Transfer of the spin of an electromagnetic wave to an ideal conductor

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It is indicated that when a circularly polarized electromagnetic wave is incident obliquely on an ideal conductor and reflected from it, the spin of the wave is partially transferred to the conductor. In this case, if the conductor rotates so that the axis of rotation is parallel to the transferred spin, then work is done. The work leads to a change in the frequency of the wave, and this can be recorded in a suitable interference experiment by shifting the interference fringes.

Key words: classic spin; circular polarization; electrodynamics
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1. Introduction

A circularly polarized electromagnetic wave carries angular momentum in the form of the angular momentum density [1,2]. Poynting [2] proposed the relationship $G = E\lambda / 2\pi$, in which E is the energy per unit volume and G represents the torque per unit area. This means that such a wave is a Weyssenhoff's spin-fluid [3], defined as "a fluid each element of which possesses besides energy and linear momentum also a certain amount of angular momentum, proportional – just as energy and the linear momentum – to the volume of the element." This is recorded in textbooks [4,5]. Since Emma Noether, this angular momentum has been described by the canonical spin tensor density [6-8]:

$$\underset{c}{\Upsilon}^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_{\alpha}\frac{\partial \mathsf{L}}{\partial(\partial_\nu A_\alpha)} = -2A^{[\lambda}F^{\mu]\nu}\,. \tag{1}$$

where $\mathsf{L} = -F_{\mu\nu}F^{\mu\nu}/4$ is the free electromagnetic field Lagrangian, A^λ is the vector potential, and $F_{\mu\nu}$ is the field-strength tensor. The spin tensor is used in the publications [9-22]. Nowadays an electromagnetic wave of circular polarization is considered as a stream of photons, the spin of which is certainly directed parallel to their momentum.

2. Experiment

It seems that the spin of wave can be detected by changing its direction by wave reflection (Fig. 1). When reflected from an ideal conductor, according to electrodynamics, the mutual orientation of the spin and momentum, that is, the helicity of the wave or photon, always changes to the opposite helicity. (The helicity of a particle is positive ("right-handed") if the direction of its spin is the same as the direction of its motion. It is negative ("left-handed") if the directions of spin and motion are opposite. So a standard clock, with its spin vector defined by the rotation of its hands, has left-handed helicity if tossed with its face directed forwards. But note tip of vector **E** pictures left spiral if the helicity is right-handed, see Fig. 1).

As can be seen from Fig. 2, the reflector receives the difference of the tangent components of the spin, $\Delta\mathbf{S} = \mathbf{S}_1 - \mathbf{S}_2$, $\Delta S = 2\hbar\sin\alpha$, where α is the angle of incidence, and, therefore, the reflector experiences a torque, τ, proportional to the flux of the incident spin and, accordingly, proportional to the power incident on the reflector, $P\cos\alpha$,

$$\tau = \frac{2P\cos\alpha\sin\alpha}{\omega} \tag{2}$$

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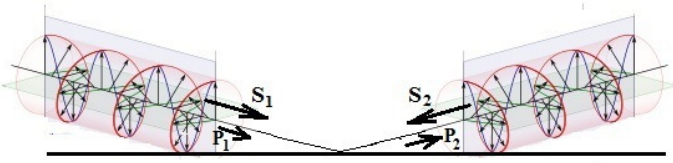

Fig. 1

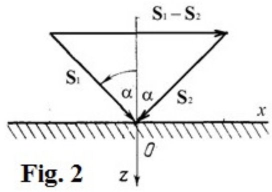

Fig. 2

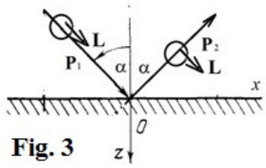

Fig. 3

Fig. 1. Reflection of a photon from a reflector.
Fig. 2. Spin transferred to the reflector
Fig. 3. Conservation of angular momentum of a ball on rebound

Significantly, a photon is different from a tennis ball. The direction of the angular momentum $\mathbf{L}$ of a rotating ball does not change when it bounces off a mirror. This is shown in Fig. 3.

If we replace the fixed reflector with a rotating cylinder so that the angular velocity of its rotation $\boldsymbol{\Omega}$ is parallel to the difference $\Delta\mathbf{S}$, i.e. to the torque τ (Fig. 4), then work will be done on the reflector. We denote the power of this work as

$$\Delta P = \tau\Omega\,. \tag{3}$$

Because of this work, the photon energy and wave frequency change, so that

$$\frac{\Delta\omega}{\omega} = \frac{\Delta P}{P\cos\alpha} = \frac{2\Omega\sin\alpha}{\omega} \tag{4}$$

The frequency shift of the reflected wave can be fixed in Lloyd's experiment (Fig. 4). If $\alpha \cong 90^o$, the frequency shift is $\Delta\omega \cong 2\Omega$. This causes a shift in the interference pattern by two fringes per cylinder revolution.

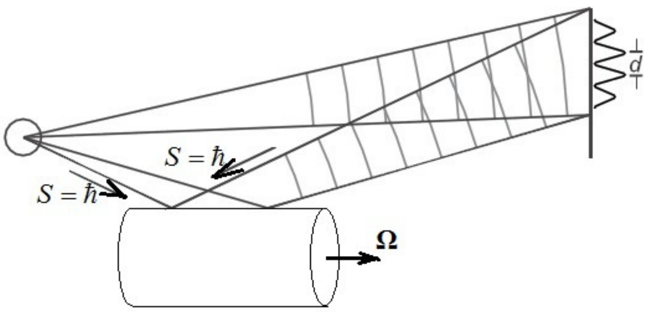

Fig. 4. Modified Lloyd's experiment

It should be noted that two waves with opposite helicite do not create constructive / destructive interference. They create linear polarization, the direction of the polarization plane of which depends on the phase difference of these waves. Therefore, Lloyd's interference fringes in this case can only be seen through a polarizer.

3. The electromagnetic waves

It seems important to make sure that the helicity of the wave actually changes to the opposite helicity when reflected from an ideal conductor at any angle of incidence.

To write the expression for a wave incident at an angle α, we use the expression for a right-hand circularly polarized electromagnetic wave incident normally on the xy-surface in the coordinates x', y', z':

$$E_1^{x'} = \cos(z'-t), \quad E_1^{y'} = -\sin(z'-t), \quad B_1^{x'} = \sin(z'-t), \quad B_1^{y'} = \cos(z'-t)\,. \tag{5}$$

For simplicity we put $\omega = k = c = \varepsilon_0 = \mu_0 = 1$ Power per unit normal area, i.e. the Poynting vector is $\frac{P}{a} = I = E^x B^y - E^y B^x = 1$.

Then the coordinate transformations

$$x' = x\cos\alpha - z\sin\alpha, \quad z' = x\sin\alpha + z\cos\alpha, \quad y' = y \tag{6}$$

gives the expressions

$$E_1^x = \cos\alpha\cos(x\sin\alpha + z\cos\alpha - t), \quad B_1^x = \cos\alpha\sin(x\sin\alpha + z\cos\alpha - t), \tag{7}$$

$$E_1^y = -\sin(x\sin\alpha + z\cos\alpha - t), \quad B_1^y = \cos(x\sin\alpha + z\cos\alpha - t), \tag{8}$$

$$E_1^z = -\sin\alpha\cos(x\sin\alpha + z\cos\alpha - t), \quad B_1^z = -\sin\alpha\sin(x\sin\alpha + z\cos\alpha - t). \tag{9}$$

for the right-hand circularly polarized wave incident at an angle α.

To write the expression for a wave reflected at an angle α, we use the expression for a left-hand circularly polarized electromagnetic wave originating along the normal from the xy-surface in the coordinates x', y', z':

$$E_2^{x'} = -\cos(z'+t), \quad E_2^{y'} = -\sin(z'+t), \quad B_2^{x'} = -\sin(z'+t), \quad B_2^{y'} = \cos(z'+t). \tag{10}$$

Then the coordinate transformations

$$x' = x\cos\alpha + z\sin\alpha, \quad z' = -x\sin\alpha + z\cos\alpha, \quad y' = y \tag{11}$$

gives the expressions

$$E_2^x = -\cos\alpha\cos(-x\sin\alpha + z\cos\alpha + t), \quad B_2^x = -\cos\alpha\sin(-x\sin\alpha + z\cos\alpha + t), \tag{12}$$

$$E_2^y = -\sin(-x\sin\alpha + z\cos\alpha + t), \quad B_2^y = \cos(-x\sin\alpha + z\cos\alpha + t), \tag{13}$$

$$E_2^z = -\sin\alpha\cos(-x\sin\alpha + z\cos\alpha + t), \quad B_2^z = -\sin\alpha\sin(-x\sin\alpha + z\cos\alpha + t). \tag{14}$$

for the wave reflected at an angle α.

One can easily see that the boundary conditions are fulfilled on the surface of the ideal conductor

$$\left[E_1^x + E_2^x\right]_{z=0} = \left[E_1^y + E_2^y\right]_{z=0} = \left[B_1^z + B_2^z\right]_{z=0} = 0. \tag{15}$$

4. Spin tensor

In this article, we consider a classical concept of electrodynamics spin. So we must obtain the result (2) for the spin flux in the frame work of classical electrodynamics, that is, by the use of spin tensor. But a problem arises here. The canonical spin tensor (1), which is used in the publications [9-22], is generally not correct.

The local sense of a spin tensor $\Upsilon^{\lambda\mu\nu}$ is as follows. The spin of the 4-volume element dV_ν is $dS^{\lambda\mu} = \Upsilon^{\lambda\mu\nu} dV_\nu$. This means, for example, that the component $dS^{xy} = dS_z$ of the spin, which is passed through the area da_z in time dt, is equal to $dS^{xy} = \Upsilon^{xyz} da_z dt$, i.e. Υ^{xyz} is the spin flux density in z-direction.

The canonical spin tensor (1) correctly describes the spin flux in the direction that coincides with the wave propagation direction. Really, the vector potential for the field (5) (without strokes) is

$$A^x = -\int E^x dt = \sin(z-t), \quad A^y = -\int E^y dt = \cos(z-t), \tag{16}$$

and the canonical spin flux density,

$$\underset{c}{\Upsilon}^{xyz} = -A^x F^{yz} + A^y F^{xz} = A^x B^x + A^y B^y = 1, \tag{17}$$

corresponds to the energy flux density, $I = 1$, just as spin $\hbar$ corresponds to energy $\hbar\omega$ $(\omega = 1)$. Therefore, the canonical spin tensor has been successfully used in [9-22]. But this tensor gives incorrect result for directions perpendicular to the direction of wave propagation. Indeed, the canonical tensor assigns a strange spin S^{zx} propagating along the direction of the *y*-axis in wave (5):

$$\underset{c}{\Upsilon}^{zxy} = -A^z F^{xy} + A^x F^{zy} = A^x B^x = \sin^2(z-t) \tag{18}$$

To fix this problem, the canonical tensor has been modified [18,19]:

$$\Upsilon^{\lambda\mu\nu} = A^\lambda \partial^\nu A^\mu - A^\mu \partial^\nu A^\lambda \tag{19}$$

It is easy to check that $\Upsilon^{xyz}=1$, $\Upsilon^{zxy}=0$. It is only necessary to take into account that, due to the signature of the metric $(+---)$, $\partial^i=-\partial_i$. We use this modified spin tensor (19) in this article.

5. Spin flux density transferred to the conductor

In accordance with Fig. 2, the S^{yz} component of the spin is transferred to the conductor. The flux density of this spin component on the conductor is given by the component

$$\Upsilon^{yzz}=A^y\partial^z A^z - A^z\partial^z A^y \qquad (20)$$

of the spin tensor, and, in the absence of interference, it is possible to calculate this component only for the incident wave and to double it. From the formula $\mathbf{A}=-\int \mathbf{E}dt$ we obtain the magnetic vector potential in the incident wave:

$$A_1^y=\cos(x\sin\alpha+z\cos\alpha-t),\quad A_1^z=-\sin\alpha\sin(x\sin\alpha+z\cos\alpha-t). \qquad (21)$$

Thus the spin flux density on the conductor is

$$\Upsilon^{yzz}=2(A_1^y\partial^z A_1^z - A_1^z\partial^z A_1^y)=\sin(2\alpha). \qquad (22)$$

It corresponds to the spin flux (2) (ω = 1).

6. Mechanism of spin absorption

Our conductor is the boundary of the wave momentum flow and, therefore, it is under pressure. The Lorentz force produces the pressure: the magnetic field of the wave interacts with the current induced in the conductor, $\mathbf{f}=\mathbf{j}\times\mathbf{B}$

Similarly, our conductor experiences a distributed torque, since it is the boundary of the flow of spin angular momentum. To obtain an expression similar to the Lorentz force, it is necessary to calculate the divergence of the spin tensor (19):

$$\partial_\nu\Upsilon^{\lambda\mu\nu}=\partial_\nu A^\lambda\partial^\nu A^\mu-\partial_\nu A^\mu\partial^\nu A^\lambda+A^\lambda\partial_\nu^\nu A^\mu-A^\mu\partial_\nu^\nu A^\lambda. \qquad (23)$$

The first pair of terms gives zero due to antisymmetry

$$\partial_\nu A^\lambda\partial^\nu A^\mu-\partial_\nu A^\mu\partial^\nu A^\lambda=2g^{\nu\sigma}\partial_\nu A^{[\mu}\partial_\sigma A^{\lambda]}=0, \qquad (24)$$

and there is a current in the second pair of terms [23 (12.123)]: $\partial_\nu^\nu A^\mu=j^\mu$

So it turns out

$$-\partial_\nu\Upsilon^{\lambda\mu\nu}=\tau_\wedge^{\lambda\mu}=2j^{[\lambda}A^{\mu]}\ \text{ or }\ \tau_\wedge=\mathbf{j}\times\mathbf{A}, \qquad (25)$$

$\tau_\wedge$ stands for torque density.

In our case, the surface current in the conductor is the boundary of the magnetic field of the wave. So:

$$j^x=B^y,\quad j^y=-B^x,\quad j^z=0. \qquad (26)$$

Interestingly, when calculating the torque density (25), it is not sufficient to double the impact of the incident wave. The total current interacts with the vector potential fields of both waves. For z = 0 it turns out

$$\begin{aligned}\tau_\wedge^{yz}&=j^yA^z=(-B_1^x-B_2^x)(A_1^z+A_2^z)\\&=-\cos\alpha\sin(x\sin\alpha-t)[-\sin\alpha\sin(x\sin\alpha-t)]-\cos\alpha\sin(x\sin\alpha-t)\sin\alpha\sin(-x\sin\alpha+t)\\&+\cos\alpha\sin(-x\sin\alpha+t)[-\sin\alpha\sin(x\sin\alpha-t)]+\cos\alpha\sin(-x\sin\alpha+t)\sin\alpha\sin(-x\sin\alpha+t)\\&=4\cos\alpha\sin\alpha\sin^2(x\sin\alpha-t);\quad <\tau_\wedge>=\sin(2\alpha)\end{aligned} \qquad (27)$$

7. Real reflectors

When a circularly polarized wave is normally incident on a reflector, the helicity of the reflected wave is opposite to that of the incident wave, regardless of whether the dielectric or conductor serves as a reflector. A difference arises as the angle of incidence increases. At the Brewster angle of the dielectric, the electric field in the plane of incidence is not reflected at all. Therefore, the reflected wave is linearly polarized. With a further increase in the angle of incidence, the circular (elliptical) polarization of the reflected wave is restored, but now its helicity coincides with the helicity of the incident wave. Therefore, interference fringes are consistently observed in Lloyd's usual experiment.

When a circularly polarized wave is reflected from an ideal conductor, the reflection coefficient of any component is equal to unity and the helicity of the reflected wave is opposite to that of the incident wave at any angle of incidence. No Brewster angle exists.

The high conductivity of metals at low frequencies is significantly reduced at optical frequencies, since the oscillation period in the wave can be equal to the electron travel time between collisions. Significantly, the complex reflection coefficient changes the phase relationship of the reflected wave components, so that the helicity of the reflected wave can coincide with the helicity of the incident wave without the existence of the Brewster angle. However, Wood discovered the Brewster angle when reflecting ultraviolet light from alkali metals.

In general, an analysis of the optical properties of metals and discussion of the possibility of using them in Lloyd's experiment with the interference of waves of opposite helicity is beyond the scope of this article. However, a reliable reversal of the helicity sign at angles of incidence on the order of 45 degrees could be used if one of the mirrors of the Michelson interferometer was replaced by a mirror rotating cylinder.

8. Conclusion

A new consequence of the concept of classical spin electrodynamics is the displacement of interference fringes in an interference experiment that uses a mirror rotating cylinder as a mirror.

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