Yukawa Potential in Klein-Gordon Equation in Cosmological Inertial Frame

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ABSTRACT

We study Yukawa potential dependent about time in cosmological inertial frame. If we solve Klein-Gordon equation, we obtain Yukawa potential dependent about time in cosmological inertial frame.

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Key words: Yukawa potential;

Klein-Gordon equation;

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1.Introduction

Our article's aim is that we make Yukawa potential theory in cosmological inertial frame.

At first, Robertson-Walker metric is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{1}$$

According to ΛCDM model, our universe's k is zero. In this time, if t_0 is cosmological time[3],

$$k = 0, t = t_0 >> \Delta t\ ,\ \ \Delta t \text{ is period of matter's motion} \tag{2}$$

Hence, the proper time is in cosmological time,

$$\begin{aligned} d\tau^2 &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dr^2 + r^2 d\Omega^2] \\ &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2] \\ &= dt^2\left(1 - \frac{1}{c^2}\Omega^2(t_0)V^2\right),\quad V^2 = \frac{dx^2 + dy^2 + dz^2}{dt^2} \end{aligned} \tag{3}$$

In this time,

$$d\overline{t} = dt, d\overline{x} = \Omega(t_0)dx, d\overline{y} = \Omega(t_0)dy, d\overline{z} = \Omega(t_0)dz \tag{4}$$

Cosmological special theory of relativity's coordinate transformations are

$$c\overline{t} = ct = \gamma\left(c\overline{t}' + \frac{v_0}{c}\Omega(t_0)\overline{x}'\right) = \gamma\left(ct + \frac{v_0}{c}\Omega(t_0)x'\Omega(t_0)\right)$$

$$\overline{x} = x\Omega(t_0) = \gamma(\overline{x}' + v_0\Omega(t_0)\overline{t}') = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t')$$

$$\begin{aligned} \overline{y} &= \Omega(t_0)y = \overline{y}' = \Omega(t_0)y', \\ \overline{z} &= \Omega(t_0)z = \overline{z}' = \Omega(t_0)z' \end{aligned}\quad ,\ \gamma = 1/\sqrt{1 - \frac{v_0^{\ 2}}{c^2}\Omega^2(t_0)} \tag{5}$$

2. Yukawa potential in Klein-Gordon equation in cosmological inertial frame

If we focus Klein-Gordon equation about Yukawa potential ϕ dependent about time,

$$\frac{m_\pi^{\ 2}c^2}{\hbar^2}\phi + \partial_\mu\partial^\mu\phi = \frac{m_\pi^{\ 2}c^2}{\hbar^{\ 2}}\phi + \frac{1}{c^{\ 2}}\frac{\partial^2}{\partial t^2}\phi - \nabla^2\phi = 0 \tag{6}$$

In this time, Yukawa potential ϕ dependent about time is.

$$\phi = -\frac{g^2}{r}\exp\left(-\frac{m_\pi rc}{\hbar}\right) + A_0 \sin\omega t$$

$$\text{Frequency}\ \ \omega = \frac{m_\pi c^2}{\hbar}\ ,\ \ m_\pi \text{ is meson's mass} \tag{7}$$

Eq(6)-Klein-Gordon equation is satisfied by Eq(7)-Yukawa potential dependent about time

In cosmological inertial frame, Klein-Gordon equation is[2]

$$-\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\phi'}{\partial t^2}+\frac{1}{\Omega(t_0)}\nabla^2\phi'=\frac{m_\pi{}^2c^2}{\hbar^2}\phi' \tag{8}$$

In this point, in cosmological inertial frame, space-time transformations in the type A of wave function and the other type B of the expanded distance are

$$\text{Type A: } r\to r\sqrt{\Omega(t_0)},\, t\to\frac{t}{\sqrt{\Omega(t_0)}},\ \text{Type B: } r\to r\Omega(t_0),\, t\to t \tag{9}$$

Space-time transformation of Yukawa potential ϕ' is depend on Type A

Hence, Yukawa potential ϕ' dependent about time is

$$\phi'=-\frac{g^2}{r\sqrt{\Omega(t_0)}}\exp[-\frac{m_\pi r\sqrt{\Omega(t_0)}c}{\hbar}]+A_0\sin(\frac{\omega t}{\sqrt{\Omega(t_0)}})$$

$$\text{Frequency } \omega=\frac{m_\pi c^2}{\hbar},\ m_\pi \text{ is meson's mass} \tag{10}$$

Eq(8)-Klein-Gordon equation is satisfied by Eq(10)-the solution.

3. Conclusion

We solve Klein-Gordon equation in cosmological inertial frame. Hence, we found Yukawa potential dependent time in cosmological inertial frame.

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