Ontological Necessity of Nonmetricity: a sketch*

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Abstract

In this short letter we present a well-defined energy-momentum tensor of spacetime (gravitational field) which will have important ontologic consequences.

The energy-momentum tensor of the gravitational field à l'Einstein[1] can be seen to be felt as an *annoying non-problem* in the current atmosphere of academia. Non-problem because field-theoretically energy density of a field must be a function of the square of the first derivatives of the field, but as the first derivative of the metric (gravitational potential) is not a tensor, it can be always set to zero by a proper choice of coordinates; so the whole idea seems to be antiquated by the standards of the paradigm of general relativity. Annoying because the question of the energy of a gravitational body is a perfectly legitimate and empirically important question.

In [1] we showed that the Einsteinian maxim *No more potentials; they must be replaced by the metric* works for geometrisation of electromagnetism by leading to the Maxwell equations in the weak-field limit.

Therefore the substitution $A_\mu \to g_{\mu\nu}$ is now perfectly justified. Accordingly by analogy with the electromagnetic energy-momentum tensor

$$T^{\mu\nu} := \frac{1}{\mu_0}\left[F^{\mu\alpha}F^{\nu}{}_{\alpha} - \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right]$$

which is constructed from the Faraday tensor

$$F_{\mu\nu} = 2!\partial_{[\mu}A_{\nu]}$$

we expect the gravitational energy-momentum tensor to be constructed from the tensor

$$\boxed{H_{\alpha\beta\gamma} := 3!\nabla_{[\alpha}g_{\beta\gamma]}} \tag{1}$$

*The reason for the hurry that is implicit in this letter is the possible internet cut-off in Iran.

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[1] We use 'à l'Einstein' to emphasise that we are working in 'the Einsteinian paradigm' and we do not consider viable 'Maxwellian' approaches like the use of 'gravitoelectric' and 'gravitomagnetic' vectors.

But this tensor is zero in the framework of the standard theory of general relativity by the imposition of the *metric-compatibility condition*

$$\nabla_\mu g_{\rho\sigma} = 0,$$

which is why any viable solution for the problem of gravitational energy-momentum tensor must inevitably go beyond standard general relativity by dispensing with the metric-compatibility condition. This is a firm indication of the ontological necessity of the *nonmetricity tensor* defined by

$$Q_{\mu\rho\sigma} := \nabla_\mu g_{\rho\sigma} \tag{2}$$

Therefore just like

$$\mathcal{L} = -\frac{1}{4\mu_0} F^{\alpha\beta} F_{\alpha\beta} - A_\alpha J^\alpha$$

we expect a Lagrangian density constructed from the $\mathbb{H}$ tensor:

$$\boxed{\mathcal{L} = A\ H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} + B\ g_{\rho\sigma} T^{\rho\sigma}} \tag{3}$$

where A and B are constants to be determined.
This Lagrangian density (hence the resulting energy-momentum tensor) satisfies the two requirements that we expect from any candidate resolution of the problem:

- *General Covariance* as it is a tensor.
- Constructed from the square of the first derivative of the field, which is in this case, the metric tensor.

As usual, application of the Hamilton principle to the Lagrangian density (3) will yield a differential equation equation which determines a metric tensor $\tilde{g}_{\mu\nu}$, but such metric tensor *by assumption* cannot be of the forms we are already familiar with, e.g. Schwarzschild metric. All the well-known metrics we know, satisfy the metric-compatibility condition. In other words such tensor does not come from Einstein Field Equations (by assumption) and we will have two completely 'independent' metrics. The physical significance of this new metric is yet to be investigated.

References

[1] Jamali, A. (2021) "Geometrisation of Electromagnetism". viXra:2107.0132