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About the field equations of gravitation with variable gravitation constant
(*Über die feldgleichungen der gravitation bei variabler gravitationskonstante*[1])

Pascual Jordan[2] **and Claus Müller**[3]

University of Göttingen[4]

Richard L. Amoroso (Trans)[5]

Noetic Advanced Studies Institute
Escalante Desert Research Station
Beryl, UT 84714 USA

amoroso@noeticadvancedstudies.us

Abstract. Dirac's well-known proposal that the gravitational constant changes in the course of cosmological evolution is studied as it relates to requiring fundamental changes to Einstein's General Theory of Relativity.

Translators Forward. This issue of IOP JPCS provides the 1st English translations of the three 1946, 1947, 1948 Jordan papers important in development of Field equations for the Kaluza hypothesis. Pascual Jordan was a well-known German theoretical physicist who is one of the founders of quantum mechanics and quantum field theory. Together with Max Born and Werner Heisenberg, Jordan coauthored an important series of papers on quantum mechanics [1,2]. He went on to pioneer early quantum field theory. In 1933 Jordan joined the Nazi party, if he had not done so; it is likely he would have won a Nobel Prize in Physics for his work with Max Born, who in 1954 with Walther Bothe.

About the field equations of gravitation with variable gravitation constant

The well-known thesis, founded by Dirac, that the so-called gravitation constant is in reality changeable in the course of cosmological development, requires a fundamental expansion of Einstein's general relativistic theory of gravity, such that $x = 8\pi f / c^2$ is introduced as a further, fifteenth field quantity in

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[1] 1947 *Zeitschrift für Naturforschung* A2 **1** 1-2; https://doi.org/10.1515/zna-1947-0102; https://www.degruyter.com/document/doi/10.1515/zna-1947-0102/html
[2] Pascual Jordan 18 October 1902 – 31 July 1980
[3] Claus Müller 20 February 1920 – 6 February 2008
[4] University of Göttingen (*Georg-August-Universität Göttingen*) Jordan earned his doctorate studying under Max Born.
[5] This work of translation generally adheres to literal meaning as stated by the author.

addition to the ten gravitational potentials g_{ik} and the electromagnetic four-fold potential. In an earlier work [3] it was shown that for this generalization of the previous theory of the vacuum field (including electromagnetic fields) a very natural basis is offered by the 5D or projective relativity theory, as it is derived from the ideas of Kaluza and later developed by a number of authors (Klein, Einstein, Mayer, Veblen, van Dantzig, Schouten, Pauli, Pais, Möller, Rosenfeld, Nörlund, Belinfante) [4]. However, in the earlier work mentioned, the new field equations of the vacuum had not yet been established; this is what this note refers to. After examining various options, we see the simplest possible option as the most probable.

The results obtained in the earlier work, which are used here, are explained so far that the present note can be understood even without the prior knowledge; the content of the previous communication will soon be made available elsewhere.

We consider the four global coordinates x^k (Latin indices always = 1 to 4) as functions of the ratios of five *projective* coordinates X^μ (Greek indices = 0 to 4). In the space of X^μ we analogize the Riemannian geometry, but with the difference that we do not require invariance against all coordinate transformations $X^\mu \to X'^\mu$, but only against homogeneous transformations; if we write $A_{|\mu}$ for the derivative from A to X^μ, then it is always.

$$X'^\mu{}_{|\nu} X^\nu = X'^\mu \tag{1}$$

The Group G_5 of the transformations (1) is isomorphic with the group of all 4D coordinate and gauge transformations [5].

The components of the 5D metric fundamental tensors should be homogeneous functions (-2[th]) degree:

$$g_{\mu\nu|\lambda}\, \mathrm{X}^\lambda = -2 g_{\mu\nu}. \tag{2}$$

Path (1) is then

$$j = g_{\mu\nu} X^\mu X^\nu. \tag{3}$$

a scalar.

The transition to a 4D representation is mediated by the derivatives

$$x^k{}_{|\nu} = g_\nu^k, \tag{4}$$

with which we define the 4D metric fundamental tensor by

$$g^{ik} = g_\mu^i g_\nu^k g^{\mu\nu}. \tag{5}$$

(One must distinguish $g_{12}^{(5)}$, the 12-components of $g_{\mu\nu}$, and $g_{12}^{(4)}$, from that of g_{ik}). For the line element one gets the formula

$$ds^2 = g_{\mu\nu} dX^\mu dX^\nu = g_{ik} dx^i dx^k + \frac{1}{J}\left(X_\mu dX^\mu\right)^2, \tag{6}$$

so that a 4D metric g_{ik} is provided in a simple manner by the 5D metric $g_{\mu\nu}$.

Specifying the 15 homogeneous functions $g_{\mu\nu} = g_{\nu\mu}$ for 5 coordinates is obviously equivalent to specifying any 15 functions of 4 coordinates; so, we are dealing with a field theory of 15 field quantities.

If one looks, as in the previous view, there are only 14 physical field quantities g_{ik}, Φ_k. To interpret geometrically, one must artificially restrict the theory under consideration, namely by the secondary condition that the mentioned scalar $J = X_\mu X^\mu$ should be constant. On the other hand, if this additional condition is dropped, one gains at the same time with a fundamentally harmonic design of the theory that it is adapted to the physical conception of a fifteenth field quantity *x*. We interpret J physically as

$$J = 2x / c^2. \tag{7}$$

With the electromagnetic field strength $F_{kl} = -F_{lk}$ we interpret the tensor

$$F^{ik} = \frac{1}{J} g_Q^i g_\sigma^k \left(X_{\mu|\nu} - X_{\nu|\mu} \right) g^{\mu Q} g^{\nu\sigma}, \tag{8}$$

which can be shown to be the rotation of a four-potential.

We want to denote the tapered 5D curvature tensor with $R_{\mu\nu}$, and the 4D one with G_{ik}. Then, we want to contrast this analogously with the basic Einsteinian equation $G_{ik} = 0$ of the pure gravitational field (with x = const.) assuming the following generalized field equation:

$$R_{\mu\nu} = 0. \tag{9}$$

The formulas developed in the earlier note can be seen that (9) is equivalent to the following 15 field equations, in which $A_{\|k}$ is to denote the covariant derivative with respect to:

$$G_{ik} + \frac{x}{c^2} F_j^k F_{kl} = -\frac{1}{2x} \left(x_{\|i\|k} - \frac{x_{\|i} x_{\|k}}{2x} \right), \tag{10}$$

$$F_{\|j}^{jk} = -\frac{3}{2} x_{\|j} F^{jk}, \tag{11}$$

$$G = -\frac{x}{2c^2} F_{jk} F^{jk} - \frac{x_{\|j} x_{\|k} g^{jk}}{x} + \frac{x_{\|j} x_{\|k} g^{ik}}{2x^2}. \tag{12}$$

The relationship between these new field equations and the previous ones is as follows: Special exact solutions of (10), (11), (12) you get when you determine pure gravitational fields (i.e., $F_{kl} = 0$) with constant *x* in such a way that they satisfy the old Einstein field equations $G_{ik} = 0$. In the presence of electromagnetic fields, on the other hand, a certain variability of *x* must always be expected, because the assumption x = const. according to (10) and (12) would lead to $G = 0$ and $F_{kl} F^{kl} = 0$. However, under all normal conditions the relative changes in *x* are so small that a sufficient approximation is obtained if one ignores (12) and with *x* = const, equations (10), (11) are simplified to the classical Einstein - Maxwellian field equations of the vacuum:

$$G_{ik} + \frac{x}{c^2} F_i^{\cdot j} \quad F_{kj} = 0, \tag{10'}$$

$$F^{jk}{}_{\|j} = 0. \tag{11'}$$

The terms of equations (10), (11), (12) containing the derivatives of *x* provide important support for the cosmological considerations which have recently been presented [6]. In the case of a cosmos expanding at constant speed, the curvature and the non-vanishing components (diagonal components) decrease inversely to the square of the age of the Earth. The other terms in (10), (12) are now such that they suggest looking for cosmological models in which *x* behaves like a certain power $t^{-\alpha}$ of the age of the world *t*, while the electromagnetic radiation density or the diagonal components of the tensors $F_i^{\cdot j} F_{kj}$, behave like $t^{\alpha-2}$ because then the individual parts in (10) are correspondingly proportional to $t^{-2} = t^{-\alpha} t^{\alpha-2} = t^{-\alpha-2} t^{\alpha} = t^{-2(\alpha+1)} t^{2\alpha}$.

The fact that more precisely $\alpha = 1$ cannot be justified without taking into account the matter in the cosmos besides electromagnetic radiation. In addition, however, other values α seem to have a certain significance for cosmological-astrophysical problems; we'll come back to that.

References

[1] Born M Jordan P 1925 Zur Quantenmechanik *Zeitschrift für Physik A Hadrons and Nuclei* **34** 1
[2] Born M Heisenberg W Jordan P 1926 Zur Quantenmechanik II *Zeitschrift für Physik A Hadrons and Nuclei* **35** 8-9, 557–615; https://doi.org/10.1007/BF01379806
[3] Jordan P 1945 *Physik. Z.* **46** (Korrektur-fahnen).
[4] Pauli P 1942 *Ann. Physik* **5** 18 305, 337; Pais A 1933 *Physica* **8** 1137
[5] Jordan P 1945 *Nachr. Ges. Wiss*. Göttingen, Math-physik. Kl. 74
[6] Jordan P 1944 *Physik. Z.* **45** 18.