Structure of Polynomial Equations and Resolution of Polynomial Equations with Rational Coefficients

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Abstract

Relationships between the coefficients of polynomial equations and the parameters that define their roots are stablished. A process is made to resolve polynomial equations with rational coefficients.

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1 Introduction

The Ruffini-Abel theorem says that polynomial equations of degree 5 or higher cannot be solved by algebraic operations and radical solving. But it is possible to solve polynomial equations of any degree with a finite procedure using identities of the coefficients of the equation as a function of parameters that define the roots of the equation. These identities and the resolution procedure are proposed here.

2 Theory

For: $F_0x^n - F_1x^{n-1} + F_2x^{n-2} - F_3x^{n-3} + F_4x^{n-4} - \ldots \pm F_{n-1}x^{n-(n-1)} \pm F_n = 0$ (Equation 1)

With roots in:

$$x_1 = \frac{b_1}{c_1}, x_2 = \frac{b_2}{c_2}, x_3 = \frac{b_3}{c_3}, \ldots, x_n = \frac{b_n}{c_n}$$

$c_1, c_2, c_3, \ldots, c_n \in \mathbb{C}$
$b_1, b_2, b_3, \ldots, b_n \in \mathbb{C}$

It holds that:
$F_0 = c_1c_2c_3c_4c_5 \ldots c_n$

$F_1 = b_1c_2c_3c_4c_5\ldots c_n + c_1b_2c_3c_4c_5\ldots c_n + c_1c_2b_3c_4c_5\ldots c_n + \ldots + c_1c_2c_3c_4c_5\ldots c_{n-2}b_{n-1}c_n + c_1c_2c_3c_4c_5\ldots .c_{n-1}b_n$

$F_2 = b_1b_2c_3c_4c_5\ldots c_n + b_1c_2b_3c_4c_5\ldots c_n + b_1c_2c_3b_4c_5\ldots c_n + \ldots + b_1c_2c_3c_4\ldots c_{n-1}b_n + c_1b_2b_3c_4c_5\ldots c_n + c_1b_2c_3b_4c_5\ldots c_n + \ldots + c_1b_2c_3c_4\ldots c_{n-1}b_n + \ldots + c_1c_2c_3c_4c_5\ldots c_{n-2}b_{n-1}b_n$

$F_3 = b_1b_2b_3c_4c_5c_6\ldots c_n + b_1b_2c_3b_4c_5c_6\ldots c_n + b_1b_2c_3c_4b_5c_6\ldots c_n + \ldots + b_1b_2c_3c_4c_5c_6\ldots c_{n-1}b_n + b_1c_2b_3b_4c_5c_6\ldots c_n + \ldots + b_1c_2b_3c_4c_5c_6\ldots c_{n-1}b_n + b_1c_2c_3b_4b_5c_6\ldots c_n + \ldots + b_1c_2c_3b_4c_5c_6\ldots c_{n-1}b_n + \ldots + b_1c_2c_3c_4c_5c_6\ldots c_{n-2}b_{n-1}b_n + c_1b_2b_3b_4c_5c_6\ldots c_n + \ldots + c_1b_2c_3c_4c_5c_6\ldots c_{n-2}b_{n-1}b_n + \ldots + c_1c_2c_3c_4c_5c_6\ldots c_{n-3}b_{n-2}b_{n-1}b_n$

$\ldots F_{n-1} = c_1b_2b_3b_4b_5\ldots b_n + b_1c_2b_3b_4b_5\ldots b_n + b_1b_2c_3b_4b_5\ldots b_n + \ldots + b_1b_2b_3b_4b_5\ldots b_{n-2}c_{n-1}b_n + b_1b_2b_3b_4b_5\ldots b_{n-1}c_n$

$F_n = b_1b_2b_3b_4b_5\ldots b_n$

It also holds that:

$$(c_1 + b_1)(c_2 + b_2)(c_3 + b_3)\ldots(c_n + b_n) = \sum_0^n F_i$$

(Equation 2)

If

$$\frac{F_0x^n - Fx^{n-1} + F_2x^{n-2} - \ldots \pm F_{n-1}x^{n-(n-1)} \pm F_n}{G_0x^k - Gx^{k-1} + G_2x^{k-2} - \ldots \pm G_{k-1}x^{k-(k-1)} \pm G_k} = \prod_1^{m=n-k}(c_mx - b_m)$$

Then

$$\prod_1^{m=n-k}(c_m + b_m) = \frac{\sum_0^n F_i}{\sum_0^k G_i}$$

3 Application

Resolution of grade n polynomial equations whose coefficients are rational numbers. The roots of equation 1 are calculated through the (c_1, b_1), (c_2, b_2),…, (c_n, b_n) pair of values that check with equation 2 or the F_0, F_1, F_2, F_3,…, F_{n-1}, F_n identities, in a finite process that starts with candidates obtained through the decomposition in n multiples (positive or negative) of the F_0 and F_n values.

To resolve equation 1: F_0, F_n and $\sum_0^n F_i$ are decomposed into prime factors. With the prime factors of F_0 it is calculated all sets of n elements whose product would be equal to F_0 (which we will call sets c). With the prime factors of F_n it is calculated all sets of n elements whose product would be equal to F_n (which we will call sets b). With the prime factors of $\sum_0^n F_i$ it is calculated all sets of n elements whose product would be equal to $\sum_0^n F_i$ (which we will call sets c+b).

Then a set c and a set b are chosen. Each element of each permutation (n of n) of the set c is assigned a different nomination from among c_1, c_2, c_3,…, c_n, always in the same sequence, achieving $n!$ configurations of c_1, c_2, c_3,…, c_n

values. Each element of the set b is assigned a different nomination from among b_1, b_2, b_3,..., b_n, then a c_1, c_2, c_3,..., c_n configuration and the b_1, b_2, b_3,..., b_n values are chosen, and the values of $c_1 + b_1$, $c_2 + b_2$, $c_3 + b_3$,..., $c_n + b_n$ are obtained. It must be corroborated if these values coincide with the values of any of the sets c+b; if there is no coincidence, the operation repeats with a different c_1, c_2, c_3,..., c_n configuration, until a coincidence is found or there are no more configurations available. Then if there is still no coincidence the operation repeats with a different pair of c and b sets, until there is a coincidence. Then each value of x is calculated using the (c_1, b_1), (c_2, b_2),..., (c_n, b_n) pair of values for which there was coincidence. If there is no coincidence with all possible pairs of sets c and sets b, then the Equation has complex roots $Z + Yi$ with $Y \neq 0$.

Another way to calculate equation 1: A c_1, c_2, c_3,..., c_n configuration and the b_1, b_2, b_3,..., b_n values are chosen, and the values of F_1, F_2, F_3,..., F_{n-1} are calculated. It must be corroborated if the values of F_1, F_2, F_3,..., F_{n-1} match those in equation 1. If there is no coincidence, the operation repeats with a different c_1, c_2, c_3,..., c_n configuration, until a coincidence is found or there are no more configurations available. Then if there is still no coincidence the operation repeats with a different pair of c and b sets, until there is a coincidence. Then each value of x is calculated using the (c_1, b_1), (c_2, b_2),..., (c_n, b_n) pair of values for which there was coincidence. If there is no coincidence with all possible pairs of sets c and sets b, then the Equation has complex roots $Z + Yi$ with $Y \neq 0$.

4 Examples

Example 1:

$x^3 - 6,5x^2 + 13,5x - 9 = 0$
$2x^3 - 13x^2 + 27x - 18 = 0$

$F_0 = 2; F_0 = 1*1*2; F_0 = -1*1*-2; \ldots; F_3 = 18(2,3,3); F_3 = 2*3*3; F_3 = 1*6*3; F_3 = 1*2*9; F_3 = -2*-3*3; \ldots; \sum_0^3 F_i = 60(2,2,3,5); \sum_0^3 F_i = 4*3*5; \sum_0^3 F_i = 1*4*15; \sum_0^3 F_i = 1*12*5; \sum_0^3 F_i = 1*6*10; \sum_0^3 F_i = 2*6*5; \sum_0^3 F_i = 1*2*30; \sum_0^3 F_i = 2*3*10; \sum_0^3 F_i = 1*3*20; \ldots$

The values $c_1 = 1, c_2 = 1, c_3 = 2, b_1 = 2, b_2 = 3, b_3 = 3$ that correspond to $F_0 = 1*1*2$ and $F_3 = 2*3*3$, are corroborated for $\sum_0^3 F_i = 4*3*5$. Then:
$x_1 = b_1/c_1 = 2/1 = 2$; $x_2 = b_2/c_2 = 3/1 = 3$; $x_3 = b_3/c_3 = 3/2 = 1,5$

Example 2:

$x^5 - 19x^4 + 133x^3 - 421x^2 + 586x - 280 = 0$

Being $n = 5$
$F_0 = c_1c_2c_3c_4c_5$
$F_1 = b_1c_2c_3c_4c_5 + c_1b_2c_3c_4c_5 + c_1c_2b_3c_4c_5 + c_1c_2c_3b_4c_5 + c_1c_2c_3c_4b_5$
$F_2 = b_1b_2c_3c_4c_5 + b_1c_2b_3c_4c_5 + b_1c_2c_3b_4c_5 + b_1c_2c_3c_4b_5 + c_1b_2b_3c_4c_5 + c_1b_2c_3b_4c_5 + c_1b_2c_3c_4b_5 + c_1c_2b_3b_4c_5 + c_1c_2b_3c_4b_5 + c_1c_2c_3b_4b_5$

$F_3 = b_1b_2b_3c_4c_5 + b_1b_2c_3b_4c_5 + b_1b_2c_3c_4b_5 + b_1c_2b_3b_4c_5 + b_1c_2b_3c_4b_5 + b_1c_2c_3b_4b_5 + c_1b_2b_3b_4c_5 + c_1b_2b_3c_4b_5 + c_1b_2c_3b_4b_5 + c_1c_2b_3b_4b_5$

$F_4 = b_1b_2b_3b_4c_5 + b_1b_2b_3c_4b_5 + b_1b_2c_3b_4b_5 + b_1c_2b_3b_4b_5 + c_1b_2b_3b_4b_5$

$F_5 = b_1b_2b_3b_4b_5$

$F_0 = 1; F_0 = 1*1*1*1*1; \ldots; F_5 = 280(2,2,2,5,7); F_5 = 1*2*4*5*7; F_5 = -1*2*4*5*-7; \ldots; \sum_0^5 F_i = 1440(2,2,2,2,2,3,3,5); \sum_0^5 F_i = 1*2*4*12*15; \sum_0^5 F_i = 2*8*6*3*5; \ldots$

The values $c_1 = 1, c_2 = 1, c_3 = 1, c_4 = 1, c_5 = 1, b_1 = 1, b_2 = 2, b_3 = 4, b_4 = 5, b_5 = 7$ that correspond to $F_0 = 1*1*1*1*1$ and $F_5 = 1*2*4*5*7$, corroborates the F_1, F_2, F_3, F_4 values according to the proposed formulas. Then:

$x_1 = b_1/c_1 = 1/1 = 1$; $x_2 = b_2/c_2 = 2/1 = 2$; $x_3 = b_3/c_3 = 4/1 = 4$; $x_4 = b_4/c_4 = 5/1 = 5$; $x_5 = b_5/c_5 = 7/1 = 7$

Also, the values $c_1 = 1, c_2 = 1, c_3 = 1, c_4 = 1, c_5 = 1, b_1 = 1, b_2 = 2, b_3 = 4, b_4 = 5, b_5 = 7$ that correspond to $F_0 = 1*1*1*1*1$ and $F_5 = 1*2*4*5*7$ are corroborated for $\sum_0^5 F_i = 2*8*6*3*5$