Exact expressions of the fine-structure constant

Stergios Pellis
sterpellis@gmail.com
http://physiclessons.blogspot.com/
Greece
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Abstract

The purpose of many sciences is to find the most accurate mathematical formula that can be found in the experimental value of fine-structure constant. Attempts to find a mathematical basis for this dimensionless constant have continued up to the present time. However,no numerological explanation has ever been accepted by the physics community. In this paper we will present the exact expressions for the fine-structure constant. A simple and absolutely accurate expression for the fine-structure constant in terms of the Archimedes constant:

$$\alpha^{-1}=2\cdot 3\cdot 11\cdot 41\cdot 43^{-1}\cdot \ln 2\cdot \pi$$

The equivalent expressions for the fine-structure constant from the madelung constant. Also a exact expression for the fine-structure constant in terms of the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot \varphi^{-2}-2\cdot \varphi^{-3}+(3\cdot \varphi)^{-5}$$

Also other expressions for the fine-structure constant. Finally we will present the continued fractions for the fine-structure constant.

Keywords

Fine-structure constant , Dimensionless physical constants , Golden ratio , Golden angle , Relativity factor , Fifth power of the golden mean , Archimedes constant , Madelung constant

1. Introduction

The fine-structure constant is one of the most fundamental constants of physics. It describes the strength of the force of electromagnetism between elementary particles in what is known as the standard model of particle physics. In particular, the fine-structure constant sets the strength of electromagnetic interaction between light (photons) and charged elementary particles such as electrons and muons. The quantity α was introduced into physics by A. Sommerfeld in 1.916 and in the past has often been referred to as the Sommerfeld fine-structure constant. In order to explain the observed splitting or fine structure of the energy levels of the hydrogen atom, Sommerfeld extended the Bohr theory to include elliptical orbits and the relativistic dependence of mass on velocity.

One of the most important numbers in physics is the fine-structure constant α which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass,length,time or electric charge . A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It’s a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons. Many eminent physicists and philosophers of science have pondered why α itself has the value that it does,because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. A similar situation occurs with the proton-electron mass ratio μ in [9],not because of its ubiquity,but rather how chemistry can be based on two key electrically charged particles of opposite electric charge that are opposite but of seemingly identical magnitude while their masses have a ratio that is quite large yet finite. These two questions have a huge bearing on why physics and chemistry behave the way they do. The product of the two quantities appears,at least at first glance,not to be so important.

There is a dream,which,albeit more often not confessed,occupies the most secret aspirations of theoreticians and is that of reducing the various constants of Physics to simple formula involving integers and transcendent numbers. The fine-structure constant plays an important role in moder

2. The search for mathematical expression for the fine-structure constant

The mystery about the fine-structure constant is actually a double mystery. The first mystery – the origin of its numerical value – has been recognized and discussed for decades. The second mystery – the range of its domain – is generally unrecognized.

— M. H. MacGregor (2.007). The Power of Alpha.

When I die my first question to the Devil will be: What is the meaning of the fine structure constant?

— Wolfgang Pauli

“God is a pure mathematician!' declared British astronomer Sir James Jeans. The physical Universe does seem to be organized around elegant mathematical relationships. And one number above all others has exercised an enduring fascination for physicists: 137,0359991.... It is known as the fine-structure constant and is denoted by the Greek letter alpha (α).”

— Paul Davies

“While twentieth-century physicists were not able to identify any convincing mathematical constants underlying the fine structure, partly because such thinking has normally not been encouraged, a revolutionary suggestion was recently made by the Czech physicist Raji Heyrovska, who deduced that the fine structure constant, ...really is defined by the [golden] ratio ....”

— Carl Johan Calleman, The Purposeful Universe: How Quantum Theory and Mayan Cosmology Explain the Origin and Evolution of Life

The fine-structure constant plays an important role in modern physics. Yet it continues to be a mystery as to exactly what it represents and why it has the mystical value it has. The elementary charge of electron e was proposed by Stoney in 1.894 and discovered by Thomson in 1.896,then Planck introduced the energy quanta $h\cdot v$ in 1.901 and explained it as photon $E=h\cdot v$ by Einstein in 1.905. Planck first noticed in 1.905 that e^2/c and h have the same dimension. In 1.909,Einstein found that there are two fundamental velocities in physics: c and e^2/h requiring explanation. He said, “It seems to me that we can conclude from $h=e^2/c$ that the same modification of theory that contains the elementary quantum e as a consequence,will also contain as a consequence the quantum structure of radiation.” Albert Einstein was the first to use a mathematical formula for the fine-structure constant α in 1.909. This expression is:

$$\alpha = \frac{7\pi}{3.000}$$

with numerical value α=0,00733038286 with an error accuracy of 0,45%. Later many scientists used other mathematical formulas for fine-structure constant but they are not at all accurate. These are Jeans 1.913,Lewis Adams 1.914,Lunn in 1.922,Peirles in 1.928 and others. Arthur Eddington was the first to focus on its inverse value and suggested that it should be an integer,that the theoretical value is $\alpha^{-1}=136$. In his original document 1.929 he applied the value:

$$\alpha^{-1}=16+1/2\times16\times(16-1)=136$$

However,the experiments themselves consistently showed that $\alpha^{-1}\approx137$. This forced him to look for an error in his original theory. He soon came to the conclusion that:

$$\alpha^{-1}=137$$

He thus argued that the extra unit was a consequence of the initial exclusion of every elementary particle pair in the universe. In the document of 1.929,Eddington considered that fine-structure constant relates in a simple way to the cosmological constants,as given by the expression:

$$\alpha=\frac{2\pi mcR_E}{h\sqrt{N}}$$

where N the cosmic number,the number of electrons and protons in the closed universe. Eddington always kept the name and the symbol α:

$$a=\frac{hc}{2\pi q_e^2}$$

The first to find an exact formula for the fine-structure constant α was the Swiss mathematician Armand Wyler in 1.969. Based on the arguments concerning the congruent group,the group consists of simple Lorentz transformations such as the space-time dimensions that leave the Maxwell equations unchanged. The first form of the Wyler constant type is:

$$\alpha_w=\left(\frac{9}{16\pi^3}\right)\left(\frac{\pi}{5!}\right)^{\frac{1}{4}}$$

With numerical value $\alpha_w=0{,}00729735252...$ At the time it was proposed,they agreed with the experiment to be within 1.5 ppm for the value α^{-1}.

3. Measurement of the fine-structure constant

The 2.018 CODATA recommended value of α is:

$$\alpha=0.0072973525693(11)$$

With standard uncertainty $0{,}0000000011\times10^{-3}$ and relative standard uncertainty $1{,}5\times10^{-10}$. For reasons of convenience,historically the value of the reciprocal of the fine-structure constant is often specified. The 2.018 CODATA recommended value is given by:

$$\alpha^{-1}=137{,}035999084(21)$$

With standard uncertainty $0{,}000000021\times10^{-3}$ and relative standard uncertainty $1{,}5\times10^{-10}$. There is general agreement for the value of α,as measured by these different methods. The preferred methods in 2.019 are measurements of electron anomalous magnetic moments and of photon recoil in atom interferometry. The most precise value of α obtained experimentally (as of 2.012) is based on a measurement of g using a one-electron so-called "quantum cyclotron" apparatus,together with a calculation via the theory of QED that involved 12.672 tenth-order Feynman diagrams:

$$\alpha^{-1}=137{,}035999174(35)$$

This measurement of α has a relative standard uncertainty of $2{,}5\times10^{-10}$. This value and uncertainty are about the same as the latest experimental results. Further refinement of this work were published by the end of 2.020,giving the value:

$$\alpha^{-1}=137{,}035999206(11)$$

with a relative accuracy of 81 parts per trillion.

4. Simple expression for the fine-structure constant in terms of the Archimedes constant

Archimedes constant π is a mathematical constant that appears in many types in all fields of mathematics and physics. It is also found in various types from other disciplines,such as Cosmology,numbers, Statistics, fractals, thermodynamics, engineering,and electromagnetism. Archimedes constant π appears in the cosmological constant, Heisenberg's uncertainty principle, Einstein's field equation of general relativity,Coulomb's law for the electric force in vacuum,Magnetic permeability of free space,Period of a simple pendulum with small amplitude,Kepler's third law of planetary motion,the buckling formula,etc. In [7] we presented exact and approximate expressions between the Archimedes constant π,the golden ratio φ,the Euler's number e and the imaginary number i.

We proposed the simple and absolutely accurate expression for the fine-structure constant in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\ln2\cdot\pi \tag{1}$$

$$\alpha^{-1}=(2.706/43)\cdot\ln2\cdot\pi \tag{2}$$

$$\alpha^{-1}=[(14^3-38)\cdot43^{-1}]\cdot\ln2\cdot\pi \tag{3}$$

The equivalent expressions for the fine-structure constant with the madelung constant $b_2(2)$ are:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot b_2(2) \tag{4}$$

$$\alpha^{-1}=-(2.706/43)\cdot b_2(2) \tag{5}$$

$$\alpha^{-1}=-[(14^3-38)\cdot43^{-1}]\cdot b_2(2) \tag{6}$$

with absolutely accurate numerical value:

$$\alpha^{-1}=137{,}035999078175526$$

$$\alpha=0{,}00729735256959$$

This accurate expression is the most impressive since it is simple and contains just a few prime numbers and the madelung constant. These prime numbers can be possibly connected to finite groups (Group of Lie type). The series representations for the fine-structure constant is:

$$\frac{2}{43}\times3\times11\times41\log(2)\,\pi=\frac{5412}{43}\,i\,\pi^2\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\frac{2706}{43}\,\pi\log(x)-\frac{2706}{43}\,\pi\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k}\quad\text{for } x<0$$

$$\frac{2}{43}\times 3\times 11\times 41\log(2)\,\pi = \frac{2706}{43}\pi\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\frac{2706}{43}\pi\log(z_0)+\frac{2706}{43}\pi\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\frac{2706}{43}\pi\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}$$

$$\frac{2}{43}\times 3\times 11\times 41\log(2)\,\pi = \frac{5412}{43}\,i\,\pi^2\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\frac{2706}{43}\pi\log(z_0)-\frac{2706}{43}\pi\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}$$

From this expression we pose:

$$\alpha_b=(2.706/43)=2\cdot 3\cdot 11\cdot 41\cdot 43^{-1}$$

This combination of prime numbers we will call the coefficient for the fine-structure constant. So the expression for the fine-structure constant can written as:

$$\alpha^{-1}=-\alpha_b\cdot b_2(2)$$

The repeating decimal of the coefficient for the fine-structure constant α_b is:

$$62,\overline{930232558139534883720}\ \text{(period 21)}$$

The continued fraction for the α_b is:

$$62+\cfrac{1}{1+\cfrac{1}{13+\cfrac{1}{3}}}$$

The egyptian fraction expansion for the α_b is:

$$62+\frac{1}{2}+\frac{1}{3}+\frac{1}{11}+\frac{1}{167}+\frac{1}{473946}$$

The repeating decimal of the α_b^{-1} is:

$$0,\overline{01589061345}\ \text{(period 10)}$$

The pattern of the continued fraction for the fine-structure constant is:

[137; 27, 1, 3, 1, 1, 16, 1, 4, 45, 12, 2, 4, 1, 6, 4, 2, 1, 155, 2, 7, 26, 1, 2, 1,
2, 10, 2, 3, 11, 2, 1, 2, 1, 4, 2, 1, 2, 7, 7, 1, 1, 1, 1, 2, 1, 5, 1, 1, 1, 1, 5, 1, 3,
1, 1, 1, 1, 1, 1, 5, 2, 9, 1, 1, 3, 2, 1, 27, 1, 11, 1, 1, 1, 11, 2, 5, 1, 3, 41, 1, 4,
1, 13, 1, 1, 38, 1, 4, 2, 1, 2, 6, 1, 2, 18, 33, 1, 1, 4, 1, 9611, 1, 6, 5, 5, 1, 5,
1, 1, 2, 2, 1, 2, 1, 2, 2, 1, 2, 12, 2, 1, 3, 8, 1, 1, 5, 2, 8, 2, 8, 111, 12, 3, 3, 2,
1, 1, 18, 1, 1, 1, 1, 15, 1, 6, 3, 7, 1, 1, 1, 1, 2, 9, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1, 3,
1, 9, 2, 2, 1, 12, 16, 1, 4, 1, 1, 3, 3, 54, 1, 1, 1, 2, 1, 2, 2, 3, 1, 9, 1, 2, 4, 1,
1, 2, 1, 6, 4, 8, 42, 1, 1, 9, 12, 1, 7, 9, 1, 1, 1, 3, 3, 3, 16, 2, 1, 2, 12, 1, 2, 2,
1, 3, 1, 5, 7, 2, 1, 3, 3, 1, 2, 1, 3, 2, 2, 6, 1, 6, 1, 2, 1, 1, 9, 1, 2, 2, 106, 1, 1,
1, 1, 8, 1, 1, 1, 27, 2, 55, 2, 1, 6, 2, 1, 3, 1, 9, 1, 3, 2, 1, 6, 4, 1, 13, 2, 2, 1,
2, 6, 1, 2, 1, 1, 3, 6, 1, 2, 189, 3, 8, 3, 4, 4, 1, 3, 5, 1, 1, 6, 565, 1, 1, 2, 7, 2,
1, 1, 4, 1, 2, 2, 3, 1, 3, 96, 9, 2, 1, 2, 67, 1, 1, 1, 1, 8, 3, 1, 4, 1, 2, 26, 1, 2,
1, 3, 5, 13, 11, 3, 1, 1, 10, 1, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1,
11, 5, 9, 14, 1, 1, 4, 2, 1, 5, 1, 9, 2, 77, 5, 1, 1, 1, 5, 4, 1, 1, 3, 1, 10, 2, 1, 5,
10, 4, 1, 6, 1, 2, 1, 2, 1, 2, 16, 708, 1, 4, 3, 1, 1, 14, 1, 3, 1, 1, 2, 38, 46, 1,
1, 4, 1, 1, 2, 3, 2, 2, 15, 1, 3, 1, 1, 20, 1, 14, 6, 2, 7, 2, 1, 2, 14, 4, 3, 1, 1, ...]

The continued fraction for the fine-structure constant is:

$$137+\cfrac{1}{27+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{16+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{45+\cfrac{1}{12+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{4+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}$$

6. Exact formula for the fine-structure constant in terms of the golden ratio

Golden ratio φ is an omnipresent number in nature,found in the architecture of living creatures as well as human buildings,music,finance,medicine,philosophy,and of course in physics and mathematics including quantum computation. It is the most irrational number known and a number-theoretical chameleon with a self-similarity property. The golden ratio can be found in nearly all domains of Science,appearing when self-organization processes are at play and expressing minimum energy configurations. Several non-exhaustive examples are given in biology (natural and artificial phyllotaxis, genetic code and DNA) physics (hydrogen bonds,chaos, superconductivity), astrophysics (pulsating stars,black holes), chemistry (quasicrystals,protein AB models),and technology (tribology, resistors, quantum computing, quantum phase transitions, photonics). The fifth power of the golden mean appears in Phase transition of the hard hexagon lattice gas model, Phase transition of the hard square lattice gas model,One-dimensional hard-boson model, Baryonic matter relation according to the E-infinity theory,Maximum quantum probability of two particles,Maximum of matter energy density,Reciprocity relation between matter and dark matter,Superconductivity phase transition, etc. Among the numbers in the Fibonacci range,the numbers 5 and 13 seem to be the most important. Whereas number 5 is involved in the definition of the golden mean,number 13 is found as a helix repetition number for instance in tubulin protein,thought to be the location from where our thinking and consciousness originates. Dr. Rajalakshmi Heyrovska has found that the golden ratio φ provides a quantitative link between various known quantities in atomic physics,research in this book chapter entitled "The golden ratio in the creations of Nature arises in the architecture of atoms and ions". While searching for the exact values of ionic radii and for the significance of the ionization potential of hydrogen,Dr. Heyrovska has found that the Bohr radius can be divided into two Golden sections pertaining to the electron and proton. More generally,it was found that φ is also the ratio of anionic to cationic radii of any atom,their sum being the covalent bond length. After that she showed,among other facts,that many bond lengths in organic and inorganic molecules behave additively,and are the sum of the covalent and the ionic radii,whether partially or fully ionic or covalent.

An interpretation and a value of the fine-structure constant α^{-1} has been discovered in terms of the golden angle. Dr. Heyrovska proposed another interpretation of α based on the observation that it is close to the golden angle. Fine-structure constant can also be formulated for the first time exclusively in terms of the golden ratio as follows:

There is a dream,which,albeit more often not confessed,occupies the most secret aspirations of theoreticians and is that of reducing the various constants of Physics to simple formula involving integers and transcendent numbers. Fine-structure constant can also be formulated in [8] exclusively in terms of the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5} \tag{7}$$

with numerical values:

$$\alpha^{-1}=137{,}03599916476564\ldots\ldots.. \tag{8}$$

$$\alpha=0{,}0072973525649829 2\ldots\ldots.. \tag{9}$$

The numerical value (7) is the average of the measurements (2),(3) and (4). We believe that the formula (10) is the exact formula for the fine-structure constant α. Another beautiful forms of the equations are:

$$\frac{1}{\alpha}=\frac{360}{\varphi^2}-\frac{2}{\varphi^3}+\frac{1}{3^5\varphi^5}$$

$$\frac{1}{\alpha}=\frac{360}{\varphi^2}-\frac{2}{\varphi^3}+\frac{3^{-5}}{\varphi^5} \tag{10}$$

Other equivalent expressions for the fine-structure constant are:

$$\alpha^{-1}=(362-3^{-4})\cdot\varphi^{-2}-(1-3^{-5})\cdot\varphi^{-1} \tag{11}$$

$$\alpha^{-1}=(362-3^{-4})+(3^{-4}+2\cdot3^{-5}-364)\cdot\varphi^{-1} \tag{12}$$

$$\alpha^{-1}=1-2\cdot\varphi^{-1}+360\cdot\varphi^{-2}-\varphi^{-3}+(3\cdot\varphi)^{-5} \tag{13}$$

$$\alpha^{-1}=\varphi^0-2\cdot\varphi^{-1}+360\cdot\varphi^{-2}-\varphi^{-3}+(3\cdot\varphi)^{-5} \tag{14}$$

$$\alpha^{-1}=\varphi^0-2\cdot\varphi^{-1}+360\cdot\varphi^{-2}-\varphi^{-3}-241\cdot3^{-5}\cdot\varphi^{-4}-(3\cdot\varphi)^{-5} \tag{15}$$

$$\alpha^{-1}=(174.474\cdot\varphi+86.995)\cdot(243\cdot\varphi^5)^{-1} \tag{16}$$

$$\alpha^{-1}=(87.480\cdot\varphi^3-486\cdot\varphi^2+1)\cdot(243\cdot\varphi^5)^{-1} \tag{15}$$

$$\frac{360}{\phi^2}-\frac{2}{\phi^3}+\frac{1}{(3\,\phi)^5}=-\frac{2}{\left(2\cos\left(\frac{\pi}{5}\right)\right)^3}+\frac{360}{\left(2\cos\left(\frac{\pi}{5}\right)\right)^2}+\frac{1}{\left(6\cos\left(\frac{\pi}{5}\right)\right)^5} \tag{16}$$

$$\frac{360}{\phi^2}-\frac{2}{\phi^3}+\frac{1}{(3\,\phi)^5}=-\frac{2}{\left(1+2\sin\left(\frac{\pi}{10}\right)\right)^3}+\frac{360}{\left(1+2\sin\left(\frac{\pi}{10}\right)\right)^2}+\frac{1}{\left(3\left(1+2\sin\left(\frac{\pi}{10}\right)\right)\right)^5} \tag{17}$$

$$\frac{360}{\phi^2}-\frac{2}{\phi^3}+\frac{1}{(3\,\phi)^5}=-\frac{2}{(2\sin(54\,°))^3}+\frac{360}{(2\sin(54\,°))^2}+\frac{1}{(6\sin(54\,°))^5} \tag{18}$$

$$\frac{360}{\phi^2}-\frac{2}{\phi^3}+\frac{1}{(3\,\phi)^5}=\frac{1}{(-6\sin(666\,°))^5}-\frac{2}{(-2\sin(666\,°))^3}+\frac{360}{(-2\sin(666\,°))^2} \tag{19}$$

$$\frac{360}{\phi^2}-\frac{2}{\phi^3}+\frac{1}{(3\,\phi)^5}=\frac{1}{(-6\cos(216\,°))^5}-\frac{2}{(-2\cos(216\,°))^3}+\frac{360}{(-2\cos(216\,°))^2} \tag{20}$$

$$\frac{360}{\phi^2}-\frac{2}{\phi^3}+\frac{1}{(3\,\phi)^5}=-\frac{2}{\left(\frac{1}{2}\csc\left(\frac{\pi}{10}\right)\right)^3}+\frac{360}{\left(\frac{1}{2}\csc\left(\frac{\pi}{10}\right)\right)^2}+\frac{1}{\left(\frac{3}{2}\csc\left(\frac{\pi}{10}\right)\right)^5} \tag{21}$$

$$\frac{360}{\phi^2}-\frac{2}{\phi^3}+\frac{1}{(3\,\phi)^5}=-\frac{2}{\left(\frac{1}{2}\sec\left(\frac{2\pi}{5}\right)\right)^3}+\frac{360}{\left(\frac{1}{2}\sec\left(\frac{2\pi}{5}\right)\right)^2}+\frac{1}{\left(\frac{3}{2}\sec\left(\frac{2\pi}{5}\right)\right)^5} \tag{22}$$

6. Continued fraction for the fine-structure constant

The pattern of the continued fraction for the fine-structure constant is:

[137; 27, 1, 3, 1, 1, 18, 1, 1, 1, 1, 132, 1, 2, 1, 1, 1, 2, 7, 6, 75, 1, 1, 2, 1, 9,
5, 1, 19, 7, 1, 5, 1, 5, 3, 7, 14, 1, 1, 4, 1, 1, 3, 2, 2, 10, 6, 1, 3, 1, 1, 19, 1, 1,
1, 26, 1, 1, 1, 6, 1, 6, 70, 3, 3, 3, 1, 8, 2, 1, 10, 13, 3, 28, 1, 6, 24, 903, 1, 4,
2, 2, 2, 1, 16, 1, 2, 12, 10, 1, 1, 4, 3, 1, 1, 2, 18, 1, 4, 1, 1, 4, 2, 7, 3, 1, 1, 4⋱
0, 1, 4, 2, 4, 12, 7, 4, 3, 10, 12, 4, 2, 1, 2, 2, 1, 1, 1, 21, 1, 3, 12, 46, 1, 3, 1,
1, 782, 3, 1, 1, 2, 7, 2, 3, 7, 1, 1, 5, 1, 1, 11, 5, 8, 43, 1, 1, 2, 4, 1, 1, 1, 1, 1⋱
0, 3, 4, 5, 1, 30, 46, 60, 2, 3, 1, 104, 1, 1, 4, 3, 1, 1, 3, 1, 9, 2, 2, 3, 2, 6, 3,
6, 4, 1, 1, 3, 3, 65, 1, 7, 2, 28, 25, 2, 1, 5, 2, 2, 225, 1, 1, 1, 1, 1, 32, 3, 8, 3,
1, 12, 1, 5, 1, 11, 1, 5, 1, 1, 4, 6, 1, 1, 18, 1, 2, 8, 24, 2, 1, 1, 4, 1, 33, 1, 8,
2, 1, 1, 3, 4, 22, 1, 1, 1, 3, 2, 1, 1, 2, 1, 2, 697, 1, 1, 1, 4, 1, 5, 1, 10, 1, 1, 2,
1, 1, 5, 1, 1, 19, 4, 1, 1, 4, 3, 1, 1, 3, 4, 1, 1, 3, 2, 2, 10, 1, 13, 2, 2, 6, 1, 5,
3, 1, 1, 2, 2, 14, 1, 1, 22, 1, 1, 1, 14, 9, 1, 5, 70, 1, 4, 1, 2, 12, 1, 3, 1, 3, 7,
2, 1, 1, 12, 1, 1, 2, 28, 1, 1, 1, 2, 1, 4, 1, 3, 41, 1, 2, 6, 1, 1, 1, 5, 1, 14, 1, 2,
1, 2, 20, 1, 4, 2, 1, 3, 1, 1, 5, 5, 1, 20, 1, 24, 74, 1, 4, 2, 1, 1, 17, 1, 28, 3, 1,
1, 1, 7, 1, 1, 4, 1, 27, 1, 1, 3, 2, 3, 80, 1, 1, 1, 15, 1, 3, 1, 93, 2, 1, 2, 1, 35,
4, 4, 1, 1, 2, 3, 18, 1, 1, 1, 1, 4, 2, 1, 9, 2, 1, 5, 1, 1, 2, 1, 47, 2, 1, 1, 1, 1, …]

The continued fraction for the fine-structure constant is:

$$137+\cfrac{1}{27+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{132+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}$$

In the notation of Carl Friedrich Gauss the fine-structure constant is:

$$\frac{1}{243\,\phi^5}+\underset{k=1}{\overset{3}{\mathrm{K}}}\frac{\begin{cases}-\frac{2}{\phi^3} & k=1\\ 180\,\phi & k=2\\ -180\,\phi & k=3\end{cases}}{1}$$

5. Other expressions for the fine-structure constant

Other formulas for the fine-structure constant are:

$$\alpha^{-2}=137^2+\pi^2-(13\cdot19/11\cdot5^2\cdot197) \tag{23}$$

$$\alpha^{-1}=\pi^4+\pi^3+\pi+\pi^{-1}+2\cdot\pi^{-3}+3\cdot\pi^{-6}+\pi^{-8}+\pi^{-9}+2\cdot\pi^{-11}+\pi^{-12}+2\cdot\pi^{-14}+\pi^{-15}+2\cdot\pi^{-16} \tag{24}$$

$$\alpha^{-1}=\varphi^{10}+2\cdot\varphi^4+\varphi^{-3}+\varphi^{-5}+\varphi^{-10}+\varphi^{-14}+\varphi^{-17}+\varphi^{-20}+\varphi^{-23}+\varphi^{-25}+\varphi^{-27}+\varphi^{-29}+\varphi^{-33}+\varphi^{-38}+\varphi^{-40} \tag{25}$$

$$\alpha^{-1}=2\cdot e^4+e^3+e^2+2\cdot e^{-2}+e^{-3}+2\cdot e^{-4}+e^{-5}+e^{-7}+e^{-8}+e^{-11}+2\cdot e^{-12}+e^{-13}+e^{-15}+2\cdot e^{-16}+e^{-19} \tag{26}$$

8. Conclusions

In this paper presented the exact expressions for the fine-structure constant. A simple and absolutely accurate expression for the fine-structure constant in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\ln2\cdot\pi$$

We presented new exact expression for the fine-structure constant α in terms of the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}$$

New interpretation and a very accurate value of the fine-structure constant has been discovered in terms of the Archimedes constant and the golden radio. These equations are simple,elegant and symmetrical in a great physical meaning. These exact expressions should be studied further.

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