

Interaction of Complex Scalar Fields and Electromagnetic Fields in Klein-Gordon-Maxwell Theory in Cosmological Inertial Frame

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ABSTRACT

We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame.

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Key words: Klein-Gordon-Maxwell Theory;

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1. Introduction

The Lagrangian L of complex scalar fields ϕ, ϕ^* and Electromagnetic fields $F^{\mu\nu}, F_{\mu\nu}$ is Klein-Gordon-Maxwell theory in special relativity theory,

$$L = (\partial_\mu \phi + ieA_\mu \phi)(\partial^\mu \phi^* - ieA^\mu \phi^*) - \frac{m^2 c^2}{\hbar^2} \phi \phi^* - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

ϕ^* is ϕ 's adjoint scalar, m is the mass of scalar fields ϕ, ϕ^*

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (1)$$

2. Equations of Interaction of Complex Scalar Fields and Electromagnetic Fields in Cosmological Inertial Frame

The Lagrangian L of interaction of complex scalar fields and Electromagnetic fields is Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$L = (\bar{\partial}_\mu \phi + ie\bar{A}_\mu \phi)(\bar{\partial}^\mu \phi^* - ie\bar{A}^\mu \phi^*) - \frac{m^2 c^2}{\hbar^2} \phi \phi^* - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad (2-1)$$

We consider Type A of wave function and Type B of expanded distance, [1],[2],[3],[4]

$$\text{Type A of wave function: } r \rightarrow r\sqrt{\Omega(t_0)} \quad , \quad t \rightarrow \frac{t}{\sqrt{\Omega(t_0)}}$$

Type B of expanded distance: $r \rightarrow r\Omega(t_0), t \rightarrow t$

$$\bar{\partial}_\mu = (\sqrt{\Omega(t_0)} \frac{\partial}{c\partial t}, \frac{1}{\sqrt{\Omega(t_0)}} \vec{\nabla}), \bar{\partial}^\mu = (\sqrt{\Omega(t_0)} \frac{\partial}{c\partial t}, -\frac{1}{\sqrt{\Omega(t_0)}} \vec{\nabla})$$

$$\bar{A}_\mu = (\phi, \vec{A}\Omega(t_0)), \bar{A}^\mu = (\phi, -\vec{A}\Omega(t_0)), \bar{F}_{\mu\nu} = F_{\mu\nu}\Omega(t_0), \bar{F}^{\mu\nu} = F^{\mu\nu}\Omega(t_0)$$

t_0 is the cosmological time. $\Omega(t_0)$ is the expanding ratio of universe in the cosmological time t_0 .

$$(2-2)$$

Complex scalar field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_\mu \left(\frac{\partial L}{\partial(\bar{\partial}_\mu \phi)} \right) - \frac{\partial L}{\partial \phi} = (\bar{\partial}_\mu - ie\bar{A}_\mu)(\bar{\partial}^\mu \phi^* - ie\bar{A}^\mu \phi^*) + \frac{m^2 c^2}{\hbar^2} \phi^* = 0 \quad (3)$$

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_\mu \left(\frac{\partial L}{\partial(\bar{\partial}_\mu \phi^*)} \right) - \frac{\partial L}{\partial \phi^*} = (\bar{\partial}^\mu + ie\bar{A}^\mu)(\bar{\partial}_\mu \phi + ie\bar{A}_\mu \phi) + \frac{m^2 c^2}{\hbar^2} \phi = 0 \quad (4)$$

If operator $\bar{\partial}_\mu, \bar{\partial}^\mu$ are in cosmological inertial frame, [1],[2],[3],[4]

$$\bar{\partial}_\mu = \left(\frac{\partial}{c\partial t}, \frac{1}{\Omega(t_0)} \bar{\nabla} \right), \bar{\partial}^\mu = \left(\frac{\partial}{c\partial t}, -\frac{1}{\Omega(t_0)} \bar{\nabla} \right)$$

$$\bar{F}^{\mu\nu} = \bar{\partial}^\mu \bar{A}^\nu - \bar{\partial}^\nu \bar{A}^\mu, \bar{F}_{\mu\nu} = \bar{\partial}_\mu \bar{A}_\nu - \bar{\partial}_\nu \bar{A}_\mu \quad (5)$$

Electromagnetic field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\begin{aligned} \bar{\partial}_\nu \left(\frac{\partial L}{\partial(\bar{\partial}_\nu \bar{A}_\mu)} \right) - \frac{\partial L}{\partial \bar{A}_\mu} &= \frac{1}{4} \bar{\partial}_\nu (\bar{\partial}^\mu \bar{A}^\nu - \bar{\partial}^\nu \bar{A}^\mu) - ie\phi(\bar{\partial}^\mu \phi^* - ie\bar{A}^\mu) \\ &= \frac{1}{4} \bar{\partial}_\nu \bar{F}^{\mu\nu} - ie\phi(\bar{\partial}^\mu \phi^* - ie\bar{A}^\mu) = 0 \end{aligned} \quad (6)$$

Hence,[5],

$$\begin{aligned} \bar{\partial}_\nu \bar{F}^{\mu\nu} &= \frac{4\pi}{c} \bar{J}^\mu = 4\bar{\phi} \bar{\partial}(\phi^* - ie\bar{A}^\mu) \\ \bar{J}^\mu &= \frac{c}{\pi} ie\bar{\phi}(\bar{\partial}^\mu \phi^* - ie\bar{A}^\mu) \end{aligned} \quad (7)$$

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\begin{aligned} \bar{\partial}^\nu \left(\frac{\partial L}{\partial(\bar{\partial}^\nu \bar{A}^\mu)} \right) - \frac{\partial L}{\partial \bar{A}^\mu} &= \frac{1}{4} \bar{\partial}^\nu (\bar{\partial}_\mu \bar{A}_\nu - \bar{\partial}_\nu \bar{A}_\mu) + ie\phi^*(\bar{\partial}_\mu \phi + ie\bar{A}_\mu) \\ &= \frac{1}{4} \bar{\partial}^\nu \bar{F}_{\mu\nu} + ie\phi^*(\bar{\partial}_\mu \phi + ie\bar{A}_\mu) = 0 \end{aligned} \quad (8)$$

Hence,[5],

$$\begin{aligned} \bar{\partial}^\nu \bar{F}_{\mu\nu} &= \frac{4\pi}{c} \bar{J}_\mu = -4\bar{\phi}^* \bar{\partial}(\phi + ie\bar{A}_\mu) \\ \bar{J}_\mu &= -\frac{c}{\pi} ie\bar{\phi}^*(\bar{\partial}_\mu \phi + ie\bar{A}_\mu) \end{aligned} \quad (9)$$

3. Conclusion

We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame.

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