next to a theoretical mass entry that is a multiple of (**1/256**)**S6h** will be shown as an integer plus a fraction with a denominator of **256**.

7. Summary

The good agreement between the values of experimental and theoretical N and Δ baryon masses illustrated in the mass spectrums presented in this paper, shows that N and Δ baryon masses are simple multiples of S6h. Specifically, they are integer multiples of power of two denominator fractions times S6h.

$$\text{N and } \Delta \text{ Mass} = \frac{n}{2^{\mathbf{x}}} \text{ S6h}$$

Where n and x are integers, S6 = $\boldsymbol{\pi}^3$, and h = 6.62607015 MeV/c^2.

8. References

[1] P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)