# Discussion of cosmological acceleration and dark energy

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**Abstract**

Following our publications, we argue that the phenomenon of cosmological acceleration has a natural explanation as a consequence of quantum de Sitter symmetry in semiclassical approximation. The explanation does not involve dark energy and other exotic concepts.

Key words: quantum de Sitter symmetry; cosmological acceleration
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# 1 Introduction

This article is unusual in the following sense. The usual practice is that the authors first announce their results in the form of a letter and then write a more detailed article. However, in my case, the situation is the opposite. The problem of cosmological acceleration was considered in my papers published in known journals, and in the book recently published by Springer.

My publications are based on large calculations. To understand them, the readers must be experts not only in quantum theory, but also in the theory of representations of Lie algebras in Hilbert spaces. Therefore, understanding my results can be a challenge for many physicists. Since the problem of cosmological acceleration and dark energy is very important, I decided to write this short note, which outlines only the ideas of my approach without calculations. I hope that after reading this note, many readers will have an interest in studying my approach which considerably differs from approaches of other authors.

# 2 History of dark energy

The history of this problem is well-known. First Einstein introduced the cosmological constant $\Lambda$ because he believed that the universe was stationary, and his equations can ensure this only if $\Lambda \neq 0$. But when Friedman found his solutions of equations of General Relativity (GR) with $\Lambda = 0$, and Hubble found that the universe was expanding, Einstein said (according to Gamow's memories) that introducing $\Lambda \neq 0$ was the greatest blunder of his life. After that, the statement that $\Lambda$ must be zero was advocated even in textbooks, and now it is accepted by most physicists.

The explanation of this statement was as follows. According to the philosophy of GR, matter creates a curvature of space-time, so when matter is absent, there should be no curvature, i.e., space-time should be the flat Minkowski space. That is why when in 1998 it was realized that the data on supernovae could be described only with $\Lambda \neq 0$, the impression was that it was a shock of something fundamental. However, the explanation has been found: the term with $\Lambda$ in the Einstein equations has been moved from the left side to the right one, and it was declared that in fact $\Lambda = 0$, but the impression that $\Lambda \neq 0$ was the manifestation of a hypothetical field. In the model where, by analogy with GR, it is assumed that $\Lambda$ is a constant, this field is called dark energy. It has been also proposed a model of time-varying dark energy called quintessence. And physicists were not confused that, if the model is based on the observational data then the energy of this field approximately equals 70% of the energy of the universe.

Let us note that currently there is no physical theory which works under all conditions. For example, it is not correct to extrapolate nonrelativistic theory to the cases when speeds are comparable to $c$, and it is not correct to extrapolate classical physics for describing energy levels of the hydrogen atom. GR is a successful classical (i.e. non-quantum) theory for describing macroscopic phenomena where large masses are present, but extrapolation of GR to the case when matter disappears is not physical. One of the principles of physics is that a definition of a physical quantity is a description how this quantity should be measured. The concepts of space and its curvature are pure mathematical. Their aim is to describe the motion of real bodies. But the concepts of empty space and its curvature should not be used in physics because nothing can be measured in a space which exists only in our imagination. Indeed, in the limit of GR when matter disappears, space remains and has a curvature (zero curvature when $\Lambda = 0$, positive curvature when $\Lambda > 0$ and negative curvature when $\Lambda < 0$) while, since space is only a mathematical concept for describing matter, a reasonable approach should be such that in this limit space should disappear too.

A common principle of physics is that when a new phenomenon is discovered, physicists should try to first explain it proceeding from the existing science. Only if all such efforts fail, something exotic can be involved. But in the case of cosmological acceleration, an opposite approach was adopted: exotic explanations with dark energy or quintessence were accepted without serious efforts to explain the data in the framework of existing science.

## 3 Elementary particles in relativistic and de Sitter-invariant theories

In the problem of cosmological acceleration, only large macroscopic bodies are involved and that is why one might think that for considering this problem, there is no need to involve quantum theory. However, ideally, the results for every classical (i.e. non-quantum) problem should be obtained from quantum theory in semiclassical approximation. We will see that considering the problem of cosmological acceleration from the point of view of quantum theory, sheds a new light on understanding this problem.

Standard particle theory is based on Poincare symmetry where elementary particles are described by irreducible representations (IRs) of the Poincare group or its Lie algebra. This description has been first given in the famous Wigner's paper [1] and then discussed in many papers and textbooks.

The representation generators of the Poincare group Lie algebra commute according to the commutation relations

$$[P^\mu, P^\nu] = 0, \quad [P^\mu, M^{\nu\rho}] = -i(\eta^{\mu\rho}P^\nu - \eta^{\mu\nu}P^\rho),$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho}M^{\nu\sigma} + \eta^{\nu\sigma}M^{\mu\rho} - \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\nu\rho}M^{\mu\sigma}) \quad (1)$$

where $\mu, \nu = 0, 1, 2, 3$, $P^\mu$ are the operators of the four-momentum and $M^{\mu\nu}$ are the operators of Lorentz angular momenta.

Although the Poincare group is the group of motions of Minkowski space, the description in terms of relations (1) does not involve Minkowski space at all, and *those relations can be treated as a definition of relativistic invariance on quantum level* (see the discussion in Refs. [2, 3]). In classical field theories, the background space (e.g., Minkowski space) is an auxiliary mathematical concept for describing real fields and bodies. In quantum theory, any physical quantity should be described by an operator, but there is no operator corresponding to the coordinate $x$ of the background space. In quantum field theory, Minkowski space also is an auxiliary mathematical concept for describing interacting fields. Here a local Lagrangian $L(x)$ is used, and $x$ is only an integration parameter. The goal of the theory is to construct the S-matrix in momentum space, and, when this construction has been accomplished, one can forget about space-time background. This is in the spirit of the Heisenberg S-matrix program according to which in quantum theory one can describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to +\infty$.

The fact that the S-matrix is the operator in momentum space does not exclude a possibility that, in semiclassical approximation, it is possible to have a space-time description with some accuracy but not with absolute accuracy (see e.g., Ref. [3] for a detailed discussion). For example, if $\mathbf{p}$ is the momentum operator of a particle then, in the nonrelativistic approximation, the position operator of this particle in momentum representation *can be defined* as $\mathbf{r} = i\hbar\partial/\partial\mathbf{p}$. In this case, $\mathbf{r}$ is a physical quantity characterizing a given particle, and is different for different particles.

In relativistic quantum mechanics, for considering a system of noninteracting particles, there is no need to involve Minkowski space. A description of a single particle is fully defined by its IR by the operators commuting according to Eq. (1) while the representation describing several particles is the tensor product of the corresponding single-particle IRs. This implies that the four-momentum and Lorenz angular momenta operators for a system are sums of the corresponding single-particle operators. In the general case, representations describing systems with interaction are not tensor products of single-particle IRs, but there is no law that the construction of such representations should necessarily involve a background space-time.

In his famous paper "Missed Opportunities" [4] Dyson notes that de Sitter (dS) and anti-de Sitter (AdS) theories are more general (fundamental) than Poincare one even from pure mathematical considerations because dS and AdS groups are more symmetric than Poincare one. The transition from the former to the latter is described by a procedure called contraction when a parameter $R$ (see below) goes to infinity. At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

The paper [4] appeared in 1972 and, in view of Dyson's results, a question arises why general theories of elementary particles (QED, electroweak theory and QCD) are still based on Poincare symmetry and not dS or AdS ones. Probably, physicists believe that, since the parameter $R$ is much greater than even sizes of stars, dS and AdS symmetries can play an important role only in cosmology and there is no need to use them for describing elementary particles. We believe that this argument is not consistent because usually more general theories shed a new light on standard concepts. The discussion in our publications and, in particular, in this paper is a good illustration of this point.

By analogy with relativistic quantum theory, the definition of quantum dS symmetry should not involve the fact that the dS group is the group of motions of dS space. If $M^{ab}$ $(a, b = 0, 1, 2, 3, 4,\ M^{ab} = -M^{ba})$ are the operators describing the system under consideration, then, *by definition of dS symmetry*, they should satisfy the commutation relations *of the dS Lie algebra* so(1,4), *i.e.*,

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \tag{2}$$

where $\eta^{ab}$ is the diagonal metric tensor such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = -\eta^{44} = 1$. The *definition* of AdS symmetry on quantum level is given by the same equations but $\eta^{44} = 1$.

The procedure of contraction from dS and AdS symmetries to Poincare one is defined as follows. If we *define* the operators $P^\mu$ as $P^\mu = M^{4\mu}/R$ where $R$ is a parameter with the dimension *length* then in the formal limit when $R \to \infty$, $M^{4\mu} \to \infty$ but the quantities $P^\mu$ are finite, Eqs. (2) become Eqs. (1). This procedure is the same for the dS and AdS symmetries.

In Refs. [3, 5, 6] it has been proposed the following

**Definition:** *Let theory A contain a finite nonzero parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. Suppose that, with any desired accuracy, theory A can reproduce any result of theory B by choosing a value of the parameter. On the contrary, when the limit is already taken, one cannot return to theory A, and theory B cannot reproduce all results of theory A. Then theory A is more general than theory B and theory B is a special degenerate case of theory A.*

As argued in Refs. [3, 5, 6], in contrast to Dyson's approach based on Lie groups, the approach to symmetry on quantum level should be based on Lie algebras. Then it has been proved that, on quantum level, dS and AdS symmetries are more general (fundamental) than Poincare symmetry, *and this fact has nothing to do with the comparison of dS and AdS spaces with Minkowski space*. It has been also proved that classical theory is a special degenerate case of quantum one in the formal limit $\hbar \to 0$, and nonrelativistic theory is a special degenerate case of relativistic one in the formal limit $c \to \infty$. In the literature the above facts are explained from physical considerations but, as shown in Refs. [3, 5, 6], they can be proved mathematically by using properties of Lie algebras.

Physicists usually understand that physics cannot (and should not) derive that $c \approx 3 \cdot 10^8$m/s and $\hbar \approx 1.054 \cdot 10^{-34}$kg$\cdot$m$^2$/s. At the same time, they usually believe that physics should derive the value of the cosmological constant $\Lambda$, and that the solution of the dark energy problem depends on this value. However, background space in GR is only a classical concept, while on quantum level symmetry is defined by a Lie algebra of basic operators.

The parameters $(c, \hbar, R)$ are on equal footing because each of them is the parameter of contraction from a more general Lie algebra to a less general one, and therefore those parameters must be finite. In particular, the formal case $c = \infty$ corresponds to the situation when the Poincare algebra does not exist because it becomes the Galilei algebra, and the formal case $R = \infty$ corresponds to the situation when the de Sitter algebras do not exist because they become the Poincare algebra.

Quantum de Sitter theories do not need the dimensionful parameters $(c, \hbar, R)$ at all. They arise in less general theories, and the question why they are as are does not arise because the answer is: $\hbar$ is as is because people want to measure angular momenta in kg$\cdot$m$^2$/s, $c$ is as is because people want to measure velocities in m/s, and $R$ is as is because people want to measure distances in meters. The values of the parameters $(c, \hbar, R)$ in $(kg, m, s)$ have arisen from people's macroscopic experience, and there is no guaranty that those values will be the same during the whole history of the universe (see e.g., Ref. [3] for a more detailed discussion). The fact that particle theories do not need the quantities $(c, \hbar)$ is often explained such that the system of units $c = \hbar = 1$ is used. However, the concept of systems of units is purely classical and is not needed in quantum theory.

It is difficult to imagine standard Poincare invariant particle theories without IRs of the Poincare algebra. Therefore, when Poincare symmetry is replaced by a more general dS one, dS particle theories should be based on IRs of the dS algebra. However, as a rule, physicists are not familiar with such IRs. The mathematical literature on such IRs is wide but there are only a few papers where such IRs are described for physicists. For example, an excellent Mensky's book [7] exists only in Russian.

# 4 Explanation of cosmological acceleration

In this section we explain that, as follows from quantum theory, the value of $\Lambda$ in classical theory must be non-zero.

Consider a system of free macroscopic bodies, i.e., we do not consider gravitational, electromagnetic and other interactions between the bodies. Suppose that the distances between the bodies are much greater than their sizes. Then the motion of each body as a whole can be formally described in the same way as the motion of an elementary particle with the same mass. In semiclassical approximation, the spin effects can be neglected, and we can consider our system in the framework of dS quantum mechanics of free particles.

The explicit expressions for the operators $M^{ab}$ in IRs of the dS Lie algebra have been derived in Ref. [8] (see also Refs. [3, 6, 9, 10]). In contrast to standard quantum theory where the mass $m$ of a particle is dimensionfull, in dS quantum theory, the mass $m_{dS}$ of a particle is dimensionless. In the approximation when Poincare symmetry works with a high accuracy, these masses in units $c = \hbar = 1$ are related as $m_{dS} = Rm$. Also, in dS quantum theory, the Hilbert space of functions in IRs is the space of functions depending not on momenta but on four-velocities $v = (v_0, \mathbf{v})$ where $v_0 = (1 + \mathbf{v}^2)^{1/2}$. Then in the spinless case, the explicit expressions for the operators $M^{ab}$ are (see e.g., Eq. (3.16) in Ref. [3]):

$$\mathbf{J} = l(\mathbf{v}), \quad \mathbf{N} = -iv_0\frac{\partial}{\partial \mathbf{v}}, \quad \mathcal{E} = m_{dS}v_0 + iv_0(\mathbf{v}\frac{\partial}{\partial \mathbf{v}} + \frac{3}{2})$$
$$\mathbf{B} = m_{dS}\mathbf{v} + i[\frac{\partial}{\partial \mathbf{v}} + \mathbf{v}(\mathbf{v}\frac{\partial}{\partial \mathbf{v}}) + \frac{3}{2}\mathbf{v}] \qquad (3)$$

where $\mathbf{J} = \{M^{23}, M^{31}, M^{12}\}$, $\mathbf{N} = \{M^{01}, M^{02}, M^{03}\}$, $\mathbf{B} = \{M^{41}, M^{42}, M^{43}\}$, $\mathbf{l}(\mathbf{v}) = -i\mathbf{v}\times\partial/\partial\mathbf{v}$ and $\mathcal{E} = M^{40}$. *The important observation is that, at this stage, we have no coordinates yet.* For describing the motion of the particle in terms of coordinates, we must define the position operator. If Poincare symmetry works with a high accuracy, the momentum of the particle can be defined as $\mathbf{p} = m\mathbf{v}$ and, as noted above, the position operator can be defined as $\mathbf{r} = i\hbar\partial/\partial\mathbf{p}$.

In semiclassical approximation, we can treat $\mathbf{p}$ and $\mathbf{r}$ as usual vectors. Then, if $E = \mathcal{E}/R$, $\mathbf{P} = \mathbf{B}/R$ and the classical nonrelativistic Hamiltonian is defined as $H = E - mc^2$, it follows from Eq. (3) that

$$H(\mathbf{P}, \mathbf{r}) = \frac{\mathbf{P}^2}{2m} - \frac{mc^2\mathbf{r}^2}{2R^2} \qquad (4)$$

Here the last term is the dS correction to the non-relativistic Hamiltonian.

The representation describing a free N-body system is a tensor product of the corresponding single-particle IRs. This means that every N-body operator $M^{ab}$ is a sum of the corresponding single-particle operators.

Consider a system of two free particles described by the quantities $\mathbf{P}_j$ and $\mathbf{r}_j$ $(j = 1, 2)$. Define standard nonrelativistic variables

$$\mathbf{P} = \mathbf{P}_1 + \mathbf{P}_2, \quad \mathbf{q} = (m_2\mathbf{P}_1 - m_1\mathbf{P}_2)/(m_1 + m_2)$$
$$\mathbf{R} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2), \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2 \qquad (5)$$

Here $\mathbf{P}$ and $\mathbf{R}$ are the momentum and position of the system as a whole, and $\mathbf{q}$ and $\mathbf{r}$ are the relative momentum and relative radius-vector, respectively. Then as follows from Eqs. (4) and (5), the internal two-body Hamiltonian is

$$H_{nr}(\mathbf{r},\mathbf{q})=\frac{\mathbf{q}^2}{2m_{12}}-\frac{m_{12}c^2\mathbf{r}^2}{2R^2} \qquad (6)$$

where $m_{12}$ is the reduced two-particle mass. Then, as follows from the Hamilton equations, in semiclassical approximation the relative acceleration is given by

$$\mathbf{a}=\mathbf{r}c^2/R^2 \qquad (7)$$

where $\mathbf{a}$ and $\mathbf{r}$ are the relative acceleration and relative radius vector of the bodies, respectively.

The fact that the relative acceleration of noninteracting bodies is not zero does not contradict the law of inertia, because this law is valid only in the case of Galilei and Poincare symmetries. At the same time, in the case of dS symmetry, noninteracting bodies necessarily repulse each other (see e.g., the discussion in Ref. [11]). In the formal limit $R\to\infty$, the acceleration becomes zero as it should be.

Equations of relative motion derived from Eq. (6) are the same as those derived from GR if $\Lambda\neq 0$. In particular, the result (7) coincides with that in GR if the curvature of dS space equals $\Lambda=3/R^2$, where $R$ is the radius of this space. Therefore the cosmological constant has a physical meaning only on classical level, and, in semiclassical approximation, the parameter of contraction from dS symmetry to Poincare one coincides with the radius of dS space.

In GR, the result (7) does not depend on how $\Lambda$ is interpreted, as the curvature of empty space (and in this case dark energy and quinessence are not needed for the explanation of cosmological acceleration) or as the manifestation of dark energy or quintessence. However, in quantum theory, there is no freedom of interpretation. Here $R$ is the parameter of contraction from the dS Lie algebra to the Poincare one, it has nothing to do with dark energy or quintessence and it must be finite because dS symmetry is more general than Poincare one.

Every dimensionful parameter cannot have the same numerical values during the whole history of the universe. For example, at early stages of the universe such parameters do not have a physical meaning because semiclassical approximation does not work at those stages. In particular, the terms "cosmological constant" and "gravitational constant" can be misleading. General Relativity successfully describes many data in the approximation when $\Lambda$ and $G$ are constants but this does not mean that those quantities have the same numerical values during the whole history of the universe.

# 5 Conclusion

The result (7) has been derived without using dS space and its geometry (metric and connection). It is simply a consequence of dS quantum mechanics in semiclassical approximation. We believe that this result is more important than the result of GR because any classical result should be a consequence of quantum theory in semiclassical approximation.

Therefore, the phenomenon of the cosmological acceleration has nothing to do with dark energy or other artificial reasons. This phenomenon is purely a kinematical consequence of dS quantum mechanics in semiclassical approximation.

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