PROOF OF GOLDBACH'S CONJECTURE AND TWIN PRIME CONJECTURE

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Abstract. In this paper, we prove Twin prime conjecture and Goldbach's conjecture. We do this in three stages by turns; one is 'Application Principle of Mathematical Induction', another is 'Proof of Twin prime conjecture' and the other is 'Proof of Goldbach's conjecture'. These three proofs are interconnected, so they help prove it. Proofs of Twin prime conjecture and Goldbach's conjecture are proved by Application Principle of Mathematical Induction. And Twin prime conjecture is based on Goldbach's conjecture. So, we can get the result, Twin prime and Goldbach's conjecture are true. The reason why we could get the result is that I use twin prime's characteristic that difference is 2 and apply this with Application Principle of Mathematical Induction. If this is proved in this way, It implies that the problem can be proved in a new way of proof.

1. Introduction

On 7 June 1742 the Prussian mathematician Christian Goldbach wrote to Leonhard Euler, suggesting the following assumptions.

'Every integer that can be written as the sum of two primes can also be written as the sum of as many primes as one wishes, until all terms are units.'

Euler replied on June 30, 1742 and recalled his previous conversation with Goldbach, changing the sentence to:

'Every positive even integer can be written as the sum of two primes.'

A modern version of Goldbach's conjecture is 'Every even integer greater than 2 can be written as the sum of two primes.'
Twin prime conjecture is 'There exist infinitely many twin primes.'
Each of them is one of the famous challenges and Hilbert's 23 problems. If these conjectures are solved, another proof based on this proof can be able to emerge. And the most important thing in this paper is Application Principle of Mathematical Induction (hereinafter we shall refer to it as AMI) that is

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utilizing original mathematical induction, processing assume-step 2 times so that it can expand the scope of proof. It is proved that the sum of the inverse of the twin primes converge by Viggo Brun, but this does not prove Twin prime conjecture. Already there are lots of tries to prove twin prime conjecture by reducing the gap between primes, but here we will prove it in a different way using AMI. First step is proving that AMI is true. Second step is proving that Twin prime conjecture by using AMI. Final step is Goldbach's conjecture.

2. Application Principle of Mathematical Induction

Suppose a set of positive integers, S satisfies two properties:

(a) 1 and 2 belong to S.
(b) If the integer k and $k+1$ belong to S, then the next integer $k+2$ also belongs to S.

Then S has all positive integers.

Proof. Let suppose that one set, T has all positive integers that do not belong to S, and T is not empty set. By well ordering principle, T contains the minimum element. Let's call the number as a. Because 1 and 2 belong to S, $a > 2$, and $0 < a-2 < a$. a is the minimum element of T, so $a-1$ and $a-2$ do not belong to T. That is, $a-1$ and $a-2$ belong to S. By assumption, S shall have $(a-2)+2 = (a-1)+1 = a$.
This is contradictory to the assumption that T has a. Therefore, T is empty set, and we conclude that S contains all positive integers.

□

3. Twin prime conjecture

Let suppose a set of twin prime is q except 3 and q_k is the k-th twin prime. And m, n is natural number.

Theorem 3.1. *q_{2m-1} is shape of $6n-1$ and q_{2m} is shape of $6n+1$. As $q_{2m-1}+2 = q_{2m}$, if q is q_{2m-1}, the next twin prime, q_{2m} exists.*

The next proposition is true.

(a) 'The twin prime, not 3, is infinite.'

Proof. Let suppose the twin prime next q_n is q_{n+1},and prove $q_n \mapsto q_{n+1}$ is true. ($\mapsto$ means the number next the symbol exists.)

For $n = 1$, $q_1 = 5$, $q_2 = 7$, $q_1 \mapsto q_2$ which is correct.

For $n = 2$, $q_2 = 7$, $q_3 = 11$, $q_1 \mapsto q_2$ which is correct.

Let us assume $q_n \mapsto q_{n+1}$ is true for $n = k$, Hence, $q_k \mapsto q_{k+1}$ is true.

(1) When $k = 2m$ (m is natural number)
$$q_{2m} \mapsto q_{2m+1}$$

By $Theorem 3.1$
$$q_{2m+1} + 2 = q_{2m+2}$$
$q_{2m+1} \mapsto q_{2m+2}$ is true.

So, $q_n \mapsto q_{n+1}$ is true for $n = 2m$

(2) When $k = 2m + 1$
$$q_{2m+1} \mapsto q_{2m+2}$$

By using AMI (Application Principle of Mathematical Induction)
Let us assume $q_n \mapsto q_{n+1}$ is true for $n = k + 1$, Hence, $q_{k+1} \mapsto q_{k+2}$ is true.
$$q_{2m+2} \mapsto q_{2m+3}$$

By $Theorem 3.1$
$$q_{2m+3} + 2 = q_{2m+4}$$
$q_{2m+3} \mapsto q_{2m+4}$ is true.

So, $q_n \mapsto q_{n+1}$ is true for $n = 2m + 1$

Thus, Twin Prime conjecture is true.

□

4. Modified Goldbach's conjecture

Let suppose prime is p (not 2), twin prime is q (shape of $6x - 1$ is q_1, shape of $6x + 1$ is q_2). And b is odd number but it isn't prime.

The next proposition is true.

(a) 'Every even integer greater than 2 can be written as the sum of prime and twin prime.'

Proof. About $P(n) : n = p + q(n \geq 3)$

For $n = 3$, $P(3) : 6 = 3 + 3$ which is correct.

For $n = 4$, $P(4) : 8 = 3 + 5$ which is correct.

Let us assume $P(n)$ is true for $n = k$, Hence, $P(k)$ is true.
$P(k) : 2k = p + q$

(1) When $q = q_1$
$$2k = p + q_1$$
$$2k + 2 = p + q_1 + 2$$
$$2k + 2 = p + q_2$$

So, $P(n)$ is true for $q = q_1$

(2) When $q = q_2$, i.e. q can't be q_1.
$$2k = p + q_2$$
$$2k = b + q_1$$
$$2k + 2 = b + q_1 + 2$$
$$2k + 2 = b + q_2$$

By using AMI
Let us assume $P(n)$ is true for $n = k + 1$, Hence, $P(k + 1)$ is true.
As $2k + 2 \neq p + q_2$,
$$2k + 2 = p + q_1$$
$$2k + 4 = p + q_1 + 2$$
$$2k + 4 = p + q_2$$

So, $P(n)$ is true for $q = q_2$

Thus, the following equation is drawn.

$$2n = p + q(n \geq 3)$$

□

5. Goldbach's conjecture

As $p + q \subset p + p$, including '$4 = 2 + 2$',

$$2n = p + p(n \geq 2)$$

The proposition, 'Every even integer greater than 2 can be written as the sum of two primes.', is true.

So Goldbach's conjecture is true.

References

[1] David M. Burton, *Elementary Number Theory*, McGraw-Hill Higher Education, 2007.

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