On the study of a Ramanujan equation. New possible mathematical connections with the Cosmological Constant and some topics of String Theory, in particular some “shift orientifolds” equations.

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Abstract

In this paper, we analyze a Ramanujan equation. We obtain new possible mathematical connections with the Cosmological Constant and some topics of String Theory, in particular some “shift orientifolds” equations.

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From:

RAMANUJAN - TWELVE LECTURES ON SUBJECTS SUGGESTED BY HIS LIFE AND WORK - *BY G. H. HARDY* - CAMBRIDGE AT THE UNIVERSITY PRESS - 1940

Now, we have that:

$$\int_0^\infty \frac{dx}{(1+x^2)(1+r^2x^2)(1+r^4x^2)\dots} = \frac{\pi}{2(1+r+r^3+r^6+r^{10}+\dots)}.$$

From the left-hand side, we obtain:

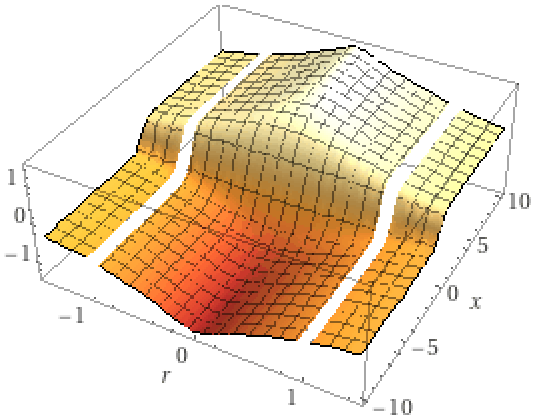
integrate1/(((1+x^2)(1+r^2x^2)(1+r^4x^2)))dx

Indefinite integral

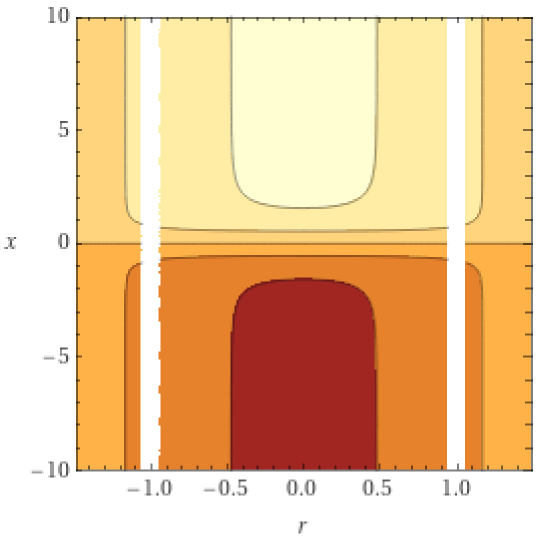
$$\int \frac{1}{(1+x^2)(1+r^2x^2)(1+r^4x^2)}\,dx = \frac{-(r^3+r)\tan^{-1}(r\,x)+r^4\tan^{-1}(r^2\,x)+\tan^{-1}(x)}{(r^2-1)^2(r^2+1)} + \text{constant}$$

$\tan^{-1}(x)$ is the inverse tangent function

3D plot **(figure that can be related to a D-brane/Instanton)**



Contour plot



Alternate forms of the integral

$$\frac{-(r^2+1)\, r \tan^{-1}(r\, x) + r^4 \tan^{-1}(r^2\, x) + \tan^{-1}(x)}{(r^2-1)^2\,(r^2+1)} + \text{constant}$$

$$\frac{-r^3 \tan^{-1}(r\, x) + r^4 \tan^{-1}(r^2\, x) - r \tan^{-1}(r\, x) + \tan^{-1}(x)}{(r-1)^2\,(r+1)^2\,(r^2+1)} + \text{constant}$$

$$\frac{-r^3 \tan^{-1}(r x) + r^4 \tan^{-1}(r^2 x) - r \tan^{-1}(r x) + \tan^{-1}(x)}{(r-1)^2 (r-i)(r+i)(r+1)^2} + \text{constant}$$

Expanded form of the integral

$$-\frac{r \tan^{-1}(r x)}{(r^2-1)^2 (r^2+1)} + \frac{\tan^{-1}(x)}{(r^2-1)^2 (r^2+1)} + \frac{r^4 \tan^{-1}(r^2 x)}{(r^2-1)^2 (r^2+1)} - \frac{r^3 \tan^{-1}(r x)}{(r^2-1)^2 (r^2+1)} + \text{constant}$$

Series expansion of the integral at x=0

$$x + \frac{1}{3}\left(-r^4 - r^2 - 1\right) x^3 + \frac{1}{5}\left(r^8 + r^6 + 2 r^4 + r^2 + 1\right) x^5 + O\left(x^6\right)$$

(Taylor series)

Series expansion of the integral at x=∞

$$-\frac{\pi\left(\left(r^2\right)^{3/2} + \sqrt{r^2} - \sqrt{r^4}\, r^2 - 1\right)}{2\left(\left(r^2-1\right)^2 \left(r^2+1\right)\right)} - \frac{1}{5 r^6 x^5} + O\left(\left(\frac{1}{x}\right)^6\right)$$

(Laurent series)

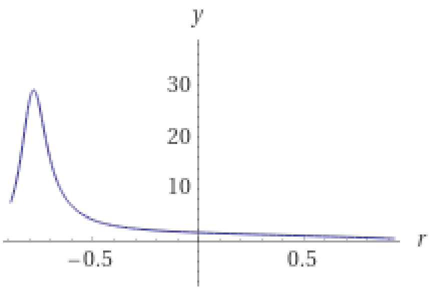
From the right-hand side, we obtain:

Pi/(2(1+r+r^3+r^6+r^10))

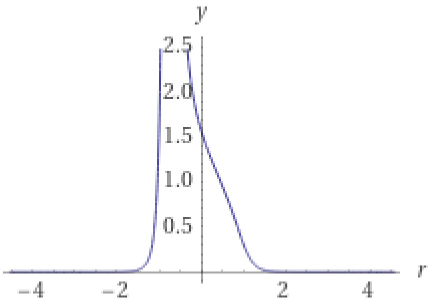
Input

$$\frac{\pi}{2\left(1 + r + r^3 + r^6 + r^{10}\right)}$$

Plots **(figures that can be related to the open strings)**



(r from −0.9 to 0.9)



(r from −4.5 to 4.5)

Alternate forms

$$\frac{\pi}{2 r^{10} + 2 r^6 + 2 r^3 + 2 r + 2}$$

$$\frac{\pi}{r\left(\left(\left(2 r^4 + 2\right) r^3 + 2\right) r^2 + 2\right) + 2}$$

Roots

(no roots exist)

Series expansion at r=0

$$\frac{\pi}{2} - \frac{\pi r}{2} + \frac{\pi r^2}{2} - \pi r^3 + \frac{3 \pi r^4}{2} + O\left(r^5\right)$$

(Taylor series)

Series expansion at r=∞

$$\frac{\pi}{2 r^{10}} + O\left(\left(\frac{1}{r}\right)^{14}\right)$$

(Laurent series)

Derivative

$$\frac{d}{dr}\left(\frac{\pi}{2\left(1 + r + r^3 + r^6 + r^{10}\right)}\right) = -\frac{\pi\left(10 r^9 + 6 r^5 + 3 r^2 + 1\right)}{2\left(r^{10} + r^6 + r^3 + r + 1\right)^2}$$

Indefinite integral

$$\int \frac{\pi}{2\left(1+r+r^3+r^6+r^{10}\right)}\, dr = \frac{1}{2}\, \pi \sum_{\{\omega:\, \omega^{10}+\omega^6+\omega^3+\omega+1=0\}} \frac{\log(r-\omega)}{10\,\omega^9+6\,\omega^5+3\,\omega^2+1} + \text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

Global maximum

$$\max\left\{\frac{\pi}{2\left(1 + r + r^3 + r^6 + r^{10}\right)}\right\} \approx 29.094 \text{ at } r \approx -0.78128$$

Limit

$$\lim_{r\to\pm\infty} \frac{\pi}{2\left(1 + r + r^3 + r^6 + r^{10}\right)} = 0$$

Definite integral

$$\int_0^\infty \frac{\pi}{2\left(1 + r + r^3 + r^6 + r^{10}\right)}\, dr \approx 1.00917144104\ldots$$

Alternative representations

$$\frac{\pi}{2\left(1+r+r^3+r^6+r^{10}\right)} = \frac{180\,°}{2\left(1+r+r^3+r^6+r^{10}\right)}$$

$$\frac{\pi}{2\left(1+r+r^3+r^6+r^{10}\right)} = -\frac{i\log(-1)}{2\left(1+r+r^3+r^6+r^{10}\right)}$$

$$\frac{\pi}{2\left(1+r+r^3+r^6+r^{10}\right)} = \frac{\cos^{-1}(-1)}{2\left(1+r+r^3+r^6+r^{10}\right)}$$

Series representations

$$\frac{\pi}{2\left(1+r+r^3+r^6+r^{10}\right)} = \frac{2\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}{1+r+r^3+r^6+r^{10}}$$

$$\frac{\pi}{2\left(1+r+r^3+r^6+r^{10}\right)} = \sum_{k=0}^{\infty} -\frac{2(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{(1+2k)\left(1+r+r^3+r^6+r^{10}\right)}$$

$$\frac{\pi}{2\left(1+r+r^3+r^6+r^{10}\right)} = \frac{\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}{2\left(1+r+r^3+r^6+r^{10}\right)}$$

Integral representations

$$\frac{\pi}{2\left(1+r+r^3+r^6+r^{10}\right)} = \frac{2}{1+r+r^3+r^6+r^{10}} \int_0^1 \sqrt{1-t^2}\, dt$$

$$\frac{\pi}{2\left(1+r+r^3+r^6+r^{10}\right)} = \frac{1}{1+r+r^3+r^6+r^{10}} \int_0^1 \frac{1}{\sqrt{1-t^2}}\, dt$$

$$\frac{\pi}{2\left(1+r+r^3+r^6+r^{10}\right)} = \frac{1}{1+r+r^3+r^6+r^{10}} \int_0^\infty \frac{1}{1+t^2}\, dt$$

Now, we have that

$$\int \frac{1}{\left(1+x^2\right)\left(1+r^2 x^2\right)\left(1+r^4 x^2\right)}\, dx = \frac{-\left(r^3+r\right)\tan^{-1}(r\, x) + r^4 \tan^{-1}\left(r^2 x\right) + \tan^{-1}(x)}{\left(r^2-1\right)^2\left(r^2+1\right)} + \text{constant}$$

for x = 2 and r = 3, from the solution of the above integral, we obtain:

(tan^(-1)(2) - (3 + 3^3) tan^(-1)(3*2) + 3^4 tan^(-1)(3^2*2))/((-1 + 3^2)^2 (1 + 3^2))

Input

$$\frac{\tan^{-1}(2) - \left(3+3^3\right)\tan^{-1}(3\times 2) + 3^4 \tan^{-1}\left(3^2\times 2\right)}{\left(-1+3^2\right)^2\left(1+3^2\right)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result

$$\frac{1}{640}\left(\tan^{-1}(2) - 30\tan^{-1}(6) + 81\tan^{-1}(18)\right)$$

(result in radians)

Decimal approximation

0.1276200668466711868000863600432077346272271815525109926433974803

...

(result in radians)

0.1276200668…

The study of this function provides the following representations:

Alternate forms

$$\frac{1}{640}\left(\tan^{-1}(2) - 30\tan^{-1}(6)\right) + \frac{81}{640}\tan^{-1}(18)$$

$$\frac{1}{640}\left(\tan^{-1}(2) - 3\left(10\tan^{-1}(6) - 27\tan^{-1}(18)\right)\right)$$

$$\frac{1}{640}\left(26\pi - \frac{1}{2}\tan^{-1}\left(58\,913\,379\,462\,895\,774\,772\,724\,778\,827\,649\,539\,057\,431\,546\,673\,162\,915\,355\,987\,398\,994\,354\,025\,134\,238\,231\,336\,405\,495\,185\,901\,438\,621\,121\,794\,846\,875\,698\,845\,540\,849\,255\,425\,358\,857\,774\,354\,320\,742\,912\,167\,917\,961\,549\,707\,013\,756\,193\,248\,205\,981\,116\,425\,025\,969\,777\,616\,718\,121\,756\,713\,935\,372\,393\,195\,989\,472\,859\,145\,459\,846\,656 / 6\,450\,833\,704\,265\,459\,357\,234\,746\,227\,776\,117\,562\,553\,648\,094\,090\,972\,229\,491\,454\,540\,812\,942\,550\,538\,191\,463\,781\,072\,269\,304\,937\,466\,348\,926\,084\,764\,084\,181\,194\,283\,974\,771\,330\,467\,686\,477\,401\,126\,899\,086\,757\,199\,598\,238\,257\,500\,975\,970\,739\,854\,839\,282\,838\,289\,412\,657\,398\,632\,235\,074\,009\,778\,411\,613\,718\,173\,790\,378\,300\,097\,744\,233\right)\right)$$

Expanded form

$$\frac{1}{640}\tan^{-1}(2) - \frac{3}{64}\tan^{-1}(6) + \frac{81}{640}\tan^{-1}(18)$$

Alternative representations

$$\frac{\tan^{-1}(2) - (3+3^3)\tan^{-1}(3\times 2) + 3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} = \frac{\text{sc}^{-1}(2\mid 0) - 30\,\text{sc}^{-1}(6\mid 0) + \text{sc}^{-1}(18\mid 0)\,3^4}{10\times 8^2}$$

$$\frac{\tan^{-1}(2) - (3+3^3)\tan^{-1}(3\times 2) + 3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} = \frac{\cot^{-1}\left(\frac{1}{2}\right) - 30\cot^{-1}\left(\frac{1}{6}\right) + \cot^{-1}\left(\frac{1}{18}\right)3^4}{10\times 8^2}$$

$$\frac{\tan^{-1}(2) - (3+3^3)\tan^{-1}(3\times 2) + 3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} = \frac{\tan^{-1}(1, 2) - 30\tan^{-1}(1, 6) + \tan^{-1}(1, 18)\,3^4}{10\times 8^2}$$

Series representations

$$\frac{\tan^{-1}(2) - (3+3^3)\tan^{-1}(3\times 2) + 3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} = \frac{13\pi}{320} + \sum_{k=0}^{\infty} -\frac{(-1)^k\,4^{-4-k}\times 9^{-2k}\left(9 - 10\times 9^k + 81^k\right)}{5(1+2k)}$$

$$\frac{\tan^{-1}(2)-(3+3^3)\tan^{-1}(3\times 2)+3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} =$$

$$\sum_{k=0}^{\infty}\left(\frac{(-1)^k 2^{-5+4k}\times 5^{-1-k}\left(1+\sqrt{\frac{21}{5}}\right)^{-1-2k} F_{1+2k}}{1+2k}+\frac{(-1)^{1+k} 3^{2+2k}\times 4^{-2+2k}\times 5^{-k}\left(1+\sqrt{\frac{149}{5}}\right)^{-1-2k} F_{1+2k}}{1+2k}+\frac{(-1)^k 2^{-5+4k}\times 5^{-1-k}\times 9^{3+2k}\left(1+\sqrt{\frac{1301}{5}}\right)^{-1-2k} F_{1+2k}}{1+2k}\right)$$

$$\frac{\tan^{-1}(2)-(3+3^3)\tan^{-1}(3\times 2)+3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} =$$

$$\frac{13}{160}\tan^{-1}(z_0)+\sum_{k=1}^{\infty}\frac{1}{1280\,k} i\left((-i-z_0)^k-(i-z_0)^k\right)\left((2-z_0)^k-30(6-z_0)^k+81(18-z_0)^k\right)(-i-z_0)^{-k}(i-z_0)^{-k}$$

for ($i z_0 \notin \mathbb{R}$ or ((not $1 \le i z_0 < \infty$) and (not $-\infty < i z_0 \le -1$)))

F_n is the n^{th} Fibonacci number

Integral representations

$$\frac{\tan^{-1}(2)-(3+3^3)\tan^{-1}(3\times 2)+3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} =$$

$$\int_0^1 \frac{2}{1+364\,t^2+13\,104\,t^4+46\,656\,t^6}\,dt$$

$$\frac{\tan^{-1}(2) - (3+3^3)\tan^{-1}(3\times 2) + 3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} = \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{i\,5^{-1-2s}\times 481^{-s}\left(18\times 5^{1+2s}\times 13^s - 729\times 37^s - 2405^s\right)\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2}{256\,\pi^{3/2}}\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{\tan^{-1}(2) - (3+3^3)\tan^{-1}(3\times 2) + 3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} = \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i\,2^{-2(4+s)}\times 9^{-2s}\left(729 - 10\times 9^{1+s} + 81^s\right)\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{5\pi\,\Gamma\left(\frac{3}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

$\Gamma(x)$ is the gamma function

Continued fraction representations

$$\frac{\tan^{-1}(2) - (3+3^3)\tan^{-1}(3\times 2) + 3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} = \frac{1}{320}\left(\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}} - \frac{90}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{36k^2}{1+2k}} + \frac{729}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{324k^2}{1+2k}}\right) =$$

$$\frac{1}{320}\left(\frac{1}{1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}} - \frac{90}{1+\frac{36}{3+\frac{144}{5+\frac{324}{7+\frac{576}{9+\dots}}}}} + \frac{729}{1+\frac{324}{3+\frac{1296}{5+\frac{2916}{7+\frac{5184}{9+\dots}}}}}\right)$$

$$\frac{\tan^{-1}(2)-\left(3+3^3\right)\tan^{-1}(3\times 2)+3^4\tan^{-1}\left(3^2\times 2\right)}{\left(-1+3^2\right)^2\left(1+3^2\right)}=$$

$$\frac{1}{320}\left(\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4(1-2k)^2}{5-6k}}-\frac{90}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{36(1-2k)^2}{37-70k}}+\frac{729}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{324(1-2k)^2}{325-646k}}\right)=$$

$$\frac{1}{320}\left(\frac{1}{1+\frac{4}{-1+\frac{36}{-7+\frac{100}{-13+\frac{196}{-19+\ldots}}}}}-\frac{90}{1+\frac{36}{-33+\frac{324}{-103+\frac{900}{-173+\frac{1764}{-243+\ldots}}}}}+\frac{729}{1+\frac{324}{-321+\frac{2916}{-967+\frac{8100}{-1613+\frac{15\,876}{-2259+\ldots}}}}}\right)$$

$$\frac{\tan^{-1}(2)-\left(3+3^3\right)\tan^{-1}(3\times 2)+3^4\tan^{-1}\left(3^2\times 2\right)}{\left(-1+3^2\right)^2\left(1+3^2\right)}=$$

$$2-\frac{1}{80\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}+\frac{81}{8\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{36\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}-\frac{59\,049}{80\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{324\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}=2-\frac{1}{80\left(3+\frac{36}{5+\frac{16}{7+\frac{100}{9+\frac{64}{11+\ldots}}}}\right)}+$$

$$\frac{81}{8\left(3+\frac{324}{5+\frac{144}{7+\frac{900}{9+\frac{576}{11+\ldots}}}}\right)}-\frac{59\,049}{80\left(3+\frac{2916}{5+\frac{1296}{7+\frac{8100}{9+\frac{5184}{11+\ldots}}}}\right)}$$

$$\frac{\tan^{-1}(2) - (3+3^3)\tan^{-1}(3\times 2) + 3^4\tan^{-1}(3^2\times 2)}{(-1+3^2)^2(1+3^2)} =$$

$$\frac{1}{320}\left(\frac{1}{5+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{8\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(3+2(-1)^k\right)(1+2k)}} - \frac{90}{37+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{72\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(19+18(-1)^k\right)(1+2k)}} + \frac{729}{325+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{648\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(163+162(-1)^k\right)(1+2k)}}\right) =$$

$$\frac{1}{320}\left(\frac{1}{5+-\frac{8}{3-\frac{8}{25-\frac{48}{7-\frac{48}{45+\ldots}}}}} - \frac{90}{37+-\frac{72}{3-\frac{72}{185-\frac{432}{7-\frac{432}{333+\ldots}}}}} + \frac{729}{325+-\frac{648}{3-\frac{648}{1625-\frac{3888}{7-\frac{3888}{2925+\ldots}}}}}\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

For r = 3, from the right-hand side of the initial expression, we obtain:

$$\frac{\pi}{2r^{10} + 2r^6 + 2r^3 + 2r + 2}$$

π/(2*3^10 + 2*3^6 + 2*3^3 + 2*3 + 2)

Input

$$\frac{\pi}{2\times 3^{10}+2\times 3^{6}+2\times 3^{3}+2\times 3+2}$$

Result

$$\frac{\pi}{119618}$$

Decimal approximation

0.0000262635443962429838190125514828830350298213429364736563140576

…

0.000026263544….

The study of this function provides the following representations:

Property

$\frac{\pi}{119618}$ is a transcendental number

Alternative representations

$$\frac{\pi}{2\times 3^{10}+2\times 3^{6}+2\times 3^{3}+2\times 3+2}=\frac{180\,°}{62+2\times 3^{6}+2\times 3^{10}}$$

$$\frac{\pi}{2\times 3^{10}+2\times 3^{6}+2\times 3^{3}+2\times 3+2}=-\frac{i\log(-1)}{62+2\times 3^{6}+2\times 3^{10}}$$

$$\frac{\pi}{2\times 3^{10}+2\times 3^{6}+2\times 3^{3}+2\times 3+2}=\frac{\cos^{-1}(-1)}{62+2\times 3^{6}+2\times 3^{10}}$$

Series representations

$$\frac{\pi}{2\times 3^{10}+2\times 3^{6}+2\times 3^{3}+2\times 3+2}=\frac{2\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}{59809}$$

$$\frac{\pi}{2\times 3^{10}+2\times 3^{6}+2\times 3^{3}+2\times 3+2}=\sum_{k=0}^{\infty}-\frac{2(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{59809(1+2k)}$$

$$\frac{\pi}{2\times 3^{10}+2\times 3^{6}+2\times 3^{3}+2\times 3+2}=\frac{\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}{119618}$$

Integral representations

$$\frac{\pi}{2\times 3^{10}+2\times 3^{6}+2\times 3^{3}+2\times 3+2}=\frac{2}{59809}\int_0^1\sqrt{1-t^2}\,dt$$

$$\frac{\pi}{2\times 3^{10}+2\times 3^{6}+2\times 3^{3}+2\times 3+2}=\frac{1}{59809}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt$$

$$\frac{\pi}{2\times 3^{10}+2\times 3^{6}+2\times 3^{3}+2\times 3+2}=\frac{1}{59809}\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

For x = 0.5 and r = 0.8, from the solution of the integral, we obtain:

(tan^(-1)(0.5) - (0.8 + 0.8^3) tan^(-1)(0.8*0.5) + 0.8^4 tan^(-1)(0.8^2*0.5))/((-1 + 0.8^2)^2 (1 + 0.8^2))

Input

$$\frac{\tan^{-1}(0.5) - (0.8 + 0.8^3)\tan^{-1}(0.8\times 0.5) + 0.8^4\tan^{-1}(0.8^2\times 0.5)}{(-1+0.8^2)^2(1+0.8^2)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

0.429452576002961480305762875181889964062236610370222718363874339
...

(result in radians)

0.42945276….

The study of this function provides the following representations:

Alternative representations

$$\frac{\tan^{-1}(0.5) - (0.8 + 0.8^3)\tan^{-1}(0.8\times 0.5) + 0.8^4\tan^{-1}(0.8^2\times 0.5)}{(-1+0.8^2)^2(1+0.8^2)} = \frac{\mathrm{sc}^{-1}(0.5\mid 0) - \mathrm{sc}^{-1}(0.4\mid 0)(0.8+0.8^3) + \mathrm{sc}^{-1}(0.5\times 0.8^2\mid 0)\,0.8^4}{(1+0.8^2)(-1+0.8^2)^2}$$

$$\frac{\tan^{-1}(0.5) - (0.8 + 0.8^3)\tan^{-1}(0.8\times 0.5) + 0.8^4\tan^{-1}(0.8^2\times 0.5)}{(-1+0.8^2)^2(1+0.8^2)} = \frac{\cot^{-1}\left(\frac{1}{0.5}\right) - \cot^{-1}\left(\frac{1}{0.4}\right)(0.8+0.8^3) + \cot^{-1}\left(\frac{1}{0.5\times 0.8^2}\right)0.8^4}{(1+0.8^2)(-1+0.8^2)^2}$$

$$\frac{\tan^{-1}(0.5)-(0.8+0.8^3)\tan^{-1}(0.8\times 0.5)+0.8^4\tan^{-1}(0.8^2\times 0.5)}{(-1+0.8^2)^2(1+0.8^2)} = \frac{\tan^{-1}(1, 0.5)-\tan^{-1}(1, 0.4)(0.8+0.8^3)+\tan^{-1}(1, 0.5\times 0.8^2)\,0.8^4}{(1+0.8^2)(-1+0.8^2)^2}$$

$sc^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function sc

Series representations

$$\frac{\tan^{-1}(0.5)-(0.8+0.8^3)\tan^{-1}(0.8\times 0.5)+0.8^4\tan^{-1}(0.8^2\times 0.5)}{(-1+0.8^2)^2(1+0.8^2)} = \sum_{k=0}^{\infty}\frac{(-1)^k\left(0.308341\,e^{-2.27887\,k}-1.23457\,e^{-1.83258\,k}+1.17623\,e^{-1.38629\,k}\right)}{0.5+k}$$

$$\frac{\tan^{-1}(0.5)-(0.8+0.8^3)\tan^{-1}(0.8\times 0.5)+0.8^4\tan^{-1}(0.8^2\times 0.5)}{(-1+0.8^2)^2(1+0.8^2)} = \sum_{k=0}^{\infty}\left(\frac{1.92713\left(-\frac{1}{5}\right)^k 0.64^{1+2k} F_{1+2k}\left(\frac{1}{1+\sqrt{1.08192}}\right)^{1+2k}}{1+2k} - \frac{6.17284\left(-\frac{1}{5}\right)^k 0.8^{1+2k} F_{1+2k}\left(\frac{1}{1+\sqrt{1.128}}\right)^{1+2k}}{1+2k} + \frac{4.70491\left(-\frac{1}{5}\right)^k 1^{1+2k} F_{1+2k}\left(\frac{1}{1+\sqrt{1.2}}\right)^{1+2k}}{1+2k}\right)$$

$$\frac{\tan^{-1}(0.5)-(0.8+0.8^3)\tan^{-1}(0.8\times 0.5)+0.8^4\tan^{-1}(0.8^2\times 0.5)}{(-1+0.8^2)^2(1+0.8^2)} = 0.459199\tan^{-1}(x)+1.92713\,\pi\left\lfloor\frac{\arg(i(0.32-x))}{2\pi}\right\rfloor - 6.17284\,\pi\left\lfloor\frac{\arg(i(0.4-x))}{2\pi}\right\rfloor + 4.70491\,\pi\left\lfloor\frac{\arg(i(0.5-x))}{2\pi}\right\rfloor + \sum_{k=1}^{\infty}\frac{1}{k}\,i\left(0.963565(0.32-x)^k-3.08642(0.4-x)^k+2.35245(0.5-x)^k\right)\left(-(-i-x)^{-k}+(i-x)^{-k}\right) \quad \text{for } (i\,x\in\mathbb{R} \text{ and } i\,x<-1)$$

F_n is the n^{th} Fibonacci number

Integral representations

$$\frac{\tan^{-1}(0.5) - (0.8 + 0.8^3)\tan^{-1}(0.8 \times 0.5) + 0.8^4 \tan^{-1}(0.8^2 \times 0.5)}{(-1 + 0.8^2)^2 (1 + 0.8^2)} = \int_0^1 \frac{122.07 - 1.77636 \times 10^{-15}\, t^4}{244.141 + 125.098\, t^2 + 20.0156\, t^4 + t^6}\, dt$$

$$\frac{\tan^{-1}(0.5) - (0.8 + 0.8^3)\tan^{-1}(0.8 \times 0.5) + 0.8^4 \tan^{-1}(0.8^2 \times 0.5)}{(-1 + 0.8^2)^2 (1 + 0.8^2)} = \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{1}{\pi^{3/2}}\, e^{-0.469053\, s} \left(-0.588114\, e^{0.24591\, s} + 0.617284\, e^{0.320633\, s} - 0.15417\, e^{0.371564\, s}\right) i\, \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s)\, \Gamma(s)^2\, ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

$$\frac{\tan^{-1}(0.5) - (0.8 + 0.8^3)\tan^{-1}(0.8 \times 0.5) + 0.8^4 \tan^{-1}(0.8^2 \times 0.5)}{(-1 + 0.8^2)^2 (1 + 0.8^2)} = \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{1}{i\, \pi\, \Gamma\left(\frac{3}{2} - s\right)} \left(0.588114\, e^{1.38629\, s} - 0.617284\, e^{1.83258\, s} + 0.15417\, e^{2.27887\, s}\right) \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s)\, \Gamma(s)\, ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

Continued fraction representations

$$\frac{\tan^{-1}(0.5) - (0.8 + 0.8^3)\tan^{-1}(0.8 \times 0.5) + 0.8^4 \tan^{-1}(0.8^2 \times 0.5)}{(-1 + 0.8^2)^2 (1 + 0.8^2)} =$$

$$\frac{0.616682}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.1024 k^2}{1+2k}} - \frac{2.46914}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.16 k^2}{1+2k}} + \frac{2.35245}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.25 k^2}{1+2k}} =$$

$$\frac{0.616682}{1 + \frac{0.1024}{3 + \frac{0.4096}{5 + \frac{0.9216}{7 + \frac{1.6384}{9 + \dots}}}}} - \frac{2.46914}{1 + \frac{0.16}{3 + \frac{0.64}{5 + \frac{1.44}{7 + \frac{2.56}{9 + \dots}}}}} + \frac{2.35245}{1 + \frac{0.25}{3 + \frac{1}{5 + \frac{2.25}{7 + \frac{4}{9 + \dots}}}}}$$

$$\frac{\tan^{-1}(0.5) - (0.8 + 0.8^3)\tan^{-1}(0.8 \times 0.5) + 0.8^4 \tan^{-1}(0.8^2 \times 0.5)}{(-1 + 0.8^2)^2 (1 + 0.8^2)} =$$

$$\frac{0.616682}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.1024 (1-2k)^2}{1.1024+1.7952 k}} - \frac{2.46914}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.16 (1-2k)^2}{1.16+1.68 k}} + \frac{2.35245}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.25 (1-2k)^2}{1.25+1.5 k}} =$$

$$\frac{0.616682}{1 + \frac{0.1024}{2.8976 + \frac{0.9216}{4.6928 + \frac{2.56}{6.488 + \frac{5.0176}{8.2832 + \dots}}}}} -$$

$$\frac{2.46914}{1 + \frac{0.16}{2.84 + \frac{1.44}{4.52 + \frac{4.}{6.2 + \frac{7.84}{7.88 + \dots}}}}} + \frac{2.35245}{1 + \frac{0.25}{2.75 + \frac{2.25}{4.25 + \frac{6.25}{5.75 + \frac{12.25}{7.25 + \dots}}}}}$$

$$\frac{\tan^{-1}(0.5) - \left(0.8 + 0.8^3\right)\tan^{-1}(0.8\times 0.5) + 0.8^4\tan^{-1}\left(0.8^2\times 0.5\right)}{\left(-1+0.8^2\right)^2\left(1+0.8^2\right)} =$$

$$0.5 - \frac{0.0631482}{3 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.1024\left(1+(-1)^{1+k}+k\right)^2}{3+2k}} + \frac{0.395062}{3 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.16\left(1+(-1)^{1+k}+k\right)^2}{3+2k}} - \frac{0.588114}{3 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.25\left(1+(-1)^{1+k}+k\right)^2}{3+2k}} =$$

$$0.5 - \frac{0.0631482}{3+\frac{0.9216}{5+\frac{0.4096}{7+\frac{2.56}{9+\frac{1.6384}{11+\ldots}}}}} + \frac{0.395062}{3+\frac{1.44}{5+\frac{0.64}{7+\frac{4.}{9+\frac{2.56}{11+\ldots}}}}} - \frac{0.588114}{3+\frac{2.25}{5+\frac{1}{7+\frac{6.25}{9+\frac{4}{11+\ldots}}}}}$$

$$\frac{\tan^{-1}(0.5) - \left(0.8 + 0.8^3\right)\tan^{-1}(0.8\times 0.5) + 0.8^4\tan^{-1}\left(0.8^2\times 0.5\right)}{\left(-1+0.8^2\right)^2\left(1+0.8^2\right)} =$$

$$\frac{0.616682}{1.1024 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.2048\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.0512+0.0512(-1)^k\right)(1+2k)}} - \frac{2.46914}{1.16 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.32\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.08+0.08(-1)^k\right)(1+2k)}} + \frac{2.35245}{1.25 + \underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.5\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.125+0.125(-1)^k\right)(1+2k)}} =$$

$$\frac{0.616682}{1.1024 + -\frac{0.2048}{3-\frac{0.2048}{5.512-\frac{1.2288}{7-\frac{1.2288}{9.9216+\ldots}}}}} - \frac{2.46914}{1.16 + -\frac{0.32}{3-\frac{0.32}{5.8-\frac{1.92}{7-\frac{1.92}{10.44+\ldots}}}}} + \frac{2.35245}{1.25 + -\frac{0.5}{3-\frac{0.5}{6.25-\frac{3}{7-\frac{3}{11.25+\ldots}}}}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

While, for r = 0.913 , from the right-hand side of the initial expression, we obtain:

π/(2*0.913^10 + 2*0.913^6 + 2*0.913^3 + 2*0.913 + 2)

Input

$$\frac{\pi}{2\times 0.913^{10}+2\times 0.913^{6}+2\times 0.913^{3}+2\times 0.913+2}$$

Result

0.42968538326429747598643358531856340696416425834304193503197577 91

...

0.4296853832….

The study of this function provides the following representations:

Alternative representations

$$\frac{\pi}{2\times 0.913^{10}+2\times 0.913^{6}+2\times 0.913^{3}+2\times 0.913+2} = \frac{180^{\circ}}{3.826+2\times 0.913^{3}+2\times 0.913^{6}+2\times 0.913^{10}}$$

$$\frac{\pi}{2\times 0.913^{10}+2\times 0.913^{6}+2\times 0.913^{3}+2\times 0.913+2} = -\frac{i\log(-1)}{3.826+2\times 0.913^{3}+2\times 0.913^{6}+2\times 0.913^{10}}$$

$$\frac{\pi}{2\times 0.913^{10}+2\times 0.913^{6}+2\times 0.913^{3}+2\times 0.913+2} = \frac{\cos^{-1}(-1)}{3.826+2\times 0.913^{3}+2\times 0.913^{6}+2\times 0.913^{10}}$$

Series representations

$$\frac{\pi}{2\times 0.913^{10}+2\times 0.913^{6}+2\times 0.913^{3}+2\times 0.913+2}=0.547092\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\frac{\pi}{2\times 0.913^{10}+2\times 0.913^{6}+2\times 0.913^{3}+2\times 0.913+2}=-0.273546+0.273546\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}$$

$$\frac{\pi}{2\times 0.913^{10}+2\times 0.913^{6}+2\times 0.913^{3}+2\times 0.913+2}=0.136773\sum_{k=0}^{\infty}\frac{2^{-k}(-6+50k)}{\binom{3k}{k}}$$

Integral representations

$$\frac{\pi}{2\times 0.913^{10}+2\times 0.913^{6}+2\times 0.913^{3}+2\times 0.913+2}=0.273546\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

$$\frac{\pi}{2\times 0.913^{10}+2\times 0.913^{6}+2\times 0.913^{3}+2\times 0.913+2}=0.547092\int_0^{1}\sqrt{1-t^2}\,dt$$

$$\frac{\pi}{2\times 0.913^{10}+2\times 0.913^{6}+2\times 0.913^{3}+2\times 0.913+2}=0.273546\int_0^{\infty}\frac{\sin(t)}{t}\,dt$$

For r = 0.913, from the solution of the integral, we obtain:

(tan^(-1)(0.913) - (0.913 + 0.913^3) tan^(-1)(0.913*0.913) + 0.913^4 tan^(-1)(0.913^2*0.913))/((-1 + 0.913^2)^2 (1 + 0.913^2))

Input

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913\times 0.913) + 0.913^4\tan^{-1}(0.913^2\times 0.913)}{(-1+0.913^2)^2\,(1+0.913^2)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

0.565257122083949777422517549153703880850147304152255135017016858
…

(result in radians)

0.565257122….

The study of this function provides the following representations:

Alternative representations

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913\times 0.913) + 0.913^4\tan^{-1}(0.913^2\times 0.913)}{(-1+0.913^2)^2\,(1+0.913^2)} = \left(\mathrm{sc}^{-1}(0.913\,|\,0) - \mathrm{sc}^{-1}(0.833569\,|\,0)\,(0.913+0.913^3) + \mathrm{sc}^{-1}(0.913\times 0.913^2\,|\,0)\,0.913^4\right)/\left((1+0.913^2)(-1+0.913^2)^2\right)$$

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913\times 0.913) + 0.913^4\tan^{-1}(0.913^2\times 0.913)}{(-1+0.913^2)^2\,(1+0.913^2)} = \frac{\cot^{-1}\left(\frac{1}{0.913}\right) - \cot^{-1}\left(\frac{1}{0.833569}\right)(0.913+0.913^3) + \cot^{-1}\left(\frac{1}{0.913\times 0.913^2}\right)0.913^4}{(1+0.913^2)(-1+0.913^2)^2}$$

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913 \times 0.913) + 0.913^4 \tan^{-1}(0.913^2 \times 0.913)}{(-1 + 0.913^2)^2 (1 + 0.913^2)}$$
$$= \left(\tan^{-1}(1, 0.913) - \tan^{-1}(1, 0.833569)(0.913 + 0.913^3) + \tan^{-1}(1, 0.913 \times 0.913^2)\, 0.913^4\right) / \left((1 + 0.913^2)(-1 + 0.913^2)^2\right)$$

$sc^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function sc

Series representations

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913 \times 0.913) + 0.913^4 \tan^{-1}(0.913^2 \times 0.913)}{(-1 + 0.913^2)^2 (1 + 0.913^2)}$$
$$= \sum_{k=0}^{\infty} \frac{(-1)^k \left(5.20595\, e^{-0.546116\,k} - 13.7377\, e^{-0.364078\,k} + 8.98825\, e^{-0.182039\,k}\right)}{0.5 + k}$$

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913 \times 0.913) + 0.913^4 \tan^{-1}(0.913^2 \times 0.913)}{(-1 + 0.913^2)^2 (1 + 0.913^2)}$$
$$= 0.20466\, i \log(2) - 6.84049\, i \log((0.761048 - i)\, i) + 16.4806\, i \log((0.833569 - i)\, i) - 9.84474\, i \log((0.913 - i)\, i) + \sum_{k=1}^{\infty} \frac{1}{k} 2^{-k} \left(-6.84049\, (0.761048 - i)^k + 16.4806\, (0.833569 - i)^k - 9.84474\, (0.913 - i)^k\right) i^{1+k}$$

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913 \times 0.913) + 0.913^4 \tan^{-1}(0.913^2 \times 0.913)}{(-1 + 0.913^2)^2 (1 + 0.913^2)}$$
$$= -0.20466\, i \log(2) + 6.84049\, i \log(-i\,(0.761048 + i)) - 16.4806\, i \log(-i\,(0.833569 + i)) + 9.84474\, i \log(-i\,(0.913 + i)) + \sum_{k=1}^{\infty} \frac{1}{k} 2^{-k} (-i)^k\, i \left(6.84049\, (0.761048 + i)^k - 16.4806\, (0.833569 + i)^k + 9.84474\, (0.913 + i)^k\right)$$

Integral representations

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913 \times 0.913) + 0.913^4\tan^{-1}(0.913^2 \times 0.913)}{(-1 + 0.913^2)^2 (1 + 0.913^2)} = \int_0^1 \frac{2.72158 - 1.42109 \times 10^{-14} t^2}{2.98092 + 6.2826 t^2 + 4.36538 t^4 + t^6} \, dt$$

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913 \times 0.913) + 0.913^4\tan^{-1}(0.913^2 \times 0.913)}{(-1 + 0.913^2)^2 (1 + 0.913^2)} = \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\pi^{3/2}} e^{-1.59077 s} \left(-4.49412 e^{0.984502 s} + 6.86885 e^{1.06318 s} - 2.60297 e^{1.13385 s}\right) i\,\Gamma\left(\frac{1}{2} - s\right)\Gamma(1-s)\,\Gamma(s)^2 \, ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913 \times 0.913) + 0.913^4\tan^{-1}(0.913^2 \times 0.913)}{(-1 + 0.913^2)^2 (1 + 0.913^2)} = \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{i\,\pi\,\Gamma\left(\frac{3}{2} - s\right)} \left(4.49412 e^{0.182039 s} - 6.86885 e^{0.364078 s} + 2.60297 e^{0.546116 s}\right) \Gamma\left(\frac{1}{2} - s\right)\Gamma(1-s)\,\Gamma(s) \, ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

Continued fraction representations

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913 \times 0.913) + 0.913^4\tan^{-1}(0.913^2 \times 0.913)}{(-1 + 0.913^2)^2 (1 + 0.913^2)} = \frac{10.4119}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.579195 k^2}{1+2k}} - \frac{27.4754}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.694837 k^2}{1+2k}} + \frac{17.9765}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.833569 k^2}{1+2k}} = \frac{10.4119}{1 + \frac{0.579195}{3 + \frac{2.31678}{5 + \frac{5.21275}{7 + \frac{9.26712}{9 + \dots}}}}} - \frac{27.4754}{1 + \frac{0.694837}{3 + \frac{2.77935}{5 + \frac{6.25354}{7 + \frac{11.1174}{9 + \dots}}}}} + \frac{17.9765}{1 + \frac{0.833569}{3 + \frac{3.33428}{5 + \frac{7.50212}{7 + \frac{13.3371}{9 + \dots}}}}}$$

$$\frac{\tan^{-1}(0.913) - \left(0.913 + 0.913^3\right)\tan^{-1}(0.913 \times 0.913) + 0.913^4 \tan^{-1}\left(0.913^2 \times 0.913\right)}{\left(-1 + 0.913^2\right)^2 \left(1 + 0.913^2\right)}$$

$$= \frac{10.4119}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.579195\,(1-2k)^2}{1.57919+0.84161\,k}} - \frac{27.4754}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.694837\,(1-2k)^2}{1.69484+0.610325\,k}} + \frac{17.9765}{1 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.833569\,(1-2k)^2}{1.83357+0.332862\,k}} =$$

$$\frac{10.4119}{1 + \frac{0.579195}{2.42081 + \frac{5.21275}{3.26242 + \frac{14.4799}{4.10403 + \frac{28.3805}{4.94564 + \dots}}}}} - \frac{27.4754}{1 + \frac{0.694837}{2.30516 + \frac{6.25354}{2.91549 + \frac{17.3709}{3.52581 + \frac{34.047}{4.13614 + \dots}}}}} + \frac{17.9765}{1 + \frac{0.833569}{2.16643 + \frac{7.50212}{2.49929 + \frac{20.8392}{2.83215 + \frac{40.8449}{3.16502 + \dots}}}}}$$

$$\frac{\tan^{-1}(0.913) - \left(0.913 + 0.913^3\right)\tan^{-1}(0.913 \times 0.913) + 0.913^4 \tan^{-1}\left(0.913^2 \times 0.913\right)}{\left(-1 + 0.913^2\right)^2 \left(1 + 0.913^2\right)}$$

$$= 0.913 - \frac{6.03052}{3 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.579195\left(1+(-1)^{1+k}+k\right)^2}{3+2k}} + \frac{19.0909}{3 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.694837\left(1+(-1)^{1+k}+k\right)^2}{3+2k}} - \frac{14.9847}{3 + \underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{0.833569\left(1+(-1)^{1+k}+k\right)^2}{3+2k}} = 0.913 -$$

$$\frac{6.03052}{3 + \frac{5.21275}{5 + \frac{2.31678}{7 + \frac{14.4799}{9 + \frac{9.26712}{11 + \dots}}}}} + \frac{19.0909}{3 + \frac{6.25354}{5 + \frac{2.77935}{7 + \frac{17.3709}{9 + \frac{11.1174}{11 + \dots}}}}} - \frac{14.9847}{3 + \frac{7.50212}{5 + \frac{3.33428}{7 + \frac{20.8392}{9 + \frac{13.3371}{11 + \dots}}}}}$$

$$\frac{\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913 \times 0.913) + 0.913^4 \tan^{-1}(0.913^2 \times 0.913)}{(-1 + 0.913^2)^2 (1 + 0.913^2)}$$

$$= \frac{10.4119}{1.57919 + \underset{k=1}{\overset{\infty}{K}} \frac{1.15839\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.2896+0.289597(-1)^k\right)(1+2k)}} - \frac{27.4754}{1.69484 + \underset{k=1}{\overset{\infty}{K}} \frac{1.38967\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.34742+0.347419(-1)^k\right)(1+2k)}} + \frac{17.9765}{1.83357 + \underset{k=1}{\overset{\infty}{K}} \frac{1.66714\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.41678+0.416785(-1)^k\right)(1+2k)}} =$$

$$\frac{10.4119}{1.57919 + -\frac{1.15839}{3. - \frac{1.15839}{7.89597 - \frac{6.95034}{7. - \frac{6.95034}{14.2128+\dots}}}}} - \frac{27.4754}{1.69484 + -\frac{1.38967}{3 - \frac{1.38967}{8.47419 - \frac{8.33805}{7 - \frac{8.33805}{15.2535+\dots}}}}} + \frac{17.9765}{1.83357 + -\frac{1.66714}{3 - \frac{1.66714}{9.16785 - \frac{10.0028}{7 - \frac{10.0028}{16.5021+\dots}}}}}$$

$\underset{k=k_1}{\overset{k_2}{K}} a_k / b_k$ is a continued fraction

We have the following equation:

x*(tan^(-1)(0.913) - (0.913 + 0.913^3) tan^(-1)(0.913*0.913) + 0.913^4 tan^(-1)(0.913^2*0.913))/((-1 + 0.913^2)^2 (1 + 0.913^2)) = π/(2*0.913^10 + 2*0.913^6 + 2*0.913^3 + 2*0.913 + 2)

Input

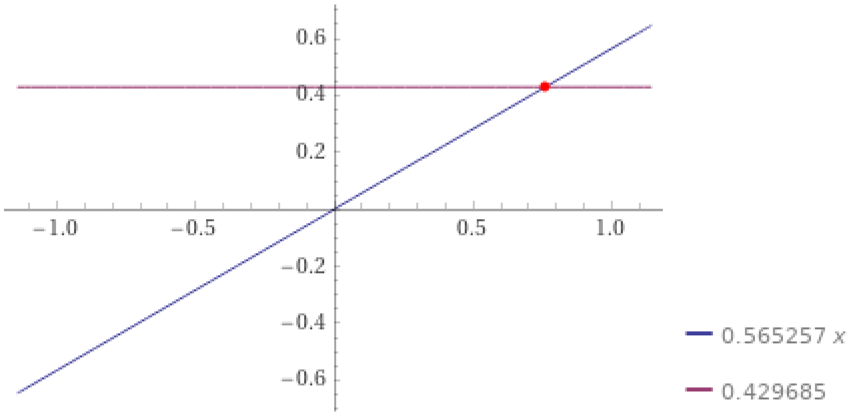
$$x \times \left(\tan^{-1}(0.913) - (0.913 + 0.913^3)\tan^{-1}(0.913 \times 0.913) + 0.913^4 \tan^{-1}(0.913^2 \times 0.913)\right) / \left((-1 + 0.913^2)^2 (1 + 0.913^2)\right) = \frac{\pi}{2 \times 0.913^{10} + 2 \times 0.913^6 + 2 \times 0.913^3 + 2 \times 0.913 + 2}$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

$0.565257 x = 0.429685$

Plot



Alternate form

$0.565257 x - 0.429685 = 0$

Alternate form assuming x is real

$0.565257 x + 0 = 0.429685$

Solution

$x \approx 0.760159$

Indeed:

e^(-1 + 1/e + 2 e) π^(1 - 2 e)*(tan^(-1)(0.913) - (0.913 + 0.913^3) tan^(-1)(0.913*0.913) + 0.913^4 tan^(-1)(0.913^2*0.913))/((-1 + 0.913^2)^2 (1 + 0.913^2))

where

$e^{-1+1/e+2e} \pi^{1-2e}$

0.7601618480162878571796051624908594110178354115967817615188329823 …

Input

$$e^{-1+1/e+2e}\,\pi^{1-2e}\times\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

0.4296868985277037012189476302079535279260252662214004347774946859
…

(result in radians)

0.4296868985277….

The study of this function provides the following representations:

Alternative representations

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=\left(\left(\operatorname{sc}^{-1}(0.913\,|\,0)-\operatorname{sc}^{-1}(0.833569\,|\,0)\left(0.913+0.913^3\right)+\operatorname{sc}^{-1}\left(0.913\times 0.913^2\,\middle|\,0\right)0.913^4\right)e^{-1+2e+1/e}\,\pi^{1-2e}\right)/\left(\left(1+0.913^2\right)\left(-1+0.913^2\right)^2\right)$$

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=\left(\left(\cot^{-1}\left(\frac{1}{0.913}\right)-\cot^{-1}\left(\frac{1}{0.833569}\right)\left(0.913+0.913^3\right)+\cot^{-1}\left(\frac{1}{0.913\times 0.913^2}\right)0.913^4\right)e^{-1+2e+1/e}\,\pi^{1-2e}\right)\bigg/\left(\left(1+0.913^2\right)\left(-1+0.913^2\right)^2\right)$$

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=$$
$$\left(\left(\tan^{-1}(1,\,0.913)-\tan^{-1}(1,\,0.833569)\left(0.913+0.913^3\right)+\tan^{-1}\left(1,\,0.913\times 0.913^2\right)0.913^4\right)e^{-1+2e+1/e}\,\pi^{1-2e}\right)/\left(\left(1+0.913^2\right)\left(-1+0.913^2\right)^2\right)$$

$sc^{-1}(x\,|\,m)$ is the inverse of the Jacobi elliptic function sc

Series representations

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=$$
$$\sum_{k=0}^{\infty}\frac{1}{0.5+k}(-1)^k\left(5.20595\,e^{-0.546116\,k}-13.7377\,e^{-0.364078\,k}+8.98825\,e^{-0.182039\,k}\right)e^{-1+1/e+2e}\,\pi^{1-2e}$$

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/$$
$$\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=\sum_{k=0}^{\infty}\frac{1}{1+2k}\left(-\frac{1}{5}\right)^k e^{-1+1/e+2e}\,\pi^{1-2e}$$
$$F_{1+2k}\left(20.8238\,e^{0.840178\,k}\left(\frac{1}{1+\sqrt{1.46336}}\right)^{1+2k}-54.9508\,e^{1.02222\,k}\left(\frac{1}{1+\sqrt{1.55587}}\right)^{1+2k}+35.953\,e^{1.20426\,k}\left(\frac{1}{1+\sqrt{1.66686}}\right)^{1+2k}\right)$$

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=0.40932\,e^{-1+1/e+2e}\,\pi^{1-2e}\tan^{-1}(x)+13.681\,e^{-1+1/e+2e}\,\pi^{2-2e}\left\lfloor\frac{\arg(i\,(0.761048-x))}{2\pi}\right\rfloor-32.9611\,e^{-1+1/e+2e}\,\pi^{2-2e}\left\lfloor\frac{\arg(i\,(0.833569-x))}{2\pi}\right\rfloor+19.6895\,e^{-1+1/e+2e}\,\pi^{2-2e}\left\lfloor\frac{\arg(i\,(0.913-x))}{2\pi}\right\rfloor+\sum_{k=1}^{\infty}\frac{1}{k}\,e^{-1+1/e+2e}\,i\,\pi^{1-2e}\left(6.84049\,(0.761048-x)^k(-i-x)^k-16.4806\,(0.833569-x)^k(-i-x)^k+9.84474\,(0.913-x)^k(-i-x)^k-6.84049\,(0.761048-x)^k(i-x)^k+16.4806\,(0.833569-x)^k(i-x)^k-9.84474\,(0.913-x)^k(i-x)^k\right)(-i-x)^{-k}(i-x)^{-k}\quad\text{for }(i\,x\in\mathbb{R}\text{ and }i\,x<-1)$$

F_n is the n^{th} Fibonacci number

Integral representations

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=\int_0^1\frac{e^{-1+1/e+2e}\,\pi^{1-2e}\left(2.72158-1.42109\times 10^{-14}\,t^2\right)}{2.98092+6.2826\,t^2+4.36538\,t^4+t^6}\,dt$$

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}e^{-1.59077\,s}\left(-4.49412\,e^{0.984502\,s}+6.86885\,e^{1.06318\,s}-2.60297\,e^{1.13385\,s}\right)e^{-1+1/e+2e}\,i\,\pi^{-1/2-2e}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds\quad\text{for }0<\gamma<\frac{1}{2}$$

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{1}{i\,\Gamma\left(\frac{3}{2}-s\right)}\left(4.49412\,e^{0.182039\,s}-6.86885\,e^{0.364078\,s}+2.60297\,e^{0.546116\,s}\right)e^{-1+1/e+2e}\,\pi^{-2e}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)\,ds\quad\text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=19.6895\,e^{-1+1/e+2e}\,\pi^{1-2e}\left(\frac{0.528805}{1+\operatorname*{K}_{k=1}^{\infty}\frac{0.579195\,k^2}{1+2k}}-\frac{1.39543}{1+\operatorname*{K}_{k=1}^{\infty}\frac{0.694837\,k^2}{1+2k}}+\frac{0.913}{1+\operatorname*{K}_{k=1}^{\infty}\frac{0.833569\,k^2}{1+2k}}\right)=14.9672$$

$$\left(\frac{0.528805}{1+\frac{0.579195}{3+\frac{2.31678}{5+\frac{5.21275}{7+\frac{9.26712}{9+\dots}}}}}-\frac{1.39543}{1+\frac{0.694837}{3+\frac{2.77935}{5+\frac{6.25354}{7+\frac{11.1174}{9+\dots}}}}}+\frac{0.913}{1+\frac{0.833569}{3+\frac{3.33428}{5+\frac{7.50212}{7+\frac{13.3371}{9+\dots}}}}}\right)$$

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=19.6895\,e^{-1+1/e+2e}\,\pi^{1-2e}\left(\frac{0.528805}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.579195(1-2k)^2}{1.57919+0.84161k}}-\frac{1.39543}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.694837(1-2k)^2}{1.69484+0.610325k}}+\frac{0.913}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.833569(1-2k)^2}{1.83357+0.332862k}}\right)=$$

$$14.9672\left(\frac{0.528805}{1+\frac{0.579195}{2.42081+\frac{5.21275}{3.26242+\frac{14.4799}{4.10403+\frac{28.3805}{4.94564+\ldots}}}}}-\frac{1.39543}{1+\frac{0.694837}{2.30516+\frac{6.25354}{2.91549+\frac{17.3709}{3.52581+\frac{34.047}{4.13614+\ldots}}}}}+\frac{0.913}{1+\frac{0.833569}{2.16643+\frac{7.50212}{2.49929+\frac{20.8392}{2.83215+\frac{40.8449}{3.16502+\ldots}}}}}\right)$$

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=$$

$$19.6895\,e^{-1+1/e+2e}\,\pi^{1-2e}\left(0.0463699-\frac{0.306281}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.579195\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}+\frac{0.9696}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.694837\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}-\frac{0.761048}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.833569\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)=$$

$$14.9672\left(0.0463699-\frac{0.306281}{3+\frac{5.21275}{5+\frac{2.31678}{7+\frac{14.4799}{9+\frac{9.26712}{11+\dots}}}}}+\frac{0.9696}{3+\frac{6.25354}{5+\frac{2.77935}{7+\frac{17.3709}{9+\frac{11.1174}{11+\dots}}}}}-\frac{0.761048}{3+\frac{7.50212}{5+\frac{3.33428}{7+\frac{20.8392}{9+\frac{13.3371}{11+\dots}}}}}\right)$$

$$\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)=$$

$$19.6895\,e^{-1+1/e+2e}\,\pi^{1-2e}\left(\frac{0.528805}{1.57919+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.15839\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.2896+0.289597(-1)^k\right)(1+2k)}}-\frac{1.39543}{1.69484+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.38967\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.34742+0.347419(-1)^k\right)(1+2k)}}+\frac{0.913}{1.83357+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.66714\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.41678+0.416785(-1)^k\right)(1+2k)}}\right)=$$

$$14.9672\left(\frac{0.528805}{1.57919+-\frac{1.15839}{3.-\frac{1.15839}{7.89597-\frac{6.95034}{7.-\frac{6.95034}{14.2128+\ldots}}}}}-\frac{1.39543}{1.69484+-\frac{1.38967}{3-\frac{1.38967}{8.47419-\frac{8.33805}{7-\frac{8.33805}{15.2535+\ldots}}}}}+\frac{0.913}{1.83357+-\frac{1.66714}{3-\frac{1.66714}{9.16785-\frac{10.0028}{7-\frac{10.0028}{16.5021+\ldots}}}}}\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

From which:

1/(((e^(-1 + 1/e + 2 e) π^(1 - 2 e)*(tan^(-1)(0.913) - (0.913 + 0.913^3) tan^(-1)(0.913*0.913) + 0.913^4 tan^(-1)(0.913^2*0.913))/((-1 + 0.913^2)^2 (1 + 0.913^2)))))^333 ((3/91)^(3/4) π)

where

$$\left(\frac{3}{91}\right)^{3/4} \pi \approx 0.243057915 7$$

Input

$$1/\left(e^{-1+1/e+2e}\, \pi^{1-2e} \times \left(\tan^{-1}(0.913) - \left(0.913 + 0.913^3\right) \tan^{-1}(0.913 \times 0.913) + 0.913^4 \tan^{-1}\left(0.913^2 \times 0.913\right)\right) / \left(\left(-1 + 0.913^2\right)^2 \left(1 + 0.913^2\right)\right)\right)^{333} \left(\left(\frac{3}{91}\right)^{3/4} \pi\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

$3.51606\ldots \times 10^{121}$

(result in radians)

$0.351606\ldots.*10^{122} \approx \Lambda_Q$

The observed value of ρ_Λ or Λ today is precisely the classical dual of its quantum precursor values ρ_Q , Λ_Q in the quantum very early precursor vacuum U_Q as determined by our dual equations

The study of this function provides the following representations:

Alternative representations

$$\frac{\left(\frac{3}{91}\right)^{3/4}\pi}{\left(\frac{e^{-1+1/e+2e}\pi^{1-2e}\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)}{\left(-1+0.913^2\right)^2\left(1+0.913^2\right)}\right)^{333}} = \left(\left(\pi\left(\frac{3}{91}\right)^{3/4}\right)\Big/\left(\left(\left(\text{sc}^{-1}(0.913\mid 0)-\text{sc}^{-1}(0.833569\mid 0)\left(0.913+0.913^3\right)+\text{sc}^{-1}\left(0.913\times 0.913^2\mid 0\right)0.913^4\right)e^{-1+2e+1/e}\pi^{1-2e}\right)\Big/\left(\left(1+0.913^2\right)\left(-1+0.913^2\right)^2\right)\right)^{333}=0\right)$$

$$\frac{\left(\frac{3}{91}\right)^{3/4}\pi}{\left(\frac{e^{-1+1/e+2e}\pi^{1-2e}\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)}{\left(-1+0.913^2\right)^2\left(1+0.913^2\right)}\right)^{333}} = \left(\left(\pi\left(\frac{3}{91}\right)^{3/4}\right)\Big/\left(\left(\left(\tan^{-1}(1, 0.913)-\tan^{-1}(1, 0.833569)\left(0.913+0.913^3\right)+\tan^{-1}\left(1, 0.913\times 0.913^2\right)0.913^4\right)e^{-1+2e+1/e}\pi^{1-2e}\right)\Big/\left(\left(1+0.913^2\right)\left(-1+0.913^2\right)^2\right)\right)^{333}=0\right)$$

$$\frac{\left(\frac{3}{91}\right)^{3/4}\pi}{\left(\frac{e^{-1+1/e+2e}\pi^{1-2e}\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)}{\left(-1+0.913^2\right)^2\left(1+0.913^2\right)}\right)^{333}} = \left(\left(\pi\left(\frac{3}{91}\right)^{3/4}\right)\Big/\left(\left(\left(\cot^{-1}\left(\frac{1}{0.913}\right)-\cot^{-1}\left(\frac{1}{0.833569}\right)\left(0.913+0.913^3\right)+\cot^{-1}\left(\frac{1}{0.913\times 0.913^2}\right)0.913^4\right)e^{-1+2e+1/e}\pi^{1-2e}\right)\Big/\left(\left(1+0.913^2\right)\left(-1+0.913^2\right)^2\right)\right)^{333}=0\right)$$

$\text{sc}^{-1}(x\mid m)$ is the inverse of the Jacobi elliptic function sc

Continued fraction representations

$$\frac{\left(\frac{3}{91}\right)^{3/4}\pi}{\left(\frac{e^{-1+1/e+2e}\pi^{1-2e}\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)}{\left(-1+0.913^2\right)^2\left(1+0.913^2\right)}\right)^{333}}$$

$$= \frac{8.101007156592956\times 10^{-433}\, e^{333(1-1/e-2e)}\,\pi^{-332+666e}}{\left(\frac{0.528805}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.579195k^2}{1+2k}}-\frac{1.39543}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.694837k^2}{1+2k}}+\frac{0.913}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.833569k^2}{1+2k}}\right)^{333}} =$$

$$\frac{1.158691846054800\times 10^{-392}}{\left(\frac{0.528805}{1+\frac{0.579195}{3+\frac{2.31678}{5+\frac{5.21275}{7+\frac{9.26712}{9+\ldots}}}}}-\frac{1.39543}{1+\frac{0.694837}{3+\frac{2.77935}{5+\frac{6.25354}{7+\frac{11.1174}{9+\ldots}}}}}+\frac{0.913}{1+\frac{0.833569}{3+\frac{3.33428}{5+\frac{7.50212}{7+\frac{13.3371}{9+\ldots}}}}}\right)^{333}}$$

$$\frac{\left(\frac{3}{91}\right)^{3/4}\pi}{\left(\frac{e^{-1+1/e+2e}\pi^{1-2e}\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)}{\left(-1+0.913^2\right)^2\left(1+0.913^2\right)}\right)^{333}}$$

$$= \frac{8.101007156592956\times 10^{-433}\, e^{333(1-1/e-2e)}\,\pi^{-332+666e}}{\left(\frac{0.528805}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.579195(1-2k)^2}{1.57919+0.84161k}}-\frac{1.39543}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.694837(1-2k)^2}{1.69484+0.610325k}}+\frac{0.913}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.833569(1-2k)^2}{1.83357+0.332862k}}\right)^{333}} =$$

$$1.158691846054800\times 10^{-392} \Bigg/ \left(\frac{0.528805}{1+\frac{0.579195}{2.42081+\frac{5.21275}{3.26242+\frac{14.4799}{4.10403+\frac{28.3805}{4.94564+\dots}}}}}-\frac{1.39543}{1+\frac{0.694837}{2.30516+\frac{6.25354}{2.91549+\frac{17.3709}{3.52581+\frac{34.047}{4.13614+\dots}}}}}+\frac{0.913}{1+\frac{0.833569}{2.16643+\frac{7.50212}{2.49929+\frac{20.8392}{2.83215+\frac{40.8449}{3.16502+\dots}}}}}\right)^{333}$$

$$\frac{\left(\frac{3}{91}\right)^{3/4}\pi}{\left(\frac{e^{-1+1/e+2e}\,\pi^{1-2e}\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)}{\left(-1+0.913^2\right)^2\left(1+0.913^2\right)}\right)^{333}}$$

$$= \left(8.101007156592956\times 10^{-433}\, e^{333(1-1/e-2e)}\,\pi^{-332+666e}\right) \Big/ \left(0.0463699-\frac{0.306281}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.579195\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}+\frac{0.9696}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.694837\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}-\frac{0.761048}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.833569\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)^{333} =$$

$$\frac{1.158691846054800\times 10^{-392}}{\left(0.0463699-\cfrac{0.306281}{3+\cfrac{5.21275}{5+\cfrac{2.31678}{7+\cfrac{14.4799}{9+\cfrac{9.26712}{11+\ldots}}}}}+\cfrac{0.9696}{3+\cfrac{6.25354}{5+\cfrac{2.77935}{7+\cfrac{17.3709}{9+\cfrac{11.1174}{11+\ldots}}}}}-\cfrac{0.761048}{3+\cfrac{7.50212}{5+\cfrac{3.33428}{7+\cfrac{20.8392}{9+\cfrac{13.3371}{11+\ldots}}}}}\right)^{333}}$$

$$\frac{\left(\frac{3}{91}\right)^{3/4}\pi}{\left(\frac{e^{-1+1/e+2e}\,\pi^{1-2e}\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)}{\left(-1+0.913^2\right)^2\left(1+0.913^2\right)}\right)^{333}}$$

$$= \left(8.101007156592956\times 10^{-433}\, e^{333(1-1/e-2e)}\,\pi^{-332+666e}\right) \Big/ \left(\frac{0.528805}{1.57919+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{1.15839\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.2896+0.289597(-1)^k\right)(1+2k)}} - \frac{1.39543}{1.69484+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{1.38967\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.34742+0.347419(-1)^k\right)(1+2k)}} + \frac{0.913}{1.83357+\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{1.66714\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1.41678+0.416785(-1)^k\right)(1+2k)}}\right)^{333} =$$

$$1.158691846054800\times 10^{-392} \Big/ \left(\frac{0.528805}{1.57919+-\frac{1.15839}{3.-\frac{1.15839}{7.89597-\frac{6.95034}{7.-\frac{6.95034}{14.2128+\ldots}}}}} - \frac{1.39543}{1.69484+-\frac{1.38967}{3-\frac{1.38967}{8.47419-\frac{8.33805}{7-\frac{8.33805}{15.2535+\ldots}}}}} + \frac{0.913}{1.83357+-\frac{1.66714}{3-\frac{1.66714}{9.16785-\frac{10.0028}{7-\frac{10.0028}{16.5021+\ldots}}}}}\right)^{333}$$

We obtain also, after some calculations:

2(1/(((e^(-1 + 1/e + 2 e) π^(1 - 2 e)*(tan^(-1)(0.913) - (0.913 + 0.913^3) tan^(-1)(0.913*0.913) + 0.913^4 tan^(-1)(0.913^2*0.913))/((-1 + 0.913^2)^2 (1 + 0.913^2))))))^8+8

Input

$$2\left(1/\left(e^{-1+1/e+2e}\,\pi^{1-2e}\times\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times0.913)+0.913^4\tan^{-1}\left(0.913^2\times0.913\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)\right)\right)^8+8$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

1729.13…

(result in radians)

1729.13….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representations

$$2\left(1/\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times0.913)+0.913^4\tan^{-1}\left(0.913^2\times0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)\right)^8+8=$$
$$8+2\left(1/\left(\left(\cot^{-1}\left(\frac{1}{0.913}\right)-\cot^{-1}\left(\frac{1}{0.833569}\right)\left(0.913+0.913^3\right)+\cot^{-1}\left(\frac{1}{0.913\times0.913^2}\right)0.913^4\right)e^{-1+2e+1/e}\,\pi^{1-2e}\right)/\left(\left(1+0.913^2\right)\left(-1+0.913^2\right)^2\right)\right)^8$$

$$2\left(1/\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times0.913)+0.913^4\tan^{-1}\left(0.913^2\times0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)\right)^8+8=$$

$$\left(8+2\left(1/\left(\left(\mathrm{sc}^{-1}(0.913\mid 0)-\mathrm{sc}^{-1}(0.833569\mid 0)\left(0.913+0.913^3\right)+\mathrm{sc}^{-1}\left(0.913\times0.913^2\mid 0\right)0.913^4\right)e^{-1+2e+1/e}\,\pi^{1-2e}\right)/\left(\left(1+0.913^2\right)\left(-1+0.913^2\right)^2\right)\right)^8=$$

$$8+\left(8.85433\times10^{-11}\,e^{8-8/e-16e}\,\pi^{-8+16e}\right)/\left(0.694837\,\mathrm{sc}^{-1}(0.761048\mid 0)-1.67405\,\mathrm{sc}^{-1}(0.833569\mid 0)+\mathrm{sc}^{-1}(0.913\mid 0)\right)^8\right)$$

$$2\left(1/\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times0.913)+0.913^4\tan^{-1}\left(0.913^2\times0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)\right)^8+8=$$

$$\left(8+2\left(1/\left(\left(\tan^{-1}(1,\,0.913)-\tan^{-1}(1,\,0.833569)\left(0.913+0.913^3\right)+\tan^{-1}\left(1,\,0.913\times0.913^2\right)0.913^4\right)e^{-1+2e+1/e}\,\pi^{1-2e}\right)/\left(\left(1+0.913^2\right)\left(-1+0.913^2\right)^2\right)\right)^8=$$

$$8+\left(8.85433\times10^{-11}\,e^{8-8/e-16e}\,\pi^{-8+16e}\right)/\left(0.694837\tan^{-1}(1,\,0.761048)-1.67405\tan^{-1}(1,\,0.833569)+\tan^{-1}(1,\,0.913)\right)^8\right)$$

$\cot^{-1}(x)$ is the inverse cotangent function

$\mathrm{sc}^{-1}(x\mid m)$ is the inverse of the Jacobi elliptic function sc

Continued fraction representations

$$2\left(1/\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^{3}\right)\tan^{-1}(0.913\times 0.913)+0.913^{4}\tan^{-1}\left(0.913^{2}\times 0.913\right)\right)\right)/\left(\left(-1+0.913^{2}\right)^{2}\left(1+0.913^{2}\right)\right)\right)^{8}+8=$$

$$8+\frac{8.85433\times 10^{-11}\,e^{8-8/e-16e}\,\pi^{-8+16e}}{\left(\frac{0.528805}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.579195\,k^{2}}{1+2k}}-\frac{1.39543}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.694837\,k^{2}}{1+2k}}+\frac{0.913}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.833569\,k^{2}}{1+2k}}\right)^{8}}=$$

$$8+\frac{7.94158\times 10^{-10}}{\left(\frac{0.528805}{1+\frac{0.579195}{3+\frac{2.31678}{5+\frac{5.21275}{7+\frac{9.26712}{9+\dots}}}}}-\frac{1.39543}{1+\frac{0.694837}{3+\frac{2.77935}{5+\frac{6.25354}{7+\frac{11.1174}{9+\dots}}}}}+\frac{0.913}{1+\frac{0.833569}{3+\frac{3.33428}{5+\frac{7.50212}{7+\frac{13.3371}{9+\dots}}}}}\right)^{8}}$$

$$2\left(1/\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)\right)^8+8=$$

$$8+\left(8.85433\times 10^{-11}\,e^{8-8/e-16e}\,\pi^{-8+16e}\right)/\left(0.913+0.694837\left(0.761048-\frac{0.440795}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.579195\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)-1.67405\left(0.833569-\frac{0.579195}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.694837\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)-\frac{0.761048}{3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.833569\left(1+(-1)^{1+k}+k\right)^2}{3+2k}}\right)^8=$$

$$8+7.94158\times 10^{-10}/\left(0.913+0.694837\left(0.761048-\frac{0.440795}{3+\frac{5.21275}{5+\frac{2.31678}{7+\frac{14.4799}{9+\frac{9.26712}{11+\ldots}}}}}\right)-1.67405\left(0.833569-\frac{0.579195}{3+\frac{6.25354}{5+\frac{2.77935}{7+\frac{17.3709}{9+\frac{11.1174}{11+\ldots}}}}}\right)-\frac{0.761048}{3+\frac{7.50212}{5+\frac{3.33428}{7+\frac{20.8392}{9+\frac{13.3371}{11+\ldots}}}}}\right)^8$$

$$2\left(1/\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)\right)^8+8=$$

$$8+\left(8.85433\times 10^{-11}\,e^{8-8/e-16e}\,\pi^{-8+16e}\right)\Bigg/\left(\frac{0.528805}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.579195(-1+2k)^2}{1+2k-0.579195(-1+2k)}}-\frac{1.39543}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.694837(-1+2k)^2}{1+2k-0.694837(-1+2k)}}+\frac{0.913}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{0.833569(-1+2k)^2}{1+2k-0.833569(-1+2k)}}\right)^8=$$

$$8+7.94158\times 10^{-10}\Bigg/\left(\frac{0.528805}{1+\frac{0.579195}{2.42081+\frac{5.21275}{3.26242+\frac{14.4799}{4.10403+\frac{28.3805}{4.94564+\ldots}}}}}-\frac{1.39543}{1+\frac{0.694837}{2.30516+\frac{6.25354}{2.91549+\frac{17.3709}{3.52581+\frac{34.047}{4.13614+\ldots}}}}}+\frac{0.913}{1+\frac{0.833569}{2.16643+\frac{7.50212}{2.49929+\frac{20.8392}{2.83215+\frac{40.8449}{3.16502+\ldots}}}}}\right)^8$$

$$2\left(1/\left(\left(e^{-1+1/e+2e}\,\pi^{1-2e}\right)\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)\right)^8+8=$$

$$8+\left(8.85433\times 10^{-11}\,e^{8-8/e-16e}\,\pi^{-8+16e}\right)\Bigg/\left(\frac{0.528805}{1.57919+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.15839\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+0.289597\left(1+(-1)^k\right)\right)(1+2k)}}-\frac{1.39543}{1.69484+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.38967\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+0.347419\left(1+(-1)^k\right)\right)(1+2k)}}+\frac{0.913}{1.83357+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{1.66714\left(1-2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\left\lfloor\frac{1+k}{2}\right\rfloor}{\left(1+0.416785\left(1+(-1)^k\right)\right)(1+2k)}}\right)^8=$$

$$8+7.94158\times 10^{-10}\Bigg/\left(\frac{0.528805}{1.57919+-\frac{1.15839}{3-\frac{1.15839}{7.89597-\frac{6.95034}{7-\frac{6.95034}{14.2128+\ldots}}}}}-\frac{1.39543}{1.69484+-\frac{1.38967}{3-\frac{1.38967}{8.47419-\frac{8.33805}{7-\frac{8.33805}{15.2535+\ldots}}}}}+\frac{0.913}{1.83357+-\frac{1.66714}{3-\frac{1.66714}{9.16785-\frac{10.0028}{7-\frac{10.0028}{16.5021+\ldots}}}}}\right)^8$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k/b_k$ is a continued fraction

(1/27((2(1/(((e^(-1 + 1/e + 2 e) π^(1 - 2 e)*(tan^(-1)(0.913) - (0.913 + 0.913^3) tan^(-1)(0.913*0.913) + 0.913^4 tan^(-1)(0.913^2*0.913))/((-1 + 0.913^2)^2 (1 + 0.913^2))))))^8+8)-1))^2-Φ

Input

$$\left(\frac{1}{27}\left(\left(2\left(1/\left(e^{-1+1/e+2e}\,\pi^{1-2e}\times\left(\tan^{-1}(0.913)-\left(0.913+0.913^3\right)\tan^{-1}(0.913\times 0.913)+0.913^4\tan^{-1}\left(0.913^2\times 0.913\right)\right)/\left(\left(-1+0.913^2\right)^2\left(1+0.913^2\right)\right)\right)\right)\right)^8+8\right)-1\right)^2-\Phi$$

$\tan^{-1}(x)$ is the inverse tangent function

Φ is the golden ratio conjugate

Result

4095.99…

(result in radians)

4095.99…. ≈ 4096 = 64^2

where 4096 and 64 are fundamental values indicated in the Ramanujan paper **"Modular equations and Approximations to π"**

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24}+g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12}+(1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

(2(1/(((e^(-1 + 1/e + 2 e) π^(1 - 2 e)*(tan^(-1)(0.913) - (0.913 + 0.913^3) tan^(-1)(0.913*0.913) + 0.913^4 tan^(-1)(0.913^2*0.913))/((-1 + 0.913^2)^2 (1 + 0.913^2))))))^8+8)^1/15

Input

$$(2(1/(e^{-1+1/e+2e}\pi^{1-2e}\times(\tan^{-1}(0.913)-(0.913+0.913^3)\tan^{-1}(0.913\times0.913)+0.913^4\tan^{-1}(0.913^2\times0.913))/((-1+0.913^2)^2(1+0.913^2))))^8+8)\,\hat{}\,(1/15)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result

1.6438233238803693576543321042556651287611541902774423952804536994...

(result in radians)

1.64382332388.... ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

From:

Open Descendants of $Z_2 \times Z_2$ Freely-Acting Orbifolds

I. Antoniadis, G. D'Appollonio, E. Dudas and A. Sagnotti - arXiv:hep-th/9907184v1 25 Jul 1999

We have the following equation:

$$\tilde{\mathcal{A}} = \frac{2^{-5}}{4}\Big\{(Q_o+Q_v)\Big[N^2vW^4+\frac{1}{v}\sum_m(D+\frac{\delta}{2}e^{2i\pi\alpha m}+\frac{\delta}{2}e^{-2i\pi\alpha m})^2P^3P_m\Big]$$
$$+\,2N(D+\delta)(Q_o-Q_v)(\frac{2\eta}{\theta_2})^2+4(Q_s+Q_c)\,(R_N^2+R_D^2)\,(\frac{2\eta}{\theta_4})^2-2R_NR_D\,(Q_s-Q_c)(\frac{2\eta}{\theta_3})^2\Big\}\,,$$

We consider:

((2η)/θ_4)^2

Input

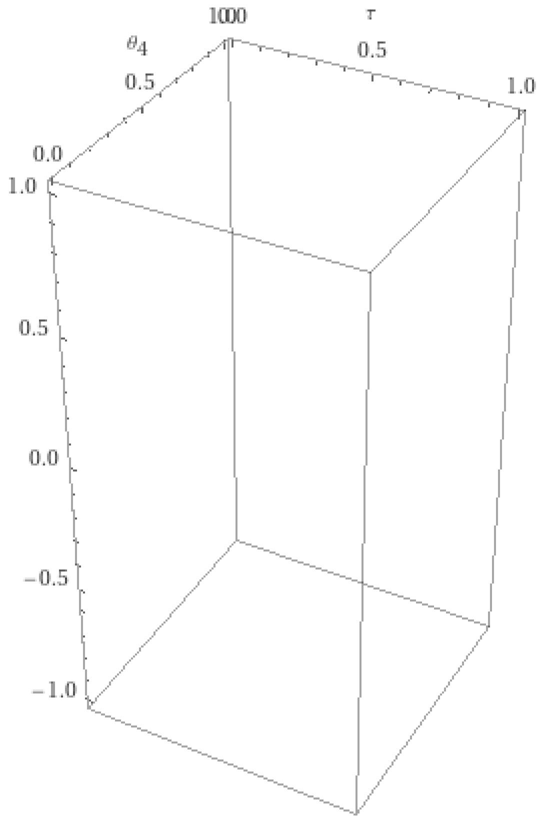
$$\left(\frac{2\,\eta(\tau)}{\theta_4}\right)^2$$

$\eta(\tau)$ is the Dedekind eta function

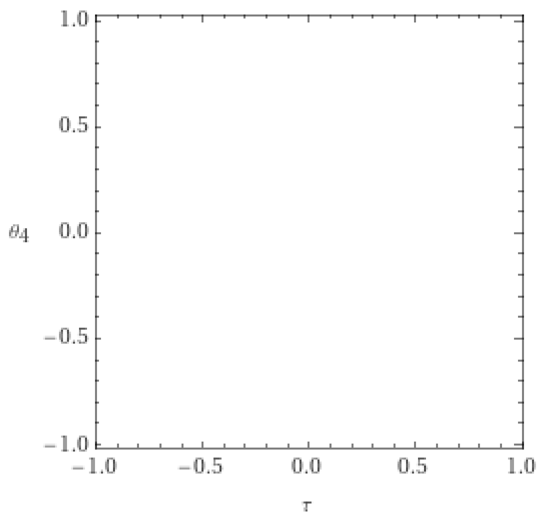
Result

$$\frac{4\,\eta(\tau)^2}{\theta_4^2}$$

3D plot



Contour plot



Series expansion at τ=0

$$e^{-(25 i \pi)/(6 \tau)} \left(\left(1 + e^{(4 i \pi)/\tau}\right) \left(\frac{4 i}{\theta_4^2 \tau} + O\left(\tau^{32}\right) \right) + e^{(2 i \pi)/\tau} \left(-\frac{8 i}{\theta_4^2 \tau} + O\left(\tau^{32}\right) \right) \right)$$

For τ = θ = 0.5 :

(4 η(1/2)^2)/0.5^2

Input

$$\frac{4 \eta\left(\frac{1}{2}\right)^2}{0.5^2}$$

$\eta(\tau)$ is the Dedekind eta function

Exact result

$$16 \eta\left(\frac{1}{2}\right)^2$$

The study of this function provides the following representations:

Series representations

$$\frac{4 \eta\left(\frac{1}{2}\right)^2}{0.5^2} = (15.4548 + 4.1411 i) e^{-2 \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} e^{i k n \pi} / k}$$

$$\frac{4 \eta\left(\frac{1}{2}\right)^2}{0.5^2} = (15.4548 + 4.1411 i) \left(\sum_{k=-\infty}^{\infty} e^{1/2 i k (1+3 k) \pi} \right)^2$$

$$\frac{4\,\eta\left(\frac{1}{2}\right)^2}{0.5^2} = (27.7128 + 16.\,i)\left(\sum_{k=0}^{\infty}(-1)^k\, e^{-2\,i\,k\,(1+k)\,\pi}\,(1+2\,k)\right)^{2/3}$$

Integral representation

$$\frac{4\,\eta\left(\frac{1}{2}\right)^2}{0.5^2} = 9.44272 \exp\left(\frac{2\,i}{\pi}\int_i^{\frac{1}{2}} \text{WeierstrassZeta}[1, \text{WeierstrassInvariants}[\{1, t\}]]\,dt\right)$$

From the result

$$16\,\eta\left(\frac{1}{2}\right)^2$$

we obtain:

16 η(1/2)^2

Input

$$16\,\eta\left(\frac{1}{2}\right)^2$$

The study of this function provides the following representations:

$\eta(\tau)$ is the Dedekind eta function

Series representations

$$16\,\eta\left(\frac{1}{2}\right)^2 = 16\,\sqrt[12]{-1}\; e^{-2\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} e^{i\,k\,n\,\pi}/k}$$

$$16\,\eta\left(\frac{1}{2}\right)^2 = 16\sqrt[12]{-1}\left(\sum_{k=-\infty}^{\infty} e^{1/2\,i\,k\,(1+3\,k)\,\pi}\right)^2$$

$$16\,\eta\left(\frac{1}{2}\right)^2 = 32\sqrt[6]{-1}\left(\sum_{k=0}^{\infty}(-1)^k\,e^{-2\,i\,k\,(1+k)\,\pi}\,(1+2\,k)\right)^{2/3}$$

Integral representation

$$16\,\eta\left(\frac{1}{2}\right)^2 = \frac{4\exp\left(\frac{2\,i}{\pi}\int_i^{\frac{1}{2}}\text{WeierstrassZeta}[1,\text{WeierstrassInvariants}[\{1,t\}]]\,dt\right)\Gamma\left(\frac{1}{4}\right)^2}{\pi^{3/2}}$$

From:

$$16\,\eta\left(\frac{1}{2}\right)^2 = 32\sqrt[6]{-1}\left(\sum_{k=0}^{\infty}(-1)^k\,e^{-2\,i\,k\,(1+k)\,\pi}\,(1+2\,k)\right)^{2/3}$$

for k = 2 :

32 (-1)^(1/6) (-1)^2 e^(-2 i *2 (1 + 2) π) (1 + 2*2)^(2/3)

Input

$$32\sqrt[6]{-1}\,(-1)^2\,e^{-2\,(i\times 2)\,(1+2)\,\pi}\,(1+2\times 2)^{2/3}$$

i is the imaginary unit

Exact result

$$32\sqrt[6]{-1}\,5^{2/3}$$

Decimal approximation

81.03275655707706725895562039584004009610274895557229787114423039…
+
46.78428381140585704810859776220676445648797816161648066276117616…
i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates

$r = 32 \times 5^{2/3}$ (radius), $\theta = \frac{\pi}{6}$ (angle)

Exact result

$32 \times 5^{2/3}$

Decimal approximation

93.568567622811714096217195524413528912975956323232961325522352322
…

93.568567622….

The study of this function provides the following representations:

Polar forms

$$32 \times 5^{2/3} \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right)\right)$$

Approximate form

$$32 \times 5^{2/3} e^{(i \pi)/6}$$

Alternate forms

$$32 \times 5^{2/3} e^{(i \pi)/6}$$

$$16 \sqrt{3}\ 5^{2/3} + 16 i\, 5^{2/3}$$

Alternative representations

$$32\sqrt[6]{-1}\ (-1)^2\ e^{-2 i\, 2 (1+2) \pi}\ (1+2\times 2)^{2/3} = 32\,(-1)^{12 i\, i}\ \sqrt[6]{-1}\ 5^{2/3}$$

$$32\sqrt[6]{-1}\ (-1)^2\ e^{-2 i\, 2 (1+2) \pi}\ (1+2\times 2)^{2/3} = 32\sqrt[6]{-1}\ 5^{2/3}\ e^{-2160 ^\circ i}$$

$$32\sqrt[6]{-1}\ (-1)^2\ e^{-2 i\, 2 (1+2) \pi}\ (1+2\times 2)^{2/3} = 32\sqrt[6]{-1}\ 5^{2/3}\ e^{12 i^2 \log(-1)}$$

Series representations

$$32\sqrt[6]{-1}\ (-1)^2\ e^{-2 i\, 2 (1+2) \pi}\ (1+2\times 2)^{2/3} = 32\sqrt[6]{-1}\ 5^{2/3} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-48 i \sum_{k=0}^{\infty} (-1)^k / (1+2 k)}$$

$$32\sqrt[6]{-1}\ (-1)^2\ e^{-2 i\, 2 (1+2) \pi}\ (1+2\times 2)^{2/3} = 32\sqrt[6]{-1}\ 5^{2/3} \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{-48 i \sum_{k=0}^{\infty} (-1)^k / (1+2 k)}$$

$$32\sqrt[6]{-1}\ (-1)^2\ e^{-2 i\, 2 (1+2) \pi}\ (1+2\times 2)^{2/3} = 32\sqrt[6]{-1}\ 5^{2/3} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-12 i \sum_{k=1}^{\infty} 4^{-k} \left(-1+3^k\right) \zeta(1+k)}$$

$\zeta(s)$ is the Riemann zeta function

Integral representations

$$32\sqrt[6]{-1}\,(-1)^2\,e^{-2i\,2(1+2)\pi}\,(1+2\times 2)^{2/3} = 32\sqrt[6]{-1}\,5^{2/3}\,e^{-24i\times\int_0^\infty 1/(1+t^2)dt}$$

$$32\sqrt[6]{-1}\,(-1)^2\,e^{-2i\,2(1+2)\pi}\,(1+2\times 2)^{2/3} = 32\sqrt[6]{-1}\,5^{2/3}\,e^{-48i\int_0^1\sqrt{1-t^2}\,dt}$$

$$32\sqrt[6]{-1}\,(-1)^2\,e^{-2i\,2(1+2)\pi}\,(1+2\times 2)^{2/3} = 32\sqrt[6]{-1}\,5^{2/3}\,e^{-24i\int_0^\infty \sin(t)/t\,dt}$$

From this other equation:

$$\frac{2^{-5}}{4}\Big\{(Q_o+Q_v)\Big[N^2vW^4+\frac{1}{v}\sum_m(D+\frac{\delta}{2}e^{2i\pi\alpha m}+\frac{\delta}{2}e^{-2i\pi\alpha m})^2P^3P_m\Big]$$
$$+\,2N(D+\delta)(Q_o-Q_v)(\frac{2\eta}{\theta_2})^2+4(Q_s+Q_c)\,(R_N^2+R_D^2)\,(\frac{2\eta}{\theta_4})^2-2R_NR_D\,(Q_s-Q_c)(\frac{2\eta}{\theta_3})^2\Big\}$$

We consider:

$N = D_1 = D_2 = D_3 = 32 \quad D = \delta$

P = W = 2 ; Q_i = 8 ; Q_j = 16 ; R_i = 16 ; v = 1

((2η)/θ_4)^2 = ((2η)/θ_3)^2 = ((2η)/θ_2)^2 = 93.5685676228117

and obtain:

1/4*1/32 [(((((32)((((32^2*2^4+(32+16*e^(2π)+16*e^(-2π))^2)*8*2))))+64*64*(16-8)* 93.5685676228117^2+4(16+8)(16^2+16^2)* ((93.5685676228117^2-2*16*16))(16-8)* 93.5685676228117^2)))]

Input interpretation

$$\frac{1}{4}\times\frac{1}{32}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)\times 8\times 2\right)+64\times 64\,(16-8)\times 93.5685676228117^2+4\,(16+8)\left(\left(16^2+16^2\right)\left(93.5685676228117^2-2\times 16\times 16\right)\right)\left((16-8)\times 93.5685676228117^2\right)\right)$$

Result

$2.2200060498872\ldots\times 10^{11}$

2.22000604….*10^11

The study of this function provides the following representations:

Alternative representations

$$\frac{1}{32\times 4}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.56856762281170000^2+4\left(16^2+16^2\right)\left(93.56856762281170000^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.56856762281170000^2\right)\right)=\frac{1}{4\times 32}\left(32\,768\times 93.56856762281170000^2+1536\times 16^2\left(-512+93.56856762281170000^2\right)93.56856762281170000^2+512\left(2^4\times 32^2+\left(32+16\,e^{-360^\circ}+16\,e^{360^\circ}\right)^2\right)\right)$$

$$\frac{1}{32\times 4}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.56856762281170000^2+4\left(16^2+16^2\right)\left(93.56856762281170000^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.56856762281170000^2\right)\right)=\frac{1}{4\times 32}\left(32\,768\times 93.56856762281170000^2+1536\times 16^2\left(-512+93.56856762281170000^2\right)93.56856762281170000^2+512\left(2^4\times 32^2+\left(32+16\,e^{-2i\log(-1)}+16\,e^{2i\log(-1)}\right)^2\right)\right)$$

$$\frac{1}{32\times 4}\Big(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+
64\times 64\,(16-8)\,93.56856762281170000^2+
4\left(16^2+16^2\right)\left(93.56856762281170000^2-2\times 16\times 16\right)
(16+8)\left((16-8)\,93.56856762281170000^2\right)\Big)=
\frac{1}{32\times 4}\Big(32\left(\left(32^2\times 2^4+\left(32+16\exp^{2\pi}(z)+16\exp^{-2\pi}(z)\right)^2\right)8\times 2\right)+
64\times 64\,(16-8)\,93.56856762281170000^2+
4\left(16^2+16^2\right)\left(93.56856762281170000^2-2\times 16\times 16\right)
(16+8)\left((16-8)\,93.56856762281170000^2\right)\Big)\quad \text{for } z=1$$

Series representations

$$\frac{1}{32\times 4}\Big(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+
64\times 64\,(16-8)\,93.56856762281170000^2+
4\left(16^2+16^2\right)\left(93.56856762281170000^2-2\times 16\times 16\right)
(16+8)\left((16-8)\,93.56856762281170000^2\right)\Big)=
5.9604644775390625\times 10^{-8}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}
\Bigg(1.717986918400000000\times 10^{10}+
6.87194767360000000\times 10^{10}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+
3.71958895314365707\times 10^{18}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+
6.87194767360000000\times 10^{10}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+
1.717986918400000000\times 10^{10}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\Bigg)$$

$$\frac{1}{32\times 4}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.56856762281170000^2+4\left(16^2+16^2\right)\left(93.56856762281170000^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.56856762281170000^2\right)\right)=$$

$$5.9604644775390625\times 10^{-8}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-16\sum_{k=1}^{\infty}\tan^{-1}(1/F_{1+2k})}\left(1.717986918400000000\times 10^{10}+6.87194767360000000\times 10^{10}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{8\sum_{k=1}^{\infty}\tan^{-1}(1/F_{1+2k})}+3.71958895314365707\times 10^{18}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{16\sum_{k=1}^{\infty}\tan^{-1}(1/F_{1+2k})}+6.87194767360000000\times 10^{10}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{24\sum_{k=1}^{\infty}\tan^{-1}(1/F_{1+2k})}+1.717986918400000000\times 10^{10}\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{32\sum_{k=1}^{\infty}\tan^{-1}(1/F_{1+2k})}\right)$$

$$\frac{1}{32\times 4}\Big(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.56856762281170000^2+4\left(16^2+16^2\right)\left(93.56856762281170000^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.56856762281170000^2\right)\Big)=$$

$$5.9604644775390625\times 10^{-8}\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{-16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\Bigg(1.717986918400000000\times 10^{10}+6.87194767360000000\times 10^{10}\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+3.71958895314365707\times 10^{18}\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{16\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+6.87194767360000000\times 10^{10}\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{24\sum_{k=0}^{\infty}(-1)^k/(1+2k)}+1.717986918400000000\times 10^{10}\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{32\sum_{k=0}^{\infty}(-1)^k/(1+2k)}\Bigg)$$

$n!$ is the factorial function

F_n is the n^{th} Fibonacci number

Integral representations

$$\frac{1}{32\times 4}\Big(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.56856762281170000^2+4\left(16^2+16^2\right)\left(93.56856762281170000^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.56856762281170000^2\right)\Big)=$$

$$2.217047782625947633\times 10^{11}+1024\,e^{-32/3\int_0^\infty \sin^3(t)/t^3\,dt}+4096\,e^{-16/3\int_0^\infty \sin^3(t)/t^3\,dt}+4096\,e^{16/3\int_0^\infty \sin^3(t)/t^3\,dt}+1024\,e^{32/3\int_0^\infty \sin^3(t)/t^3\,dt}$$

$$\frac{1}{32\times 4}\Big(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+ 64\times 64\,(16-8)\,93.56856762281170000^2+ 4\left(16^2+16^2\right)\left(93.56856762281170000^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.56856762281170000^2\right)\Big)= 2.217047782625947633\times 10^{11}+1024\,e^{-160/11\int_0^\infty \sin^6(t)/t^6\,dt}+ 4096\,e^{-80/11\int_0^\infty \sin^6(t)/t^6\,dt}+4096\,e^{80/11\int_0^\infty \sin^6(t)/t^6\,dt}+1024\,e^{160/11\int_0^\infty \sin^6(t)/t^6\,dt}$$

$$\frac{1}{32\times 4}\Big(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+ 64\times 64\,(16-8)\,93.56856762281170000^2+ 4\left(16^2+16^2\right)\left(93.56856762281170000^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.56856762281170000^2\right)\Big)= 2.217047782625947633\times 10^{11}+1024\,e^{-1536/115\int_0^\infty \sin^5(t)/t^5\,dt}+ 4096\,e^{-768/115\int_0^\infty \sin^5(t)/t^5\,dt}+ 4096\,e^{768/115\int_0^\infty \sin^5(t)/t^5\,dt}+1024\,e^{1536/115\int_0^\infty \sin^5(t)/t^5\,dt}$$

From which:

1/128 [((((((32)((((32^2*2^4+(32+16*e^(2π)+16*e^(-2π))^2)*8*2))))+64*64*8* 93.56856762^2+4(16+8)(16^2+16^2)* ((93.56856762^2-2*16*16))8* 93.56856762^2)))]*(731 ζ(3) - 876)

where

$731\,\zeta(3)-876\approx 2.703596209663$

Input interpretation

$$\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)\times 8\times 2\right)+64\times 64\times 8\times 93.56856762^2+ 4\,(16+8)\left(\left(16^2+16^2\right)\left(93.56856762^2-2\times 16\times 16\right)\right)\left(8\times 93.56856762^2\right)\right)(731\,\zeta(3)-876)$$

$\zeta(s)$ is the Riemann zeta function

Result

$6.00199994\ldots \times 10^{11}$

6.00199994…*10^11 result almost equal to the value of Sneutrino mass as possible heavy Dark Matter particle ≈ 6.002*10^11 eV

The study of this function provides the following representations:

Alternative representations

$$\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\times 8\times 93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left(8\times 93.5686^2\right)\right)(731\,\zeta(3)-876)=\frac{1}{128}\left(32768\times 93.5686^2+1536\times 16^2\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16\,e^{-2\pi}+16\,e^{2\pi}\right)^2\right)\right)(-876+731\,\zeta(3,1))$$

$$\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\times 8\times 93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left(8\times 93.5686^2\right)\right)(731\,\zeta(3)-876)=\frac{1}{128}\left(-876+731\,S_{2,1}(1)\right)\left(32768\times 93.5686^2+1536\times 16^2\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16\,e^{-2\pi}+16\,e^{2\pi}\right)^2\right)\right)$$

$$\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\times 8\times 93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left(8\times 93.5686^2\right)\right)(731\,\zeta(3)-876)=\frac{1}{128}\left(-876-\frac{731\,\mathrm{Li}_3(-1)}{\frac{3}{4}}\right)\left(32768\times 93.5686^2+1536\times 16^2\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16\,e^{-2\pi}+16\,e^{2\pi}\right)^2\right)\right)$$

$\zeta(s, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

$\text{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\times 8\times 93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left(8\times 93.5686^2\right)\right)(731\,\zeta(3)-876)=748\,544\,e^{-4\pi}\left(1+4\,e^{2\pi}+2.16509\times 10^8\,e^{4\pi}+4\,e^{6\pi}+e^{8\pi}\right)\left(-1.19836+\sum_{k=1}^{\infty}\frac{1}{k^3}\right)$$

$$\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\times 8\times 93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left(8\times 93.5686^2\right)\right)(731\,\zeta(3)-876)=998\,059.\,e^{-4\pi}\left(1+4\,e^{2\pi}+2.16509\times 10^8\,e^{4\pi}+4\,e^{6\pi}+e^{8\pi}\right)\left(-0.898769-\sum_{k=1}^{\infty}\frac{(-1)^k}{k^3}\right)$$

$$\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\times 8\times 93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left(8\times 93.5686^2\right)\right)(731\,\zeta(3)-876)=855\,479.\,e^{-4\pi}\left(1+4\,e^{2\pi}+2.16509\times 10^8\,e^{4\pi}+4\,e^{6\pi}+e^{8\pi}\right)\left(-1.04856+\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^3}\right)$$

Integral representations

$$\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\times 8\times 93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left(8\times 93.5686^2\right)\right)(731\,\zeta(3)-876)=-\frac{1}{\Gamma(3)}897\,024\,e^{-4\pi}\left(1+4\,e^{2\pi}+2.16509\times 10^8\,e^{4\pi}+4\,e^{6\pi}+e^{8\pi}\right)\left(\Gamma(3)-0.834475\int_0^{\infty}\frac{t^2}{-1+\mathcal{A}^t}\,dt\right)$$

$$\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\times 8\times 93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left(8\times 93.5686^2\right)\right)(731\,\zeta(3)-876)=-\frac{1}{\Gamma(3)}897024\,e^{-4\pi}\left(1+4\,e^{2\pi}+2.16509\times 10^8\,e^{4\pi}+4\,e^{6\pi}+e^{8\pi}\right)\left(\Gamma(3)-1.11263\int_0^\infty \frac{t^2}{1+\mathcal{A}^t}\,dt\right)$$

$$\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\times 8\times 93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left(8\times 93.5686^2\right)\right)(731\,\zeta(3)-876)=-\frac{1}{\Gamma(3)}897024\,e^{-4\pi}\left(1+4\,e^{2\pi}+2.16509\times 10^8\,e^{4\pi}+4\,e^{6\pi}+e^{8\pi}\right)\left(\Gamma(3)-0.476843\int_0^\infty t^2\operatorname{csch}(t)\,dt\right)$$

We obtain also:

(1/128*1/(728+1/8+π^2+2*2^(1/3)))(([(((((32)((((32^2*2^4+(32+16*e^(2π)+16*e^(-2π))^2)*8*2))))+64*64*(16-8)* 93.568567^2+4(16+8)(16^2+16^2)* ((93.568567^2-2*16*16))(16-8)* 93.568567^2)))]))

Input interpretation

$$\left(\frac{1}{128}\times\frac{1}{728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}}\right)\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)\times 8\times 2\right)+64\times 64\,(16-8)\times 93.568567^2+4\,(16+8)\left(\left(16^2+16^2\right)\left(93.568567^2-2\times 16\times 16\right)\right)\left((16-8)\times 93.568567^2\right)\right)$$

Result

$2.997924\ldots\times 10^8$

$2.997924\ldots*10^8 \approx$ c = speed of light

The study of this function provides the following representations:

Alternative representations

$$\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.5686^2\right)\right)\Big/\left(128\left(728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}\right)\right)=\left(32\,768\times 93.5686^2+1536\times 16^2\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16\,e^{-360^\circ}+16\,e^{360^\circ}\right)^2\right)\right)\Big/\left(128\left(728+2\sqrt[3]{2}+\frac{1}{8}+(180^\circ)^2\right)\right)$$

$$\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.5686^2\right)\right)\Big/\left(128\left(728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}\right)\right)=\left(32\,768\times 93.5686^2+1536\times 16^2\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16\,e^{-2i\log(-1)}+16\,e^{2i\log(-1)}\right)^2\right)\right)\Big/\left(128\left(728+2\sqrt[3]{2}+\frac{1}{8}+(-i\log(-1))^2\right)\right)$$

$$\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.5686^2\right)\right)\Big/\left(128\left(728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}\right)\right)=\left(32\left(\left(32^2\times 2^4+\left(32+16\exp^{2\pi}(z)+16\exp^{-2\pi}(z)\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.5686^2\right)\right)\Big/\left(128\left(728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}\right)\right)\quad\text{for } z=1$$

Integral representations

$$\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.5686^2\right)\right)\Big/\left(128\left(728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}\right)\right)=\left(512\,e^{-8\int_0^\infty \sin(t)/t\,dt}\left(1+4\,e^{4\int_0^\infty \sin(t)/t\,dt}+2.16509\times 10^8\,e^{8\int_0^\infty \sin(t)/t\,dt}+4\,e^{12\int_0^\infty \sin(t)/t\,dt}+e^{16\int_0^\infty \sin(t)/t\,dt}\right)\right)\Big/\left(365.322+2\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^2\right)$$

$$\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.5686^2\right)\right)\Big/\left(128\left(728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}\right)\right)=\left(512\,e^{-8\times\int_0^\infty 1/(1+t^2)\,dt}\left(1+4\,e^{4\times\int_0^\infty 1/(1+t^2)\,dt}+2.16509\times 10^8\,e^{8\times\int_0^\infty 1/(1+t^2)\,dt}+4\,e^{12\times\int_0^\infty 1/(1+t^2)\,dt}+e^{16\times\int_0^\infty 1/(1+t^2)\,dt}\right)\right)\Big/\left(365.322+2\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2\right)$$

$$\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)8\times 2\right)+64\times 64\,(16-8)\,93.5686^2+4\left(16^2+16^2\right)\left(93.5686^2-2\times 16\times 16\right)(16+8)\left((16-8)\,93.5686^2\right)\right)\Big/\left(128\left(728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}\right)\right)=\left(512\,e^{-16\int_0^1\sqrt{1-t^2}\,dt}\left(1+4\,e^{8\int_0^1\sqrt{1-t^2}\,dt}+2.16509\times 10^8\,e^{16\int_0^1\sqrt{1-t^2}\,dt}+4\,e^{24\int_0^1\sqrt{1-t^2}\,dt}+e^{32\int_0^1\sqrt{1-t^2}\,dt}\right)\right)\Big/\left(365.322+8\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2\right)$$

And again, after some calculations:

(1/2(√(√((1/128*1/(728+1/8+π^2+2*2^(1/3)))(([(((((32)((((32^2*2^4+(32+16*e^(2π) +16*e^(-2π))^2)*8*2))))+64*64*8* 93.568567^2+4(24)(16^2+16^2)*(93.568567^2-2*16*16)8* 93.568567^2)))])))))-2e+π-2Φ))^2-4+Φ

Input interpretation

$$\left(\frac{1}{2}\left(\sqrt{\left(\sqrt{\left(\left(\frac{1}{128}\times\frac{1}{728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}}\right)(32\left((32^2\times 2^4+(32+16\,e^{2\pi}+16\,e^{-2\pi})^2)\times 8\times 2\right)+64\times 64\times 8\times 93.568567^2+4\times 24\left((16^2+16^2)(93.568567^2-2\times 16\times 16)\right)(8\times 93.568567^2))\right)}\right)}-2\,e+\pi-2\,\Phi\right)\right)^2-4+\Phi$$

Φ is the golden ratio conjugate

Result

4096.0484…

4096.0484…. ≈ 4096 = 64^2

27sqrt((1/2(√(√((1/128*1/(728+1/8+π^2+2*2^(1/3)))(([(((((32)((((32^2*2^4+(32+16 *e^(2π)+16*e^(-2π))^2)*8*2))))+4096*8* 93.56856^2+4(24)*512*(93.56856^2-2*16*16)8*93.56856^2)))]))))-2e+π-2Φ))^2-4+Φ)+1

Input interpretation

$$27\sqrt{\left(\frac{1}{2}\left(\sqrt{\left(\sqrt{\left(\left(\frac{1}{128}\times\frac{1}{728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}}\right)(32\left((32^2\times 2^4+(32+16\,e^{2\pi}+16\,e^{-2\pi})^2)\times 8\times 2\right)+4096\times 8\times 93.56856^2+4\times 24\times 512\,(93.56856^2-2\times 16\times 16)(8\times 93.56856^2))\right)}\right)}-2\,e+\pi-2\,\Phi\right)\right)^2-4+\Phi}+1$$

Φ is the golden ratio conjugate

Result

1729.010…

1729.010….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(27sqrt((1/2(√(√((1/128*1/(728+1/8+π^2+2*2^(1/3)))(([(((((32)((((32^2*16+(32+16*e^(2π)+16*e^(-2π))^2)*16))))+4096*8* 93.56856^2+4(24)*512*(93.56856^2-512)8*93.56856^2)))]))))-2e+π-2Φ))^2-4+Φ)+1)^1/15

Input interpretation

$$\left(27\sqrt{\left(\frac{1}{2}\left(\sqrt{\left(\sqrt{\left(\left(\frac{1}{128}\times\frac{1}{728+\frac{1}{8}+\pi^2+2\sqrt[3]{2}}\right)\left(32\left(\left(32^2\times 16+\left(32+16e^{2\pi}+16e^{-2\pi}\right)^2\right)\times 16\right)+4096\times 8\times 93.56856^2+4\times 24\times 512\left(93.56856^2-512\right)\left(8\times 93.56856^2\right)\right)\right)}-2e+\pi-2\Phi\right)}\right)^2-4+\Phi\right)}+1\right)\text{^}(1/15)$$

Φ is the golden ratio conjugate

Result

1.6438159…

1.6438159…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

From the previous expression, we obtain also:

(1/128 [(((((32)((((32^2*2^4+(32+16*e^(2π)+16*e^(-2π))^2)*16))))+4096*8* 93.56856762^2+4(16+8)*512*((93.56856762^2-2*16*16))8*93.56856762^2)))])^11

Input interpretation

$$\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)\times 16\right)+4096\times 8\times 93.56856762^2+4\,(16+8)\times 512\left(93.56856762^2-2\times 16\times 16\right)\left(8\times 93.56856762^2\right)\right)\right)^{11}$$

Result

$6.4549924\ldots\times 10^{124}$

$6.4549924\ldots*10^{124}$

From which:

1/(-e^(2 + 2/e + π) π^(3 - 3 e) sec(e π))((1/(1/128 [(((((32)((((32^2*2^4+(32+16*e^(2π)+16*e^(-2π))^2)*16))))+4096*8* 93.56856762^2+4(16+8)*512*((93.56856762^2- 2*16*16))8*93.56856762^2)))])^11))^1/3

where

$-e^{2+2/e+\pi}\,\pi^{3-3e}\sec(e\,\pi)\approx 1.54238036976 6$

Input interpretation

$$-\frac{1}{e^{2+2/e+\pi}\,\pi^{3-3e}\sec(e\,\pi)}\left(1\middle/\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)\times 16\right)+4096\times 8\times 93.56856762^2+4\,(16+8)\times 512\left(93.56856762^2-2\times 16\times 16\right)\left(8\times 93.56856762^2\right)\right)\right)^{11}\right)\hat{}\,(1/3)$$

$\sec(x)$ is the secant function

Result

$1.61625519\ldots\times 10^{-42}$

$1.61625519\ldots*10^{-42}$

The study of this function provides the following representations:

Alternative representations

$$-\left(\left(\left(1\Big/\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)\right)^{11}\right)\text{^}(1/3)\right)\Big/\left(e^{2+2/e+\pi}\,\pi^{3-3e}\sec(e\,\pi)\right)\right)=-\frac{1}{\frac{e^{2+\pi+2/e}\,\pi^{3-3e}}{\cos(e\,\pi)}}\left(\left(1\Big/\left(\frac{1}{128}\left(32768\times 93.5686^2+393216\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16\,e^{-2\pi}+16\,e^{2\pi}\right)^2\right)\right)\right)^{11}\right)\text{^}(1/3)\right)$$

$$-\left(\left(\left(1\Big/\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)\right)^{11}\right)\text{^}(1/3)\right)\Big/\left(e^{2+2/e+\pi}\,\pi^{3-3e}\sec(e\,\pi)\right)\right)=-\frac{1}{\frac{e^{2+\pi+2/e}\,\pi^{3-3e}}{\cosh(e\,i\,\pi)}}\left(\left(1\Big/\left(\frac{1}{128}\left(32768\times 93.5686^2+393216\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16\,e^{-2\pi}+16\,e^{2\pi}\right)^2\right)\right)\right)^{11}\right)\text{^}(1/3)\right)$$

$$-\left(\left(\left(1\Big/\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)\right)^{11}\right)\text{^}(1/3)\right)\Big/\left(e^{2+2/e+\pi}\,\pi^{3-3e}\sec(e\,\pi)\right)\right)=-\frac{1}{\frac{e^{2+\pi+2/e}\,\pi^{3-3e}}{\cosh(-i\,e\,\pi)}}\left(\left(1\Big/\left(\frac{1}{128}\left(32768\times 93.5686^2+393216\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16\,e^{-2\pi}+16\,e^{2\pi}\right)^2\right)\right)\right)^{11}\right)\text{^}(1/3)\right)$$

Series representations

$$-\left(\left(\left(1\middle/\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)^{11}\right)\right)^\wedge(1/3)\right)\middle/\left(e^{2+2/e+\pi}\,\pi^{3-3e}\sec(e\,\pi)\right)\right)=\frac{16\,777\,216\times 2^{2/3}\,e^{-2-2/e-\pi}\sqrt[3]{\frac{1}{\left(2.83782\times 10^{13}+131\,072\,e^{-4\pi}\left(1+e^{2\pi}\right)^4\right)^{11}}}\,\pi^{-3+3e}}{\sum_{k=1}^{\infty}(-1)^k\,q^{-1+2k}}$$

for $q=e^{i\,e\,\pi}$

$$-\left(\left(\left(1\middle/\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)^{11}\right)\right)^\wedge(1/3)\right)\middle/\left(e^{2+2/e+\pi}\,\pi^{3-3e}\sec(e\,\pi)\right)\right)=-\frac{8\,388\,608\times 2^{2/3}\,e^{-2-2/e-\pi}\sqrt[3]{\frac{1}{\left(2.83782\times 10^{13}+131\,072\,e^{-4\pi}\left(1+e^{2\pi}\right)^4\right)^{11}}}\,\pi^{-2+3e}}{\sum_{k=0}^{\infty}\frac{(-1)^k(1+2k)}{-4e^2+(1+2k)^2}}$$

$$-\left(\left(\left(1\middle/\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)^{11}\right)\right)^\wedge(1/3)\right)\middle/\left(e^{2+2/e+\pi}\,\pi^{3-3e}\sec(e\,\pi)\right)\right)=-\frac{33\,554\,432\times 2^{2/3}\,e^{-2-2/e-\pi}\sqrt[3]{\frac{1}{\left(2.83782\times 10^{13}+131\,072\,e^{-4\pi}\left(1+e^{2\pi}\right)^4\right)^{11}}}\,\pi^{-3+3e}}{\sum_{k=-\infty}^{\infty}(-1)^k\,\mathcal{A}^{e\,i\,(1+2k)\pi}\,(-1+2\,\theta(k))}$$

Multiple-argument formulas

$$-\left(\left(\left(1\Big/\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)\right)^{11}\right)\hat{}\,(1/3)\right)\Big/\left(e^{2+2/e+\pi}\,\pi^{3-3e}\,\sec(e\,\pi)\right)\right)=-33\,554\,432\times 2^{2/3}\,e^{-2-2/e-\pi}\sqrt[3]{\frac{1}{\left(2.83782\times 10^{13}+131\,072\,e^{-4\pi}\left(1+e^{2\pi}\right)^4\right)^{11}}}\,\pi^{-3+3e}\,T_e(\cos(\pi))$$

$$-\left(\left(\left(1\Big/\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)\right)^{11}\right)\hat{}\,(1/3)\right)\Big/\left(e^{2+2/e+\pi}\,\pi^{3-3e}\,\sec(e\,\pi)\right)\right)=\frac{1}{\sec^3\left(\frac{e\,\pi}{3}\right)}33\,554\,432\times 2^{2/3}\,e^{-2-2/e-\pi}\sqrt[3]{\frac{1}{\left(2.83782\times 10^{13}+131\,072\,e^{-4\pi}\left(1+e^{2\pi}\right)^4\right)^{11}}}\,\pi^{-3+3e}\left(-4+3\sec^2\left(\frac{e\,\pi}{3}\right)\right)$$

$$-\left(\left(\left(1\middle/\left(\frac{1}{128}\left(32\left(\left(32^2\times2^4+\left(32+16e^{2\pi}+16e^{-2\pi}\right)^2\right)16\right)+4096\times8\times93.5686^2+4\times8\times93.5686^2(16+8)\left(512\left(93.5686^2-2\times16\times16\right)\right)\right)\right)^{11}\right)\hat{}(1/3)\right)\middle/\left(e^{2+2/e+\pi}\,\pi^{3-3e}\sec(e\,\pi)\right)\right)=\frac{1}{\sec^2\left(\frac{e\pi}{2}\right)}33\,554\,432\times2^{2/3}\,e^{-2-2/e-\pi}\sqrt[3]{\frac{1}{\left(2.83782\times10^{13}+131\,072\,e^{-4\pi}\left(1+e^{2\pi}\right)^4\right)^{11}}}\,\pi^{-3+3e}\left(-2+\sec^2\left(\frac{e\pi}{2}\right)\right)$$

$T_n(x)$ is the Chebyshev polynomial of the first kind

We note that the result 1.61625519…*10^{-42} is equal to the Planck's electric inductance :

$$L_P=\frac{E_P}{I_P^2}=\frac{m_P l_P^2}{q_P^2}=\sqrt{\frac{G\hbar}{16\pi^2\varepsilon_0^2c^7}}$$

sqrt(((6.67408*10^-11 * 1.054571817*10^-34) / (16*π^2*(8.8541878176*10^-12)^2*(299792458)^7)))

Input interpretation

$$\sqrt{\frac{6.67408\times10^{-11}\times1.054571817\times10^{-34}}{16\pi^2\left(8.8541878176\times10^{-12}\right)^2\times299\,792\,458^7}}$$

Result

1.61623… × 10^{-42}

1.61623…*10^{-42}

From the initial expression, we obtain also:

(1/128 [((((((32)((((32^2*2^4+(32+16*e^(2π)+16*e^(-2π))^2)*16))))+4096*8*93.56856762^2+4(16+8)*512*((93.56856762^2-2*16*16))8*93.56856762^2)))])^11*(log(2)/(16 e^4 log^4(3)))

Input interpretation

$$\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16 e^{2\pi}+16 e^{-2\pi}\right)^2\right)\times 16\right)+4096\times 8\times 93.56856762^2+4\,(16+8)\times 512\left(93.56856762^2-2\times 16\times 16\right)\left(8\times 93.56856762^2\right)\right)\right)^{11}\times\frac{\log(2)}{16\, e^4 \log^4(3)}$$

log(x) is the natural logarithm

Result

$3.5159726\ldots\times 10^{121}$

$0.35159726\ldots*10^{122}\approx\Lambda_Q$

The observed value of ρ_Λ or Λ today is precisely the classical dual of its quantum precursor values ρ_Q , Λ_Q in the quantum very early precursor vacuum U_Q as determined by our dual equations. **With regard the Cosmological constant, fundamental are the following results: $\Lambda = 2.846 * 10^{-122}$ and $\Lambda_Q = 0.3516 * 10^{122}$ (New Quantum Structure of the Space-Time -** *Norma G. SANCHEZ* - arXiv:1910.13382v1 [physics.gen-ph] 28 Oct 2019)

The study of this function provides the following representations:

Alternative representations

$$\frac{1}{16 e^4 \log^4(3)}\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16 e^{2\pi}+16 e^{-2\pi}\right)^2\right) 16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)\right)^{11}\log(2)=$$

$$\frac{1}{16 e^4 \log_e^4(3)}\log_e(2)\left(\frac{1}{128}\left(32768\times 93.5686^2+393216\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16 e^{-2\pi}+16 e^{2\pi}\right)^2\right)\right)\right)^{11}$$

$$\frac{1}{16\,e^4\log^4(3)}\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)\right)^{11}\log(2)=$$

$$\left(\log(a)\log_a(2)\left(\frac{1}{128}\left(32\,768\times 93.5686^2+393\,216\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16\,e^{-2\pi}+16\,e^{2\pi}\right)^2\right)\right)\right)^{11}\right)\Big/\left(16\,e^4\,(\log(a)\log_a(3))^4\right)$$

$$\frac{1}{16\,e^4\log^4(3)}\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)16\right)+4096\times 8\times 93.5686^2+4\times 8\times 93.5686^2\,(16+8)\left(512\left(93.5686^2-2\times 16\times 16\right)\right)\right)\right)^{11}$$

$$\log(2)=\frac{1}{16\,e^4\left(2\coth^{-1}(2)\right)^4}2\coth^{-1}(3)\left(\frac{1}{128}\left(32\,768\times 93.5686^2+393\,216\left(-512+93.5686^2\right)93.5686^2+512\left(2^4\times 32^2+\left(32+16\,e^{-2\pi}+16\,e^{2\pi}\right)^2\right)\right)\right)^{11}$$

Series representations

$$\frac{1}{16 e^4 \log^4(3)} \left(\frac{1}{128} \left(32 \left(\left(32^2 \times 2^4 + \left(32 + 16 e^{2\pi} + 16 e^{-2\pi}\right)^2\right) 16\right) + 4096 \times 8 \times 93.5686^2 + 4 \times 8 \times 93.5686^2 (16+8) \left(512 \left(93.5686^2 - 2 \times 16 \times 16\right)\right)\right)\right)^{11} \log(2) = \left(\left(2.83782 \times 10^{13} + 512 \left(16384 + \left(32 + 16 e^{-2\pi} + 16 e^{2\pi}\right)^2\right)\right)^{11} \left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)\right) \Big/ \left(2417851639229258349412352 e^4 \left(2 i \pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k}\right)^4\right) \text{ for } x < 0$$

$$\frac{1}{16 e^4 \log^4(3)} \left(\frac{1}{128} \left(32 \left(\left(32^2 \times 2^4 + \left(32 + 16 e^{2\pi} + 16 e^{-2\pi}\right)^2\right) 16\right) + 4096 \times 8 \times 93.5686^2 + 4 \times 8 \times 93.5686^2 (16+8) \left(512 \left(93.5686^2 - 2 \times 16 \times 16\right)\right)\right)\right)^{11} \log(2) = \left(\left(2.83782 \times 10^{13} + 512 \left(16384 + \left(32 + 16 e^{-2\pi} + 16 e^{2\pi}\right)^2\right)\right)^{11} \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)\right) \Big/ \left(2417851639229258349412352 e^4 \left(\log(z_0) + \left\lfloor \frac{\arg(3-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k}\right)^4\right)$$

$$\frac{1}{16 e^4 \log^4(3)} \left(\frac{1}{128}\left(32\left(\left(32^2 \times 2^4 + \left(32 + 16 e^{2\pi} + 16 e^{-2\pi}\right)^2\right) 16\right) + 4096 \times 8 \times 93.5686^2 + 4 \times 8 \times 93.5686^2 (16+8)\left(512\left(93.5686^2 - 2 \times 16 \times 16\right)\right)\right)\right)^{11} \log(2) =$$

$$\left(\left(2.83782 \times 10^{13} + 512\left(16384 + \left(32 + 16 e^{-2\pi} + 16 e^{2\pi}\right)^2\right)\right)^{11} \left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k}\right)\right) \Bigg/ \left(2417851639229258349412352 e^4 \left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{3}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k}\right)^4\right)$$

Integral representations

$$\frac{1}{16 e^4 \log^4(3)} \left(\frac{1}{128}\left(32\left(\left(32^2 \times 2^4 + \left(32 + 16 e^{2\pi} + 16 e^{-2\pi}\right)^2\right) 16\right) + 4096 \times 8 \times 93.5686^2 + 4 \times 8 \times 93.5686^2 (16+8)\left(512\left(93.5686^2 - 2 \times 16 \times 16\right)\right)\right)\right)^{11} \log(2) =$$

$$\frac{8.11296 \times 10^{31} e^{-4-44\pi} \left(1 + 4 e^{2\pi} + 2.16509 \times 10^8 e^{4\pi} + 4 e^{6\pi} + e^{8\pi}\right)^{11} \int_1^2 \frac{1}{t} \, dt}{\left(\int_1^3 \frac{1}{t} \, dt\right)^4}$$

$$\frac{1}{16 e^4 \log^4(3)} \left(\frac{1}{128} \left(32 \left(\left(32^2 \times 2^4 + \left(32 + 16 e^{2\pi} + 16 e^{-2\pi}\right)^2\right) 16\right) + 4096 \times 8 \times 93.5686^2 + 4 \times 8 \times 93.5686^2 (16+8) \left(512 \left(93.5686^2 - 2 \times 16 \times 16\right)\right)\right)\right)^{11} \log(2) =$$

$$\left(\left(2.83782 \times 10^{13} + 512 \left(16384 + \left(32 + 16 e^{-2\pi} + 16 e^{2\pi}\right)^2\right)\right)^{11} i^3 \pi^3 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \, ds\right) \Big/ \left(302231454903657293676544 \, e^4 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} \, ds\right)^4\right)$$

for $-1 < \gamma < 0$

From this other equation:

$$\frac{2^{-5}}{4}\left\{16(Q_s+Q_c)\ (R_N^2+R_D^2+R_\delta^2)\ (\frac{\eta}{\theta_4})^2 - 8R_N(R_D+R_\delta)\ (Q_s-Q_c)(\frac{\eta}{\theta_3})^2\right\}$$

we consider:

$$N = D_1 = D_2 = D_3 = 32$$

$$D = \delta$$

P = W = 2 ; Q_i = 8 ; Q_j = 16 ; R_i = 16 ; v = 1

((η)/θ_4)^2 = ((η)/θ_3)^2 = 46.7842838114

and we obtain:

1/128 (16(16+8)(256+256+256) 46.7842838114^2-8*16*32*8*46.7842838114^2)

Input interpretation

$$\frac{1}{128}\left(16 (16+8) (256+256+256) \times 46.7842838114^2 - 8 \times 16 \times 32 \times 8 \times 46.7842838114^2\right)$$

Result

$4.4825993456550381788568 7808 \times 10^6$

$4.48259934565503817885687808 \times 10^6$

Dividing the two results, we obtain:

1/4.48259934565 × 10^6(1/128 [(((((32)((((32^2*2^4+(32+16*e^(2π)+16*e^(-2π))^2)*16))))+64*64*8*93.56856762^2+4(16+8)512*((93.56856762^2-2*16*16))8*93.56856762^2)))])

Input interpretation

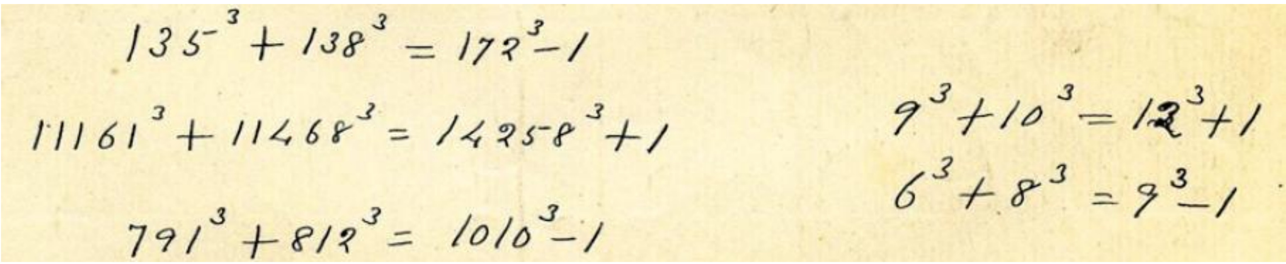
$$\frac{1}{4.48259934565 \times 10^6}\left(\frac{1}{128}\left(32\left(\left(32^2 \times 2^4 + \left(32 + 16 e^{2\pi} + 16 e^{-2\pi}\right)^2\right) \times 16\right) + 64 \times 64 \times 8 \times 93.56856762^2 + 4(16+8)\left(512\left(93.56856762^2 - 2 \times 16 \times 16\right)\right)\left(8 \times 93.56856762^2\right)\right)\right)$$

Result

49524.9715…

49524.9715….. ≈ 49525

From the following Ramanujan taxicab numbers:



$135^3 + 138^3 = 172^3 - 1$

$11161^3 + 11468^3 = 14258^3 + 1$

$791^3 + 812^3 = 1010^3 - 1$

$9^3 + 10^3 = 12^3 + 1$

$6^3 + 8^3 = 9^3 - 1$

we note that:

11468 × 4 + 1010 + 1729 + 791 + 135 = 49537

11468 × 4 + 1010 + 1729 + 812 + 138 = 49561

where

(11468 × 4 + 1010 + 1729 + 791 + 135) - 12

Input

(11 468 × 4 + 1010 + 1729 + 791 + 135) − 12

Result

49 525

49525

Or:

(11468 × 4 + 1010 + 1729 + 812 + 138) - 36

Input

(11 468 × 4 + 1010 + 1729 + 812 + 138) − 36

Result

49 525

49525

From the above expressions, after some calculations, we obtain:

(1/4(sqrt(1/4.48259934565 × 10^6(1/128 [(((((32)((((32^2*2^4+(32+16*e^(2π)+16*e^(-2π))^2)*16))))+64*64*8*93.56856762^2+4(16+8)512*((93.56856762^2-2*16*16))8*93.56856762^2)))]))+34))^2-18+Φ

Input interpretation

$$\left(\frac{1}{4}\left(\sqrt{\left(\frac{1}{4.48259934565\times 10^6}\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)\times 16\right)+64\times 64\times 8\times 93.56856762^2+4\,(16+8)\left(512\left(93.56856762^2-2\times 16\times 16\right)\right)\left(8\times 93.56856762^2\right)\right)\right)\right)}+34\right)\right)^2-18+\Phi$$

Φ is the golden ratio conjugate

Result

4095.98254…

4095.98254…. ≈ 4096 = 64^2

27*√((1/4(sqrt(1/4.48259934565 × 10^6(1/128 [(((((32)((((32^2*2^4+(32+16*e^(2π)+16*e^(-2π))^2)*16))))+64*64*8*93.56856762^2+4(16+8)512*((93.56856762^2-2*16*16))8*93.56856762^2)))]))+34))^2-18+Φ)+1

Input interpretation

$$27\sqrt{\left(\left(\frac{1}{4}\left(\sqrt{\left(\frac{1}{4.48259934565\times 10^6}\left(\frac{1}{128}\left(32\left(\left(32^2\times 2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)\times 16\right)+64\times 64\times 8\times 93.56856762^2+4\,(16+8)\left(512\left(93.56856762^2-2\times 16\times 16\right)\right)\left(8\times 93.56856762^2\right)\right)\right)\right)}+34\right)\right)^2-18+\Phi\right)+1}$$

Φ is the golden ratio conjugate

Result

1728.996317…

1728.996317…. ≈ 1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = $8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(27√((1/4(sqrt(1/4.48259934565e+6(1/128 [(((((32)((((32^2*2^4+(32+16*e^(2π)+16*e^(-2π))^2)*16))))+64*64*8*93.56856762^2+4(16+8)512*((93.56856762^2-2*16*16))8*93.56856762^2)))]))+34))^2-18+Φ)+1)^1/15

Input interpretation

$$\left(27\sqrt{\left(\left(\frac{1}{4}\left(\sqrt{\left(\frac{1}{4.48259934565\times10^6}\left(\frac{1}{128}\left(32\left(\left(32^2\times2^4+\left(32+16\,e^{2\pi}+16\,e^{-2\pi}\right)^2\right)\times 16\right)+64\times64\times8\times93.56856762^2+4\,(16+8)\left(512\left(93.56856762^2-2\times16\times16\right)\right)\left(8\times93.56856762^2\right)\right)\right)\right)}+34\right)\right)^2-18+\Phi\right)}+1\right)\verb|^|(1/15)$$

Φ is the golden ratio conjugate

Result

1.6438149953…

1.6438149953….. ≈ $\zeta(2) = \frac{\pi^2}{6}$ = 1.644934 ... (trace of the instanton shape)

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