Plane wave spin work

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A simple calculation shows that when an electromagnetic wave of circular polarization is absorbed, the rotation of the absorber changes the flow of electromagnetic energy to the absorber by the amount of mechanical work performed by the spin of this wave. The result obtained shows the need to replace the currently used definition of the classical spin of electromagnetic radiation.

The idea of Sadovsky [1] and Poynting [2] is that if electromagnetic radiation is circularly polarized, then, in addition to the energy and momentum flux density, it also contains the angular momentum flux density, which is now recognized as the spin flux density. But this means that when absorbing such radiation, the absorber experiences, in addition to heating and pressure, also the density of the distributed torque $\tau_\wedge$ [N∗m/m^2]. And if the absorber rotates at the same time, then the radiation produces mechanical work. From this, the heating during rotation decreases. We show below that the energy balance is maintained in this case.

Indeed, if the absorber rotates, for certainty, in the same direction as the electromagnetic vectors, then the radiation frequency observed by the absorber decreases by the angular velocity of rotation of the absorber Ω. It is done $\omega' = \omega - \Omega$, instead of ω. (Note that the frequency of a linear polarized radiation does not change) This decreases the observed energy of the absorbed radiation quanta, while the number of the quanta n is obviously conserved. This leads to a decrease in the heating of the absorber. The electromagnetic energy flux density observed by the absorber is made

$$I' = n\hbar\omega' = n\hbar\omega - n\hbar\Omega = I - I\Omega/\omega$$

instead of I.

At the same time, the density of torque is equal to $\tau_\wedge = I/\omega$, and the density of mechanical power produced by it is just $\tau_\wedge \Omega = I\Omega/\omega$

The presented simple calculation is important because it is not possible within the framework of the modern concept of electrodynamics spin. Here are some details.

The idea of Sadovsky and Poiting is simply expressed. There is an angular momentum of the electromagnetic field, independent of the linear momentum: "If we put *E* for the energy in unit volume and *G* for the torque per unit area, we have $G = E\lambda/2\pi$" [2, p. 565]. This statement proclaims the existence of a *density* of angular momentum that is not related to linear momentum.

Keeping in mind this angular momentum independent of the linear momentum, **Weyssenhoff** introduced the concept of a spin liquid [3]: "By spin-fluid we mean a fluid each element of which possesses besides energy and linear momentum also a certain amount of angular momentum, proportional – just as energy and the linear momentum – to the volume of the element". Thus, according to Poynting, circularly polarized electromagnetic radiation is a spin liquid.

Within the framework of the Lagrangian formalism, electrodynamics spin is described by the canonical spin tensor [4-6]

$$\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_{\alpha}\frac{\partial \mathsf{L}}{\partial(\partial_\nu A_\alpha)} = -A^\lambda F^{\mu\nu} + A^\mu F^{\lambda\nu} = -2A^{[\lambda}F^{\mu]\nu},$$

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where A^{λ} and $F_{\mu\nu}$ are the magnetic vector potential and the field-strength tensor of the electromagnetic field, respectively. The spin tensor is now successfully used to calculate the spin angular momentum of light [7-17].

Contrary to Sadovsky and Poiting, according to the modern definition, the total angular momentum of electromagnetic radiation is equal to a moment of a linear momentum of the electromagnetic field $\varepsilon_0\mathbf{E}\times\mathbf{B}$ [18-22]

$$\mathbf{J}=\varepsilon_0\int\mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV \ .$$

It is proclaimed that the spin $\mathbf{S}$ of electromagnetic radiation is a part of this angular momentum

$$\mathbf{J}=\varepsilon_0\int\mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV=\mathbf{L}+\mathbf{S} \ .$$

So a plane wave has no angular momentum at all.

Heitler W: "A plane wave travelling in z-direction and with infinite extension in the xy-directions can have no angular momentum about the z-axis, because $(\mathbf{E}\times\mathbf{B})$ is in the z-direction and $(\mathbf{r}\times(\mathbf{E}\times\mathbf{B}))_z=0$" [22].

The result obtained in this article shows the need to replace this currently used definition of the classical spin of electromagnetic radiation.

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