

Proper Space Time Coordinates of Events – Completing and Disproving the Special Theory of Relativity

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Abstract

The special theory of relativity (STR) is perhaps one of the most controversial theories in the history of science. Countless arguments have been made against STR by researchers, prominent scientists and lay persons. This paper reveals for the first time that most of the confusions arise from incomplete development of the theory, rather than from possible fundamental contradictions. This has caused STR not being not fully understood even by professional physicists, who have been applying it inconsistently. For example, the GPS is being cited wrongly to support STR. Proponents and opponents of relativity theory have been arguing over an incomplete theory for more than a century. In this paper we introduce a new concept called ‘proper space time coordinates of events’ that will complete the STR, clearing the contradictions and paradoxes. It is shown that the existing concepts of proper length and proper time are incomplete. The new formulation not only results in a complete version of STR that is mathematically and logically consistent, but also, ironically, leads to its ultimate disproof. It turns many of the light speed experiments, such as the GPS and the Lunar Laser Ranging (LLR,), against STR. These add to the already existing experimental evidences contradicting with relativity theory, such as the Silvertooth, the Marinov and the Roland De Witte experiments.

Introduction

The special theory of relativity (STR) has been perhaps one of the most controversial theories in the history of science. Countless arguments have been made against STR researchers, prominent scientists and laypersons, for more than a century. So many internal contradictions and paradoxes of the theory have been debated. The debates on whether or not given light speed experiments agree with STR is still going on.

This paper reveals for the first time that all the confusions in the STR have been caused by incompleteness of the theory, rather than fundamental contradictions. A new concept called ‘proper space time coordinates of events’ is proposed which makes STR a complete theory that is mathematically and logically consistent. The new formulation not only makes STR a complete theory, but also leads to its ultimate disproof. It will be shown that many of the experiments usually cited to support SRT actually contradict it. These add to the already existing experimental evidences against relativity, including the Marinov, the Silvertooth, the Roland De Witte and the Miller experiments.

The problems with the STR theory are:

1. It is incomplete in its current form , and therefore marred by contradictions
2. It has not been applied with sufficient rigor

If physicists had pursued a rigorous application of STR, then they would have discovered its contradictions. This was in fact the case when I tried a rigorous application of STR to a hypothetical light speed experiment [1].

Proper space time coordinates of events- a new concept to complete the special theory of relativity

The concepts of proper time and proper length are briefly introduced in STR books in the analysis of the Michelson-Morley experiment. However, the standard application of STR does not usually refer to them elsewhere, such as in the analysis of the GPS and the Sagnac effect. This is caused by the difficulty in the application of proper length, proper distance and proper time to all light speed experiments in general. This paper reveals that this difficulty arises due to the incompleteness of the concepts. In this paper, we introduce a more fundamental concept of *proper space-time coordinates of events* that will bring an end to the confusions in the theory of special relativity.

Proper length of an arm of the Michelson-Morley interferometer is the length of the arm as measured in the rest frame of the apparatus. The proper round trip time of one of the light beams is the round trip time as measured in the rest frame of the experiment. Proper lengths and proper times are used to determine/predict the experimental outcome (the fringe shift).

In this paper we use the more fundamental *proper space time coordinates of events* (light emission, reflection, transmission, detection) instead of proper length and proper time in the relativistic analysis of experiments.

We define the proper space time coordinates of events as follows. We define four events that exist in all light speed experiments: emission, reflection, transmission, detection.

Emission of light:

The proper space time coordinates of emission of light is the space time coordinates of the event (emission) in the rest frame of the source.

Reflection of light:

The proper space time coordinates of reflection of light from a mirror is the space time coordinates of the event (reflection) in the reference frame of the mirror.

Transmission of light:

The proper space time coordinates of transmission of light in glass is the space time coordinates of the event (transmission) in the reference frame of the glass.

Detection of light:

The proper space time coordinates of detection of light is the space time coordinates of the event (detection) in the reference frame of the detector.

Therefore, in the analysis of any light speed experiment, we start by defining the reference frames of the source, the mirrors and the detectors. We start at $t = t' = t'' = \dots = 0$, when the origins of all the reference frames coincide and when the source emits a light pulse. Then we start by determining the proper space time coordinates of emission of light, which is the space time coordinates of the event (emission) in the reference frame of the source. Then we use Lorentz Transformations (LT) to determine the space time coordinates of this event in the reference frame of the next event, for example, reflection from a beam-splitter. Then the proper space time coordinates of this event is determined in the rest frame of the beam-splitter. Then, the coordinates of this event (reflection from beam splitter) is determined in the reference frame of the next event, for example, reflection from the transverse mirror by applying LT, and so on, until the proper space time coordinates of detection of light, which is the space time coordinates of the event (detection) in the reference frame of the detector, is determined. This procedure is completed for each light beam. The experimental outcome is determined from the coordinates of the event of light detection in the frame of the detector. Next we illustrate this concept by applying it to various light speed experiments.

The new concept is that, in a typical light speed experiment, light is emitted in one reference frame, reflected from mirrors in different frames and detected in yet another reference frame. Only this new concept / procedure enables a consistent application of STR to avoid contradictions and confusions.

The Michelson-Morley and the Kennedy- Thorndike experiments

The analysis of these experiments is straightforward since the proper space time coordinates of the events (light emission, reflection, transmission, detection) are the space time coordinates of the events in the rest frame of the apparatus. Since all the components of the experiment, the light source, the beam-splitter, the mirrors and the detector are at rest relative to each other, these experiments can also be analyzed by using the traditional concepts of proper length and proper time.

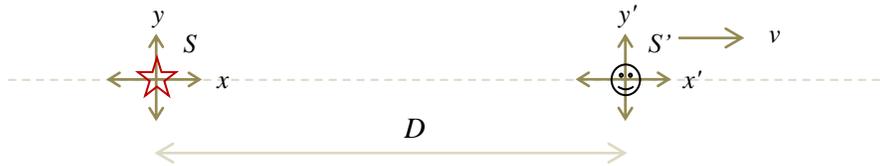
However, in my paper [1] I applied the standard or existing practice of application of STR which led to a contradiction: the fringe shifts determined in two reference frames disagreed. In this paper, I will show that the contradiction arose because of incompleteness of the special theory of relativity, and not due to fundamental contradictions in the theory.

The STR can be applied consistently in light speed experiments in which all the components (source, mirrors, beam-splitters, detector) are at rest relative to each other. However, confusion arises with light speed experiments in which the source, mirrors, detectors etc. are in relative motion such as in the GPS, the Lunar Laser Ranging (LLR) and moving observer experiments.

Moving observer thought experiment

The following is a simple thought experiment to illustrate the source of existing confusions in the theory of special relativity.

Consider an inertial reference frame S with a light source at the origin. At $t = 0$, the source emits a light pulse. At $t = 0$ an observer/detector is at $x = D$, moving with velocity v in the $+x$ direction, in reference frame S . The question is: what is the time of detection of the light by the detector, relative to S , according to STR?



The immediate answer according to current understanding would be:

$$\frac{D \left(1 + \frac{v}{c} \right)}{c}$$

which is wrong.

The correct and rigorous analysis is as follows.

Let the reference frame of the moving observer/detector be S' . At $t = 0$ the origins of S and S' coincide and the clocks are synchronized.

As we have stated already, we start by determining the proper space time coordinates of the first event: emission of light. This is the space time coordinates of the event (emission) in the reference frame of the source.

$$x = 0 \quad \text{and} \quad t = 0$$

The next step is to determine the proper space-time coordinates of the detection of light, which is the coordinates of the event (detection) in the reference frame of the detector, which is S' .

For this we first need to determine the coordinates of the emission of light in frame S' . For this, we apply Lorentz Transformations.

$$x' = \gamma (x - vt) = \gamma (0 - v * 0) = 0$$

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) = \gamma \left(0 - \frac{v * 0}{c^2} \right) = 0$$

Now that we have determined the coordinates of emission of light in frame S', we can determine the proper space time coordinates of the light detection event in S'.

$$x' = D \quad \text{and} \quad t' = 0 + \frac{D}{c} = \frac{D}{c}$$

where D is the position of the detector in the reference frame in which it is at rest, frame S', *at the instant of light emission*. Note that our initial formulation of the problem itself was strictly incorrect according to STR (the proposed correct version) because we started by assuming that D is the position of the source in S at $t = 0$.

Therefore, it turns out that the time of light detection in the reference frame of the observer is not affected by the observer's velocity, which is in contradiction with experience as we will also see in the GPS analysis.

Since we set out to determine the coordinates of the event of light detection in frame S, we use LT for this:

$$x = \gamma (x' + vt') = \gamma \left(D + v \frac{D}{c} \right) = \gamma D \left(1 + \frac{v}{c} \right)$$

$$t = \gamma \left(t' + \frac{vx'}{c^2} \right) = \gamma \left(\frac{D}{c} + \frac{vD}{c^2} \right) = \gamma \frac{D}{c} \left(1 + \frac{v}{c} \right)$$

which differs from the original prediction by a factor of gamma.

We can now also determine the position of the observer in S at $t = 0$.

$$x = \gamma (x' + vt') = \gamma (D + v * 0) = \gamma D$$

where D is the position of the detector in S' as we have already stated. However, this difference can be considered negligible in some cases, such as in our argument ahead against the accepted view that the GPS agrees with STR because the GPS debate is about a much larger first order effect, and not a second order effect.

The Twin paradox solved!

As we have already stated, the current incomplete theory of special relativity leads to endless confusions when applied to physical or thought experiments in which the component parts of the experiment (source, mirrors, beam-splitters, detectors) are in relative motion.

In the twin paradox, the problem concerns the age of the travelling twin (twin B) in relation to the age of the twin at home (twin A), during the time he/she is travelling and after he returns to Earth.

Consider two reference frames S and S' . S is the reference frame of twin A and S' is that of twin B. S' is moving with velocity near the speed of light relative to S, in the + x direction. Twins A and B are at the origins of S and S', respectively, together with their clocks. At $t = 0$, the origins of S and S' coincide and the two clocks are synchronized.

We introduce a new component to the thought experiment which is lacking in almost all formulations of the problem. The twins agree before the journey that twin B to transmit a light signal towards twin A when he/she is exactly 20 years old.

As discussed already we start by asking: what is the proper time of the age of the travelling twin? The proper time of the age of the travelling twin is as measured by the clock at rest in the frame of the travelling twin.

A light signal is emitted towards twin A when the proper age of twin B is 20 years. The proper space time coordinates of this event will be:

$$x' = 0 \quad t' = 20 \text{ years}$$

The next event is the detection of this signal in the reference frame of twin A, frame S.

For this we first need to determine the coordinates of light emission in frame S. Therefore, we use LT from S' to S.

$$x = \gamma (x' + vt') = \gamma (0 + v * 20) = 20 v\gamma$$

$$t = \gamma \left(t' + \frac{vx'}{c^2} \right) = \gamma \left(20 + \frac{v * 0}{c^2} \right) = 20 \gamma = 80 \text{ years}$$

Therefore, STR predicts that when the proper age of twin B is 20 years, twin A 'sees' the age of twin B to be 80 years.

But there is a physical meaning to this and there is no contradiction ! The apparent difference between the ages of the twins is because of one of the principles of STR itself: information cannot travel faster than the speed of light!

Twin A cannot see the age of twin B instantaneously, according to STR. The information about twin B has to travel at the speed of light to reach twin A. The apparent paradox arises when one thinks about twin A 'seeing' twin B's age without any physical means. Therefore, the age of twin B as seen by twin A should be interpreted as 'the age of B as *physically measured* by twin A.

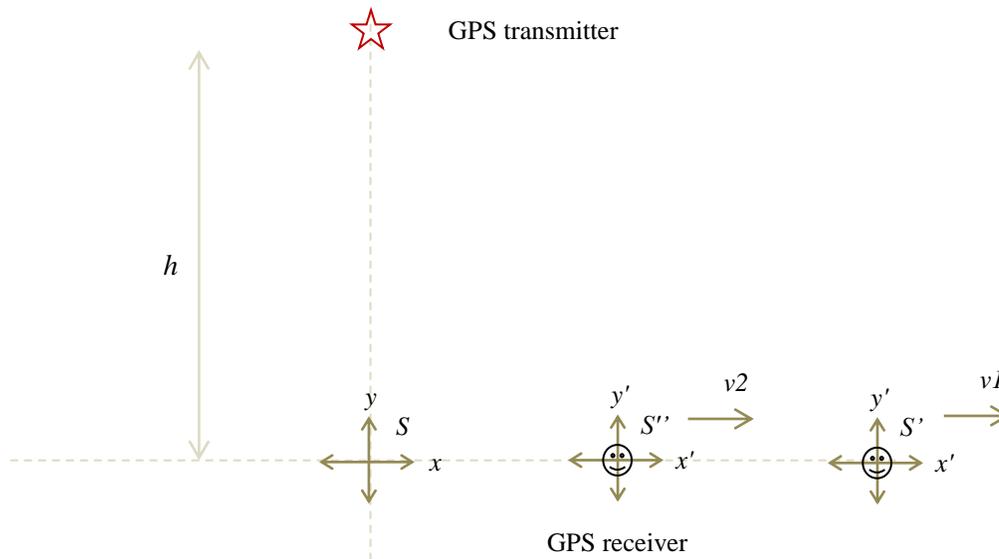
Therefore, out of the 80 years, 60 years is the time taken for information to reach twin A! There is no paradox! To think that the twin paradox is a paradox would contradict one of the fundamental principles of STR itself: information cannot travel faster than the speed of light. Therefore, the twins are always the same age during the entire time twin B is travelling and after he/she returns home.

The same applies to the age of twin A as seen by twin B.

The Global Positioning System

One of the most important consequences of the new concept of 'proper space time coordinates of events' is that it changes the accepted relation between the GPS and the STR: that the GPS supports or agrees with STR. The new analysis brings an end to this view. The GPS is a direct disproof of STR.

Consider the GPS transmitter in space and the GPS receiver on the Earth's surface in the ECI frame. S is the ECI frame, S' is the inertial frame of the GPS transmitter and S'' is the inertial frame of the GPS receiver. S' and S'' are moving with velocities $v1$ and $v2$, respectively, relative to S in the + x direction.



Suppose that at $t = 0$, the origins of S, S' and S'' coincide at the center of the Earth, and the clocks are synchronized. At $t = 0$ the source emits a short light (EM) pulse. As before, we start from the proper space time coordinates of the emission event, which is the space time coordinates of the emission event in the reference frame of the source, S'.

$$t' = 0 \quad x' = 0 \quad y' = h$$

where h is the y-coordinate of the transmitter in the reference frame of the source S' , not strictly in the ECI frame! However, this difference can be considered negligible in our GPS argument. Note that we have assumed a GPS transmitter direct overhead, for simplicity.

Next we determine the proper space time coordinates of the detection event, which is in reference frame S'' . Therefore, we first use LT between reference frames S' and S'' to determine the space time coordinates of the emission event in S'' .

The velocity of S' relative to S'' is $u = v_1 - v_2$ in the $+x''$ direction.

$$t'' = 0 \quad x'' = 0 \quad y'' = y' = h$$

where h is the position of the GPS transmitter *at the instant of transmission*.

Now that we have determined the coordinates of light emission in the reference frame of the detector (S''), we can determine the space and time coordinates of light detection in S'' .

$$t'' = \frac{h - R}{c} \quad x'' = 0 \quad y'' = 0$$

where R is the Earth's radius, assuming the Earth is a perfect sphere.

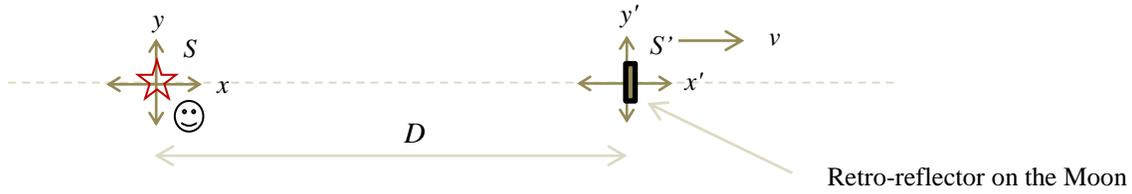
We can see that, according to the new version of STR proposed in this paper, the velocity of the receiver does not affect the time of GPS signal reception on Earth ! But we know that motion of the receiver is corrected for in GPS and physicists have been referring to this as ' the GPS Sagnac effect' and have been wrongly citing GPS as one of the evidences for STR.

Lunar Laser Ranging experiment

The new version of STR will change the perceived relation between many light speed experiments and STR. One such controversial experiment is the Lunar Laser Ranging (LLR) experiment. A paper [2] has brought to light the data from Lunar Laser Ranging experiment by NASA, claiming that the data disproves STR. Although the author's arguments are based on the current incomplete version of STR and therefore will not be effective, the data itself is invaluable.

We formulate the problem as follows. A light source on Earth's surface emits a short light pulse towards the retro-reflector on the Moon. The light reflected is detected on Earth, and the time interval τ between emission and detection computed.

Let S and S' be the inertial frames of the light source (and the detector, since both are on Earth) and the inertial frame of the retro-reflector, respectively. The source is at the origin of S . At $t = 0$, the origins of S and S' coincide at the source and the source emits a light pulse, and the clocks are synchronized. S' is moving with velocity v in frame S in the $+x$ direction.



With the same procedure we have been following, we start by determining the proper space time coordinates of the event of light emission, which is in frame S, the reference frame of the source.

At any instant of time we know the position of the light retro-reflector (of the Moon) in the reference frame S. We will use this position/distance although it is not strictly accurate at the level of second order. Since the claimed violation of STR is a much larger first order effect, we ignore this second order difference.

Therefore, the proper space time coordinates of the emission event is the coordinates of the same event in the reference frame of the source, which is S.

$$t = 0 \quad x = 0$$

The next event is the reflection of the light pulse from the retro-reflector on the Moon. The proper space time coordinates of this event is the coordinates of this event in the reference frame of the retro-reflector (or the Moon), frame S'.

For this we first need to determine the coordinates of light emission in the reference frame of the retro-reflector, by applying LT.

$$x' = \gamma (x - vt) = \gamma (0 - v * 0) = 0$$

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) = \gamma \left(0 - \frac{v * 0}{c^2} \right) = 0$$

Now that we have determined the coordinates of light emission in S', we can determine the proper space time coordinate of light reflection, which is in the frame of the retro-reflector, S'.

$$\text{time of reflection in } S' = \text{time of emission in } S' + \frac{\text{distance of emission in } S'}{c}$$

$$x' = D \quad \text{and} \quad t' = 0 + \frac{D}{c} = \frac{D}{c}$$

where D is the position of the retro-reflector (the Moon) in the Earth's frame *at the instant of light emission*.

The next step is to determine the proper coordinates of the event of light detection, which is in S . For this we need to first determine the coordinates of light reflection in S , by applying LT between S' and S .

$$x = \gamma (x' + vt') = \gamma \left(D + v \frac{D}{c} \right) = \gamma D \left(1 + \frac{v}{c} \right)$$

$$t = \gamma \left(t' + \frac{vx'}{c^2} \right) = \gamma \left(\frac{D}{c} + \frac{vD}{c^2} \right) = \gamma \frac{D}{c} \left(1 + \frac{v}{c} \right)$$

Now that we have determined the coordinates of light reflection in S , we can determine the proper time of light detection on Earth.

$$\text{time of detection in } S = \text{time of reflection in } S + \frac{\text{distance of reflection in } S}{c}$$

$$t = \gamma \frac{D}{c} \left(1 + \frac{v}{c} \right) + \frac{\gamma D \left(1 + \frac{v}{c} \right)}{c} = 2\gamma \frac{D}{c} \left(1 + \frac{v}{c} \right)$$

Remember that D is the distance of the retro-reflector from Earth *at the instance of light emission*, at $t = 0$.

But the distance of the retro-reflector *at the instance of light reflection* is:

$$D' = D \left(1 + \frac{v}{c} \right) \Rightarrow D = \frac{D'}{\left(1 + \frac{v}{c} \right)}$$

Therefore,

$$t = \frac{2\gamma}{c} D \left(1 + \frac{v}{c} \right) = \frac{2\gamma}{c} \frac{D'}{\left(1 + \frac{v}{c} \right)} \left(1 + \frac{v}{c} \right) = \gamma \frac{2D'}{c}$$

Ignoring the gamma factor,

$$t = \tau = \frac{2D'}{c}$$

which is the round trip time.

Therefore, STR predicts that the round trip time is not affected by the relative motion between the Moon and the Earth. However, the paper[2] has showed that the velocity of the Moon relative to the Earth is evident in the round trip time data. This is another direct disproof of STR.

Correct analysis of the LLR experiment

I have proposed a new theory called Apparent Source Theory (AST) of the speed of light [3] that can explain most of the usually problematic light speed experiments, including the Michelson-Morley and the Kennedy-Thorndike experiments, the Sagnac effect, Stellar aberration, the Silvertooth and the Marinov experiments, the Venus planet radar ranging experiment ‘anomaly’ and others. Here we apply it to the LLR experiment.



According to AST:

$$\tau = \frac{D}{c} + \frac{D}{c + 2v} = \frac{2D (c + v)}{c(c + 2v)}$$

where D is the distance of the retro-reflector (of the Moon) from Earth *at the time of reflection*, τ is the round trip time of the light pulse and v the relative velocity of the detector and the retro-reflector.

The average speed of light will be:

$$c' = \frac{2D}{\tau} = \frac{2D}{\frac{2D (c + v)}{c(c + 2v)}} = \frac{c(c + 2v)}{c + v} \cong c + v \quad , \quad \text{for } v \ll c$$

This is the same value of the average speed of light reported in the paper [2]. It should be noted that the classical ballistic (emission) theory also makes the same prediction.

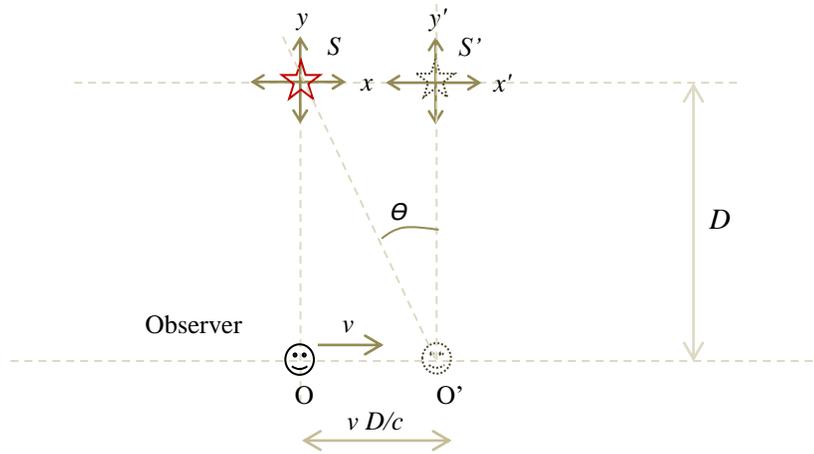
The Bryan G Wallace Venus planet radar ranging experiment anomaly

Ironically, this experiment was done to test the gravitational time dilation predicted by the general theory of relativity, by emitting RF pulses grazing the Sun towards Venus planet and analyzing the round trip time. As Bryan G Wallace reported, however, there was a large first order effect/variation in the round trip time of the radar signal, which was related to the relative velocity of the Earth and the Venus planet. The analysis is similar as in LLR above. This is yet another evidence against STR.

Stellar aberration

Next we apply the new version of STR to the phenomenon of stellar aberration.

Let S and S' be the inertial reference frames of the star and the observer on Earth, respectively. Let the origins of S and S' coincide at the star at $t = 0$ which is the instant of light emission. S' moves with velocity v relative to S in the $+x$ direction. We assume a star directly overhead.



We start by determining the proper space time coordinates of the emission event, which is in S .

$$x = 0 \quad y = 0 \quad t = 0$$

Next we determine the space time coordinates of the light detection/observation event. For this we first determine the emission event in the reference frame of the observer (S').

$$x' = \gamma (x + vt) = \gamma (0 + v * 0) = 0$$

$$y' = y = 0$$

$$t' = \gamma \left(t + \frac{vx}{c^2} \right) = \gamma \left(0 + \frac{v * 0}{c^2} \right) = 0$$

Now that we have determined the coordinates of the emission event in S' , we can determine the proper coordinates of light detection which is in S' .

$$x' = 0 \quad y' = y \quad t' = \frac{D}{c}$$

At the instant of light emission, the observer is at point O, and at the instant of light detection the observer is at point O'. In the reference frame of the moving observer, light comes from the origin of S', whereas for an observer stationary at point O', light comes from the origin of S. In the observer's frame, the position (distance and direction) of the star at the instant of observation is the same as the position of the star at the instant of emission.

The time of light detection is independent of observer's motion, which contradicts with experience as we have seen in the GPS analysis. However, STR agrees with the phenomenon of stellar aberration, which is only about the direction of light and not about the time delay.

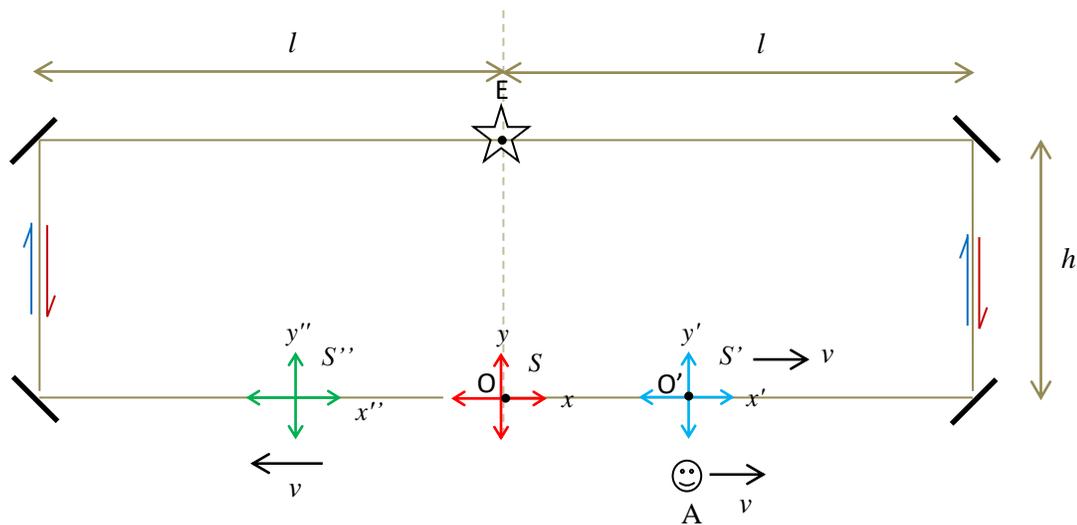
The angle of aberration is determined from:

$$\sin \theta \cong \frac{D \frac{v}{c}}{\sqrt{D^2 + (D \frac{v}{c})^2}} \cong \frac{D \frac{v}{c}}{D} = \frac{v}{c}$$

A hypothetical experiment

Now we present a re-analysis of a hypothetical experiment I proposed in my paper [1] in which I showed that the special theory of relativity as it is known leads to a contradiction. In this paper, we present a new analysis that is free of contradictions.

The mirrors are at rest in the lab frame. The detector/observer looking at the interference fringes moves with velocity v to the right and the source moves with velocity v to the left. At $t = 0$, the positions are as shown below, the origins of S and S' coincide at O and the clocks of S and S' are synchronized, and the source emits light. S is the lab frame, S' is the observer frame, S'' is the source frame.



Counterclockwise beam

Emission from source in S'':

$$y'' = h \quad t = 0$$

Emission from source in S:

$$y = h \quad t = 0$$

Emission from source in S':

$$y' = h \quad t = 0$$

Reflection from M4 in frame S:

$$t = \frac{L+h}{c} \quad x = -L$$

Reflection from M4 in frame S' :

$$x' = \gamma (x - vt) = \gamma \left(-L - v \frac{L+h}{c} \right)$$

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) = \gamma \left(\frac{L+h}{c} + \frac{vL}{c^2} \right)$$

Detection in S'

$$\text{time of detection in } S' = \text{time of emission in } S' + \frac{\text{distance of emission in } S'}{c}$$

$$\begin{aligned} t' &= \gamma \left(\frac{L+h}{c} + \frac{vL}{c^2} \right) + \frac{\gamma}{c} \left(L + \frac{v}{c} (L+h) \right) \\ &= \frac{\gamma}{c} \left(1 + \frac{v}{c} \right) (2L+h) \end{aligned}$$

Clockwise beam

Emission is the same as for the counterclockwise beam.

Reflection from M1 in S:

$$t = \frac{L+h}{c} \quad x = L$$

Reflection from M1 in S' :

$$x' = \gamma (x - vt) = \gamma \left(L - v \frac{L+h}{c} \right)$$

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) = \gamma \left(\frac{L+h}{c} - \frac{vL}{c^2} \right)$$

Detection in S'

$$\begin{aligned} t' &= \gamma \left(\frac{L+h}{c} - \frac{vL}{c^2} \right) + \frac{\gamma}{c} \left(L - \frac{v}{c} (L+h) \right) \\ &= \frac{\gamma}{c} \left(1 - \frac{v}{c} \right) (2L+h) \end{aligned}$$

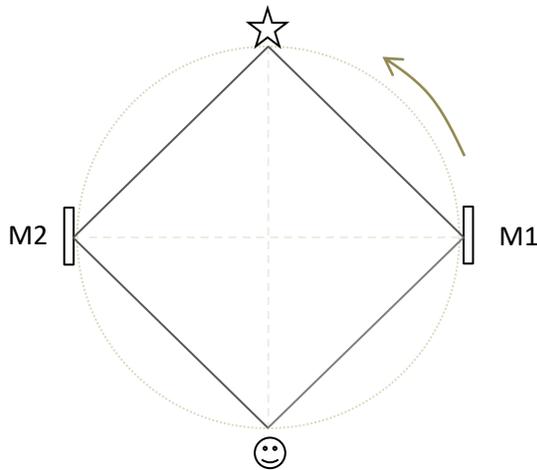
The time difference between the CCW and CW beams is:

$$\begin{aligned} t_{ccw} - t_{cw} &= \frac{\gamma}{c} \left(1 + \frac{v}{c} \right) (2L+h) - \frac{\gamma}{c} \left(1 - \frac{v}{c} \right) (2L+h) \\ &= \frac{\gamma}{c} (2L+h) \frac{2v}{c} \end{aligned}$$

The Sagnac effect

The Sagnac effect has been perhaps the most challenging experiment to analyze using STR. It has been an experiment that has been often cited against STR.

However, ironically, it turns out that the Sagnac effect agrees with STR. However, this does not prove STR because this agreement is not unique as the Sagnac effect also agrees with ether theory and absolute motion theory.



S is the inertial frame of the source, moving with velocity $v = \omega R$ in the lab frame to the left

S' is the inertial frame of the mirror M1, moving with velocity $v = \omega R$ in the lab frame downwards

S'' is the inertial frame of the mirror M2, moving with velocity $v = \omega R$ in the lab frame upwards

S''' is the inertial frame of the detector, moving with velocity $v = \omega R$ in the lab frame right

At $t = 0$, the source emits a short light pulse and the origins of S, S' and S'' coincide. The source is at the origin of S.

The clockwise light beam

Emission in S

The proper coordinates of emission are the coordinates of the event in S

$$t = 0 \quad x = 0$$

Reflection in M1

For this we first determine the coordinates of the emission event in the frame of M1, which is S'.

$$x' = \gamma (x - vt) \quad , \quad t' = \gamma \left(t - \frac{vx}{c^2} \right)$$
$$\Rightarrow x' = \gamma (0 - v * 0 = 0) \quad , \quad t' = \gamma \left(0 - \frac{v * 0}{c^2} \right) = 0$$

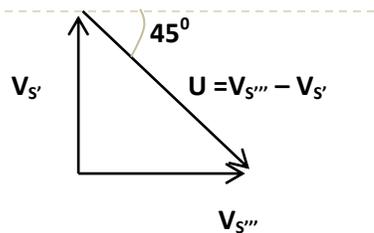
Now we can determine the proper coordinates of the reflection event in S':

$$x' = R \quad , \quad y' = -R \quad , \quad t' = \frac{R\sqrt{2}}{c}$$

Detection in S'''

The proper space time coordinates of the detection event are the coordinates of the event in the reference frame of the detector.

But first we have to determine the coordinates of the reflection event in S''', by applying LT from S' to S'''. For this, first we need to determine the velocity of S''' relative to S'.



Since $V_{S'} = V_{S''} = v = \omega R$, the velocity of S''' relative to S' (u) will be:

$$u = v\sqrt{2}$$

with its direction shown in the diagram above.

We use the generalized Lorentz transformation equation:

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = B(\boldsymbol{\nu}) \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

where

$$\mathbf{B}(\boldsymbol{\nu}) = \begin{bmatrix} \gamma & -\gamma\beta_x & -\gamma\beta_y & -\gamma\beta_z \\ -\gamma\beta_x & 1 + (\gamma - 1)\frac{\beta_x^2}{\beta^2} & (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2} & (\gamma - 1)\frac{\beta_x\beta_z}{\beta^2} \\ -\gamma\beta_y & (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2} & 1 + (\gamma - 1)\frac{\beta_y^2}{\beta^2} & (\gamma - 1)\frac{\beta_y\beta_z}{\beta^2} \\ -\gamma\beta_z & (\gamma - 1)\frac{\beta_x\beta_z}{\beta^2} & (\gamma - 1)\frac{\beta_y\beta_z}{\beta^2} & 1 + (\gamma - 1)\frac{\beta_z^2}{\beta^2} \end{bmatrix}$$

Since there is no relative motion in the z - direction, the Lorentz transformation matrix will be:

$$\begin{bmatrix} \gamma & -\gamma\beta_x & -\gamma\beta_y & 0 \\ -\gamma\beta_x & 1 + (\gamma - 1)\frac{\beta_x^2}{\beta^2} & (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2} & 0 \\ -\gamma\beta_y & (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2} & 1 + (\gamma - 1)\frac{\beta_y^2}{\beta^2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

From the generalized Lorentz transformation matrix,

$$(ct') = \gamma (ct) - \gamma \beta_x (x) - \gamma \beta_y (y)$$

$$(x') = -\gamma\beta_x (ct) + \left(1 + (\gamma - 1)\frac{\beta_x^2}{\beta^2}\right) (x) + (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2} (y)$$

$$(y') = (-\gamma\beta_y)(ct) + (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2}(x) + \left(1 + (\gamma - 1)\frac{\beta_y^2}{\beta^2}\right)(y)$$

We first need to determine β_x , β_y and γ .

$$\beta_x = \frac{v_x}{c} = \frac{u \cos 45^\circ}{c} = \frac{(v\sqrt{2})\left(\frac{\sqrt{2}}{2}\right)}{c} = \frac{v}{c}$$

$$\beta_y = \frac{v_y}{c} = -\frac{u \sin 45^\circ}{c} = -\frac{(v\sqrt{2})\left(\frac{\sqrt{2}}{2}\right)}{c} = -\frac{v}{c}$$

$$\beta = \frac{u}{c} = \frac{v\sqrt{2}}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where

$$v = \omega R$$

Therefore,

$$\frac{\beta_x^2}{\beta^2} = \frac{\frac{v^2}{c^2}}{\frac{2v^2}{c^2}} = \frac{1}{2} \quad \text{and} \quad \frac{\beta_x\beta_y}{\beta^2} = \frac{-\frac{v^2}{c^2}}{\frac{2v^2}{c^2}} = -\frac{1}{2} \quad \text{and} \quad \frac{\beta_y^2}{\beta^2} = \frac{\frac{v^2}{c^2}}{\frac{2v^2}{c^2}} = \frac{1}{2}$$

Therefore, applying the generalized LT from S' to S''',

$$(ct''') = \gamma(ct') - \gamma\beta_x(x') - \gamma\beta_y(y')$$

$$\Rightarrow (ct''') = \gamma\left(c\frac{R\sqrt{2}}{c}\right) - \gamma\frac{v}{c}(R) - \gamma\left(-\frac{v}{c}\right)(-R)$$

$$\Rightarrow t''' = \frac{\gamma R}{c}\left(\sqrt{2} - \frac{2v}{c}\right) \cong \frac{R}{c}\left(\sqrt{2} - \frac{2v}{c}\right)$$

$$(x''') = -\gamma\beta_x(ct') + \left(1 + (\gamma - 1)\frac{\beta_x^2}{\beta^2}\right)(x') + (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2}(y')$$

$$(x''') = -\gamma\frac{v}{c}\left(c\frac{R\sqrt{2}}{c}\right) + \left(1 + (\gamma - 1)\left(\frac{1}{2}\right)\right)(R) + (\gamma - 1)\left(\frac{-1}{2}\right)(-R)$$

$$\Rightarrow x''' = \gamma R\left(-\frac{v\sqrt{2}}{c} + 1\right) \cong R\left(-\frac{v\sqrt{2}}{c} + 1\right)$$

$$(y''') = (-\gamma\beta_y)(ct') + (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2}(x') + \left(1 + (\gamma - 1)\frac{\beta_y^2}{\beta^2}\right)(y')$$

$$(y''') = -\gamma\left(-\frac{v}{c}\right)\left(c\frac{R\sqrt{2}}{c}\right) + (\gamma - 1)\left(\frac{-1}{2}\right)(R) + \left(1 + (\gamma - 1)\frac{1}{2}\right)(-R)$$

$$\Rightarrow y''' = \gamma R\left(\frac{v\sqrt{2}}{c} - 1\right) \cong R\left(\frac{v\sqrt{2}}{c} - 1\right)$$

Detection of CW beam in S''' frame

$$t''' = \frac{R}{c}\left(\sqrt{2} - \frac{2v}{c}\right) + \frac{1}{c}\sqrt{R^2\left(-\frac{v\sqrt{2}}{c} + 1\right)^2 + \left(R\left(\frac{v\sqrt{2}}{c} - 1\right) + 2R\right)^2}$$

$$t''' \cong \frac{2R}{c}\left(\sqrt{2} - \frac{v}{c}\right), \text{ ignoring second order term}$$

The counterclockwise light beam

Emission in S

The proper coordinates of emission are the coordinates of the event in S

$$t = 0 \quad x = 0$$

Reflection in M2

For this we first determine the coordinates of the emission event in the frame of M2, which is S''.

$$x'' = \gamma(x - vt) \quad , \quad t'' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$\Rightarrow x'' = \gamma(0 - v \cdot 0 = 0) \quad , \quad t'' = \gamma\left(0 - \frac{v \cdot 0}{c^2}\right) = 0$$

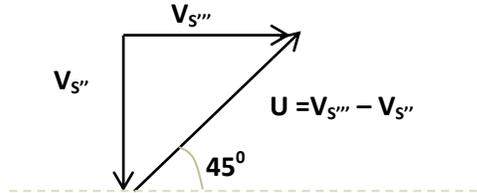
Now we can determine the proper coordinates of the reflection event in S'':

$$x'' = -R \quad , \quad y'' = -R \quad , \quad t'' = \frac{R\sqrt{2}}{c}$$

Detection in S'''

The proper space time coordinates of the detection event are the coordinates of the event in the reference frame of the detector.

But first we have to determine the coordinates of the reflection event in S''', by applying LT from S'' to S'''. For this, first we need to determine the velocity of S''' relative to S''.



Since $V_{S''} = V_{S'''} = v = \omega R$, the velocity of S''' relative to S' (\mathbf{U}) will be:

$$u = v\sqrt{2}$$

with its direction shown in the diagram above.

From the generalized Lorentz transformation matrix,

$$(ct') = \gamma (ct) - \gamma \beta_x (x) - \gamma \beta_y (y)$$

$$(x') = -\gamma \beta_x (ct) + (1 + (\gamma - 1) \frac{\beta_x^2}{\beta^2}) (x) + (\gamma - 1) \frac{\beta_x \beta_y}{\beta^2} (y)$$

$$(y') = (-\gamma \beta_y)(ct) + (\gamma - 1) \frac{\beta_x \beta_y}{\beta^2} (x) + \left(1 + (\gamma - 1) \frac{\beta_y^2}{\beta^2}\right) (y)$$

We first need to determine β_x , β_y and γ .

$$\beta_x = \frac{v_x}{c} = \frac{u \cos 45^\circ}{c} = \frac{(v\sqrt{2}) \left(\frac{\sqrt{2}}{2}\right)}{c} = \frac{v}{c}$$

$$\beta_y = \frac{v_y}{c} = \frac{u \sin 45^\circ}{c} = \frac{(v\sqrt{2}) \left(\frac{\sqrt{2}}{2}\right)}{c} = \frac{v}{c}$$

$$\beta = \frac{u}{c} = \frac{v\sqrt{2}}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where

$$v = \omega R$$

Therefore,

$$\frac{\beta_x^2}{\beta^2} = \frac{\frac{v^2}{c^2}}{\frac{2v^2}{c^2}} = \frac{1}{2} \quad \text{and} \quad \frac{\beta_x\beta_y}{\beta^2} = \frac{-\frac{v^2}{c^2}}{\frac{2v^2}{c^2}} = -\frac{1}{2} \quad \text{and} \quad \frac{\beta_y^2}{\beta^2} = \frac{\frac{v^2}{c^2}}{\frac{2v^2}{c^2}} = \frac{1}{2}$$

Therefore, applying the generalized LT from S'' to S''',

$$(ct''') = \gamma(ct'') - \gamma\beta_x(x'') - \gamma\beta_y(y'')$$

$$\Rightarrow (ct''') = \gamma\left(c\frac{R\sqrt{2}}{c}\right) - \gamma\frac{v}{c}(-R) - \gamma\left(-\frac{v}{c}\right)(-R)$$

$$\Rightarrow t''' = \frac{\gamma R}{c}\left(\sqrt{2} + \frac{2v}{c}\right) \cong \frac{R}{c}\left(\sqrt{2} + \frac{2v}{c}\right)$$

$$(x''') = -\gamma\beta_x(ct'') + \left(1 + (\gamma - 1)\frac{\beta_x^2}{\beta^2}\right)(x'') + (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2}(y'')$$

$$(x''') = -\gamma\frac{v}{c}\left(c\frac{R\sqrt{2}}{c}\right) + \left(1 + (\gamma - 1)\frac{1}{2}\right)\left(\frac{1}{2}\right)(-R) + (\gamma - 1)\frac{1}{2}(-R)$$

$$\Rightarrow x''' = -\gamma R\left(\frac{v\sqrt{2}}{c} + 1\right) \cong -R\left(\frac{v\sqrt{2}}{c} + 1\right)$$

$$(y''') = (-\gamma\beta_y)(ct'') + (\gamma - 1)\frac{\beta_x\beta_y}{\beta^2}(x'') + \left(1 + (\gamma - 1)\frac{\beta_y^2}{\beta^2}\right)(y'')$$

$$\begin{aligned}
(y''') &= -\gamma \left(\frac{v}{c}\right) \left(c \frac{R\sqrt{2}}{c}\right) + (\gamma - 1) \left(\frac{1}{2}\right) (-R) + \left(1 + (\gamma - 1) \frac{1}{2}\right) (-R) \\
\Rightarrow y''' &= -\gamma R \left(\frac{v\sqrt{2}}{c} + 1\right) \cong -R \left(\frac{v\sqrt{2}}{c} + 1\right)
\end{aligned}$$

Detection of CCW beam in S''' frame

$$\begin{aligned}
t''' &= \frac{R}{c} \left(\sqrt{2} + \frac{2v}{c}\right) + \frac{1}{c} \sqrt{R^2 \left(\frac{v\sqrt{2}}{c} + 1\right)^2 + \left(-R \left(\frac{v\sqrt{2}}{c} + 1\right) + 2R\right)^2} \\
t''' &= \frac{2R}{c} \left(\sqrt{2} + \frac{v}{c}\right)
\end{aligned}$$

Time difference of the CCW and CW beams:

$$\Delta\tau = \frac{2R}{c} \left(\sqrt{2} + \frac{v}{c}\right) - \frac{2R}{c} \left(\sqrt{2} - \frac{v}{c}\right) = \frac{4vR}{c^2} = \frac{4\omega R^2}{c^2}$$

The area of the square formed by the light beams is:

$$\begin{aligned}
A &= (R\sqrt{2})^2 = 2R^2 \\
\Delta\tau &= \frac{4\omega R^2}{c^2} = \frac{2A\omega}{c^2}
\end{aligned}$$

This is half the value of the well-known Sagnac formula. This is expected because the beams of our modified Sagnac device travel half the distance of the normal Sagnac device. Therefore, the Sagnac effect agrees with STR.

Conclusion

The special theory relativity has been perhaps one of the most controversial theories in the history of science. This paper has revealed for the first time the root cause of the problem: the incompleteness of the theory or its interpretation ever since it was introduced in 1905. This in turn resulted in sloppy applications of the theory both by its opponents and its proponents, leading to endless confusions and arguments. As I showed in my previous paper [1], if physicists had tried to apply the theory with as much rigor as possible, they would have long discovered a fundamental contradiction that could have put the theory under scrutiny. In this paper, we have seen a new concept of 'proper space time coordinates of events' that will make STR a complete theory with no contradictions. The new concept first makes STR a complete theory and then ultimately disproves it because the new 'version' of STR no longer agrees with experiments that are traditionally considered as supporting it, such as the GPS and Lunar Laser Ranging (LLR) experiments.

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References

1. A Contradiction in the Special theory of Relativity, by Henok Tadesse

https://www.researchgate.net/publication/361464699_A_Contradiction_in_Special_Relativity_Theory

2. Lunar Laser Ranging Test of the Invariance of c , by Daniel Y. Gezar

<https://arxiv.org/vc/arxiv/papers/0912/0912.3934v1.pdf>

3. Apparent Constancy of the Speed of Light and Apparent Change of Position and Time of Light Emission Relative to an Inertial Observer in Absolute Motion, by Henok Tadesse

<https://vixra.org/abs/2201.0131>