Simultaneity and Synchronization by rods in Relativity as a simple geometry problem

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Abstract

In this paper it is reconsidered the physical meaning of the one-way speed of light, that has also been addressed in other recent papers, complemented with a simple geometric approach. Usually in the standard Special Relativity what is considered is the Einstein one-way speed of light that has been introduced by Einstein in the 1905 article "by definition". In the standard interpretation the one-way speed of light is not considered since the Einstein speed of light is considered the speed of light. However, in our previous work we have shown that this is a terminological confusion. Now we explain why this is so with a very simple geometric Pythagorean approach that complement recent papers about the same subject. With this approach the measurement of the one-way speed of light and the solution of the conventionality of simultaneity and synchronization controversy is addressed. This proposed formulation is based on the existence of a gap of "synchronizations" with a rod that standard formulation is unable to detect.

Introduction

In our previous works [1-16] a broad approach of Special Theory of Relativity (STR) has been formulated. The implications of this approach in the interpretation and experimental determination of the one-way light speed is consider. In Special Relativity the problem of the physical meaning and the experimental determination of the one-way speed of light has been debated since the emergence of the theory when Maxwell discovered the wave equation in his equations of the Electromagnetic Field. The similitude of the value of the speed of propagation of the waves obtained theoretically with the experimental value early obtained by Römer, Bradley, Fizeau, Foucault, naturally convinced Maxwell that the speed of light must be related with the theoretically description he obtained. This is the origin of the idea of the independence of the speed of light of the speed of the source sometimes misinterpreted as implying that the speed of light is the same in every frame. For sure one of the postulates of STR based on experience and theoretical reasoning is that the speed of light is isotropic in vacuum independently of the speed of the source in one frame that we previously designate by Einstein Frame (EF) [6, 7]. Another postulate of special relativity based on the experiences of Michelson-Morley-Miller is that the two-way speed of light in every frame is the same in every direction in vacuum with the value c obtained experimentally (although the experiment has been originally performed in air and does not give a null result, but it has been assumed initially that air does not interfere [17] (see Irvine experiment)). Therefore, the value of the one-way speed of light in EF is also c. From these postulates without invoking the constancy of the one-way speed of light Special Relativity has been constructed initially by Fitzgerald, Larmor, Poincaré and Lorentz with a constructive theory based on experience interpreted with the assumption of a privileged frame where the one-way speed of light have the value c [9-13, 16, 18]. In our previous works based on these postulates we conciliate the analysis of Einstein based on a Principle theory [19] with the Lorentz-Poincaré approach [1-16]. Several works, some very recent, point out the importance of this discussion about the foundations of Relativity, Quantum Mechanics, Cosmology and Biophysics [1-85].

In section I we consider two rods designated by S and S´ moving relatively to each other longitudinally and we obtain the one-way speed of light and the Einstein speed of light.

In Ia we consider the rod S´ with length l_1 moving with speed v_1 in relation to EF where is located rod S with proper length l_0. The rod S´ is moving longitudinally in the same direction defined by the rods. Since the rod S´ is Lorentz contracted (S is the EF, see Ib) [2] we know l_1 when the extremities of the rods pass by each other simultaneously. This is the most primitive notion of simultaneity that Special relativity does not rule out [2-16, 18-29, 73]. Consequently, we can calculate the one-way speed of light in S´ confirming that it is not c. Of course Einstein one way speed of light is c by definition since the Lorentz clock at the extremity of the rod is desynchronized of the clock synchronized, the clock has been desynchronized conveniently with the condition that light arrives to the extremity of the rod where a clock is waiting marking l_1/c. Therefore, both values of the "speed of light" are true and must be observed if not the theory collapse.

In Ib we approach the problem geometrically. Since we assume that in every frame the two-way speed of light is c (based on the experiments of Michelson-Morley-Miller [22, 55]), the harmonic mean of the speeds of light is c. We impose geometrically this condition and we obtain geometrically the relativistic one-way speed of light. From this analysis emerge the meaning of the desynchronization of the clocks that preserve the c condition. Particularly the Einstein one-way speed of light that the standard formulation affirm is the only speed of light.

In IIa we consider another rod S´´ moving with speed v_2 in relation to EF (S). We obtain the length l_2 that satisfies the condition of simultaneity with bar S with length l_0 and the relation between l_2 and l_1 the length of S´ considered at Ia. We previously discovered [2-4] that this relation is no more the Lorentz-Fitzgerald contraction although a relation formally identical exist [11, 13, 16] and originates a gap of "synchronizations" as previously pointed out by Mansouri and Sexl [16, 30]. This gap of "synchronizations" is consistent with the gap revealed by the geometrical approach.

In section IIb based on the results obtained in IIa we introduce a heuristic method of "synchronization" designed to detect the synchronization condition that permit to conceive the experimental determination of the one-way speed of light. The conventional thesis of standard formulation about synchronization and simultaneity is clarified by the acknowledgement of the existence of a spatial gap detected with rods and cannot subsist since the simultaneity of the passing of the extremities of the rods is not conventional.

I. One-Way Speed of Light

Ia. Consider a rod S´ with proper length l_1 moving with speed v_1 in relation to EF where is located another bar S with proper length l_0 (Fig.1).

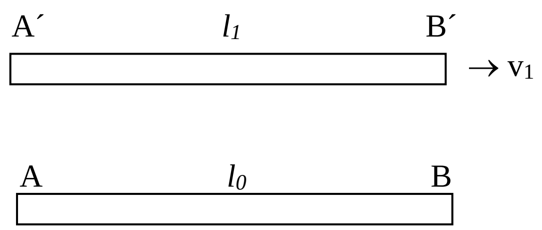

Fig. 1 Rod S´ is moving with speed v_1 in relation to rod S at rest in EF. The extremities of the rods coincide simultaneously and therefore, can synchronize clocks at A, A´ and B, B´

The rod S´ is moving with speed v_1. Since the bar S´ is Lorentz contracted (since S is at rest in the EF) we know l_1 when the extremities of the rods pass by each other simultaneously, when A´ coincide with A and B´ with B as represented in the figure 1. This is the most primitive notion of simultaneity that Special Relativity does not ruled out. However standard interpretation induce to think that it is impossible to synchronize clocks because it is not possible to send a signal from A´ to B´ with infinite speed and since the one-way speed of light was not known in frame S´ Einstein postulate that the one-way speed of light is also *c* in S´ [3, 16, 19]. In this context this affirmation must be ruled out [4].

Indeed, we can calculate the one-way speed of light at S´.

We have

$$l_1 = \frac{l_0}{\sqrt{1-\frac{v_1^2}{c^2}}} \quad (1)$$

From the origin of S´ (A´) it is emitted a ray of light in the direction of the extremity B´ of S´ when A´ pass by A. This ray of light moves in the EF with speed *c*. Therefore, we can calculate the coordinate *x* where the ray of light intercepts the extremity B´

$$x = l_0 + v_1 t \quad (2)$$

$$x = ct \quad (3)$$

$$t = \frac{l_0}{c-v_1} \quad (4)$$

Since S´ is moving with speed v_1 in relation to EF we have the Larmor time dilation [11, 13, 16, 18]

$$t' = t\sqrt{\left(1-\frac{{v_1}^2}{c^2}\right)} \quad (5)$$

From (4) and (5)

$$t' = \frac{l_0}{c-v_1}\sqrt{\left(1-\frac{{v_1}^2}{c^2}\right)} \qquad (6)$$

Therefore, we obtain the one-way speed of light in S´

$$c_+ = \frac{l_0}{\sqrt{\left(1-\frac{v_1^2}{c^2}\right)}} \times \frac{c-v_1}{l_0} \times \frac{1}{\sqrt{\left(1-\frac{v_1^2}{c^2}\right)}} \qquad (7)$$

$$c_+ = \frac{c-v_1}{1-\frac{v_1^2}{c^2}} = \frac{c}{1+\frac{v_1}{c}} \qquad (8)$$

As expected, the one-way speed of light is not c. Only in a first order approximation is c and we obtain the Galileo approximation $c - v_1$for a second order approximation.

Consider now Einstein's one-way speed of light. By definition Einstein has defined "Einstein synchronization" by a clock at *x´* (the generic coordinate of B´) marking *x´/c* and awaiting the arrival of the ray of light emitted at $(x' = 0, t_L' = 0)$ to initiate. This time is the Lorentzian time t_L' and of course $x'/t_L' = c$, it cannot be otherwise [2-4]. Since $t' = x'/c_+' = (x'/c)(1+v_1/c)$ we have $t_L' = t' - (v_1/c^2)\, x'$. Since the clocks marking *t´* are synchronized the clocks marking t_L' are desynchronized. Note that the one-way speed of light that preserve the value *c* for the two way of light is the harmonic mean of c_+' and c_-' [30] given by

$$c_\pm' = \frac{c}{1 \pm \alpha\frac{v_1}{c}} \qquad (9)$$

with $\alpha \in [0, 1]$ as we are going to explain at Ib and II (see also the novel "synchronization" result obtained, Appendix B).

Lorentz was right, t_L' is the local time and SR can be formulated with the synchronized time [11]. Clearly this analysis address the problem of conventionalism [64] since the simultaneous passing of the extremities of the rods is not a convention.

Ib. One-way speed of light as a simple geometry problem

Consider now a more generic problem without the conditions imposed strictly to light. The condition we impose in this generic case is that the two-way speed of a whatever "object" that is moving from A´ to B´ or from B´ to A´ has a generic value c. We have

$$c = \frac{2\,d}{T} \qquad (10)$$

where T is the time measured by a clock at A´ and *d* is the distance between A´and B´.

We can write

$$T = t_+^{'} + t_-^{'} \qquad (11)$$

where $t_+^{'}$ is a "time" associated to the trip A´B´ and $t_-^{'}$ is a "time" associated the trip B´A´. We impose also $t_+^{'} > 0$ and $t_-^{'} > 0$ with

$$t_+^{'} = \frac{d}{c_+^{'}} \qquad (12)$$

$$t_-^{'} = \frac{d}{c_-^{'}} \qquad (13)$$

We have two "speeds", $c_+^{'}$ and $c_-^{'}$ satisfying (see Fig. 2)

$$c = \frac{2\,d}{T} = \frac{2\,d}{t_+^{'}+t_-^{'}} = \frac{2\,d}{\frac{d}{c_+^{'}}+\frac{d}{c_-^{'}}} = \frac{2}{\frac{1}{c_+^{'}}+\frac{1}{c_-^{'}}} = \frac{2\,c_+^{'}c_-^{'}}{c_+^{'}+c_-^{'}} = \frac{G^2}{A} = H \qquad (14)$$

Therefore, *c* is the harmonic mean of the "speeds". The problem is the meaning of "times" and "speeds". As a simple example we can consider that in the trip + the "object" is a "Camel" and for the trip – the object is a "F1 car" [11, 71, 72]. If the condition c is imposed, we can study the problem of the determination of "times" and "speeds" available to answer the problem in the real conditions given by the "objects" "Camel" and "F1 car". And for light the problem is well defined and must be also be answered with a simple geometric formulation that eventually can help to understand the physical meaning of the quantities involved. Since in vacuum based on the Michelson-Morley-Miller experiments the two-way speed of light in every frame is a constant *c* the one-way speed of light in the EF, the Pythagorean means, arithmetic, geometric and harmonic must reveal geometrically the relativistic one-way speed of light and also the one-way Einstein speed of light by definition. Einstein has introduced the meaning of "his definition" of "synchronism" [19] with an assumption

"We assume that is possible for this definition of synchronism to be free of contradictions …" [19]. And based on this assumption Einstein obtain the Lorentz transformation. However, there are other "synchronizations" without contradictions.

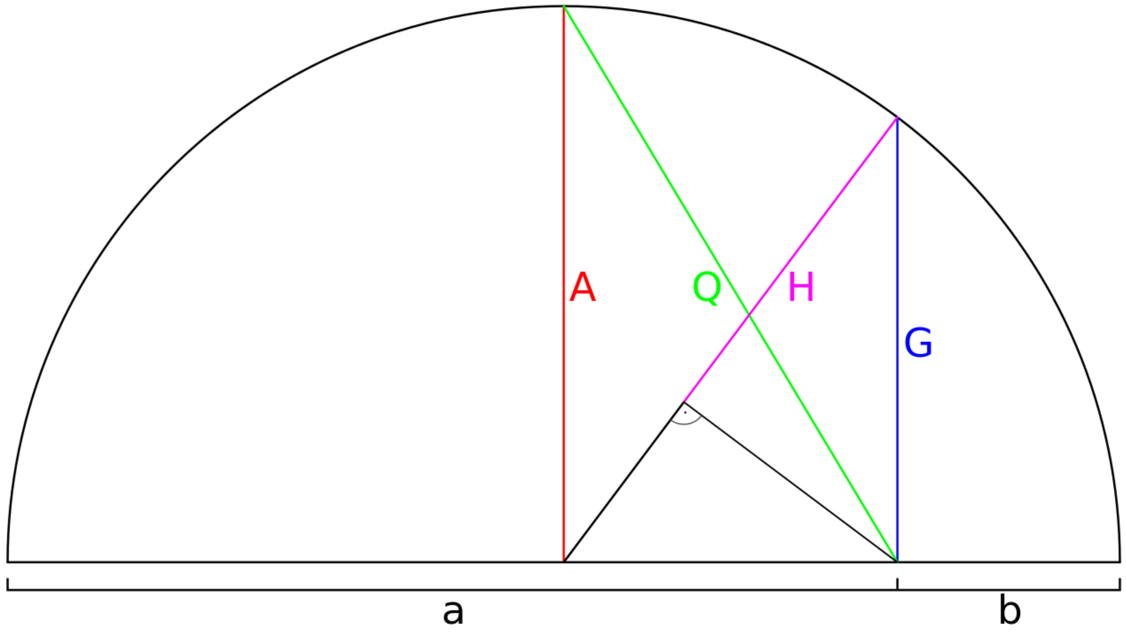

Fig. 2

A geometric construction of the three Pythagorean means of two numbers, *a* and *b*. The harmonic mean is denoted by *H* in purple. *Q* denotes a fourth mean, the quadratic mean. Since a hypotenuse is always longer than a leg of a right triangle, the diagram shows that *Q* > *A* > *G* > *H* (Wikipedia)

We obtain geometrically the several "speeds" and the several "times" satisfying $H = c$.

From Fig. 2 we obtain easily Fig. 3 imposing the constancy of the harmonic mean $H = c$.

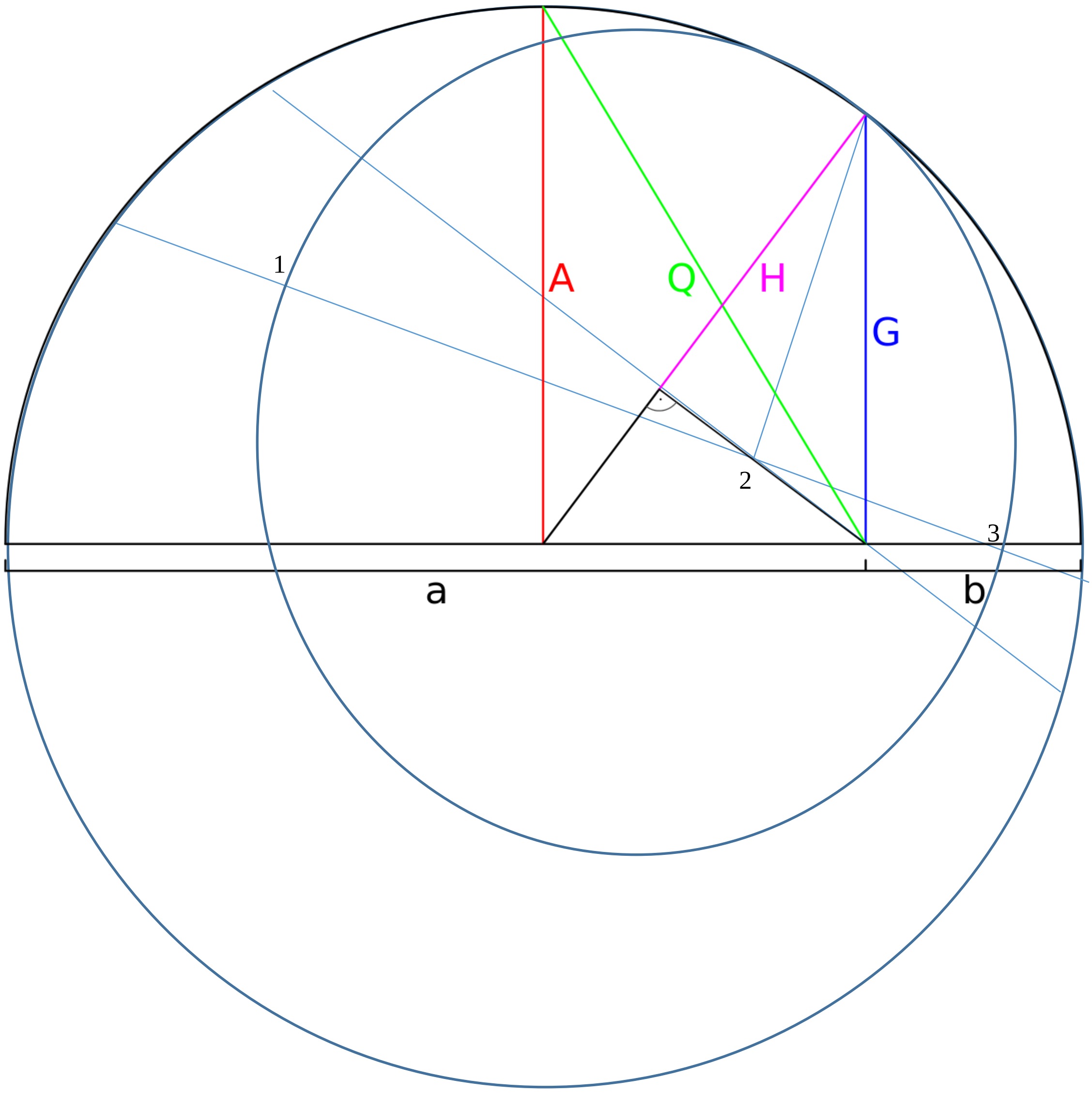

Fig. 3

A new geometric construction where we obtain the distance between the points 1 and 2 and between 2 and 3 as the new numbers that have the same harmonic mean *H* (see Fig. 4 and 5 and Appendix A).

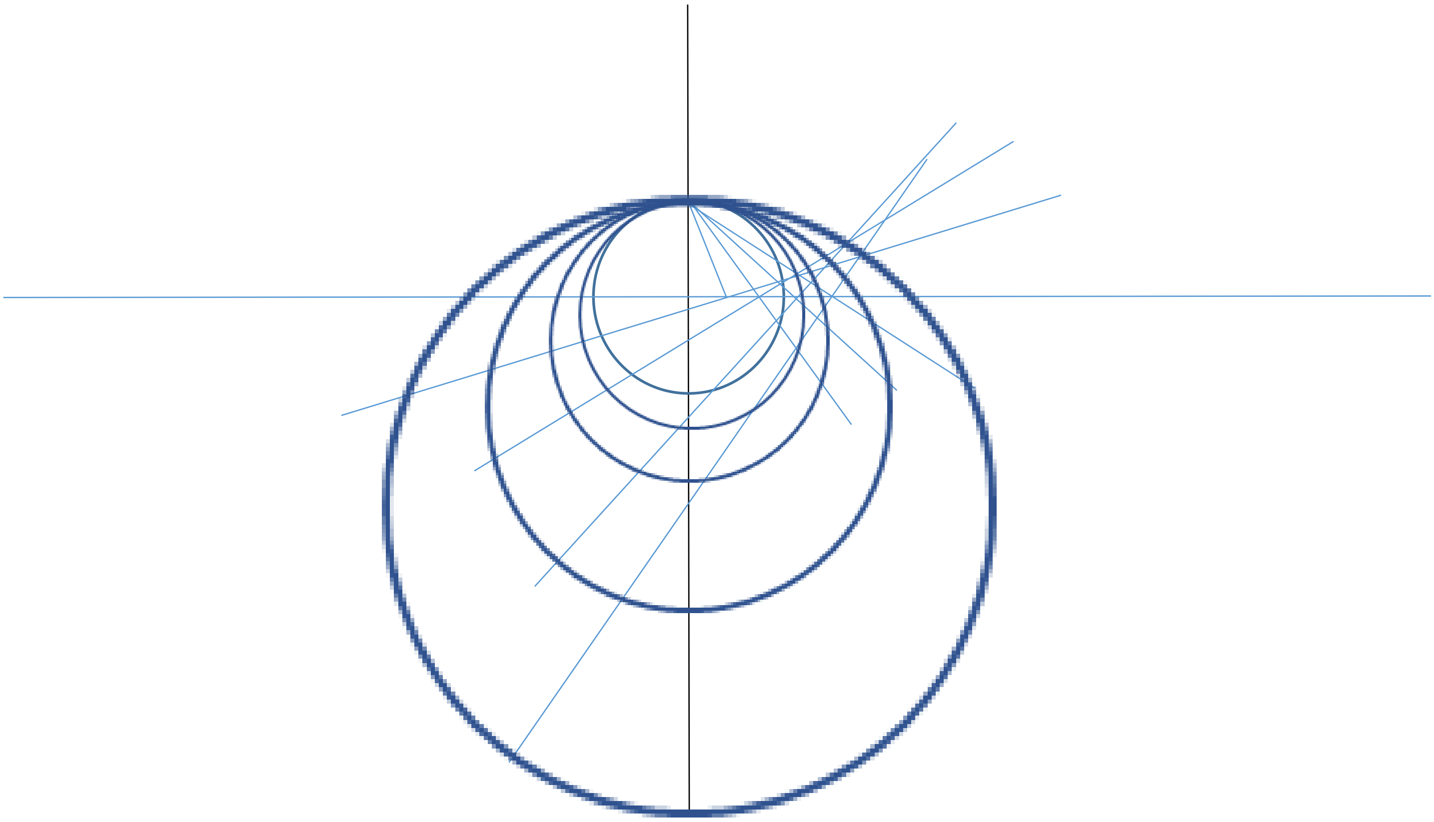

Fig. 4

As we saw in Fig. 3, 4 and 5 we obtain geometrically the values given by (9), the several "speeds" are obtained, particularly the speeds for $\alpha = 1$. And we obtain also the gap (see Ib) between $\alpha = 0$ and $\alpha = 1$ (Appendix A).

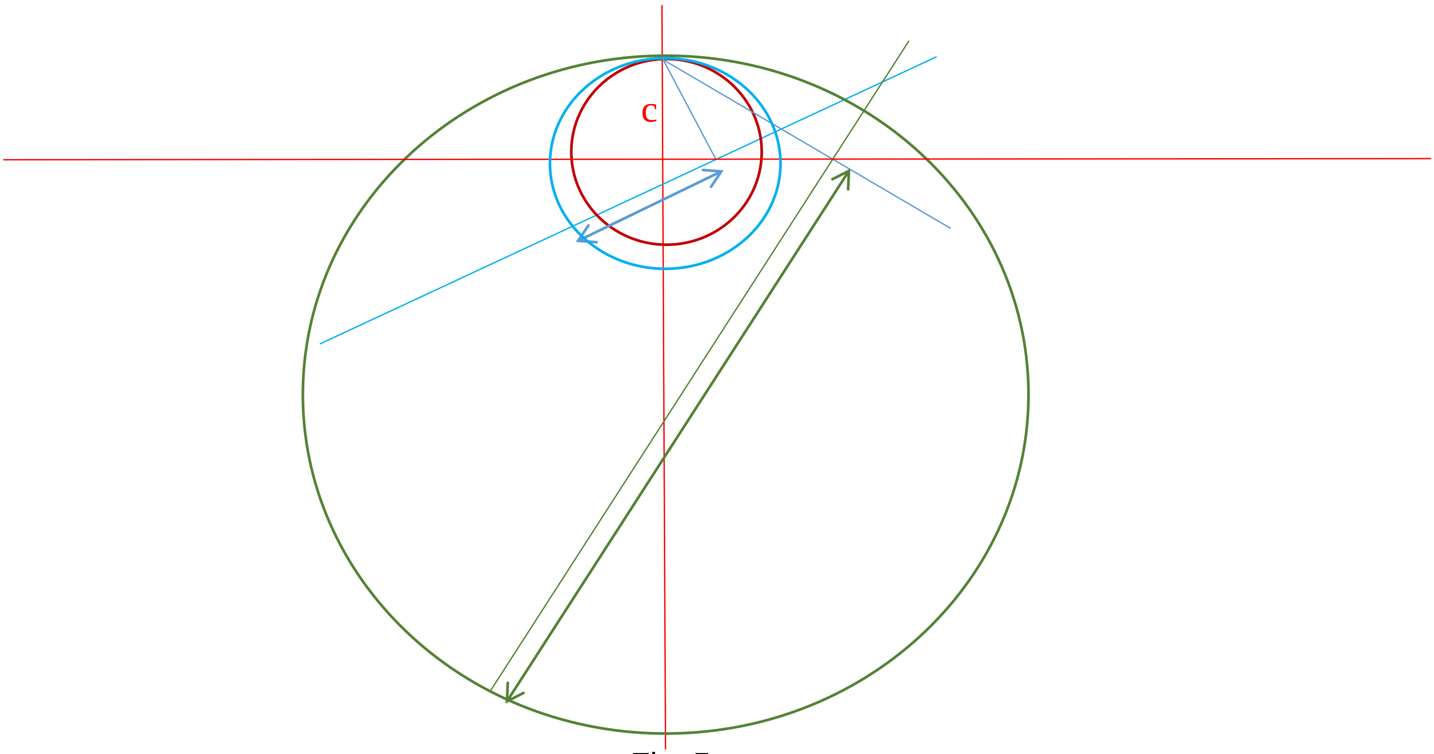

Fig. 5

Two points on the horizontal axis indicate by one of the extremes of the arrows, green and blue correspondent to $\alpha = 1$. $\alpha = 0$ correspond to the intersection of the axis. The lengths of the arrows blue and green correspond to a speed of light backward greater than c (Appendix A).

In this case we have, as a result of the existence of the speed limit c for light in the EF, the speeds limit in the frame moving with absolute speed v_1 with the respective times for the trips. For a given $v_1$we have only one answer to the speeds and times ($\alpha = 1$). And we have also a clear answer to the meaning of the other "speeds" in particular for Einstein speed of light c, by definition, the other extreme, for the gap ($\alpha = 0$) that preserve the c value of the two-way speed of light for a given v_1.

We can go deeper trying to understand, as a consequence of this geometrical approach the recent attained result with a synchronization method different of the standard method [16] established by Einstein and considered by the standard formulation the only method that satisfies Special Relativity. For that we consider another rod S´´.

IIa. Two rods moving in relation to EF

We introduce now a third rod S´´ with length l_2.

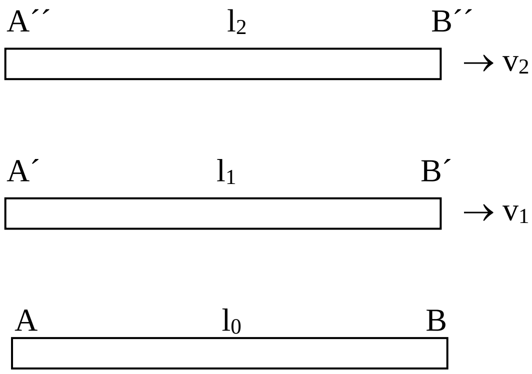

Fig. 6 A third rod S´´ is moving with speed v_2 in relation to EF passing also simultaneously from the extremities of S.

The rod S´´ is moving with speed v_2 in relation to EF. The rod has proper length l_2

$$l_2 = \frac{l_0}{\sqrt{(1-\frac{v_2^2}{c^2})}} \qquad (15)$$

From (1) we have

$$l_2 = l_1 \frac{\sqrt{(1-\frac{v_1^2}{c^2})}}{\sqrt{(1-\frac{v_2^2}{c^2})}} \qquad (16)$$

It easy to obtain from (16)

$$l_2 = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\left(1+\frac{v_1\, v'_E}{c^2}\right) \qquad (17)$$

since Einstein's speed of rod S´´ in relation to S´ is given by [11]

$$v'_E = \frac{v_2 - v_1}{1-\frac{v_1 v_2}{c^2}} \qquad (18)$$

The correct evaluation of the distance l_2 is crucial as we pointed out in several previous works [2, 6, 13-15] and used to solve the Twin Paradox in a one-way trip [15] analysing the approach of Φ. Grϕn [45].

We see from (17) that we can consider two lengths

$$l_2 = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} \qquad (19)$$

and

$$l_2 = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\left(1 + \frac{v_1\, v'_E}{c^2}\right) \qquad (20)$$

When $v_1 = 0$ (S´ is at rest in EF) we have only for l_2 the value given by (19). However, there are several values of l_2, a gap, between the values given by (19) and (20). We can define this gap by

$$g = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\left(\alpha\, \frac{v_1\, v'_E}{c^2}\right) \qquad (21)$$

with $\alpha \in [0, 1]$ and in the following figure l_2 for $\alpha = 0$ correspond to Einstein synchronization.

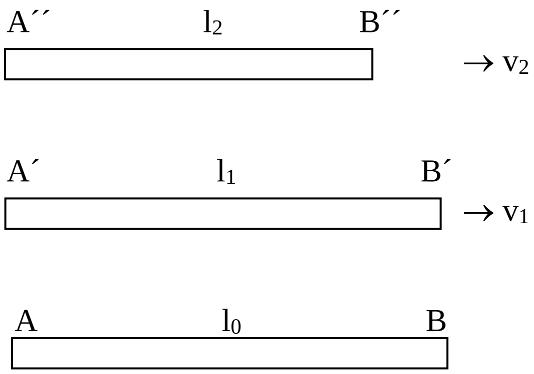

Fig. 7 S´´ with the proper length l_2 ($\alpha = 0$) that “synchronize” a Lorentzian clock located at B´ marking zero when B´´ pass by B´

Indeed when $t'_L = 0$ at B´ (l_2 is given by (19)) B´´ coincide with B´ [25] (as the Principle of Relativity in a restricted sense determine [11]). Since we know v'_E we know l_2 for $\alpha = 0$. This eventually can be experimentally tested with the other method of “synchronization” by light signalling. When A´´ pass by A´ light is emitted from A´ to B´. Since B´ has been previously “synchronized” ($t'_L = 0$ at B´) by the passing of B´´ if the theory is correct the arrival of light at B´ is $t'_L = x'/c$. This method of synchronization corresponds to an external “synchronization” [11, 30]. Rod S´´ establish the connection with the EF, rod S, the “external synchronization” [30].

IIb. A method to determinate experimentally the one-way speed of light

The crucial matter is what is revealed by Fig. 8

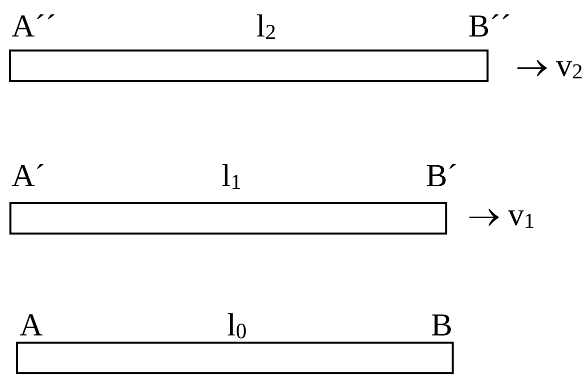

Fig. 8 Bar S´´ has proper length that exceeds the range of “synchronizations”

From the previous analysis corresponding to Einstein synchronization but now by rods (figure 7) we can increment the length l_2 (for several values of $\alpha > 0$) but it has no meaning $\alpha > 1$ (figure 8). For those values the extremity B´´ pass by B´ previously to the passing of A´´ to A´. This violates the principle of causality corresponding to a speed of signalling “superior to infinity” that has no meaning (Fig. 8). Therefore, we can conceive the tentative determination of the condition $\alpha = 1$ and corresponding value of v_1 that satisfies the value c for the two-way of light. We can begin incrementing the length of rod S´´ from the length given by (19). The clock at B´ is set to zero and begin working when B´´ pass by B´. When A´ pass by A´´ light is emitted to B´ that measure the arrival of light with the clock previously “synchronized”. Light can also be emitted from A´´ to B´´ when A´ pass by A´´. The chronometer at B´´ is set to zero and begin working by the passing of B´ and therefore can measure the time of the arrival of light emitted by A´´. When the length of rod S´´ is given by (20) we obtain the synchronization of clock B´´ with the clock at A´´ and the synchronization of clock at B´ with the clock at A´. Since the clocks at A´, A´´ has been synchronized with the clocks at B´, B´´ respectively we obtain from the measurements of the times of the arrival of light at B´ and B´´ v_1 and v_2 that satisfies $v_E^{'}$ given by (18). Indeed this times are given by $t_+^{'} = l_1/c_+^{'} = (l_1/c)\,(1 + v_1/c)$ and $t_+^{''} = l_2/c_+^{''} = (l_2/c)\,(1 + v_2/c)$. Indeed we can calculate for the several lengths of rod S´´ given by (21) the time of arrival of light at the clock B´´ if the clock at B´´ is set to zero when pass by B´ (note that this is not the Lorentzian time coordinate nor the synchronized time coordinate except for $(\alpha = 1)$ (this new method of “synchronization” is intentionally designed to detect synchronization $(\alpha = 1)$). The solution is (Appendix B)

$$t_N^{''}(\alpha) = \frac{l_2}{c}\left(1 + \frac{\alpha v_1 + v'_E}{1 + \frac{\alpha v_1 v'_E}{c^2}}\,\frac{1}{c}\right) \quad (22)$$

and

$$t'_N(\alpha) = \frac{l_1}{c}\left(1 + \alpha v_1 \frac{1}{c}\right) \qquad (23)$$

that is independent of v'_E and N refer to new synchronization (Appendix B).

For α = 1 as expected we obtain from (18) and (22) and (23) the synchronized solution

$$\frac{l_{1,2}}{c}\left(1 + v_{1,2} \frac{1}{c}\right) \qquad (24)$$

When $v'_E \to 0$, as also expected it is obtained the classical solution with $l_2 \to l_1$. Therefore for small values of v'_E the gap tend to zero and we can tentatively conceive synchronization with a rod S´´ with lengths $l_2 = l_1$ in a first order approximation, since the value of the time measured by the clock at B´ gives v_1 and the clock at B´´ gives v_2 that tends to v_1 and this eventually can be observable because it is dependent of v_1. We also know that for $v_1 = 0$ this also must be true since the gap is zero independently of the value of v'_E (see our comment on "The Motion Paradox from Einstein´s Relativity of Simultaneity" [24, 25]). Note that we can control tentatively the condition ($\alpha = 1$) emitting light from B´ to A´ when B´´ pass by B´ since light can arrive at A´ before the arriving of A´´ if the length of rod S´´ exceed the condition. This is a very simple condition that avoid the observation at B´´. Therefore, it seems justified (Appendix B) the simple idea of synchronization of the clocks of S´ with another rod with "equal" length moving with very slow speed in relation to S´, independently of the value v_1. Or with a "Bell´s spaceships" configuration.

Indeed when the length of rod S´´ exceed the gap if light emitted from B´ (when B´´ pass by B´) arrive at A´ when the clock at A´ is not yet working because A´´ has not arrived yet at A´(see Fig 4.) we can detect this condition at B´. The length of rod S´´ exceed the length given by (20) the limit of the gap, $\alpha = 1$. This excess of length d'' can be quantified. The speed of light from B´ to A´ is given from (9)

$$c'_- = \frac{c}{1-\frac{v_1}{c}} \qquad (25)$$

Therefore, the travel time for light (B´A´) is given by

$$\frac{l_1}{c}\left(1 - \frac{v_1}{c}\right) \qquad (26)$$

Since any point of S´´ has the same speed v' through S´ the distance d' in S´ that correspond to d'' is

$$d' = v'\frac{l_1}{c}\left(1 - \frac{v_1}{c}\right) = \frac{v'_E}{1+\frac{v_1 v'_E}{c^2}}\frac{l_1}{c}\left(1 - \frac{v_1}{c}\right) \qquad (27)$$

where the speed $v'$is given by [9, 11, 13]

$$v' = \frac{v_2 - v_1}{1 - \frac{v_1^2}{c^2}} = \frac{v'_E}{1 + \frac{v_1 v'_E}{c^2}} \qquad (28)$$

Since

$$\frac{d''}{d'} = \frac{l_2(\alpha=1)}{l_1} = \frac{1 + \frac{v_1 v'_E}{c^2}}{\sqrt{1 - \frac{v'^2_E}{c^2}}} \qquad (29)$$

$$d'' = \frac{l_1}{\sqrt{1 - \frac{v'^2_E}{c^2}}} \frac{v'_E}{c}\left(1 - \frac{v_1}{c}\right) = \frac{l_1}{\sqrt{1 - \frac{v'^2_E}{c^2}}} \frac{v'_E}{c} - \frac{l_1}{\sqrt{1 - \frac{v'^2_E}{c^2}}} \frac{v_1 v'_E}{c^2} \qquad (30)$$

Therefore $d'' \to 0$ when $v'_E \to 0$. Note that the gap (inexistent when $v_1 = 0$)

$$\frac{l_1}{\sqrt{1 - \frac{v'^2_E}{c^2}}} \frac{v_1 v'_E}{c^2} \qquad (31)$$

tend to zero when v'_E tend to zero (however, the time for the trip of B´´ in S´ correspondent to the Lorentz synchronization is only dependent of v_1, $l_1 \frac{v_1}{c^2}$ (see Fig. 7) as expected). With this novel method of synchronization emerge the physical meaning of the two-way speed of light measured with the harmonic mean of the "speeds" for a given v_1 (eq. (9)) and of course this does not contradict that the two-way speed of light is *c* measured with one clock in a two-way trip after reflection that is completely independent of the "synchronizations" and independent of v_1 [36, 37].

The new length of rod S´´ with d'' is given by adding the length given by (20) with d'' given by (22). It is independent of v_1

$$l_2 = \frac{l_1}{\sqrt{\left(1 - \frac{v'^2_E}{c^2}\right)}}\left(1 + \frac{v'_E}{c}\right) \quad (32)$$

The length of the rod change when $d'' \to 0$ and therefore the "times" of arrival of light emitted by A´ and B´ tend to $\frac{l_1}{c}\left(1 + \frac{v_1}{c}\right)$ and $\frac{l_1}{c}\left(1 - \frac{v_1}{c}\right)$.

We can eventually detect this condition experimentally reading the clock B´ after receiving light from A´. Since the length of S´´ is given by (32) although d' is not known because (30) is dependent of $v_1$we can decrease the length d'' by *n* decreasing the length

d' by n and measuring at B´ the arrival of light emitted by A´. The time marked by the clock A´ is

$$t'_- = (1 - \frac{1}{n})\frac{l_1}{c}\left(1 - v_1 \frac{1}{c}\right) \quad (33)$$

and the time marked by B´is

$$t'_+ = \tau'_{B'} + \frac{l_1}{c}\left(1 + v_1 \frac{1}{c}\right) \quad (34)$$

where $\tau'_{B'}$ is the proper time change of the clock B´

$$\tau'_{B'} = (\frac{1}{n})\frac{l_1}{c}\left(1 - v_1 \frac{1}{c}\right) \quad (35)$$

correspondent to the emission of light by A´ and $\frac{l_1}{c}\left(1 + v_1 \frac{1}{c}\right)$ is the proper time change correspondent to the trip of light A´B´

$$t'_+ = \frac{l_1}{c}[1 + \frac{1}{n} + \left(1 - \frac{1}{n}\right) v_1 \frac{1}{c}] \quad (36)$$

Therefore

$$T = t'_- + t'_+ = \frac{2l_1}{c} \quad (37)$$

However, this new "times" (given by (33) and (36)) can be defined through the new "speeds" of light c'_- and c'_+

$$t'_- = \frac{l_1}{c'_-} \quad (38)$$

$$t'_+ = \frac{l_1}{c'_+} \quad (39)$$

We obtain

$$c'_- = \frac{c}{\left(1 - \frac{1}{n}\right) - \left(1 - \frac{1}{n}\right) v_1 \frac{1}{c}} \quad (40)$$

$$c'_+ = \frac{c}{\left(1 + \frac{1}{n}\right) + \left(1 - \frac{1}{n}\right) v_1 \frac{1}{c}} \quad (41)$$

that does not have the same form of the "speeds" into the gap. This eventually permit from the data obtained experimentally measure the one-way speed of light since we obtain the same values when $\alpha \to 1$ and $n \to \infty$. And after acquired the meaning of synchronization by rods (the algorithms for the two "zones") it is possible to obtain experimentally the data with light signalling through the drift of the "time" marked by one of the clocks (Appendix C).

Another way to check the theory is for considering $v_2 = -v_1$. Indeed when $v_2 = -v_1$equations (16), or (18) and (20), gives synchronization with $l_2 = l_1$when the speeds of light are the same in both frames, only dependent of v_1. Although this is obvious and motivates Einstein to introduce synchronization by rods (p. 129 of [16]) [see also 50, 51] the reader is invited to check it formally.

Conclusion

It is our firm belief that physics should assume itself as the heir of natural philosophy. And thus question, with no fear nor prejudice, the postulates or hypothesis at the origin of each theory. Only in this way is it possible to claim that to understand a physical theory goes much beyond the simple knowledge of how to perform the calculations. Unfortunately, special relativity is presented in most textbooks and papers by passing too swiftly over the discussion of its postulates [11].

It is beyond doubt that different types of clocks synchronization simply provide time coordinates to describe the same reality. In addition, the words "time", "speed" and "simultaneity", wich we use to attribute a precise physical meaning, actually refer to different notions when different types of clocks are used. Since different descriptions made with various types of clocks, are mathematically equivalent, this latter issue is mainly a question of language. Nonetheless it is an important one and likely to originates several misunderstandings because the physical concepts underlying each of these descriptions are quite different. Many disputes and hot debates around special relativity are related to the problem of using the same word to designate different concepts. For this reason, it is of major importance to know what kind of clocks one ends up after performing synchronization, p.40 and 41 [11]. This is the conclusion of our article about the meaning of The Principle of Relativity and the Indeterminacy of Special Relativity [11]. This is also the conclusion of another recent experimental article, Misconception Regarding Conventional Coupling of Fields and Particles in XFEL Codes, p.11 and 12 [26, 29].

Therefore, perhaps it is now clear what is happen with standard special relativity. Light is moving with one-way speed $c_+^{\prime}$ and $c_-^{\prime}$ given by

$$c_{\pm}^{\prime} = \frac{c}{1 \pm \frac{v_1}{c}}$$

Standard relativity affirms that the one-way speed of light is c for both trips in every frame. Of course, it isn´t. c is the one-way Einstein speed of light, as a result of its very definition in every frame. End of the mystery. However, this is an important matter because we can conceive the experimental determination of the one-way speed of light. The important point is that for $\alpha = 0$ the "speeds" for the trip + and trip – are equal and independent of the absolute speed of the frame and it is useful since it is operational. This, perhaps ironically, open the door to conceive the experimental detection of Einstein´s frame.

In sections I and II we consider several configurations of three rods designated by S, S´ and S´´ moving relatively to each other longitudinally. The idea is that rods moving in relation to each other can reveal the movement in relation to the frame where the one-way speed of light is isotropic with value c, the frame that we designate by Einstein Frame (EF) because the movement of the rods in relation to EF can affect differently each rod and this effect can be observable. A similar idea has been defended recently by Espen Haug [23-25] with several pertinent questions in relation to the difficulty to conceive "absolute simultaneity".

In section Ia we consider a rod S´ moving with speed v_1 in relation to EF where rod S is at rest. Since S´ is Lorentz-Fitzgerald contracted it is easy to obtain the one-way speed of light in the frame of S´. The Einstein's one-way speed of light is c by definition.

In Ib we approach the problem geometrically. Since we assume that in every frame the two-way speed of light is c (based on the experiments of Michelson-Morley-Miller), the harmonic mean of the speeds of light are c. We impose geometrically this condition and we obtain geometrically the relativistic one-way speed of light. Because of this analysis emerge the meaning of the desynchronization of the clocks that preserve the c condition for a given value v_1. Particularly the Einstein one-way speed of light that the standard formulation affirm is the only speed of light.

In section IIa we consider a third rod moving with speed v_2 in relation to EF and we obtain the relation between the proper lengths of the rods that reveals a gap of possible "synchronizations" that preserve the value c for the two-way speed of light for a given v_1.

In section IIb based on the results obtained in IIa we introduce a novel method of "synchronizations" designed to detect the synchronization condition that permit to conceive the experimental determination of the one-way speed of light.

As a conclusion the remark that the condition of the constancy of the two-way speed of light that we observe experimentally is a result of the existence of EF as we point out in our previous work. Exactly the contrary of what is affirmed by standard relativity.

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Appendix A

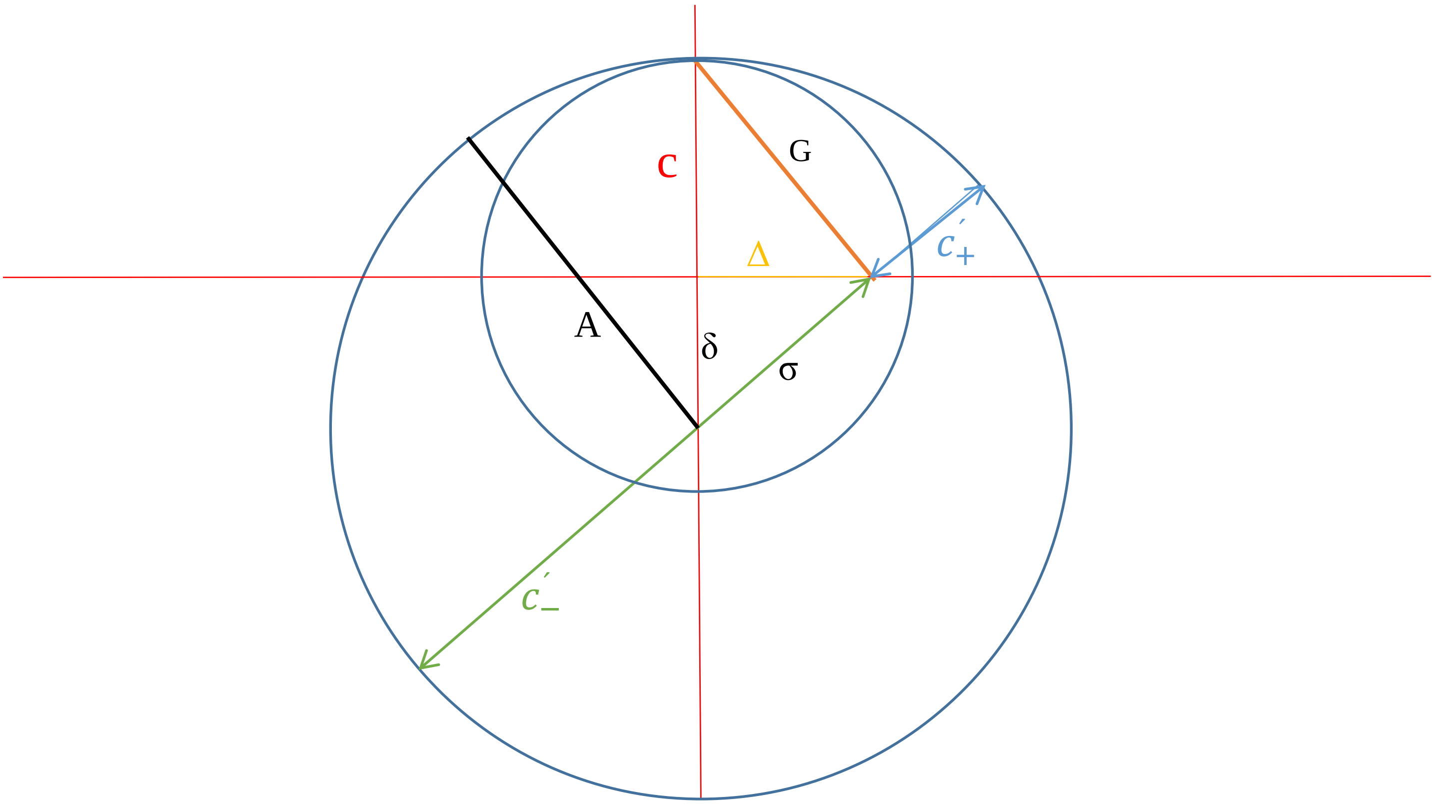

Fig. 9 Several parameters are indicated, that can be calculated with the Pythagorean approach, particularly Δ that are related to the gap of synchronizations for a given speed of S´

As is well known, since G is the height of the right triangle in the circle with hypotenuse of length $c'_- + c'_+$, the diameter, we have

$$G^2 = c'_- \times c'_+ \quad (1)$$

For the other right triangle with height Δ we have

$$\Delta^2 = c \times \delta \qquad (2)$$

Since

$$\delta = A - c \qquad (3)$$

where

$$A = (c_{-}^{'} + c_{+}^{'})/2 \qquad (4)$$

$$\Delta^2 = c \times (A - c) = cA - c^2 = c(c_{-}^{'} + c_{+}^{'})/2 - c^2 \qquad (5)$$

$$c_{-}^{'} + c_{+}^{'} = \frac{c}{1-\frac{v_1}{c}} + \frac{c}{1+\frac{v_1}{c}} = \frac{2c}{1-\frac{v_1^2}{c^2}} \qquad (6)$$

$$G^2 = AH = \frac{c^2}{1-\frac{v_1^2}{c^2}} \qquad (7)$$

$$A = \frac{c}{1-\frac{v_1^2}{c^2}} \qquad (8)$$

$$\Delta = \frac{v_1}{\sqrt{1-\frac{v_1^2}{c^2}}} \qquad (9)$$

$$\delta = A - c = \frac{c}{1-\frac{v_1^2}{c^2}} - c = c\frac{\frac{v_1^2}{c^2}}{1-\frac{v_1^2}{c^2}} \qquad (10)$$

We clearly see with the geometric representation the relativistic correction emerging from the constancy of the two-way speed of light. When $v_1 \to c$ we see from Fig. 4, 5, 9 and eq. (8) and (9) of the article and also (8) and (9) and (10) of this appendix the rising of A, Δ *and* δ and the tendency of $c_{+}^{'}$ to $\frac{c}{2}$ and the tendency of $c_{-}^{'}$to ∞.

Appendix B

Obtention of times at $l_2(\alpha), t_N^{''}(\alpha)$ and $l_1, t_N^{'}(\alpha)$ (time at B´´ and time at B´) when light emitted from A´´ and A´ arrive, with the new synchronization that emerge from the existence of the gap.

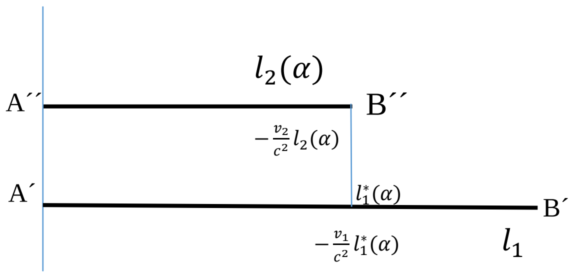

Fig. 10 Lorentzian times are indicated at

B´´ and B´at the departure.

$$l_2(\alpha) = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\left(1+\alpha\frac{v_1\, v'_E}{c^2}\right) \qquad (1)$$

$$l_2(\alpha=1) = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\left(1+\frac{v_1\, v'_E}{c^2}\right) \qquad (2)$$

$$\frac{l_2(\alpha=1)}{l_1} = \frac{l_2(\alpha)}{l_1^*} = \frac{\sqrt{\left(1-\frac{v_1^2}{c^2}\right)}}{\sqrt{\left(1-\frac{v_2^2}{c^2}\right)}} = \frac{1+\frac{v_1\, v'_E}{c^2}}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} \qquad (3)$$

$$\frac{\tau''(\alpha=1)}{\tau'(\alpha=1)} = \frac{0}{0} = \frac{\tau''(\alpha)}{\tau'(\alpha)} = \frac{\sqrt{\left(1-\frac{v_2^2}{c^2}\right)}}{\sqrt{\left(1-\frac{v_1^2}{c^2}\right)}} = \frac{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}{1+\frac{v_1\, v'_E}{c^2}} \qquad (4)$$

where $\tau''(\alpha) and\ \tau'(\alpha)$ are proper times.

$$l_1^* = l_2(\alpha)\frac{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}{1+\frac{v_1\, v'_E}{c^2}} = \frac{l_1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}\left(1+\alpha\frac{v_1\, v'_E}{c^2}\right)\frac{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}{1+\frac{v_1\, v'_E}{c^2}} \qquad (5)$$

$$l_1^* = \frac{l_1}{1+\frac{v_1\, v'_E}{c^2}}\left(1+\alpha\frac{v_1\, v'_E}{c^2}\right) \qquad (6)$$

$$l_1 - l_1^* = l_1\frac{v_1\, v'_E}{c^2}\frac{(1-\alpha)}{1+\frac{v_1\, v'_E}{c^2}} \qquad (9)$$

$$l_1 - l_1^* = 0 \quad (\alpha = 1) \qquad (10)$$

$l_1 - l_1^* = l_1 \frac{v_1 v'_E}{c^2} \frac{1}{1+\frac{v_1 v'_E}{c^2}} \quad (\alpha = 0)$ (11)

From (9)

$\tau''(\alpha) = \frac{l_1 - l_1^*}{v'_E} \sqrt{\left(1 - \frac{v'^2_E}{c^2}\right)} = l_1 \frac{v_1}{c^2} \frac{(1-\alpha)}{1+\frac{v_1 v'_E}{c^2}} \sqrt{\left(1 - \frac{v'^2_E}{c^2}\right)}$ (12)

From (4) and (12)

$\tau'(\alpha) = l_1 \frac{v_1}{c^2} (1-\alpha)$ (13)

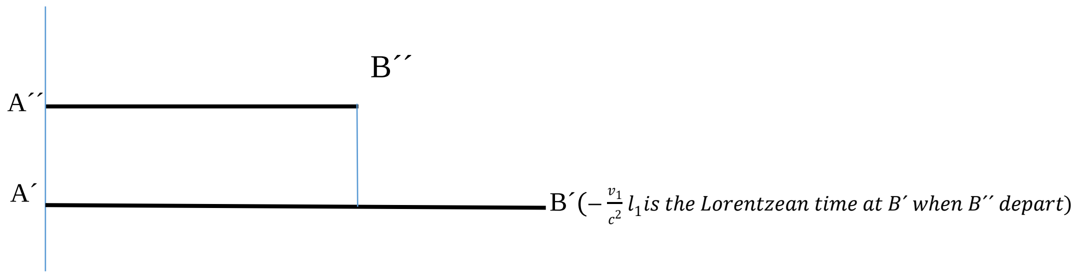

Fig. 11

When B´´ pass at B´ the time for the trip at S´is given by (13). Therefore the Lorentzian time at B´ is for this event, for a generic α is

$t'_L(\alpha) = -\frac{v_1}{c^2} l_1 + \tau'(\alpha) = -\frac{v_1}{c^2} l_1 + \frac{v_1}{c^2} l_1 (1-\alpha)$ (14)

$t'_L(\alpha) = -\frac{v_1}{c^2} l_1 \alpha$ (15)

$t'_L = 0 \quad (\alpha = 0)$ (16)

$t'_L = -\frac{v_1}{c^2} l_1 \quad (\alpha = 1)$ (17)

The synchronized time t' is

$t' = t'_L + \frac{v_1}{c^2} l_1$ (18)

$t'(\alpha) = -\frac{v_1}{c^2} l_1 \alpha + \frac{v_1}{c^2} l_1 = \frac{v_1}{c^2} l_1 (1-\alpha)$ (19)

If at B´ we consider a a synchronized clock from (19) the clock is marking $\frac{v_1}{c^2} l_1 (1-\alpha)$ therefore a clock marking zero is desynchronized by this clock the same quantitity (in absolute value). Therefore when light arrives to B´ the synchronized clock mark $\frac{l_1}{c}(1 + \frac{v_1}{c})$ and the other clock mark (with the new synchronization labeled with N)

$t'_N(\alpha) = \frac{l_1}{c}\left(1 + \frac{v_1}{c}\right) - \frac{v_1}{c^2} l_1 (1-\alpha) = \frac{l_1}{c}\left(1 + \alpha \frac{v_1}{c}\right)$ (20)

Similarly for B´´ we have from (12) and (19)

$$t'_L(\alpha) = -\frac{v_2}{c^2} l_2(\alpha) + \tau''(\alpha) = -\frac{v_2}{c^2} l_2 + l_1 \frac{v_1}{c^2} \frac{(1-\alpha)}{1+\frac{v_1 v'_E}{c^2}} \sqrt{\left(1 - \frac{v'^2_E}{c^2}\right)} \qquad (21)$$

But now l_2 is not constant and change with α. From (1) and (21)

$$t'_L(\alpha) = -\frac{v_2}{c^2} \frac{l_1\,(1+\alpha\frac{v_1 v'_E}{c^2})}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} + l_1 \frac{v_1}{c^2} \frac{(1-\alpha)}{1+\frac{v_1 v'_E}{c^2}} \sqrt{\left(1 - \frac{v'^2_E}{c^2}\right)} \qquad (22)$$

Since

$$v'_E = \frac{v_2 - v_1}{1-\frac{v_1 v_2}{c^2}} \qquad (23)$$

$$v_2 = \frac{v_1 + v'_E}{1+\frac{v_1 v'_E}{c^2}} \qquad (24)$$

Substituting (24) in (22) we obtain

$$t'_L(\alpha) = -(\alpha v_1 + v'_E) \frac{l_1}{c^2} \frac{1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} \qquad (25)$$

$\alpha = 0$

$$t'_L(\alpha) = -v'_E \frac{l_1}{c^2} \frac{1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} \qquad (26)$$

$\alpha = 1$

$$t'_L(\alpha = 1) = -(v_1 + v'_E) \frac{l_1}{c^2} \frac{1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} = -v_2 \left(1 + \frac{v_1 v'_E}{c^2}\right) \frac{l_2}{c^2} \frac{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} \frac{1}{1+\frac{v_1 v'_E}{c^2}} \qquad (27)$$

$$t'_L = -\frac{v_2}{c^2} l_2 \qquad (28)$$

Therefore the clock that mark zero when B´´ pass by B´ (with the new synchronization labeled with N) when light arrives at B´´ mark

$$t''_N(\alpha) = \frac{l_2}{c} + (\alpha v_1 + v'_E) \frac{l_1}{c^2} \frac{1}{\sqrt{\left(1-\frac{v'^2_E}{c^2}\right)}} \qquad (29)$$

that can be writen from (2)

$$t''_N(\alpha) = \frac{l_2}{c} \left(1 + \frac{\alpha v_1 + v'_E}{1+\frac{\alpha v_1 v'_E}{c^2}} \frac{1}{c}\right) \qquad (30)$$

An interesting exercise for acquire the approach proposed is to consider the case $v_1 = 0$ when there is no gap and aplly the proceedings.

Appendix C

Consider a chronometer at B´ marking zero. When light is emitted to A´the chronometer begin working. When light arrives at A´ an identical chronometer also marking zero begin working and A´ emit light to B´. When light arrives at B´ the clock at B´ mark $2l_1/c$. Of course, after the chronometers are working, we can drift the time marked by clock B´ and reproduce the "times" obtained with the rod S´´. This is for the zone forbidden where the rod S´´, the extremity B´´, "invert" causality. B´´ exceed the gap of synchronizations. B´´ first. But is operational. And eventually can be observed. And perhaps gives a new meaning for the "everyday synchronization" that can not be reciprocal [83].

The same is true for the gap. From A´, the chronometer begin working and send light to B´ that has a chronometer desynchronized marking Lorentzian time. After light arrives at B´ the clock marking l_1/c and emit light to A´ that receive light at the time $2l_1/c$. After the clock at B´ is working can be "drifted" and the "times" obtained by rods are reproduced by light signalling with clear physical meaning since it is operational and eventually can be experimentally implemented.