Exact expressions of the fine structure constant α

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Abstract

In this paper are the exact formulas for the fine-structure constant. A simple and absolutely accurate expression for the fine-structure constant in terms of the Archimedes constant π. We propose the exact formula for the fine-structure constant α in terms of the golden angle,relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}$$

Also we will present the continued fraction for the fine-structure constant.

Keywords

Fine-structure constant , Dimensionless physical constants , Golden ratio , Golden angle , Relativity factor , Fifth power of the golden mean

1. Introduction

Golden ratio φ is an omnipresent number in nature,found in the architecture of living creatures as well as human buildings,music,finance,medicine,philosophy,and of course in physics and mathematics including quantum computation. It is the most irrational number known and a number-theoretical chameleon with a self-similarity property. The golden ratio can be found in nearly all domains of Science,appearing when self-organization processes are at play and expressing minimum energy configurations. Several non-exhaustive examples are given in biology (natural and artificial phyllotaxis, genetic code and DNA) physics (hydrogen bonds,chaos, superconductivity), astrophysics (pulsating stars,black holes), chemistry (quasicrystals,protein AB models),and technology (tribology, resistors, quantum computing, quantum phase transitions, photonics). The fifth power of the golden mean appears in Phase transition of the hard hexagon lattice gas model, Phase transition of the hard square lattice gas model,One-dimensional hard-boson model, Baryonic matter relation according to the E-infinity theory,Maximum quantum probability of two particles,Maximum of matter energy density,Reciprocity relation between matter and dark matter,Superconductivity phase transition, etc. Among the numbers in the Fibonacci range,the numbers 5 and 13 seem to be the most important. Whereas number 5 is involved in the definition of the golden mean,number 13 is found as a helix repetition number for instance in tubulin protein,thought to be the location from where our thinking and consciousness originates.

One of the most important numbers in physics is the fine-structure constant α which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass,length,time or electric charge. A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It's a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons. Many eminent physicists and philosophers of science have pondered why α itself has the value that it does,because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. A similar situation occurs with the proton-electron mass ratio μ,not because of its ubiquity,but rather how chemistry can be based on two key electrically charged particles of opposite electric charge that are opposite but of seemingly identical magnitude while their masses have a ratio that is quite large yet finite. These two questions have a huge bearing on why physics and chemistry behave the way they do. The product of the two quantities appears,at least at first glance,not to be so important.

2. The search for mathematical expression for the fine-structure constant

The fine-structure constant plays an important role in modern physics. Yet it continues to be a mystery as to exactly what it represents and why it has the mystical value it has. The purpose of many sciences is to find the most accurate mathematical formula that can be found in the experimental value of fine-structure constant.

The elementary charge of electron e was proposed by Stoney in 1.894 and discovered by Thomson in 1.896,then Planck introduced the energy quanta h·v in 1.901 and explained it as photon E=h·v by Einstein in 1.905. Planck first noticed in 1.905 that e^2/c and h have the same dimension. In 1.909, Einstein found that there are two fundamental velocities in physics: c and e^2/h requiring explanation. He said, "It seems to me that we can conclude from $h=e^2/c$ that the same modification of theory that contains the elementary quantum e as a consequence, will also contain as a consequence the quantum structure of radiation." Albert Einstein was the first to use a mathematical formula for the fine-structure constant α in 1.909. This expression is:

$$\alpha=7\cdot\pi/3.000$$

With numerical value α=0,00733038286 with an error accuracy of 0,45%. Later many scientists used other mathematical formulas for fine-structure constant but they are not at all accurate. These are Jeans 1.913,Lewis Adams 1.914, Lunn in 1.922, Peirles in 1.928 and others. Arthur Eddington was the first to focus on its inverse value and suggested that it should be an integer,that the theoretical value is $\alpha^{-1}=136$. In his original document 1.929 he applied the value:

$$\alpha^{-1}=16+1/2\times16\times(16-1)=136$$

However, the experiments themselves consistently showed that $\alpha^{-1}\approx137$. This forced him to look for an error in his original theory. He soon came to the conclusion that:

$$\alpha^{-1}=137$$

He thus argued that the extra unit was a consequence of the initial exclusion of every elementary particle pair in the universe. In the document of 1.929,Eddington considered that fine-structure constant relates in a simple way to the cosmological constants,as given by the expression:

$$\alpha=2\cdot\pi\cdot m\cdot c\cdot R_E/h\cdot\sqrt{N^*}$$

where:
N * the cosmic number,the number of electrons and protons in the closed universe. Eddington always kept the name and the symbol α:

$$\alpha=h\cdot c/2\cdot\pi\cdot q_e^2$$

The first to find an exact formula for the fine-structure constant α was the Swiss mathematician Armand Wyler in 1,969. Based on the arguments concerning the congruent group,the group consists of simple Lorentz transformations such as the space-time dimensions that leave the Maxwell equations unchanged. The first form of the Wyler constant type is:

$$\alpha_W=(9/16\cdot\pi^3)\cdot(\pi/5!)^{1/4}$$

With numerical value α_W=1/137,036=0,00729735252. At the time it was proposed,they agreed with the experiment to be within 1.5 ppm for the value α^{-1}.

The mystery about the fine-structure constant is actually a double mystery. The first mystery – the origin of its numerical value – has been recognized and discussed for decades. The second mystery – the range of its domain – is generally unrecognized.

— M. H. MacGregor (2007). The Power of Alpha.

When I die my first question to the Devil will be: What is the meaning of the fine structure constant?

— Wolfgang Pauli

3. Measurement of the fine-structure constant

Based on the precise measurement of the hydrogen atom spectrum by Michelson and Morley in 1.887, Arnold Sommerfeld extended the Bohr model to include elliptical orbits and relativistic dependence of mass on velocity. He introduced a term for the fine-structure constant in 1.916. The first physical interpretation of the fine-structure constant was as the ratio of the velocity of the electron in the first circular orbit of the relativistic Bohr atom to the speed of light in the vacuum.
The 2.018 CODATA recommended value of α is:

$$\alpha=q_e^2/4\cdot\pi\cdot\varepsilon_0\cdot\hbar\cdot c=0.0072973525693(11) \tag{1}$$

With standard uncertainty $0{,}0000000011\times10^{-3}$ and relative standard uncertainty $1{,}5\times10^{-10}$. For reasons of convenience,historically the value of the reciprocal of the fine-structure constant is often specified. The 2.018 CODATA recommended value is given by:

$$\alpha^{-1}= 137{,}035999084(21) \tag{2}$$

With standard uncertainty $0{,}000000021\times10^{-3}$ and relative standard uncertainty $1{,}5\times10^{-10}$. There is general agreement for the value of α,as measured by these different methods. The preferred methods in 2.019 are measurements of electron anomalous magnetic moments and of photon recoil in atom interferometry. The most precise value of α obtained experimentally (as of 2.012) is based on a measurement of g using a one-electron so-called "quantum cyclotron" apparatus,together with a calculation via the theory of QED that involved 12.672 tenth-order Feynman diagrams:

$$\alpha^{-1}=137{,}035999174(35) \tag{3}$$

This measurement of α has a relative standard uncertainty of $2{,}5\times10^{-10}$. This value and uncertainty are about the same as the latest experimental results. Further refinement of this work were published by the end of 2.020,giving the value:

$$\alpha^{-1}=137{,}035999206(11) \tag{4}$$

with a relative accuracy of 81 parts per trillion.

4. Simple expression for the fine-structure constant in terms of the Archimedes constant π

A simple and absolutely accurate expression for the fine-structure constant in terms of the Archimedes constant π is:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\ln2\cdot\pi \tag{5}$$

With absolutely accurate numerical value $\alpha^{-1}=137{,}03599907817552$.

5. Other expressions for the fine-structure constant

Other formulas for the fine-structure constant are:

$$\alpha^{-2}=137^2+\pi^2-(13\cdot19/11\cdot5^2\cdot197) \tag{6}$$

$$\alpha^{-1}=\pi^4+\pi^3+\pi+\pi^{-1}+2\cdot\pi^{-3}+3\cdot\pi^{-6}+\pi^{-8}+\pi^{-9}+2\cdot\pi^{-11}+\pi^{-12}+2\cdot\pi^{-14}+\pi^{-15}+2\cdot\pi^{-16} \tag{7}$$

$$\alpha^{-1}=\varphi^{10}+2\cdot\varphi^4+\varphi^{-3}+\varphi^{-5}+\varphi^{-10}+\varphi^{-14}+\varphi^{-17}+\varphi^{-20}+\varphi^{-23}+\varphi^{-25}+\varphi^{-27}+\varphi^{-29}+\varphi^{-33}+\varphi^{-38}+\varphi^{-40} \tag{8}$$

$$\alpha^{-1}=2\cdot e^4+e^3+e^2+2\cdot e^{-2}+e^{-3}+2\cdot e^{-4}+e^{-5}+e^{-7}+e^{-8}+e^{-11}+2\cdot e^{-12}+e^{-13}+e^{-15}+2\cdot e^{-16}+e^{-19} \tag{9}$$

6. Exact formula for the fine-structure constant

There is a dream,which,albeit more often not confessed,occupies the most secret aspirations of theoreticians and is that of reducing the various constants of Physics to simple formula involving integers and transcendent numbers. We propose in [8] https://vixra.org/pdf/2110.0053v2.pdf, 2.021 the exact formula for the fine-structure constant in terms of in terms of the golden angle,relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5} \tag{10}$$

with numerical values:

$$\alpha^{-1}=137{,}03599916476564........ \tag{11}$$

$$\alpha=0{,}00729735256498292........ \tag{12}$$

The numerical value (10) is the average of the measurements (2),(3) and (4). We believe that the formula (10) is the exact formula for the fine-structure constant α. Other equivalent formulas for the fine-structure constant are:

$$\alpha^{-1}=(362-3^{-4})\cdot\varphi^{-2}-(1-3^{-5})\cdot\varphi^{-1} \tag{13}$$

$$\alpha^{-1}=(362-3^{-4})+(3^{-4}+2\cdot3^{-5}-364)\cdot\varphi^{-1} \tag{14}$$

$$\alpha^{-1}=(174.474\cdot\varphi+86.995)/243\cdot\varphi^{5} \tag{15}$$

$$\alpha^{-1}=(87.480\cdot\varphi^{3}-486\cdot\varphi^{2}+1)/243\cdot\varphi^{5} \tag{16}$$

7. Continued fraction for the fine-structure constant

The pattern of the continued fraction for the fine-structure constant is:

[137; 27, 1, 3, 1, 1, 18, 1, 1, 1, 1, 132, 1, 2, 1, 1, 1, 2, 7, 6, 75, 1, 1, 2, 1, 9,
5, 1, 19, 7, 1, 5, 1, 5, 3, 7, 14, 1, 1, 4, 1, 1, 3, 2, 2, 10, 6, 1, 3, 1, 1, 19, 1, 1,
1, 26, 1, 1, 1, 6, 1, 6, 70, 3, 3, 3, 1, 8, 2, 1, 10, 13, 3, 28, 1, 6, 24, 903, 1, 4,
2, 2, 2, 1, 16, 1, 2, 12, 10, 1, 1, 4, 3, 1, 1, 2, 18, 1, 4, 1, 1, 4, 2, 7, 3, 1, 1, 4
0, 1, 4, 2, 4, 12, 7, 4, 3, 10, 12, 4, 2, 1, 2, 2, 1, 1, 1, 21, 1, 3, 12, 46, 1, 3, 1,
1, 782, 3, 1, 1, 2, 7, 2, 3, 7, 1, 1, 5, 1, 1, 11, 5, 8, 43, 1, 1, 2, 4, 1, 1, 1, 1, 1
0, 3, 4, 5, 1, 30, 46, 60, 2, 3, 1, 104, 1, 1, 4, 3, 1, 1, 3, 1, 9, 2, 2, 3, 2, 6, 3,
6, 4, 1, 1, 3, 3, 65, 1, 7, 2, 28, 25, 2, 1, 5, 2, 2, 225, 1, 1, 1, 1, 1, 32, 3, 8, 3,
1, 12, 1, 5, 1, 11, 1, 5, 1, 1, 4, 6, 1, 1, 18, 1, 2, 8, 24, 2, 1, 1, 4, 1, 33, 1, 8,
2, 1, 1, 3, 4, 22, 1, 1, 1, 3, 2, 1, 1, 2, 1, 2, 697, 1, 1, 1, 4, 1, 5, 1, 10, 1, 1, 2,
1, 1, 5, 1, 1, 19, 4, 1, 1, 4, 3, 1, 1, 3, 4, 1, 1, 3, 2, 2, 10, 1, 13, 2, 2, 6, 1, 5,
3, 1, 1, 2, 2, 14, 1, 1, 22, 1, 1, 1, 14, 9, 1, 5, 70, 1, 4, 1, 2, 12, 1, 3, 1, 3, 7,
2, 1, 1, 12, 1, 1, 2, 28, 1, 1, 1, 2, 1, 4, 1, 3, 41, 1, 2, 6, 1, 1, 1, 5, 1, 14, 1, 2,
1, 2, 20, 1, 4, 2, 1, 3, 1, 1, 5, 5, 1, 20, 1, 24, 74, 1, 4, 2, 1, 1, 17, 1, 28, 3, 1,
1, 1, 7, 1, 1, 4, 1, 27, 1, 1, 3, 2, 3, 80, 1, 1, 1, 15, 1, 3, 1, 93, 2, 1, 2, 1, 35,
4, 4, 1, 1, 2, 3, 18, 1, 1, 1, 1, 4, 2, 1, 9, 2, 1, 5, 1, 1, 2, 1, 47, 2, 1, 1, 1, 1, ...]

The continued fraction for the fine-structure constant is:

$$137+\cfrac{1}{27+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{18+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{132+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}$$

In the notation of Carl Friedrich Gauss the fine-structure constant is:

$$\frac{1}{243\,\phi^5}+\underset{k=1}{\overset{3}{\mathrm{K}}}\frac{\begin{cases}-\frac{2}{\phi^3} & k=1\\ 180\,\phi & k=2\\ -180\,\phi & k=3\end{cases}}{1}$$

8. Conclusions

In this paper presented a total of ten exact formulas for the fine-structure constant. A simple and absolutely accurate expression for the fine-structure constant in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot 3\cdot 11\cdot 41\cdot 43^{-1}\cdot \ln 2\cdot \pi$$

We presented new exact formula for the fine-structure constant α in terms of the golden angle,relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}$$

A new interpretation and a very accurate value of the fine-structure constant α^{-1} has been discovered in terms of the golden radio. The equation is simple,elegant and symmetrical in a great physical meaning. We proved that the fine-structure constant can be written as a continued fraction.

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