Wrong Prediction of Special Relativity on GPS Sagnac Correction

Henok Tadesse
Email: *entkidmt@yahoo.com*

03 March 2021

Abstract

The application of special relativity theory, Lorentz transformation equations, length contraction and time dilation concepts to such problems as the Sagnac effect, the GPS Sagnac correction and moving observer experiments has been a source of endless debates that have never been settled to this date. There are still confusions regarding, for example, the application of special relativity theory to non-relativistic problems. It is usually claimed that special relativity gives the same answer as classical physics for non-relativistic problems. In a previous paper, " A Simple Challenge to Relativity ", I proposed an apparently simple problem that has not been treated in many cases so far strictly according to Lorentz transformation equations. In this paper, a new analysis of moving observer experiments by applying Lorentz transformation equations is proposed. It is found that special relativity (wrongly) predicts half of the value used for GPS Sagnac correction.

Introduction

The application of special relativity theory, Lorentz transformation equations, length contraction and time dilation concepts to such problems as the Sagnac effect, the GPS Sagnac correction and moving observer experiments has been a source of endless debates that have never been settled to this date. For example, there are still confusions regarding the application of special relativity theory to non-relativistic problems. It is usually claimed that special relativity gives the same answer as classical physics for non-relativistic problems. In a previous paper [1], I proposed an apparently simple problem that has not been treated in many cases so far strictly according to Lorentz transformation equations. In this paper, Lorentz transformation equations are applied to the problem of moving observer experiments.

A moving observer experiment

Consider a simple problem of a light source and an observer moving directly away from the light source. The light source emits a short light pulse at the time instant that the distance between the source and the observer is D. The problem is to find the time instant of light detection by the observer. We analyze the experiment according to Lorentz transformations as follows.

We have two inertial reference frames, S and S', with origins O and O', respectively. The coordinates of S are (x,t) and the coordinates of S' are (x',t'). At $t = t' = 0$, the origins of S and S' coincide and the clocks in the two frames are synchronized. A light source is at the origin of S and an observer is at the origin of S'. Reference frame S' moves with velocity v relative to S in the $+x$ direction.

Electrical engineer, BSc, Debrezeit, Ethiopia Mobile: +251 910 751339
or +251 912 228639 Alternate email: *wchmar@gmail.com*

The light source emits a short light pulse just at the time instant the distance between the source and the observer is equal to *D*, relative to frame S.

We have two events:

Event 1 (emission of light) has coordinates (x_1, t_1) and (x_1', t_1') in S and S', respectively.
Event 2 (detection of light) has coordinates (x_2, t_2) and (x_2', t_2') in S and S', respectively.

Next we determine the coordinates of the two events relative to S and then relative to S', use Lorentz transformations.

<u>Event 1 (emission of light)</u>

The coordinates of Event 1 in frame S will be:

$$x_1 \; = \; 0 \quad , \quad t_1 \; = \; \frac{D}{v}$$

The coordinates of Event 1 in frame S' is determined by using Lorentz transformations:

$$t' \; = \; \gamma \; (\; t \; - \; \frac{vx}{c^2} \;)$$

$$x' \; = \; \gamma \; (\; x \; - \; vt \;)$$

where

$$\gamma \; = \; \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Therefore,

$$x_1{}' \; = \; \gamma \; (\; x_1 - \; vt_1 \;) = \gamma \; \left(\; 0 - \; v \, \frac{D}{v} \; \right) \; = \; - \gamma D$$

and

$$t_1{}' \; = \; \gamma \; \left(\; t_1 \; - \; \frac{v \, x_1}{c^2} \; \right) = \gamma \; \left(\frac{D}{v} \; - \; \frac{v * 0}{c^2} \right) = \gamma \; \frac{D}{v}$$

Event 2 (detection of light)

It can be shown that the coordinates of Event 2 in frame S are:

$$x_2 \ = \ D\,\frac{c}{c-V} \quad , \quad t_2 \ = \ t_1 \ + \ \frac{D}{c\ - v} \ = \ \frac{D}{v} \ + \ \frac{D}{c\ - v} \ = \ D\,\frac{c}{v(c\ - v)}$$

Therefore, the coordinates of Event 2 in frame S' will be:

$$x_2{}' \ = \ \gamma\ (\ x_2 - \ v\,t_2\ ) = \gamma\ \left(\ D\,\frac{c}{c-V} - \ v\ D\,\frac{c}{v(c\ - v)}\ \right) \ = \ 0$$

and

$$t_2{}' \ = \ \gamma\left(t_2 \ - \ \frac{v\,x_2}{c^2}\ \right) = \gamma\ \left(\ D\,\frac{c}{v(c\ - v)} \ - \ \frac{v\,D\,\frac{c}{c-V}}{c^2}\ \right) = \gamma\,\frac{Dc}{c-v}\ \left(\frac{1}{v} \ - \ \frac{v}{c^2}\ \right) = \gamma D\,\frac{c+v}{cv}$$

Now let us compare the prediction of relativity with classical prediction and with experimental facts.

We have the time instant of emission (t_1) in frame S and the time instant of detection ($t_2{}'$) in frame S'. Although these time instants are registered in different frames, we can use the difference between the two as a test of special relativity theory. After all, these are the time instants that can practically be measured and used in real experiments, such as the GPS and moving observer experiments.

In principle, the time instants ($t_1{}'$) and (t_2) can also be measured but this is not practical. For this, we would need infinite (or large) number of detectors and associated synchronized clocks at every point in each frame. For example, the space and time coordinate of Event 1 (emission) in S' can be obtained by using the recorded data of large number of detectors in frame S'. The point of light emission in S' can be found experimentally based on the principle that it is the same as the point of that detector that has registered the event first (that has detected the light first), earlier than all detectors in S'. The time of the emission event in S' is also the time recorded by the associated clock of that detector.

Therefore,

$$t_2{}' - \ t_1 \ = \ \gamma D\,\frac{c+v}{cv} \ - \ \frac{D}{v} \ = \ \frac{D}{v}\left(\ \sqrt{\frac{c+v}{c-v}} \ - \ 1\ \right) = \frac{D}{v}\left(\ \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \ - \ 1\ \right)$$

Using Taylor series expansion:

$$\sqrt{\frac{1+x}{1-x}} \ = 1 + x \ + \ \frac{x^2}{2} + \ \frac{x^3}{2} \ + \ \frac{3x^4}{8} \ + \ \frac{3x^5}{8} \ + \ \frac{5x^6}{16} \ + \ \frac{5x^7}{16} \ + \ . \ . \ .$$

Therefore:

$$\sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} = 1 + \frac{v}{c} + \frac{v^2}{2c^2} + \frac{v^3}{2c^3} + \frac{3v^4}{8c^4} + \ldots$$

From which:

$$t_2{}' - t_1 = \frac{D}{v}\left(\sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} - 1 \right) = \frac{D}{v} \left(\frac{v}{c} + \frac{v^2}{2c^2} + \frac{v^3}{2c^3} + \frac{3v^4}{8c^4} + \ldots \right)$$

$$= \left(\frac{D}{c} + \frac{D\,v}{2c^2} + \frac{Dv^2}{2c^3} + \ldots \right)$$

$$= \left(\frac{D}{c} + \frac{D}{c}\frac{v}{2c} + \frac{D}{c}\frac{v^2}{2c^2} + \ldots \right)$$

$$= \frac{D}{c}\left(1 + \frac{v}{2c} + \frac{v^2}{2c^2} + \ldots \right)$$

$$\approx \frac{D}{c}\left(1 + \frac{v}{2c} \right) \ , \ for \ \ v \ll c$$

If we calculate the speed of light based on $t_2{}' - t_1$:

$$c' = \frac{D}{t_2{}' - t_1} \approx \frac{D}{\frac{D}{c}\left(1 + \frac{v}{2c} \right)} = \frac{c}{\left(1 + \frac{v}{2c} \right)}$$

Using the expansion:

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \ldots$$

$$c' = \frac{D}{t_2{}' - t_1} = \frac{c}{\left(1 + \frac{v}{2c} \right)} = c \left(1 - \frac{v}{2c} + \frac{v^2}{4c^2} - \frac{v^3}{8c^3} + \ldots \right)$$

$$c' \approx c \left(1 - \frac{v}{2c} \right) = \left(c - \frac{v}{2} \right) \quad , for \ v \ll c$$

This is the prediction of special relativity that can be tested experimentally.

But classical physics predicts the time difference between the two events as:

$$t_2 - t_1 = D \frac{c}{v(c - v)} - \frac{D}{v} = \frac{D}{c - v}$$

And the speed of light in the observer's reference frame will be:

$$c' = c - v$$

Experiments, such as the GPS Sagnac correction, confirm the classical prediction.

Conclusion

By consistent application of Lorentz transformation equations, we have been able to clear the confusion that special relativity gives the same answer as classical physics for $v << c$. The current, intuitive understanding is that special relativity gives $c \pm v$, which has been shown to be wrong. The correct value has been shown to be: $c \pm v/2$. However, this is disproved by the GPS Sagnac correction which uses $c \pm v$. This paper demonstrates that rigorous, consistent application of Lorentz transformation equations is necessary to conclusively defend or refute the claims of special relativity. This same approach should be used for other debated light speed problems also, instead of just intuitive assertions.

Thanks to Almighty God Jesus Christ and His Mother Our Lady Saint Virgin Mary

References

1. A Simple Challenge to Relativity, by Henok Tadesse, www.viXra.org