The problem of shuffle in mahjong:Identification with the disordered system

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Abstract

I studied mechanism of shuffle in mahjong, which is a kind of table game. This research is the first research that focuses on the physical aspect, not the victory or defeat of mahjong. In physics terms, it is a study of how disordered systems can be created. I considered whether there is a difference between a completely random shuffle with random numbers and a shuffle with human hands. I implemented human shuffle with a new mathematical model. It was found that the equilibrium reached by human hand shuffle is different from that of completely random shuffle in the number of shuffles that can be normally assumed. This means that creating a random state can be difficult in some cases.

1 Introduction

Mahjong is a kind of table game, which is non-perfect information game. In mahjong, there are 4 tiles with 3 types of 1 to 9 numbers, and 4 tiles with the remaining 7 types. If there are three consecutive numbers of the same type of tiles, or if there are three tiles of the same type and numbers, a face is formed, and the points are given when they are aligned. In mahjong, it is important to arrange the tiles as soon as possible according to the rules. It is not just a matter of aligning them, but the ones that are stochastically difficult to align will get higher scores, and four people will compete for this.

In mahjong, before starting the game, arrange the tiles as randomly as possible before starting. Various methods such as human hands, machines, and simulations are used to create as random a state as possible. However, the differences between how they are mixed, that is, how they are shuffled, have not been studied in detail so far. It is known that shuffling in playing cards, when mixed about 7 times, shifts to a rapidly mixed state.[1]

In Ch.2, mahjong is equated with a generalization of the Ising model, and its energy is equated with the degree of mixing. This replaces the problem of table games with the problem of physics. In Ch.3, I quantitatively evaluate the mixed state produced by a completely random shuffle. In Ch.4,The operation of shuffling the tiles by hand is

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mathematically modeled, the mixing condition at that time is quantitatively evaluated, and the case of random shuffling is compared. In Ch.5, I conclude.

2 Mahjong tile pile and generalization of Ising model

There are four types of mahjong tiles, but if you focus on the score, they are almost unrelated to each other. The important thing is when the tiles of the same type have same or close numbers. In mahjong tiles, four piles of tiles are shuffled before the game starts. Therefore, it is sufficient to investigate the mixing condition of the tiles in these four tile piles. One pile of tiles contains 34 tiles, so consider the case where they are lined up in a row. At this time, when the tiles of the same type are between two adjacent tiles, the mixing degree is calculated based on the difference between the written numbers. This is equivalent to calculating the energy of the Ising model. The local binding energy is defined as $9 - d$, where d is the difference between adjacent numbers for tiles of same type. The local binding energy between tiles of different type is defined as 0. The sum of the local binding energies of 33 tiles is taken as the energy of the pile of the tiles, and the energy of the four piles is calculated.

The lower the energy, the better the degree of mixture.

3 A completely random shuffle mix

Using Python's random module, we randomly generated four tile piles 3000 times and calculated the average value of their energies.

When the average energy was calculated in order from the mountain with the lowest energy, it was 33.4,44.0,53.9,66.9. The standard deviation is about 10. In Fig.1, probability distribution of tile pile energy when mixed completely randomly is plotted.

The average value of the energy of the four tile mountains was 49.5. The standard deviation is about 18. In Fig.1, probability distribution of average tile pile energy when mixed completely randomly is plotted.

4 Modeling and mixing of shuffle by human hands

The initial state is that the first pile of tiles is arranged in sequence, one for each type. Consider shuffling from this state. For each pile of tiles, remove x tiles from the 17th of the 34 tiles almost evenly to the left and right, and shuffle them randomly. Then, slide the tiles left on each pile and add the mixed tiles to the beginning.

We investigated the change in energy when shuffling 30 and 3000 times with 100 trials. The figure below shows the results when the number of tiles removed from each tile pile in one shuffle is $cut = x$.

In Fig.3,4, I plotted the mixing condition when mixing at 1/3 of the tile pile, that is, x = 11. The energy for random shuffle is also plotted at the same time. The energy value asymptotics to 114. In this case, it have not yet asymptotically approached the energy of a random shuffle. This value is not within the 3σ range of the probability distribution in the random case.

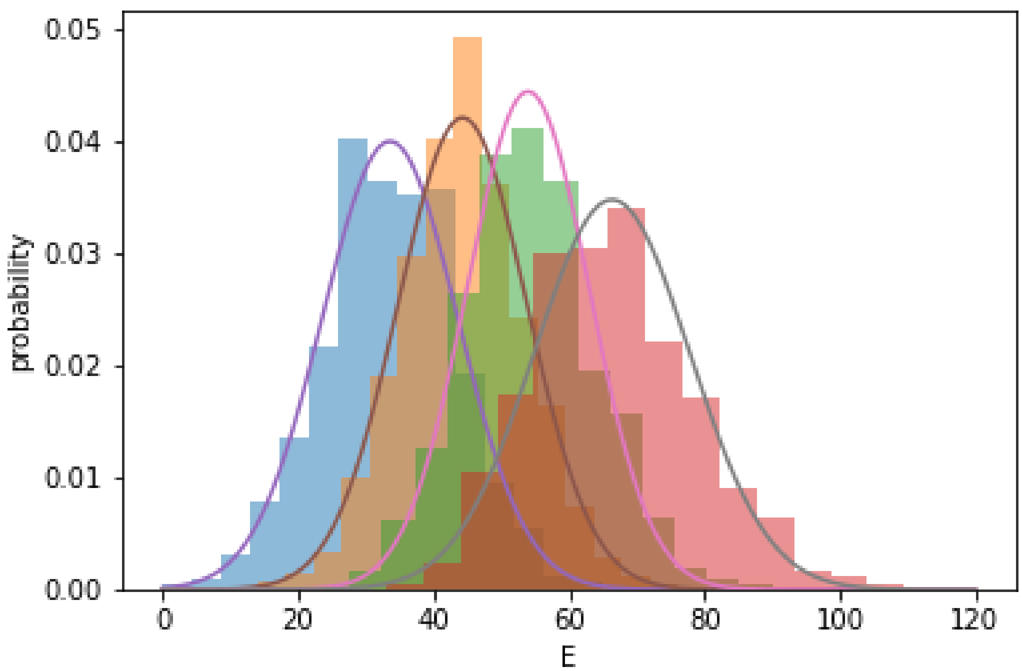

Figure 1: Probability distribution of each tile pile energy when mixed completely randomly

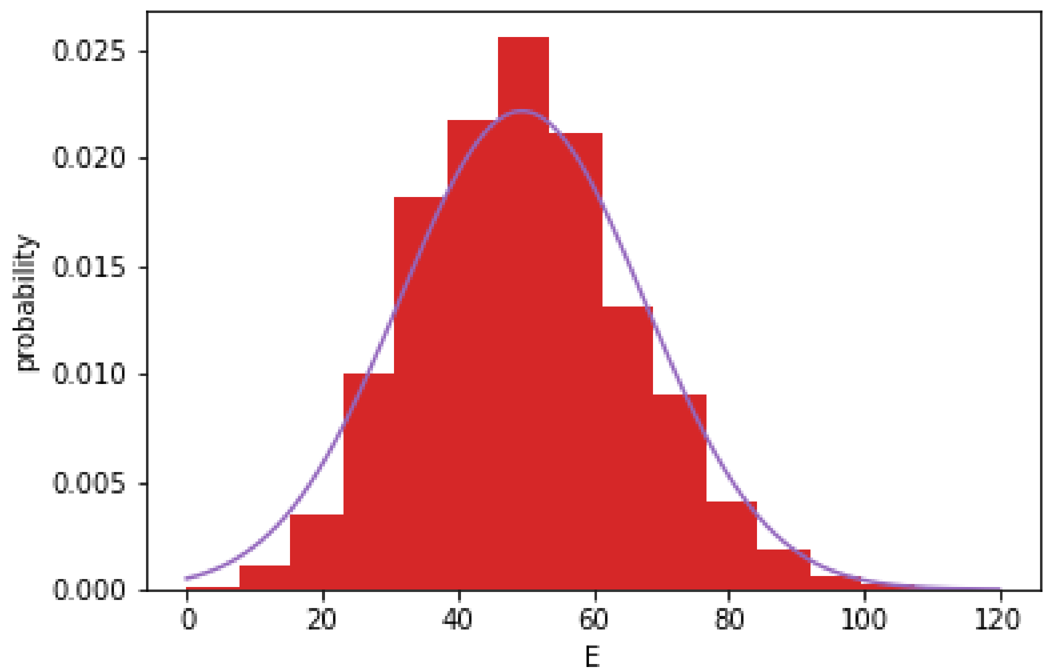

Figure 2: Probability distribution of average tile pile energy when mixed completely randomly

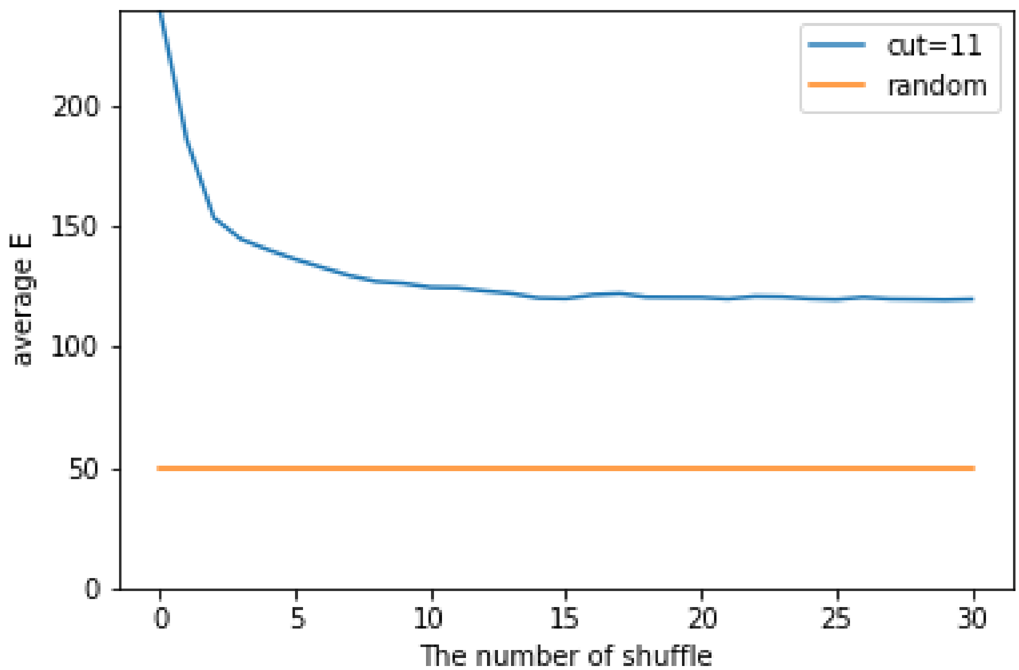

Figure 3: Equilibrium state by shuffle model when cut = 11

It can be seen that the asymptotic state is reached by mixing about 10 times.

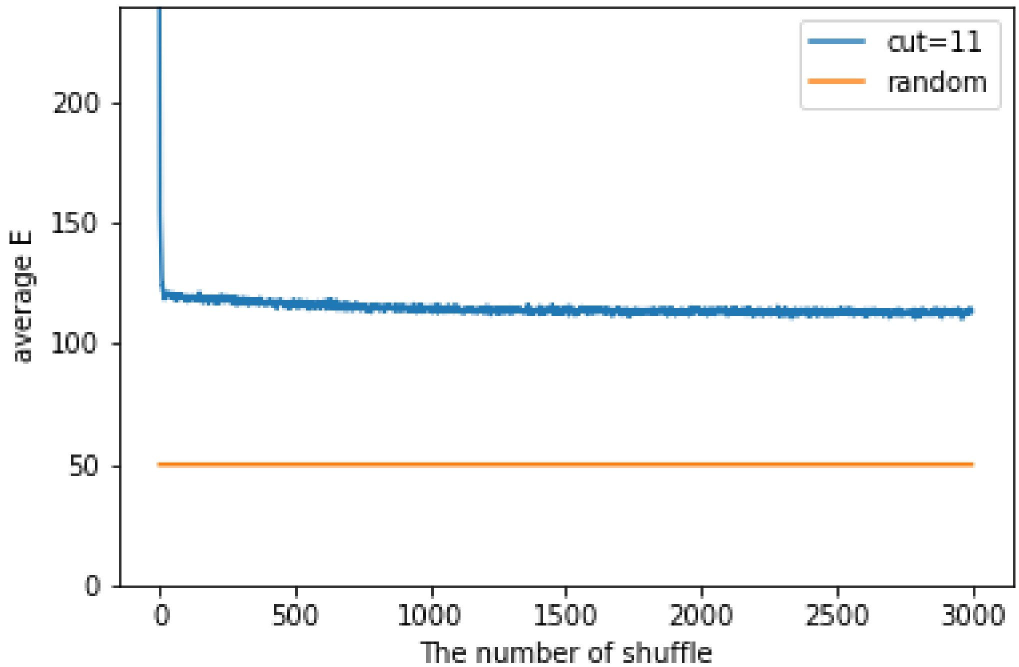

Figure 4: Equilibrium state by shuffle model when cut = 11

In Fig.5,6, I plotted the mixing condition when mixing at 60% of the tile pile, that is, x = 20. The energy for random shuffle is also plotted at the same time. The energy value asymptotics to 89. In this case, it has not yet asymptotically approached the energy of a random shuffle. This value is not within the 2σ range of the probability distribution in the random case.

It can be seen that the asymptotic state is reached after mixing a few times.

In Fig.7,8, I plotted the mixing condition when mixing at 89% of the tile pile, that is, x = 30. The energy for random shuffle is also plotted at the same time. The energy value asymptotics to 49.

It can be seen that the asymptotic state is reached after mixing a few times.

In this case, it asymptotics to the energy of a random shuffle.

5 Conclusion

The result of numerical simulation suggests that it was found that when mixing by normal human hands, that is, by stirring 1/3 of the whole, the mixing condition in the

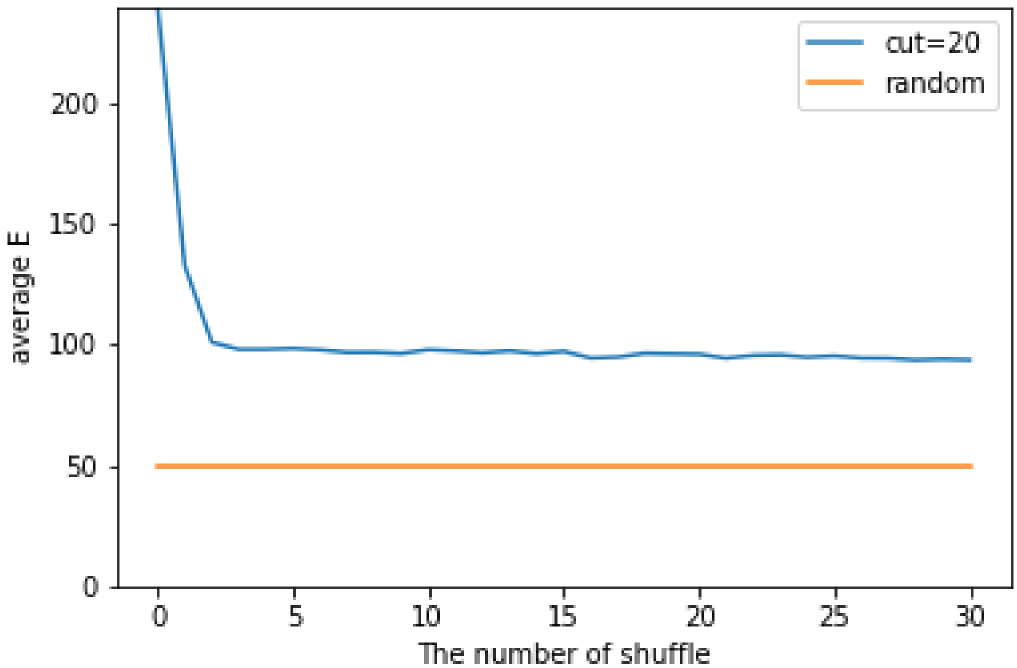

Figure 5: Equilibrium state by shuffle model when cut = 20

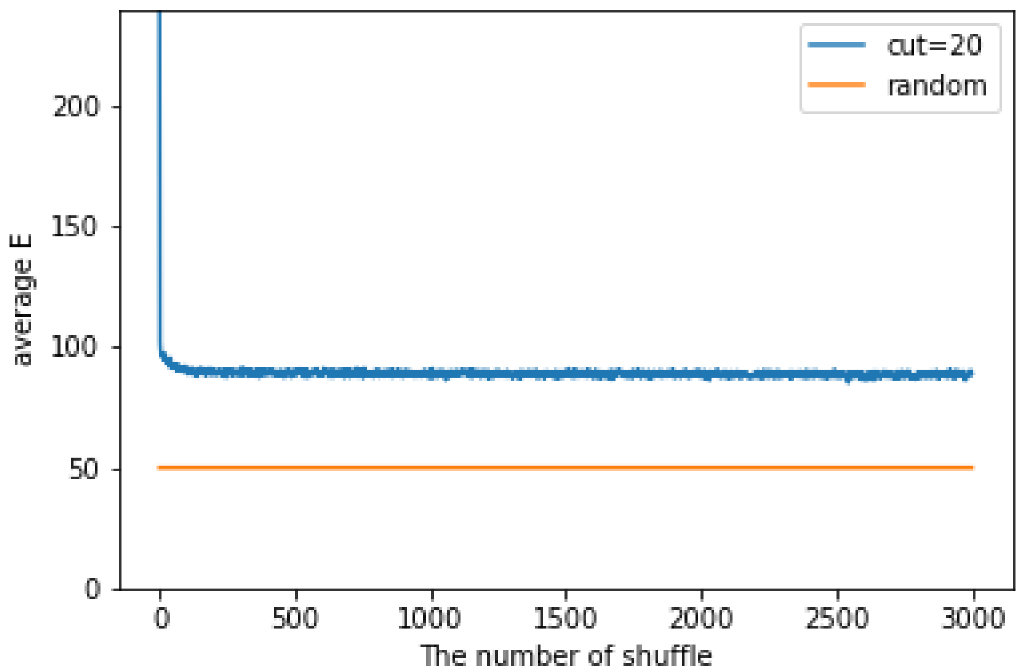

Figure 6: Equilibrium state by shuffle model when cut = 20

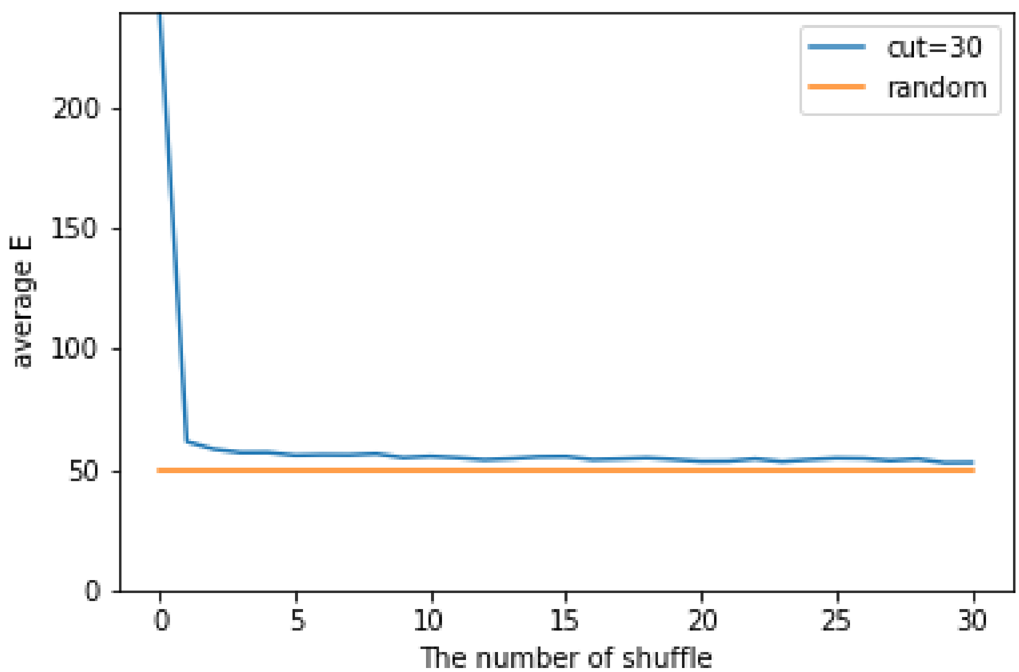

Figure 7: Equilibrium state by shuffle model when cut = 30

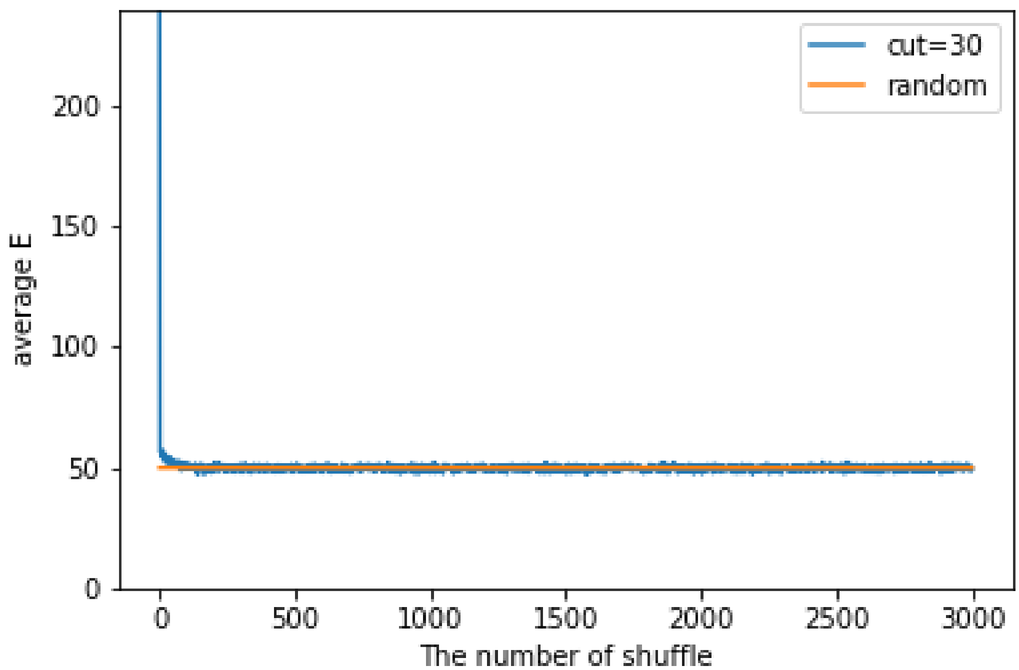

Figure 8: Equilibrium state by shuffle model when cut = 30

random case cannot be reached even if it is sufficiently mixed. This result means that the ease of matching mahjong tiles depends on how they are mixed. It was found that the equilibrium state reached by shuffling differs between human hands, machines, and random simulations.

Acknowledgments

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References

[1] D. Bayer and P. Diaconis, Trailing the dovetail shuffle to its lair, Ann. Appl. Probab. 2 (1992), 294-313.