L^1 from Special Relativity

Alireza Jamali*

Senior Researcher

Natural Philosophy Department, Hermite Foundation†

alireza.jamali.mp@gmail.com

November 21, 2021

Abstract

As a result of my recent work on (special) Relativity in Function Spaces, here I present a derivation of the L^1-norm distance from the analogue of Minkowski metric (of special relativity) for a function space.

Keywords— relativity in function spaces, metric space, function space, functional analysis

Following my insights in [1] we attempt to define the analogue of the Minkowski metric for a 'good' enough function space. Let us begin by

$$d_M\big(f(x), g(y)\big)(x, y) := \sqrt{\big(f(x) - g(y)\big)^2 - (x - y)^2} \tag{1}$$

But this d_M is –let us name it– a *hybrid metric* or a *duplex metric*, for it is not a proper metric on a function space *solely*; it also takes into consideration the 'inputs' of functions, something which is not considered at all in the current theory of functional analysis, to the best of my knowledge. It is easy to see this point: let

$$f(x) = g(y), \quad x \neq y$$

If this was a proper metric on a function space, we would expect to have

$$d_M\big(f(x), f(y)\big) \stackrel{?}{=} 0,$$

yet in this case (1) yields

$$d_M = |x - y|$$

*Corresponding author

†3rd Floor - Block No. 6 - Akbari Alley - After Dardasht Intersection - Janbazané Sharghi - Tehran - Iran

signifying the hybrid nature.
To turn (1) into a proper metric for the function space solely, it is natural to try to 'get rid' of the 'inputs' by integrating over them. Therefore we have to define the *differential* of d_M. The first guess would be

$$d^2 d_M\big(f(x), g(y)\big) = \sqrt{\big(f(x) - g(y)\big)^2 - (x-y)^2}\ dxdy,$$

but this faces a problem: we expect (1) to pass us over to its functional aspect by letting $x = y$, which results in

$$d^2 d_M\big(f(x), g(y)\big) = |f(x) - g(x)|dxdx,$$

as we are going to have to finally integrate this differential, –quite roughly-speaking– we are going to need one of the dxes on the right-hand-side, so

$$\frac{d^2}{dx} d_M = |f(x) - g(x)|dx,$$

the only way to make sense of the 'fraction' on the left-hand-side which would allow integration seems to be

$$d(\frac{dd_M}{dx}) = |f(x) - g(x)|dx,$$

upon integration it gives

$$\frac{dd_M}{dx} = \int |f(x) - g(x)|dx,$$

which is absurd as the right-hand-side is *not a function of* x; rendering it not well-defined. *Even* **if** we ignore this problem –rather silly but just to make sure–, changing the integration variable to t to avoid confusion, and letting the domain of integration to be $[a, b]$, we have

$$\int_a^b |f(t) - g(t)|dt = A = constant,$$

so

$$\frac{dd_M}{dx} = A \Rightarrow d_M = Ax + C,$$

which is an absurdity as this d_M is not even a metric. So it seems that we are having a 'surplus' of differentials: on the left-hand-side of

$$\frac{d^2}{dx} d_M = |f(x) - g(x)|dx,$$

there is a 'surplus' dx and one 'surplus' d [again quite roughly-speaking]. Everything would be perfect had we had

$$dd_M = |f(x) - g(x)|dx,$$

yielding the L^1-norm distance $d = \int |f(x) - g(x)|dx$.
So let us define

$$\boxed{dd_M\big(f(x), g(y)\big) = \sqrt{\big(f(x) - g(y)\big)^2 - (x-y)^2}\sqrt{dxdy}} \tag{2}$$

and give it a chance! Let

$$x = y; \tag{3}$$

Before continuing, recall our expectation: $x = y$ should 'wash away' the 'input-dependent' aspect of (1) and yield a proper metric for function space. Now assuming

$$\sqrt{dx\ dx} = \sqrt{(dx)^2} = dx \tag{4}$$

then

$$d\ d_M\big(f(x), g(x)\big) = \sqrt{\big(f(x) - g(x)\big)^2}dx = |f(x) - g(x)|\ dx$$

$$\Rightarrow \boxed{d\big(f(x), g(x)\big) = \int |f(x) - g(x)|\ dx} \tag{5}$$

It might seem that (1) is the product metric of space of functions and $\mathbb{R}$ but notice that

$$d = \sqrt{d_1^2(f, g) + d_2^2(x, y)}$$

'glues' two metrics together while respecting their 'independence': f, g and x, y in the above (product) metric are 'dummies'. You do not need to know anything about d_2 to find d_1. This is not at all the case for (1) as it is in fact 'intertwining' functions and their inputs.

A full theory is under construction and will be presented as soon as possible.

References

[1] Alireza Jamali. Relativity in function spaces (preprint). *viXra:2111.0074*, 2021.