Resolution of Polynomial Equation with Rational Coefficients

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Abstract

Identities of the coefficients of the polynomial equations are created in function of the parameters that define their roots, and with their application it is created a process to solve polynomial equations with rational coefficients.

Introduction

The roots of the equation $F_0x^n - F_1x^{n-1} + F_2x^{n-2} - F_3x^{n-3} + \ldots \pm F_{n-1}x^{n-(n-1)} \pm F_n = 0$ are defined in this paper as the result of the division of one of the multiples of F_n between one of the multiples of F_0. The sets of multiples of F_n y F_0 are determinable and of finite amount.

Proposal of Theory

Being $x_1 = \frac{b_1}{c_1}, x_2 = \frac{b_2}{c_2}, x_3 = \frac{b_3}{c_3}, \ldots, x_n = \frac{b_n}{c_n}$ the roots of the equation $F_0x^n - F_1x^{n-1} + F_2x^{n-2} - F_3x^{n-3} + F_4x^{n-4} - \ldots \pm F_{n-1}x^{n-(n-1)} \pm F_n = 0$, where $c_1, c_2, c_3, \ldots, c_n$ and $b_1, b_2, b_3, \ldots, b_n \in \mathbb{C}$.

Then we will have:
$F_0 = c_1c_2c_3c_4c_5 \ldots c_n$
$F_1 = b_1c_2c_3c_4c_5 \ldots c_n + c_1b_2c_3c_4c_5 \ldots c_n + c_1c_2b_3c_4c_5 \ldots c_n + \ldots + c_1c_2c_3c_4c_5 \ldots c_{n-2}b_{n-1}c_n + c_1c_2c_3c_4c_5 \ldots .c_{n-1}b_n$
$F_2 = b_1b_2c_3c_4c_5 \ldots c_n + b_1c_2b_3c_4c_5 \ldots c_n + b_1c_2c_3b_4c_5 \ldots c_n + \ldots + b_1c_2c_3c_4 \ldots c_{n-1}b_n + c_1b_2b_3c_4c_5 \ldots c_n + c_1b_2c_3b_4c_5 \ldots c_n + \ldots + c_1b_2c_3c_4 \ldots c_{n-1}b_n + \ldots + c_1c_2c_3c_4c_5 \ldots c_{n-2}b_{n-1}b_n$
$F_3 = b_1b_2b_3c_4c_5c_6 \ldots c_n + b_1b_2c_3b_4c_5c_6 \ldots c_n + b_1b_2c_3c_4b_5c_6 \ldots c_n + \ldots + b_1b_2c_3c_4c_5c_6 \ldots c_{n-1}b_n + b_1c_2b_3b_4c_5c_6 \ldots c_n + \ldots + b_1c_2b_3c_4c_5c_6 \ldots c_{n-1}b_n + b_1c_2c_3b_4b_5c_6 \ldots c_n + \ldots + b_1c_2c_3b_4c_5c_6 \ldots c_{n-1}b_n + \ldots + b_1c_2c_3c_4c_5c_6 \ldots c_{n-2}b_{n-1}b_n + c_1b_2b_3b_4c_5c_6 \ldots c_n + \ldots + c_1b_2c_3c_4c_5c_6 \ldots c_{n-2}b_{n-1}b_n + \ldots + c_1c_2c_3c_4c_5c_6 \ldots c_{n-3}b_{n-2}b_{n-1}b_n$
$\ldots F_{n-1} = c_1b_2b_3b_4b_5 \ldots b_n + b_1c_2b_3b_4b_5 \ldots b_n + b_1b_2c_3b_4b_5 \ldots b_n + \ldots + b_1b_2b_3b_4b_5 \ldots b_{n-2}c_{n-1}b_n + b_1b_2b_3b_4b_5 \ldots b_{n-1}c_n$
$F_n = b_1b_2b_3b_4b_5 \ldots b_n$

And also have the equation:

$$\prod_{i=1}^{n}(c_i + b_i) = \sum_{i=0}^{n} F_i$$

Application

Resolution of polynomial equations with rational coefficients. The coefficients of the equation are converted to whole numbers with a greatest common divisor of 1. Then we will have $F_0x^n - F_1x^{n-1} + F_2x^{n-2} - F_3x^{n-3} + F_4x^{n-4} - \ldots \pm F_{n-1}x^{n-(n-1)} \pm F_n = 0$, where F_0, F_1, F_2, F_3,..., F_n are whole numbers.

We will call sets c to all sets of n elements whose product would be equal to F_0 (without obviating elements of the type $[Z + Yi]$, with $Y \neq 0$).

We will call sets b to all sets of n elements whose product would be equal to F_n (without obviating elements of the type $[Z + Yi]$, with $Y \neq 0$).

Being the case that $(Z+Yi)(Z-Yi) = Z^2+Y^2$, it is useful for this procedure to make a table of $Z^2 + Y^2$ values, as big as needed, and then make a list with of those values ordered from lowest to highest alongside their associated Z and Y values.

A set c and a set b are chosen. Each element of each permutation (n of n) of the set c is assigned a different nomination from among c_1, c_2, c_3,..., c_n, always in the same sequence, achieving that way $n!$ configurations of values of c_1, c_2, c_3,..., c_n. Each element of the set b is assigned a different nomination from among b_1, b_2, b_3,..., b_n. A configuration of c_1, c_2, c_3,..., c_n and the values of b_1, b_2, b_3,..., b_n are chosen, and the values of F_1, F_2, F_3,..., F_{n-1} are calculated. It must be verified if the values of F_1, F_2, F_3,..., F_{n-1} coincide with the coefficients of the equation. If there is no coincidence, the operation repeats with a different c_1, c_2, c_3,..., c_n configuration, until a coincidence is found or there are no more configurations available. If there is still no coincidence, the operation repeats with a different pair of c and b sets until there is a coincidence. Then each value of x is calculated using the (c_1, b_1), (c_2, b_2),..., (c_n, b_n) pair of values for which there was coincidence.

For higher degree equations, it may be convenient to filtrate all the sets of values of $c_1, c_2, c_3, ..., c_n$ and $b_1, b_2, b_3, ..., b_n$ through the following equation:

$$\prod_{i=1}^{n}(c_i + b_i) = \sum_{i=0}^{n} F_i$$

The sets of values of $c_1, c_2, c_3, ..., c_n$ and $b_1, b_2, b_3, ..., b_n$ that do not verify this equation can be discarted as candidates to be verified in the F_1, F_2,...,F_{n-1} identities.

Examples of Equation Resolution

1) $x^5 - 19x^4 + 133x^3 - 421x^2 + 586x - 280 = 0$

Being $n = 5$

$F_0 = c_1c_2c_3c_4c_5$

$F_1 = b_1c_2c_3c_4c_5 + c_1b_2c_3c_4c_5 + c_1c_2b_3c_4c_5 + c_1c_2c_3b_4c_5 + c_1c_2c_3c_4b_5$

$F_2 = b_1b_2c_3c_4c_5 + b_1c_2b_3c_4c_5 + b_1c_2c_3b_4c_5 + b_1c_2c_3c_4b_5 + c_1b_2b_3c_4c_5 + c_1b_2c_3b_4c_5 + c_1b_2c_3c_4b_5 + c_1c_2b_3b_4c_5 + c_1c_2b_3c_4b_5 + c_1c_2c_3b_4b_5$

$F_3 = b_1b_2b_3c_4c_5 + b_1b_2c_3b_4c_5 + b_1b_2c_3c_4b_5 + b_1c_2b_3b_4c_5 + b_1c_2b_3c_4b_5 + b_1c_2c_3b_4b_5 + c_1b_2b_3b_4c_5 + c_1b_2b_3c_4b_5 + c_1b_2c_3b_4b_5 + c_1c_2b_3b_4b_5$

$F_4 = b_1b_2b_3b_4c_5 + b_1b_2b_3c_4b_5 + b_1b_2c_3b_4b_5 + b_1c_2b_3b_4b_5 + c_1b_2b_3b_4b_5$

$F_5 = b_1b_2b_3b_4b_5$

$F_0 = 1$

$F_0 = 1*1*1*1*1$; $F_0 = -1*-1*1*1*1$; $F_0 = -1*i*i*1*1$; ...

$F_5 = 280\ (2,2,2,5,7)$

$F_5 = 1*2*4*5*7$; $F_5 = -1*2*4*5*-7$; $F_5 = (1+i)*(1-i)*4*5*7$; $F_5 = 2*4*(1+2i)*(1-2i)*7$; $F_5 = (1+i)*(1-i)*(1+2i)*(1-2i)*28$; $F_5 = (1+i)*(1-i)*(2+i)*(2-i)*28$; $\ldots$

The values $c_1 = 1, c_2 = 1, c_3 = 1, c_4 = 1, c_5 = 1, b_1 = 1, b_2 = 2, b_3 = 4, b_4 = 5, b_5 = 7$ that correspond to $F_0 = 1*1*1*1*1$ and $F_5 = 1*2*4*5*7$ verify the values of F_1, F_2, F_3, F_4 according to the proposed formulas. Then:

$x_1 = b_1/c_1 = 1/1 = 1$; $x_2 = b_2/c_2 = 2/1 = 2$; $x_3 = b_3/c_3 = 4/1 = 4$; $x_4 = b_4/c_4 = 5/1 = 5$; $x_5 = b_5/c_5 = 7/1 = 7$

2) $21x^6 - 838x^5 + 10799x^4 - 69988x^3 + 231891x^2 - 367190x + 217833 = 0$

Being $n = 6$

$F_0 = c_1c_2c_3c_4c_5c_6$

$F_1 = b_1c_2c_3c_4c_5c_6 + c_1b_2c_3c_4c_5c_6 + c_1c_2b_3c_4c_5c_6 + c_1c_2c_3b_4c_5c_6 + c_1c_2c_3c_4b_5c_6 + c_1c_2c_3c_4c_5b_6$

$F_2 = b_1b_2c_3c_4c_5c_6 + b_1c_2b_3c_4c_5c_6 + b_1c_2c_3b_4c_5c_6 + b_1c_2c_3c_4b_5c_6 + b_1c_2c_3c_4c_5b_6 + c_1b_2b_3c_4c_5c_6 + c_1b_2c_3b_4c_5c_6 + c_1b_2c_3c_4b_5c_6 + c_1b_2c_3c_4c_5b_6 + c_1c_2b_3b_4c_5c_6 + c_1c_2b_4c_4b_5c_6 + c_1c_2b_3c_4c_6b_6 + c_1c_2c_3b_4b_5c_6 + c_1c_2c_3b_4c_5b_6 + c_1c_2c_3c_4b_5b_6$

$F_3 = b_1b_2b_3c_4c_5c_6 + b_1b_2c_3b_4c_5c_6 + b_1b_2c_3c_4b_5c_6 + b_1b_2c_3c_4c_5b_6 + b_1c_2b_3b_4c_5c_6 + b_1c_2b_3c_4b_5c_6 + b_1c_2b_3c_4c_5b_6 + b_1c_2c_3b_4b_5c_6 + b_1c_2c_3b_4c_5b_6 + b_1c_2c_3c_4b_5b_6 + c_1b_2b_3b_4c_5c_6 + c_1b_2b_3c_4b_5c_6 + c_1b_2b_3c_4c_5b_6 + c_1b_2c_3b_4b_5c_6 + c_1b_2c_3b_4c_5b_6 + c_1b_2c_3c_4b_5b_6 + c_1c_2b_3b_4b_5c_6 + c_1c_2b_3b_4c_5b_6 + c_1c_2b_3c_4b_5b_6 + c_1c_2c_3b_4b_5b_6$

$F_4 = b_1b_2b_3b_4c_5c_6 + b_1b_2b_3c_4b_5c_6 + b_1b_2b_3c_4c_5b_6 + b_1b_2c_3b_4b_5c_6 + b_1b_2c_3b_4c_5b_6 + b_1b_2c_3c_4b_5b_6 + b_1c_2b_3b_4b_5c_6 + b_1c_2b_3b_4c_5b_6 + b_1c_2b_3c_4b_5b_6 + b_1c_2c_3b_4b_5b_6 + c_1b_2b_3b_4b_5c_6 + c_1b_2b_3b_4c_5b_6 + c_1b_2b_3c_4b_5b_6 + c_1b_2c_3b_4b_5b_6 + c_1c_2b_3b_4b_5b_6$

$F_5 = b_1b_2b_3b_4b_5c_6 + b_1b_2b_3b_4c_5b_6 + b_1b_2b_3c_4b_5b_6 + b_1b_2c_3b_4b_5b_6 + b_1c_2b_3b_4b_5b_6 + c_1b_2b_3b_4b_5b_6$

$F_6 = b_1b_2b_3b_4b_5b_6$

$F_0 = 21\ (3,7)$

$F_0 = 1*1*1*1*7*3$; $F_0 = 1*1*1*1*1*21$; $F_0 = 1*1*1*1*-7*-3$; $F_0 = 1*1*1*-1*7*-3$; $F_0 = 1*1*1*-1*-7*3$; $F_0 = 1*1*-1*-1*7*3$; $F_0 = 1*1*1*1*-1*-21$; $F_0 = 1*1*1*-1*-1*21$; ...

$F_6 = 217833\ (3,7,11,23,41)$

$F_6 = 1*3*7*11*23*41$; $F_6 = 1*1*1*1*1*217833$; $F_6 = 1*1*1*1*3*72611$; $F_6 = 1*1*1*3*7*10373$; $F_6 = 1*1*3*7*11*943$; $F_6 = 3*7*11*23*(5+4i)*(5-4i)$; $F_6 = 3*7*11*23*(4+5i)*(4-5i)$; ...

The values $c_1 = 7$, $c_2 = 3$, $c_3 = 1$, $c_4 = 1$, $c_5 = 1$, $c_6 = 1$, $b_1 = 11$, $b_2 = 7$, $b_3 = 3$, $b_4 = 23$, $b_5 = (5+4i)$, $b_6 = (5-4i)$ that correspond to $F_0 = 1*1*1*1*3*7$ and $F_6 = 3*7*11*23*(5+4i)*(5-4i)$ verify the values of F_1, F_2, F_3, F_4, F_5 according to the proposed formulas. Then:

$x_1 = \frac{b_1}{c_1} = \frac{11}{7}$; $x_2 = \frac{b_2}{c_2} = \frac{7}{3}$; $x_3 = \frac{b_3}{c_3} = \frac{3}{1} = 3$; $x_4 = \frac{b_4}{c_4} = \frac{23}{1} = 23$; $x_5 = \frac{b_5}{c_5} = \frac{(5+4i)}{1} = (5+4i)$; $x_6 = \frac{b_6}{c_6} = \frac{(5-4i)}{1} = (5-4i)$