The vacuum energy, compact dimensions and consequences of a fractal hypothesis

Abstract

The fractal hypothesis of vacuum energy allows to investigate the theoretical compact dimensions of string theory, and its relationship with the nature of the quantum of action.

Keywords: Vacuum energy, compact dimensions, relative fractal dimension, isotropic fractal

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1. Mathematical approach

The mathematical approach will be very simple, treat the vacuum energy in a similar way to how we treat fractal lines. These form curves while the vacuum energy determines the geometric structure of the vacuum. In the absence of such energy, the vacuum would be completely "flat", and the particles would follow true classical trajectories instead of pseudo trajectories that are fractals of dimension 2 [1]. In fractal lines we work with scalars that represent one-dimensional segments, in quantum fluctuations with scalars representing the energy of the vacuum.

2. Relative fractal dimension and distance dependence

Suppose a fractal surface with dimension D = 2.356. The value of the dimension that exceeds 2 gives us a measure of the irregularity of the fractal and we will call it ε. Then, the fractal dimension $\boxed{D= \delta + \varepsilon}$ (topological or apparent dimension plus dimensional coefficient ε). The dimensional coefficient ε, in a way, offers us an idea of the fractal's ability to occupy part of the third dimension and, therefore, of space. We can have another fractal with the same dimensional value and yet be much more irregular than the first: for example, a curve that almost fills the space. It can have the same dimension, but it is much more irregular because its topological dimension is 1, unlike the fractal surface whose topological dimension is 2. Thus, we see that the dimension of a fractal does not give a real idea of its irregularity if it does not we compare with its topological dimension.

For variables with a topological dimension greater than unity, it is convenient to speak of the quotient D / δ, which we will call the relative fractal dimension, and not its fractal dimension [2]. We reduce the dispersion of results and can compare them with simple examples such as one-dimensional trajectories. We will have:

The relative fractal dimension = $D/\delta = (\delta + \varepsilon)/\delta$. (1)

This expression helps us understand how the characteristics of a fractal are modulated by modifying the geometry of space.

There is a very interesting property that continuous fractal curves exhibit as the Koch curve or the Brownian motion. In the case of Brownian motion, its dimension is 2 because it can cover a surface: this indicates that this motion to move away N effective steps from any point needs to travel N^2 total steps. This ability to "wander" is closely related to the fractal dimension [3]. Generalizing:

$$\boxed{(\text{Effective distance})^{\textbf{fractal dimension}} = \text{total distance over the fractal}} \quad (2)$$

The relative fractal dimension reduces any continuous fractal of topological dimension greater than unity to an equivalent fractal curve. The more isotropic the fractal, the more faithful the conversion will be, because it does not retain the directional properties of the original fractal. Once the conversion is done, we can apply expression (2), with care, according to the characteristics of the fractal. We will substitute in expression (2) the fractal dimension for the relative fractal dimension.

3. Add or subtract dimensions

The dimensional coefficient ε is added to the topological dimension. We can ask the following question: Is there a phenomenon that represents a subtraction of dimensions?[4] Of course, if we roll up a surface along one of its dimensions until it becomes a line, we will have passed from a 2-dimensional object to a 1-dimensional one, we will have subtracted one dimension. This geometric operation represents a subtraction of dimensions while the irregularity of a fractal, expressed by the dimensional coefficient ε, represents an addition to the topological dimension of the object. We do opposite geometric operations.

4. Inverse dependence on distance

If the dimension of a fractal is $\delta + \varepsilon$, its relative fractal dimension will be:

Relative fractal dimension = $(\delta + \varepsilon) / \delta$. (3)

If we subtract a value equal to ε from the number of topological dimensions, the new value of significant dimensions will be $(\delta - \varepsilon)$, and:

The new value of the relative fractal dimension = $\delta / (\delta - \varepsilon)$ (4)

There is a significant difference between expression (3) and expression (4). The first can only be positive, but the second can also be negative. We are interested that its value is (-1), that is, the inverse dependence of the fractal with the distance. In that case: $\delta / (\delta - \varepsilon) = -1$ which is satisfied for the value of the new significant dimensions equal to $\varepsilon / 2$.

5. Application to vacuum energy

The vacuum energy depends on the inverse of the distance, applying the expression $\delta / (\delta - \varepsilon) = -1$ and substituting the value δ for 3, which are the spatial dimensions of our universe, we obtain a value of 6 for the dimensional coefficient ε and for compact dimensions.

In our universe there would be 6 compact dimensions and the 3 dimensions that we recognize. The vacuum energy would have a fractal dimension 9 (topological dimension 3, plus a dimensional coefficient $\varepsilon = 6$).

Just as a fractal curve of dimension 3 (with topological dimension 1 and dimensional coefficient $\varepsilon = 2$) can cover a 3-dimensional space, the vacuum energy with a topological dimension 3 and a dimensional coefficient $\varepsilon = 6$ would cover all nine dimensions of our universe, six of them compact.

6. Generalization

In the fractal called Koch curve, or Koch snowflake, its fractal dimension is equal to (log 4) / log 3 because in the first iteration a segment of value 3 is replaced by 4 segments of value 1. In the case of the vacuum energy, we have a similar quotient in this case between the logarithms of two energies, where n is the distance:

Relative fractal dimension of vacuum energy = $(\log x)/\log n^{-1} = -1$

To achieve equality the value of $x = n$, then the energy represented in the numerator must be proportional to the distance. In the Kock curve the rectilinear segment of value 3 is defined in one dimension, but the four segments that replace it are defined in the plane. Similarly, the energy of the numerator is defined in 9 dimensions and that of the denominator, linked to the quantum of action, is defined in the ordinary 3 dimensions.

The energy of the vacuum depends on the inverse of the distance due to the very nature of the quantum of action: Suppose that we introduce a fictitious factor f to the expression of the quantum of action:

$$\Delta E \, \Delta t^{f} = \text{constant} \qquad (5)$$

Now the energy does not depend on the inverse of the distance, it depends on the inverse of the distance raised to f, since the expression of the relative dimension would be $(\log n) / \log 1 / n^{f}$. Then we could set the expression (4) equal to the value $- (1 / f)$, that is:

$\delta / (\delta - \varepsilon) = - (1/f)$ (in our universe f = 1, logically)

It can be expressed more simply:

$$(\varepsilon - \delta) / \delta = f \qquad (6)$$

This simple expression relates the number of ordinary dimensions, the number of compact dimensions, and the fictitious weight factor f, linked to the nature of the quantum.

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Note added. The approach is more particular than the one represented by the Nottale [5] scale relativity. We study the vacuum energy as an isotropic fractal and (with the relative fractal dimension) we analyze its simplified behavior as a fractal trajectory.

References[i]

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