Modeling that matches, augments, and unites data about physics properties, elementary particles, cosmology, and astrophysics

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Abstract

This essay shows modeling that - across four facets of physics - matches and predicts data. The facets are elementary particles, properties of elementary particles and other objects, cosmology, and astrophysics. Regarding elementary particles, our modeling matches all known particles and suggests new particles. New particles include zero-charge quark-like particles, a graviton, an inflaton, and other elementary particles. Some models split gravitational fields in ways similar to the splitting of electromagnetic fields into electric fields and magnetic fields. Regarding properties, our modeling suggests a new property - isomer. An isomer is a near copy of a set of most elementary particles. Our modeling includes a parameter that catalogs charge, mass, spin, and other properties. Regarding cosmology and astrophysics, the elementary particles and the new property seem to explain dark matter. Most dark matter has bases in five new isomers of the Standard Model elementary particles. More than eighty percent of dark matter is cold dark matter. Some dark matter has similarities to ordinary matter. Regarding cosmology, our modeling points to a basis for the size of recent increases in the rate of expansion of the universe. Our modeling suggests five eras in the evolution of the universe. Two eras would precede inflation. Regarding astrophysics, our modeling explains ratios of dark matter to ordinary matter. One ratio pertains to densities of the universe. Some ratios pertain to galaxy clusters. Some ratios pertain to galaxies. One ratio pertains to depletion of cosmic microwave background radiation. The modeling seems to offer insight about galaxy formation. That our work seems to explain cosmology data and astrophysics data might confirm some of our work regarding properties and elementary particles. Our work augments and does not disturb centuries of useful physics. Our modeling has roots in discrete mathematics. Our modeling unites itself and widely-used physics modeling.

Keywords: Beyond the Standard Model, Dark matter, Galaxy evolution, Rate of expansion of the universe, Inflation, Quantum gravity

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1. Introduction

1.1. Overview

This essay suggests advances regarding two sets of physics challenges. One set features describing elementary particles that people have yet to find and describing dark matter. One set features explaining cosmology and astrophysics data that people have yet to explain and predicting cosmology and astrophysics data that people have yet to obtain.

Our explanations regarding cosmology and astrophysics data offer the possibility that our descriptions of new particles and dark matter have merit.

1.2. Context

Three opportunities provide context for our work. Each opportunity opened up at least 80 years ago. One opportunity associates with elementary particles. One associates with astrophysics. One associates with cosmology.

The next three paragraphs describe the opportunities.

Describe all elementary particles. This opportunity stems from observations - before 1900 - regarding the electron.

Describe dark matter. Or, find an explanation - that does not involve dark matter - for observations that might imply the existence of gravity that would associate with objects that people cannot see. This astrophysics opportunity stems from observations - before and during the 1930s - regarding a galaxy and regarding a galaxy cluster.

Explain phenomena related to the moving apart from each other of distant objects. This cosmology opportunity stems from observations - before 1930 - regarding distant galaxies.

This essay describes work that addresses the three opportunities.

We offer united modeling that seems to capture those opportunities.

Compared to other attempts to address the opportunities, the following notions seem to pertain. Our work seems to feature more reuse of extant concepts and modeling. Our modeling seems to feature simpler mathematics. Our work seems to explain otherwise unexplained data.

1.3. Inspirations

When we started this work, we were aware of the notion of three eras regarding the so-called expansion of the universe. An early brief era would feature rapid expansion. A multi-billion-year era features continued expansion, but with decreasing rate of expansion. A recent multi-billion-year era features continued expansion, with increasing rate of expansion. We decided to explore a notion that people could model gravity based on so-called components. Paralleling electrostatics, gravity might have at least a monopole component and a dipole component. The monopole component of gravity might somewhat parallel the notion of an electrostatic interaction with charge. A dipole component of gravity might somewhat echo the notion of a magnetostatic interaction with magnetic dipole moment. We think that the gravitational dipole moment associates with - regarding modeling based on general relativity - rotational frame dragging. We also found that, at least, quadrupole and octupole interactions might pertain regarding gravity. Octupole repulsion governed the brief era of rapid expansion. Quadrupole attraction governed the era of decreasing rate of expansion. Dipole repulsion governs the recent era of increasing rate of expansion.

When we started this work, we were aware of three densities of the universe. The ratio of dark matter density to ordinary matter density is somewhat more than five. The ratio of dark energy density to the sum of dark matter density and ordinary matter density is between two and three. We decided to explore a notion that the universe might feature near copies of a set of most elementary particles. Ordinary matter and some dark matter would associate with one copy. Most dark matter would associate with five near copies. Dark energy might associate with some number - an integer multiple of six - of near copies. Eventually, we adopted the word isomer to associate with the notion of near copy.

While we were pursuing this work, we noted possible numerical relationships between physics constants.

One numerical relationship seemed to link the ratio of the mass of the tauon to the mass of the electron, m_τ/m_e, with the ratio of electrostatic repulsion to gravitational attraction between two electrons, $((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2)$. The relationship is $(m_\tau/m_e)^{12} = (3/4)\times((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2)$. While we were doing our work, people refined experimental results regarding the gravitational constant. The value - that our relationship would predict - for the tauon mass stayed within one standard deviation of experimental results. The error - that our relationship would predict - for the tauon mass decreased. Eight calculated standard deviations fit within one experimental standard deviation.

One numerical relationship pertains to the masses of the weak interaction bosons. When people were starting to pinpoint the mass of the Higgs boson, we estimated a Higgs mass by extrapolating from the relationship for the weak interaction bosons. Over time, the experimental mass for the Higgs boson hovered near our extrapolation, which is $(17/9)^{1/2}$ times the mass of the Z boson. Our extrapolation associated with differences of less than two measured standard deviations from the nominal experimental results.

Before we started intensively into this work, we were aware of a possible opportunity that associates with a contrast between two integers. Some physics uses two harmonic oscillators to model excitations of photons. Modeling regarding each oscillator can associate with one spatial dimension. The notation $|0>$ symbolizes the ground state for each oscillator. Some physics uses the notions that the number of spatial dimensions is three and the number of temporal dimensions is one. We considered the possibility that modeling photons based on four - not two - harmonic oscillators might create opportunities. And, we knew of an extension to traditional mathematics that might keep the modeling palatable regarding physics. The notation $|-1>$ symbolizes the ground state - and the only state - for the third spatial oscillator.

When we were aware that one set of models seems to interrelate some properties of elementary particles, components of gravity, isomers, the density of the universe ratio of dark matter to ordinary matter, and eras in the expansion of the universe, we were aware of a possible opportunity to unite the models based on extensions to the modeling - for photons - that features four harmonic oscillators. Exploring this opportunity led to modeling that matches all known elementary particles and suggests new elementary particles.

When we were aware that one set of models seems to interrelate elementary particles, some properties of elementary particles, components of gravity, isomers, the density of the universe ratio of dark matter to ordinary matter, and eras in the expansion of the universe, we found that the work explains various inferred ratios - other than densities of the universe - of dark matter to ordinary matter. For example, some of those ratios pertain to galaxies. And, we found that the work seems to be compatible with aspects of concordance cosmology. For example, our work suggests that most dark matter is cold dark matter.

1.4. Methods

We blend two sets of work - extant modeling and proposed modeling.

We use the two-word term extant modeling to describe models developed by people other than us. We divide the models into two categories. We use the word core and the word unverified to discuss that division. The word core means that people have found that the models match data. The word unverified points to other extant modeling.

We use the two-word term proposed modeling to describe our work. We divide the models into two categories. We use the word core and the word supplementary to discuss that division. Core proposed modeling addresses properties of elementary particles and dark matter. Core proposed modeling also suggests explanations for cosmology and astrophysics data. Supplementary proposed modeling features suggested supplements to core extant modeling kinematics models.

This essay unites core extant modeling and core proposed modeling. Core extant modeling provides models for the motions of and changes to objects. Core proposed modeling suggests and interrelates properties of objects.

Proposed modeling augments core extant modeling. Proposed modeling does not disturb core extant modeling. Some relevant core extant modeling features the principle of stationary action and has bases in functions of continuous variables. Proposed modeling has bases in discrete mathematics. Proposed modeling has bases in a principle for which we use the term double-entry arithmetic.

Some extant modeling uses space-time coordinates. Core proposed modeling has bases that do not use space-time coordinates. Core proposed modeling does not disturb core extant modeling that people might associate with notions of space-time.

Proposed modeling suggests limits regarding the usefulness of some extant modeling models.

Goals, results, and key concepts

(Proposed modeling)

Goals and results	Key concepts
Embrace core extant modeling.	Emphasize objects and their properties. Accept core extant modeling regarding motion and other changes in properties.
Explain ratios of large (that is: ordinary matter, dark matter, and dark energy) densities of the universe.	Posit isomers of elementary particles.
Explain three eras in the rate of expansion of the universe.	Posit multipole components of gravity (similar to multipole components for static electromagnetism).
List elementary particles and their properties.	Develop an analog – for elementary particles – to the periodic table for chemical elements.
Explain and predict ratios (regarding, for example, galaxies or CMB * depletion) of dark matter to ordinary matter.	Use the notions of isomers and multipole components of G-family ** elementary particles.
Suggest how to close various possible gaps between extant modeling and observations.	Use aspects of proposed modeling.
Interrelate physics properties.	Use aspects of proposed modeling.

Core extant modeling

Proposed modeling

* CMB – Cosmic microwave background radiation

** G-family elementary particles – photon (spin one), graviton (spin two), a particle having spin three, and a particle having spin four

Figure 1: Goals, results, and key concepts

1.5. Results

We preview some results that this essay discusses.

Figure 1 summarizes goals of our work and results that our work seems to achieve. Figure 6 discusses the notion of isomer.

Figure 2 shows physics results that core proposed modeling might add to physics results that associate with core extant modeling. Results accumulate downward. (Results that associate with a specific one of the four types of modeling include results that pertain for types of modeling that the figure shows above the specific type of modeling.)

Proposed modeling matches all known elementary particles. Proposed modeling suggests elementary particles that people have yet to find.

Figure 3 summarizes some information about elementary particles. The figure alludes to all known elementary particles. The figure alludes to elementary particles that proposed modeling suggests and that people have yet to find. Each row discusses one value of Σ. The symbol Σ equals $2S$. The symbol S denotes spin as per the extant modeling expression $S(S+1)\hbar^2$ regarding angular momentum.

Proposed modeling includes two complementary techniques, each of which suggests subfamilies of elementary particles and suggests limits on subfamilies of elementary particles.

Figure 4 shows outputs - from one of the two techniques - that associate with known and suggested elementary particles. The outputs associate with all elementary particles to which figure 3 alludes. The word solutions associates with double-entry arithmetic solutions to some equations. The expression $n_{ETA0} = -1$ associates with the notion that the elementary particles always model as entangled. The expression $n_{ETA0} = 0$ associates with the notion that the elementary particles can model as not entangled. We defer - to elsewhere in this essay - further discussing the two complementary techniques.

Proposed modeling predicts masses for some elementary particles. Formulas for masses of elementary particles include aspects that reflect charge and spin.

For the Higgs boson and the weak interaction bosons, proposed modeling suggests that the ratios of squares of masses $(m_{\mathrm{Higgs}})^2 : (m_{\mathrm{Z}})^2 : (m_{\mathrm{W}})^2$ are $17 : 9 : 7$. Details include the following. Start from $17 = 4^2 + 1$ for the Higgs boson and $10 = 3^2 + 1$ for the weak interaction bosons. If $S = 1$, subtract one. If the magnitude of the charge is $|q_e|$, subtract two. The symbol q_e denotes the charge of the electron.

Proposed modeling suggests a formula for the masses of the elementary fermions. The formula yields values of $\log(m/m_e)$. The symbol m_e denotes the mass of the electron. The fine-structure constant - α or $((q_e)^2/(4\pi\varepsilon_0))/(\hbar c)$ - appears in the formula. An aspect - $\hbar$ - related to spin appears in α. An aspect - q_e - related to charge appears in α.

Figure 5 shows rest energies that proposed modeling suggests for some elementary fermions. Unverified extant modeling suggests that measurements show indirectly that at least one neutrino rest energy differs from the rest energies of the other two neutrinos. Proposed modeling can comport with the notion of

Extant modeling and proposed modeling

(Incremental results that associate with of various types of modeling)

Modeling	ι_I	Descriptions and explanations regarding ...	Subtleties regarding ...
Baseline	NR	Phenomena that are known and explained	-
PR1ISP	1	New elementary particles	Internal symmetries
		One type of dark matter	-
		Possible eras early in the development of the universe	Known eras regarding the rate of expansion of the universe
PR6ISP	6	More types of dark matter	-
		Ratios of dark matter effects to ordinary matter effects	Galaxy formation and evolution
		Objects, smaller than galaxies, that feature dark matter	-
		-	Eras regarding the rate of expansion of the universe
		-	Spans (of G-family elementary particles)
		-	Ranges of applicability of some extant modeling kinematics models
PR36ISP	36	Possible dark energy stuff	Dark energy density of the universe
		-	Spans (of G-family elementary particles)

Core extant modeling

Core proposed modeling

ι_I – Number of isomers of simple particles (that is, elementary particles other than G-family elementary particles)

G-family elementary particles – Photon, graviton, a similar spin-three boson, and a similar spin-four boson

NR – Not relevant

Figure 2: Extant modeling and proposed modeling

Elementary particles

(Elementary-particle subfamilies)

m>0			Σ=2S *		|q|=0		
In hadron-like		|q|		m		In hadron-like	G family
	0H	0	0	0	0I		
1Q	1C	>0	1	>0	1N	1R	
	2W	≥0	2	0	2J	2U	2G
			4	0			4G
			6	0			6G
			8	0			8G

Known | Suggested

m – mass | * S – spin, as in $S(S+1)\hbar^2$ | q – charge

0H: Higgs boson. 1Q: quarks. 1C: charged leptons. 2W: weak-interaction (Z and W) bosons.

1N: neutrinos. 2U: gluons. 2G: photon.

0I: inflaton. 1R: zero-charge analogs to quarks. 2J: modeling analog to the Pauli exclusion force.

4G: graviton. 6G: interacts with (and strength differs by) fermion generation. 8G: interacts with angular momentum.

Figure 3: Subfamilies of elementary particles

Solutions that associate with elementary particles

(Based on GFC – or, G-family component – modeling)

Subfamily: subset	G-family solution(s)	For Σ=0, Σ=...	n_{ETA0}	Note
0H	0G2468	|+2−4−6+8|	0	$n_{ETA0}=0$ associates with "the object can model as not entangled."
0I	0G2468[[16]]	|+2−4−6−8+16|	−1	$n_{ETA0}=-1$ associates with "the object models as entangled."
1Q: one charge	0G2468[[12]][[16]]	|+2−4+6−8−12+16|	−1	12∈Γ associates with "elementary fermions."
1Q: the other charge	0G2468[[12]][[16]]	|−2−4−6+8−12+16|	−1	16∈Γ implies $n_{ETA0}=-1$.
One of 1C or 1N	0G246[[12]]	|−2−4−6+12|	0	
The other of 1C or 1N	0G268[[12]]	|+2−6−8+12|	0	
One of 1R or N/R	0G268[[12]][[16]]	|−2+6−8−12+16|	−1	N/R denotes not relevant.
The other of 1R of N/R	0G246[[12]][[16]]	|−2+4−6−12+16|	−1	
2W: one of Z or W	0G268	|−2−6+8|	0	
2W: the other one of Z or W	0G246	|−2−4+6|	0	
2J	0G268[[16]]	|−2−6−8+16|	−1	
2U	0G∅	|$\sum_{\emptyset}$|	−1	$\sum_{\emptyset}$ denotes a sum over the empty set.
2G	2GΓ		0	ΣGΓ – with Σ≥2 – implies that more than one solution pertains.
4G	4GΓ		0	
6G	6GΓ		0	
8G	8GΓ		0	

Extant modeling	Proposed modeling

Figure 4: G-family solutions that associate with elementary particles

unequal neutrino rest energies. Proposed modeling might also comport with the notion that the three neutrino rest energies equal each other. For either case, proposed modeling suggests that some interactions - for example with 8G - might explain extant modeling notions that suggest differences between squares of neutrino masses. In general, 4G interacts with rest energy. 4G catalyzes neutrino oscillations. Regarding elementary fermions, 6G interacts with generation.

Proposed modeling suggests that most dark matter has bases in isomers of most - but not all - elementary particles.

Proposed modeling suggests that nature includes six isomers of a set of elementary particles. (Here, we discuss PR6ISP modeling. See figure 2. We postpone discussing PR36ISP modeling.) Proposed modeling calls the isomers isomer zero, isomer one, . . ., and isomer five. Stuff that measures as ordinary matter is most of - but not all of - the stuff that has bases in isomer zero.

Regarding each isomer, the set of elementary particles includes all elementary particles except G-family elementary particles. Except for charged leptons, the elementary particles in one isomer might be nearly identical to the elementary particles in each other isomer. For charged leptons, pairings of rest energy and generation can differ between isomers. We provide an example. For isomer zero, the electron is a charged lepton that associates with generation one. For isomer zero, the muon is a charged lepton that associates with generation two. For isomer one, a charged lepton that has the mass of the isomer zero electron associates with generation three. For isomer one, a charged lepton that has the mass of the isomer zero muon associates with generation one.

Proposed modeling suggests that - in the early universe - jay bosons catalyze roughly equal - across isomers - populations of stuff.

Each isomer has its own analog of the extant modeling notion of the photon. Each isomer can scarcely detect photons emitted by other isomers. (We postpone further discussing the proposed modeling notion that 2G intermediates some interactions between isomers.)

Each isomer forms, based on the isomer's arcs (or, 1R elementary fermions) and gluons, hadron-like particles. We use the symbol 1R⊗2U to denote these hadron-like particles. These hadron-like particles have no (non-virtual) charged components. Isomer zero 1R⊗2U hadron-like particles do not interact with isomer zero photons. Isomer zero 1R⊗2U hadron-like particles measure as being dark matter.

Figure 6 shows a proposed modeling explanation for the inferred ratio - five-plus to one - of dark matter density of the universe to ordinary matter density of the universe.

Proposed modeling suggests insight regarding eras in the evolution of the universe.

Proposed modeling suggests phenomena that govern changes in the rate of expansion of the universe.

Proposed modeling models include a decomposition of the gravitational field that an object produces.

Suggested rest energies for some elementary fermions

(Calculated approximate rest energies)

Subfamily	Elementary particle	Approximate rest energy		Note
1C	Tauon	1776.8400±0.0115	MeV	The standard deviation reflects the standard deviation of measurements of G_N.
1Q	Up (quark)	2.335	MeV	
1Q	Down (quark)	4.479	MeV	
1Q	Charm (quark)	1.178×10^3	MeV	
1Q	Strange (quark)	1.006×10^2	MeV	
1Q	Top (quark)	1.695×10^5	MeV	
1Q	Bottom (quark)	4.232×10^3	MeV	
1R	Arc – generation one	8.593	MeV	
1R	Arc – generation two	8.593	MeV	
1R	Arc – generation three	1.0566×10^2	MeV	This rest energy equals the muon rest energy.
1N	Neutrinos – each of at least two (of the three) mass eigenstates	3.4475×10^{-2}	eV	Regarding the possibility that this result pertains for all three mass eigenstates, measurements that people interpret as implying that neutrino masses differ by eigenstate might reflect effects of interactions between neutrinos and 6G.
1N	Neutrinos – no more than one mass eigenstate	4.1629×10^{-6}	eV	Might instead equal 4.4305×10^{-4} eV.

Known	Known particle	Suggested value or
Suggested	Suggested particle	more accurate value

Figure 5: Suggested rest energies for some elementary fermions

Dark matter and ordinary matter

(Relative densities of the universe – dark matter : ordinary matter :: 5^+ : 1)

Isomer *	Fraction that is dark matter
0	Approx. 1/17
1	1
2	1
3	1
4	1
5	1

* Isomer – An instance (or, copy) of all elementary particles, except the graviton (4G) (This definition omits some nuances regarding charged leptons, the photon (2G), and each other G-family elementary particle.)

1	1R⊗2U hadron-like particles (These particles contain no charged elementary particles.)
0	Ordinary matter (Isomer 0 1R⊗2U {hadron-like} particles measure as being dark matter.)
1	Dark matter (Stuff made from isomer 1 or isomer 4 evolves into cold dark matter.)
1	Dark matter (Stuff made from isomer 2 or isomer 5 evolves into cold dark matter.)
1	Dark matter (Stuff made from isomer 3 evolves similarly to ordinary matter.)

Known (ordinary matter)

Suggested (dark matter)

Figure 6: Dark matter and ordinary matter

The components of gravity (or, 4G) have parallels to components that extant modeling (for example, Maxwell's equations) attributes to electromagnetic fields. For a stationary object, extant modeling points to a spatial monopole component - of 2G - that reflects the charge of the object. A spatial dipole component reflects the magnetic dipole moment of the object.

We explore the proposed modeling decomposition of gravity. The rest energy of an object is non-negative. For 4G, proposed modeling points, as people might expect, to an attractive spatial monopole component of gravity. A dipole component dilutes overall attraction. (We think that the dipole component has similarities to some aspects of the extant modeling general relativity notion of rotational frame dragging.) For objects that are adequately massive and adequately close to each other, dipole repulsion can exceed monopole attraction. Modeling for 4G also includes - at least - an attractive quadrupole component and two repulsive octupole components. The monopole component of 4G intermediates attractive interactions between elementary particles in any one of the six isomers and elementary particles in any of the six isomers. We say that the monopole component has a span of six isomers. The quadrupole component of 4G intermediates attractive interactions between stuff in any isomer and stuff in (only) the same isomer. We say that the quadrupole component has a span of one isomer. (In effect, each isomer has its own quadrupole component.) Each of the two octupole components of 4G intermediates repulsive interactions between stuff in any isomer and stuff in (only) the same isomer. We say that each octupole component has a span of one isomer. (In effect, each isomer has its own pair of octupole components.) The dipole component of 4G intermediates repulsive interactions between stuff in any isomer and stuff in (only) the same isomer and one other isomer. We say that the dipole component has a span of two isomers. (Among the six isomers, three instances of the dipole component exist.)

Proposed modeling might resolve seeming inabilities of extant modeling to explain unexpectedly large increases in the rate of expansion of the universe during the most recent some billions of years. The proposed modeling explanation has bases in the notion of isomers and in the notion of the repulsive dipole component of 4G.

The difference between span-one for the quadrupole component of 4G and span-two for the dipole component of 4G might resolve the following seeming problem regarding unverified extant modeling. People develop extant modeling for the kinematics of large clumps and for equations of state for large regions. (Large clumps might include filaments and galaxy clusters.) People tune models to account for phenomena during the multi-billion-year period during which the rate of expansion decreases. People say that applying the models to the current era of increasing rate of expansion underestimates current increases in the rate. Proposed modeling suggests that such extant modeling models underestimate the dominant repulsive effect by - in effect - a factor of two. The factor of two reflects the ratio of the span of the dipole component of 4G to the span of the quadrupole component of 4G.

Proposed modeling suggests insight regarding the early universe.

Unverified extant modeling suggests an era that people call inflation and a related elementary particle - the inflaton. The proposed modeling list of elementary particles includes a candidate - the aye (or, 0I) elementary boson - for the inflaton.

Proposed modeling suggests insight regarding two possible eras that would precede inflation.

Figure 7 catalogs eras regarding the evolution of the universe. Proposed modeling suggests aspects regarding each of five eras.

Proposed modeling suggests insight regarding various inferred ratios of dark matter to ordinary matter.

Based on notions of isomers and spans, proposed modeling suggests details regarding galaxy formation scenarios and galaxy evolution scenarios. Details suggest that galaxies tend to evolve toward some specific ratios of dark matter stuff to ordinary matter stuff.

Figure 8 lists some seemingly prevalent inferred ratios of dark matter to ordinary matter. We use the word seemingly because we are aware of at least one set - but not necessarily many sets - of measurements that yield each observed ratio. Generally, we are not aware of measurements that produce seemingly as-significant other ratios. Ratios regarding galaxy clusters seem to reflect ratios regarding densities of the universe. The one-to-one ratio regarding some absorption of CMB (or, cosmic microwave background radiation) might confirm aspects regarding the notion of isomers and the notion of spans for components of electromagnetism (or, 2G). People attribute half of the absorption to ordinary matter hydrogen atoms. A seemingly relevant component of 2G has a span of two. Hydrogen atom like objects in one isomer that does not feature ordinary matter would account for the other half of the absorption. Proposed modeling galaxy evolution scenarios suggest explanations for numbers - in figure 8 - that pertain for individual galaxies. Proposed modeling galaxy evolution scenarios reflect notions of isomers and notions of spans for components of gravity (or, 4G).

Proposed modeling catalogs some properties of objects. A catalog features an index λ. The notion of λ

Eras regarding the rate of separation of large clumps

(Eras regarding “the rate of expansion of the universe”)

Time	Force between two large clumps *	RSDF **	Span per instance of force ***	Rate of separation	Name of era ****	Duration	Note
Early universe				?	TBD	?	Speculative
...	Attractive	r^{-6}	ι_{I}	Is negative	TBD	Fraction of a second	Isomers of 2J form *****
...	Repulsive	r^{-5}	1	Is positive	TBD	Fraction of a second	Isomers of 0I form *****
...	Repulsive	r^{-5}	1	Increases	Inflation	Fraction of a second	
...	Attractive	r^{-4}	1	Decreases	TBD	Billions of years	Isomers of non-0I form *****
Recent past	Repulsive	r^{-3}	2	Increases	TBD	Billions of years	
...	(Attractive)	(r^{-2})	6				Speculative

* Force – A component of 4G (or, gravity).

** RSDF – Radial Spatial Dependence of Force (assuming the use of modeling that has bases in Newtonian kinematics) that dominates between two large clumps. The notion of components of 4G, each with its own RSDF, parallels the 2G (or, electromagnetism) notions – regarding stationary, non-rotating objects – of r^{-2} for an electrostatic monopole force and r^{-3} for a magnetic dipole force.

*** Span – The number of isomers between which an instance of the component of 4G force intermediates interactions. For a component of 4G, the span times the number of instances is ι_{I}. The symbol ι_{I} denotes the total number of isomers and associates with PRι_{I}ISP modeling.

**** TBD – To be determined.

***** Isomer – An instance (or, copy) of all elementary particles, except the graviton (4G) (This definition omits some nuances regarding charged leptons, the photon (2G), and each other G-family elementary particle.) Isomers of most elementary particles might form early in the era that associates with the attractive r^{-4} component of 4G.

Known

Suggested

Some extant models assume such particles first arise after inflation.

Figure 7: Eras regarding the evolution of the universe

Ratios of dark matter to ordinary matter

(Seemingly prevalent approximate ratios)

Dark matter	Ordinary matter	Phenomenon
5^{+}	1	Density of the universe - observed
5^{+}	1	Some galaxy clusters - observed
1	1	Some absorption of CMB *
		- Observed ratio; possibly, dark matter associates with half of the overall observation
0^{+}	1	Some galaxies
		- Observed regarding early galaxies
		- Observed regarding later galaxies
4	1	Some galaxies
		- Seemingly possible regarding somewhat early galaxies
		- Observed regarding later galaxies
1	0^{+}	Dark matter galaxies
		- Observed regarding early galaxies (inferred based on properties of later galaxies)
		- Observed regarding later galaxies

* CMB – Cosmic microwave background radiation

Figure 8: Seemingly prevalent approximate ratios of dark matter to ordinary matter

Some properties of objects

(A catalog that include an index λ and an index λ_T)

λ_T	λ	Examples of symbols	Note
	0	n_{USA0}	Parameter – charge or no charge (for elementary fermions), mass or no mass (otherwise)
	2	q^2	Charge
	4	E^2	Energy (passive gravitational energy)
	6	$(E_F)^2$, M'' *	Freeable energy (generally), related to generations (for elementary fermions) *
	8	$S(S+1)\hbar^2$	Intrinsic angular momentum
	10	$J(J+1)\hbar^2$	Total angular momentum
	12	$I(i_2; i_4)$	Isomers of mass (or gravity) – up to 6: $i_4 = 0, 1, 2, 3, 4$, or 5
	14	$I(i_2; i_4)$	Isomers of charge – up to 6: $i_2 = 0, 1, 2, 3, 4$, or 5
	16	P^2	Momentum
0		n_{UTA0}	Parameter – the object models as entangled or can model as not entangled
2		$I(i_2; i_4)$	Isomers of charge – up to 6: $i_2 = 0, 1, 2, 3, 4$, or 5
4		$I(i_2; i_4)$	Isomers of mass (or gravity) – up to 6: $i_4 = 0, 1, 2, 3, 4$, or 5
6		r, b, g	Color charge (for entangled elementary fermions), clear (otherwise)
8 - 16		(perhaps) $T_{\mu\nu}$ **	10 components associating with passive gravitation ***

Extant modeling	* Energy above ground state or above energy for lowest-energy relevant elementary fermion
Proposed modeling	** Might be (for general relativity) a contribution to a component of the stress-energy tensor
Conjecture	*** Might be (in general) a contribution related to one of 10 isomer-span-related components of gravity

Figure 9: A catalog of some properties of objects

has uses beyond the use as an index. For example, for some modeling, $\lambda = 2$ pairs with electromagnetism and $\lambda = 4$ pairs with gravity.

Figure 9 shows a catalog of some properties of objects. The series $\lambda = 2$, $\lambda = 4$, $\lambda = 8$, and $\lambda = 16$ associates with, respectively, charge, mass or rest energy, intrinsic angular momentum, and momentum. Each of $\lambda = 4$ and $\lambda = 6$ relates to aspects that associate with energy. Each of $\lambda = 8$ and $\lambda = 10$ relates to angular momentum. Each of $\lambda = 12$ and $\lambda = 14$ relates to isomers that the object includes. Each of $\lambda = 2$ and $\lambda = 16$ relates to charge.

Figure 9 echoes the notion that proposed modeling interrelates some properties of objects. For example, models regarding elementary bosons interrelate mass, spin, and charge.

Figure 9 alludes to the notion that proposed modeling includes a parameter that associates with charge or no charge for elementary fermions and with mass or no mass for other objects. This parameter associates with aspects of figure 4.

Figure 9 alludes to the notion that proposed modeling includes a parameter that associates with whether an object models as entangled or can model as not entangled. This parameter associates with aspects of figure 4.

2. Methods

This unit addresses the following opportunities. Motivate and develop methods that proposed modeling uses. Use the methods. Develop and show results from using the methods. Discuss the methods and results.

2.1. Mathematics that underlies proposed modeling

We discuss mathematics that underlies much of proposed modeling.

2.1.1. Double-entry arithmetic

We discuss mathematics for which we use the two-element term double-entry arithmetic.

For each of accounting and proposed modeling, equation (1) associates with two sets of numbers. The letter a associates with a type of modeling. For each type of modeling, the letter b associates with two choices. (The notation $\{x| \ldots\}$ denotes a set of x such that the conditions that $\ldots$ states pertain. The symbol $\in$ denotes the four-word phrase is a member of.)

$$abA = \{n_{abA\iota} | 1 \leq \iota \leq N_{abA}, n_{abA\iota} \in \{\text{real numbers}\}\} \tag{1}$$

The following notions pertain regarding accounting.

For the letter a, we use the letter B, as in bookkeeping.

One set associates b with debits and with the letter D. Here, N_{BDA} is a number of asset accounts. The value of $n_{BDA\iota}$ denotes a monetary amount that associates with the asset account ι. One set associates b with credits and with the letter C. Here, N_{BCA} is a number of liability accounts. (In effect and without loss of relevance for this essay, this discussion includes - within liabilities - shareholders equity.) The value of $n_{BCA\iota}$ denotes a monetary amount that associates with credit account ι. For each of debits and credits, a total - of the form that equation (2) shows - pertains. (A construct of the form $x \equiv y$ denotes that y provides the definition of x.)

$$A_{abA} \equiv \sum_{\{n_{abA\iota} | n_{abA\iota} \in abA\}} n_{abA\iota} \tag{2}$$

The following notions pertain regarding accounting and regarding proposed modeling.

Equation (3) associates with the accounting notion of double-entry bookkeeping and with a proposed modeling notion for which we use the two-element term double-entry arithmetic. For example, increasing an $n_{aDA\iota}$ by one requires either decreasing a different $n_{aDA\iota'}$ by one or increasing one $n_{aCA\iota}$ by one.

$$0 = A_{aA} \equiv A_{aDA} - A_{aCA} \tag{3}$$

The following notions pertain regarding proposed modeling.

We set the letter a to denote a type of modeling within the realm of proposed modeling. (Table 6 shows types of modeling.)

Regarding the notion of abA, we use the symbol aXA. XA can be either one of TA and SA. For some extant modeling, TA associates with the word temporal and SA associates with the word spatial. We extend uses of the words temporal and spatial to associate with modeling that does not associate with notions of time and space.

Regarding equation (3), aDA associates with aTA. Also, aCA associates with aSA.

For each of some - but not all - types (that table 6 denotes by a) of modeling, the notion of quantum excitation pertains. For these types, proposed modeling uses mathematics that associates with the two-word term harmonic oscillators.

For much of proposed modeling, subsets of the set of integers replace - in equation (1) - the set of real numbers.

2.1.2. Two aspects of mathematics that associates with harmonic oscillators

We provide perspective about harmonic oscillator mathematics.

We point to two types of expressions that represent aspects of solutions to equations that pertain within mathematics for one-dimensional harmonic oscillators. (See equation (4) and equation (5).) Regarding mathematics that associates with each type of expression, we assign a name.

PDE mathematics features solutions that feature sums of terms of the form that equation (4) shows. The symbol x denotes a continuous variable. The one-element term PDE abbreviates the three-word phrase partial differential equation.

$$x^{\nu} \exp(x^{-2}) \tag{4}$$

ALG mathematics features solutions that feature sums of terms of the form that equation (5) shows. The occupation number n is an integer. The one-element term ALG abbreviates the word algebraic.

$$|n> \tag{5}$$

We use the terms PDE and ALG in the context of cataloging types of modeling that our work uses. Generally, such work features modeling that associates with the notion of multi-dimensional isotropic harmonic oscillators.

2.1.3. ALG mathematics

We explore mathematics underlying ALG modeling. The math differs - in at least two ways - from math that extant modeling uses. Proposed modeling ALG modeling embraces notions that extant modeling might characterize by the three-word phrase below ground state. Proposed modeling ALG modeling features double-entry arithmetic and, thereby, features equations that so-called solutions solve.

We discuss aspects of ALG modeling that associates with isotropic harmonic oscillators.

For the letter a, we use the letter A, as in ALG. (See equation (1).)

Equation (6) shows an extant modeling representation for states for a one-dimensional harmonic oscillator. The symbol $|\_>$ associates with the notion of quantum state. (See equation (5).) Equation (7) shows the extant modeling representation for a raising operator. Equation (8) shows the extant modeling representation for a lowering operator. People use the two-word term ladder operators to refer to the raising operator and the lowering operator. In extant modeling, n is a nonnegative integer.

$$|n> \tag{6}$$

$$a^{+}|n>= (1+n)^{1/2}|n+1> \tag{7}$$

$$a^{-}|n>= n^{1/2}|n-1> \tag{8}$$

Proposed modeling extends the domain associating with equation (6) from the extant modeling domain of $n \geq 0$ to the proposed modeling domain that includes negative integers. For aspects of proposed modeling that involve ladder operators, the domain of $n \geq -1$ pertains, equation (9) pertains, and equation (10) pertains.

$$a^{+}|-1>= 0|0> \tag{9}$$

$$a^{-}|0>= 0|-1> \tag{10}$$

In the context of a one-dimensional harmonic oscillator, equation (9) and equation (10) isolate $|-1>$ from the states $|n>$ for which n is non-negative. In the context of harmonic oscillators for which the number of dimensions exceeds one, isolation does not necessarily pertain.

Proposed modeling posits that equations (11) and (12) have relevance for the domain $-1 \leq n \leq 0$.

$$b^{+}|n>= n^{1/2}|n+1> \tag{11}$$

$$b^{-}|n>= (1+n)^{1/2}|n-1> \tag{12}$$

Equation (13) and equation (14) show an extant modeling representation for states for an isotropic harmonic oscillator. In each equation, each use of the symbol $AXA\iota$ associates with a one-dimensional harmonic oscillator.

$$|n_{AXA}>= \prod_{\{AXA\iota\}} |n_{AXA\iota}> \tag{13}$$

$$n_{AXA} \equiv \sum_{\{AXA\iota\}} n_{AXA\iota} \tag{14}$$

Equation (15), equation (16), and equation (17) show operators that extant modeling would associate with an isotropic harmonic oscillator. For XA being SA, the multiplicative product of a scale factor that has dimensions of energy and equation (17) associates with extant modeling notions of energies that associate with each of the various states $|n_{ASA\iota}>$.

$$a^{+}_{AXA}|n_{AXA}>= (1+n_{AXA})^{1/2}|n_{AXA}+1> \tag{15}$$

$$a^{-}_{AXA}|n_{AXA}>= {n_{AXA}}^{1/2}|n_{AXA}-1> \tag{16}$$

$$A_{AXA} \equiv \sum_{\{AXA\iota\}} (a^{+}_{AXA\iota}a^{-}_{AXA\iota} + (1/2)) \tag{17}$$

We discuss all ALG modeling. This discussion pertains regarding ALG modeling that associates with isotropic harmonic oscillators and that might include ladder operators. This discussion pertains regarding ALG modeling that does not associate with harmonic oscillators.

Equation (18) pertains. Equation (13) and equation (14) pertain. Equation (19) might or might not pertain.

Table 1: Terms associating with a PSA PDE equation (assuming that $(\xi'_{PSA}/2) = 1$ and $\eta_{PSA} = 1$)

Term/ $\exp(-r^2/2)$	Symbol for term	Change in power of r	Non-zero unless ...	Notes
$-r^{\nu_{PSA}+2}$	K_{+2}	$+2$	-	Cancels V_{+2}
$(D + \nu_{PSA})r^{\nu_{PSA}}$	K_{0a}	0	$D + \nu_{PSA} = 0$	-
$\nu_{PSA} r^{\nu_{PSA}}$	K_{0b}	0	$\nu_{PSA} = 0$	-
$-\nu_{PSA}(\nu_{PSA} + D - 2)r^{\nu_{PSA}-2}$	K_{-2}	-2	$\nu_{PSA} = 0$ or $(\nu_{PSA} + D - 2) = 0$	Cancels V_{-2}
$\Omega_{PSA} r^{\nu_{PSA}-2}$	V_{-2}	-2	$\Omega_{PSA} = 0$	Cancels K_{-2}
$r^{\nu_{PSA}+2}$	V_{+2}	$+2$	-	Cancels K_{+2}

$$0 = A_{AA} \equiv A_{ATA} - A_{ASA} \tag{18}$$

$$n_{AXA} \geq 0 \tag{19}$$

2.1.4. PDE mathematics

We explore mathematics underlying PDE modeling.

For the letter a, we use the letter P, as in PDE. (See equation (1).)

Equations (20) and (21) associate with an isotropic quantum harmonic oscillator. Here, r denotes the radial coordinate and has dimensions of length. The parameter η_{PSA} has dimensions of length. The parameter η_{PSA} is a non-zero real number. The magnitude $|\eta_{PSA}|$ associates with a scale length. Each of ξ_{PSA} and ξ'_{PSA} is an as-yet unspecified constant. The symbol $\Psi(r)$ denotes a function of r. The symbol ${\nabla_r}^2$ denotes a Laplacian operator. The symbol Ω_{PSA} is a constant. We associate the term PSA with this use of symbols and mathematics. We anticipate that the symbols used associate with spatial aspects of some physics modeling. We anticipate that PTA symbols and mathematics pertain for - and associate with temporal aspects of - some modeling.

$$\xi_{PSA}\Psi(r) = (\xi'_{PSA}/2)(-(\eta_{PSA})^2{\nabla_r}^2 + (\eta_{PSA})^{-2}r^2)\Psi(r) \tag{20}$$

$${\nabla_r}^2 = r^{-(D-1)}(\partial/\partial r)(r^{D-1})(\partial/\partial r) - \Omega_{PSA} r^{-2} \tag{21}$$

We explore solutions that pertain for the range that equation (22) shows.

$$0 < r < \infty \tag{22}$$

We consider solutions of the form that equation (23) shows.

$$\Psi(r) \propto (r/\eta_{PSA})^{\nu_{PSA}} \exp(-r^2/(2(\eta_{PSA})^2)), \text{ with } (\eta_{PSA})^2 > 0 \tag{23}$$

Table 1 provides details that lead to solutions that equations (24) and (25) characterize. We consider equations (20), (21), and (23). The table assumes, without loss of generality, that $(\xi'_{PSA}/2) = 1$ and that $\eta_{PSA} = 1$. More generally, we assume that each of the four terms $K\_$ and each of the two terms $V\_$ includes appropriate appearances of $(\xi'_{PSA}/2)$ and η_{PSA}. The term V_{+2} associates with the rightmost term in equation (20). The term V_{-2} associates with the rightmost term in equation (21). The four $K\_$ terms associate with the other term to the right of the equals sign in equation (21). The sum of the two $K_0\_$ terms associates with the factor $D + 2\nu_{PSA}$ in equation (24).

Equations (24) and (25) characterize solutions. The parameter η_{PSA} does not appear in these equations.

$$\xi_{PSA} = (D + 2\nu_{PSA})(\xi'_{PSA}/2) \tag{24}$$

$$\Omega_{PSA} = \nu_{PSA}(\nu_{PSA} + D - 2) \tag{25}$$

We explore the topic of normalization regarding $\Psi(r)$.

In extant modeling, people consider that $\Psi(r)$ normalizes if and only if equation (26) pertains. The symbol $(\Psi(r))^*$ denotes the complex conjugate of $(\Psi(r))$.

$$\int_0^\infty (\Psi(r))^* \Psi(r) r^{D-1} dr < \infty \tag{26}$$

Our work embraces somewhat the same concept - as extant modeling embraces - regarding normalization. The difference in the domain for r (that is, $0 < r < \infty$ for our work versus $0 \leq r < \infty$ for extant modeling) is not material for this essay. For essentially the entire remainder of this essay, we assume that equation (27) pertains. (For a complex number z, the expression $z = \Re(z) + i\Im(z)$ pertains. The expression $\Re(z)$ denotes the real part of z. The expression $\Im(z)$ denotes the imaginary part of z. The symbol i denotes the positive square root of the number -1.) We take the liberty to assume that the normalization criterion that equation (26) defines pertains for any real number D.

$$\Im(D) = 0 \tag{27}$$

For essentially the entire remainder of this essay, we assume that equation (28) pertains.

$$\Im(\nu_{PSA}) = 0 \tag{28}$$

Equation (29) associates with the domains of D and ν_{PSA} for which normalization pertains for $\Psi(r)$. For $D + 2\nu_{PSA} = 0$, normalization pertains in the limit $(\eta_{PSA})^2 \to 0^+$. Regarding mathematics relevant to normalization for $D + 2\nu_{PSA} = 0$, the delta function that equation (30) shows pertains. Here, x^2 associates with r^2 and 4ϵ associates with $(\eta_{PSA})^2$. (Reference [1] provides equation (30).) The difference in domains, between $-\infty < x < \infty$ and equation (22), is not material here. (Our use of this type of modeling features normalization. Considering normalization leads to de-emphasizing possible concerns, regarding singularities as r approaches zero, regarding some $\Psi(r)$.)

$$D + 2\nu_{PSA} \geq 0 \tag{29}$$

$$\delta(x) = \lim{}_{\epsilon \to 0^+} (1/(2\sqrt{\pi\epsilon})) e^{-x^2/(4\epsilon)} \tag{30}$$

We use the one-element term volume-like to describe solutions for which $D + 2\nu_{PSA} > 0$. The term volume-like pertains regarding behavior with respect to the coordinate or coordinates that underlie modeling. (For extant modeling, generally, the word coordinates - as in r plus angular coordinates - can be appropriate.) We use the one-element term point-like to describe solutions for which $D + 2\nu_{PSA} = 0$. For a point-like solution, $\Psi(r)$ is effectively zero for all $r > 0$. The term point-like pertains regarding behavior with respect to the coordinate or coordinates that underlie modeling.

We explore some relationships regarding and between solutions.

We explore modeling regarding cases for which ν_{PSA} is not necessarily an integer, j is an integer, and $j\nu_{PSA}$ is an integer. We develop a process for transforming fractional-integer-ν_{PSA} modeling into integer-ν_{PSA} modeling. We anticipate using such modeling for cases for which $D + 2\nu_{PSA} \geq 0$, $j = 2$, and $j\nu_{PSA}$ satisfies one of $j\nu_{PSA} = -1$ and $j\nu_{PSA} = -3$. (See, for example, table 5b.) People might also find interest in, for example, cases for which $j = 2$, $\nu_{PSA} > 0$, $j\nu_{PSA}$ is an integer, and ν_{PSA} is not an integer. (Extant modeling does not necessarily consider cases for which $2\nu_{PSA}$ is a positive integer and ν_{PSA} is not an integer.)

We start with equation (31), which re-expresses equation (25). Equation (32) defines, for integer k, D_{k+1} in terms of D_k. Equation (33) pertains. Equation (33) associates with an equivalent of equation (25). (Some uses of equation (33) may associate with, in effect, absorbing the factor - in the rightmost term in the equation - of j^{-2} into the term $\xi'_{PSA}/2$.)

$$\Omega_{PSA} = (1/j^2)(j\nu_{PSA})((j\nu_{PSA} + jD_1 - 2j) \tag{31}$$

$$D_{k+1} = j(D_k - 2) + 2 \tag{32}$$

$$\Omega_{PSA} = (1/j^2)(j\nu_{PSA})(j\nu_{PSA} + (j(D_1 - 2) + 2) - 2) = (1/j^2)(j\nu_{PSA})(j\nu_{PSA} + D_2 - 2) \tag{33}$$

Adding the assumption that $D_2 > 0$ yields equation (34).

$$D_1 > 2(1 - (1/j)) \tag{34}$$

Table 2: Some results of recursive applications of equation (32), assuming that $j = 2$

D_1	D_2	D_3	D_4	D_5	$D_{...}$
$\cdots$					
-1	-4	-10	-22	-46	$\cdots$
0	-2	-6	-14	-30	$\cdots$
1	0	$\cdots$			Note the case for which $D_1 = 0$.
2	2	$\cdots$			2
3	4	6	10	18	$\cdots$
4	$\cdots$				Note the case for which $D_2 = 4$.
5	8	14	26	50	$\cdots$
$\cdots$					

Table 3: Steps to avoid problems to which equation (39) seems to point

Possible steps
• Use a transformation from $D_1 = 3$ to $D_2 = 4$. (See equation (37).)
• Split a set of four (as in, $D_2 = 4$) oscillators into two sets, each consisting of a pair of oscillators.
• Develop appropriate modeling that associates with at least one of the two sets of a pair of oscillators.

Adding the assumptions that D_1 is an integer and that $j > 0$ yields equation (35).

$$D_1 \geq 2 \tag{35}$$

For the case $j = 2$, equation (36) pertains for $D_1 \geq 2$.

$$D_2 = 2D_1 - 2 \tag{36}$$

For the case $j = 2$ and $D_1 = 3$, equation (37) pertains.

$$D_2 = 2D_1 - 2 = 4 \tag{37}$$

Table 2 shows, for $j = 2$, results D_2 from applying equation (32) once to some values of D_1 and results D_k (for $k > 2$)) of reapplying equation (32).

We explore modeling that considers angular coordinates for the sub-case for which $D_1 = 3$, $j = 2$, and $\nu_{PSA} = 1/2$. Here, ν_{PSA} is positive and the possibly (that is, for example, for extant modeling) so-called total angular momentum $l\hbar$ associates with $l = \nu_{PSA} = 1/2$. Equation (38) shows the angular factor in $\Psi(r) = \phi(r)Y_{l,m}(\theta, \phi)$. Equations (39) and (40) pertain. In extant modeling, people use notions of two-component spinors and four-component spinors to avoid problems to which the non-equality in equation (39) seems to point.

$$Y_{1/2,\pm 1/2}(\theta, \phi) = \exp(\pm i(1/2)\phi), \text{ for } 0 \leq \phi \leq 2\pi \tag{38}$$

$$Y_{l,m}(\theta, 2\pi) = \exp(\pm i\pi) = -1 \neq 1 = Y_{l,m}(\theta, 0) \tag{39}$$

$$Y_{l,m}(\theta, j(2\pi)) = Y_{l,m}(\theta, 0) \tag{40}$$

Table 3 list steps - other than deploying mathematics associating with spinors - that proposed modeling suggests to avoid problems to which equation (39) seems to point.

We explore some modeling that considers angular coordinates. Regarding equation (25), we explore mathematics for which equation (41) pertains for some choice of σ_{PSA}, S_{PSA}, and D'_{PSA}. Equation (42) restates equation (25). Combining equations (41) and (42) yields equation (43).

$$\Omega_{PSA} = \sigma_{PSA} S_{PSA}(S_{PSA} + D'_{PSA} - 2), \text{ for } \sigma_{PSA} = \pm 1 \tag{41}$$

$$D = 2 - \nu_{PSA} + (\nu_{PSA})^{-1}\Omega_{PSA} \tag{42}$$

$$D = 2 - \nu_{PSA} + (\nu_{PSA})^{-1}\sigma_{PSA} S_{PSA}(S_{PSA} + D'_{PSA} - 2) \tag{43}$$

Table 4: A process for transforming a solution that is appropriate for $D_1 = D$ dimensions into a solution that is appropriate for $D_2 = D'_{PSA}$ dimensions

Steps
• Choose values of ν_{PSA}, σ_{PSA}, S_{PSA}, and D'_{PSA}.
• Determine (a first value of) D via equation (43). Let D_1 denote this value of D.
• Embrace the radial dependence of $\Psi(r)$ that equation (23) implies and set any dependence on angular coordinates to a non-zero constant.
• Combine the radial dependence with an angular dependence appropriate to a solution (to equations (20) and (21)) for which (a second value of) D (in equation (21)) satisfies $D = D'_{PSA}$. Let D_2 denote this (second) value of D. (The value of D_2 is not necessarily the same as the value of D_1.)
• Thereby, produce a $\Psi(r)$ that (may have angular dependence and) pertains regarding D'_{PSA} dimensions.

Table 4 shows a process for transforming a solution that is appropriate for $D_1 = D$ dimensions into a solution that is appropriate for $D_2 = D'_{PSA}$ dimensions. (See equation (43).)

We anticipate using PDE modeling that combines PTA aspects and PSA aspects. The following equations define the operators A_{PTA} and A_{PSA}. The symbol $\Psi(t,r)$ denotes a solution.

$$A_{PTA}\Psi(t,r) = \xi_{PTA}\Psi(t,r) = (\xi'_{PTA}/2)(-(\eta_{PTA})^2{\nabla_t}^2 + (\eta_{PTA})^{-2}t^2)\Psi(t,r) \tag{44}$$

$${\nabla_t}^2 = t^{-(D_{PTA}-1)}(\partial/\partial t)(t^{D_{PTA}-1})(\partial/\partial t) - \Omega_{PTA}t^{-2} \tag{45}$$

$$A_{PSA}\Psi(t,r) = \xi_{PSA}\Psi(t,r) = (\xi'_{PSA}/2)(-(\eta_{PSA})^2{\nabla_r}^2 + (\eta_{PSA})^{-2}r^2)\Psi(t,r) \tag{46}$$

$${\nabla_r}^2 = r^{-(D_{PSA}-1)}(\partial/\partial r)(r^{D_{PSA}-1})(\partial/\partial r) - \Omega_{PSA}r^{-2} \tag{47}$$

For core proposed modeling, we assume that equation (48) pertains. (Perhaps, compare with equation (18).)

$$0 = A_{PA} \equiv A_{PTA} - A_{PSA} \tag{48}$$

Some of our work features the numbers of dimensions that equations (49) and (50) show.

$$D^*_{PSA} = 3 \tag{49}$$

$$D^*_{PTA} = 1 \tag{50}$$

We anticipate using equations (51) and (52). Here, each of $2S$ and $2S_{PTA}$ is a nonnegative integer. (We de-emphasize using the symbol S_{PSA} instead of the symbol S.) The case that features equation (51), $\sigma_{PSA} = +1$, and $S = \nu_{PSA}$ is a restating of equation (25).

$$\Omega_{PSA} = \sigma_{PSA}S(S + D_{PSA} - 2) = \sigma_{PSA}S(S+1), \text{ for } \sigma_{PSA} = \pm 1 \tag{51}$$

$$\Omega_{PTA} = \sigma_{PTA}S_{PTA}(S_{PTA} + D_{PTA} - 2) = \sigma_{PTA}S_{PTA}(S_{PTA} - 1), \text{ for } \sigma_{PTA} = \pm 1 \tag{52}$$

Along with mathematics associating with three dimensions and $D^*_{PSA} = 3$ and with mathematics associating with one dimension and $D^*_{PTA} = 1$, we anticipate needing mathematics associating with two dimensions and a case that we denote by $D'' = 2$.

Table 5 shows some relationships between some PDE parameters. The symbol XA can denote either SA or TA. Here, we associate with D'' the symbols S'', ν'', Ω'', and σ''. Each of S'', ν'', Ω'', and σ'' does not necessarily associate with uses of S, ν_{PSA}, Ω_{PSA}, σ_{PSA}, S_{PTA}, ν_{PTA}, Ω_{PTA}, or σ_{PTA}. For $\Omega'' = 0$, the table uses the letters NR to denote that the sign of σ'' is not relevant. For table 5b, we use equation (42) to develop the relevant expressions for D_{PSA} and to calculate values of D_{PSA}. Similar methodologies pertain regarding $D_{...}$ in tables 5c, 5d, and 5e. (When considering tables 5b, 5c, 5d, and 5e, perhaps note that calculations of $D_{...}$ do not involve values of D^*_{PSA}, D^*_{PTA}, and D''.)

The following notions pertain regarding uses - in this essay - of PDE mathematics.

Table 5: Relationships between some PDE parameters

(a) Relationships relevant to D^*_{PXA} and D'' (with the leftmost four columns showing inputs to calculations; with the rightmost two columns showing outputs from calculations; and with XA denoting either SA or TA)

D^*_{PXA}	ν_{PXA}	D''	ν''	$D^*_{PXA}+2\nu_{PXA}$	$D''+2\nu''$
1	$-1/2$			0	
1	-1			-1	
1	$-3/2$			-2	
		2	-1		0
3	$-1/2$			2	
3	-1			1	
3	$-3/2$			0	

(b) PSA relationships, for $\sigma_{PSA} = +1$ (with the leftmost three columns showing inputs; and with * denoting a possible cause for concern regarding a possible lack of normalization)

ν_{PSA}	S	σ_{PSA}	Ω_{PSA}	Formula for D_{PSA}	D_{PSA}	$D_{PSA}+2\nu_{PSA}$	$D^*_{PSA}+2\nu_{PSA}$
-1	0	$+1$	0	$3-\Omega_{PSA}$	3	1	1
$-1/2$	$1/2$	$+1$	$3/4$	$(5-4\Omega_{PSA})/2$	1	0	2
$-3/2$	$1/2$	$+1$	$3/4$	$(21-4\Omega_{PSA})/6$	3	0	0
-1	1	$+1$	2	$3-\Omega_{PSA}$	1	-1*	1

(c) PTA relationships, for $\sigma_{PTA} = +1$ (with the leftmost three columns showing inputs; and with * denoting a possible cause for concern regarding a possible lack of normalization)

ν_{PTA}	S_{PTA}	σ_{PTA}	Ω_{PTA}	Formula for D_{PTA}	D_{PTA}	$D_{PTA}+2\nu_{PTA}$	$3+2\nu_{PTA}$
-1	0	$+1$	0	$3-\Omega_{PTA}$	3	1	1
$-1/2$	$3/2$	$+1$	$3/4$	$(5-4\Omega_{PTA})/2$	1	0	2
$-3/2$	$3/2$	$+1$	$3/4$	$(21-4\Omega_{PTA})/6$	3	0	0
-1	1	$+1$	2	$3-\Omega_{PTA}$	1	-1*	1

(d) PSA relationships, for $\sigma_{PSA} = -1$ (with the leftmost three columns showing inputs)

ν_{PSA}	S	σ_{PSA}	Ω_{PSA}	Formula for D_{PSA}	D_{PSA}	$D_{PSA}+2\nu_{PSA}$
$-1/2$	$1/2$	-1	$-3/4$	$(5-4\Omega_{PSA})/2$	4	3
$-1/2$	$3/2$	-1	$-15/4$	$(5-4\Omega_{PSA})/2$	10	$\cdots$
$-1/2$	$\cdots$	$\cdots$	$\cdots$	$(5-4\Omega_{PSA})/2$	$\cdots$	$\cdots$
-1	0	-1	0	$3-\Omega_{PSA}$	3	1
-1	1	-1	-2	$3-\Omega_{PSA}$	5	3
-1	2	-1	-6	$3-\Omega_{PSA}$	9	$\cdots$
-1	$\cdots$	$\cdots$	$\cdots$	$3-\Omega_{PSA}$	$\cdots$	$\cdots$
$-3/2$	$1/2$	-1	$-3/4$	$(21-4\Omega_{PSA}\Omega)/6$	4	1
$-3/2$	$3/2$	-1	$-15/4$	$(21-4\Omega_{PSA}\Omega)/6$	6	$\cdots$
$-3/2$	$\cdots$	$\cdots$	$\cdots$	$(21-4\Omega_{PSA}\Omega)/6$	$\cdots$	$\cdots$

(e) Relationships between some parameters, for $D'' = 2$ and $D'' + 2\nu'' = 0$ (with the leftmost three columns showing inputs; and with NR denoting that the sign of σ'' is not relevant)

ν''	S''	σ''	Ω''	Formula for D	D	$D+2\nu''$
-1	1	$+1$	1	$3-\Omega''$	2	0
-1	0	NR	0	$3-\Omega''$	3	1
-1	1	-1	-1	$3-\Omega''$	4	2
-1	2	-1	-4	$3-\Omega''$	7	5
-1	3	-1	-9	$3-\Omega''$	12	10
-1	4	-1	-16	$3-\Omega''$	19	17
-1	5	-1	-25	$3-\Omega''$	28	26
-1	6	-1	-36	$3-\Omega''$	39	37
-1	7	-1	-49	$3-\Omega''$	52	50
-1	8	-1	-64	$3-\Omega''$	67	65
-1	9	-1	-81	$3-\Omega''$	84	82

People might want to consider the extent to which equation (49) associates with extant modeling notions of three spatial dimensions.

Equation (53) associates with the case that features equation (51), $\sigma_{PSA} = +1$, and $S = \nu_{PSA}$. That equation and that case associate with some aspects of extant modeling KIN modeling and with some aspects of proposed modeling FIP modeling. (Perhaps, see table 6.)

$$S(S+1)\hbar^2, \text{ for nonnegative integer } 2S \tag{53}$$

People might want to consider the extent to which equation (50) associates with extant modeling notions of one temporal dimension.

The case that features equation (51) and $\sigma_{PSA} = -1$ associates with some aspects of proposed modeling models. (Perhaps, note table 5e. Perhaps, see table 35b and discussion related to equation (107).)

2.1.5. Some mathematics that associates with harmonic oscillators and groups

We discuss some notions related to harmonic oscillator mathematics and to group theory.

Modeling for a j-dimensional isotropic harmonic oscillator can feature j linear coordinates x_k - each with a domain $-\infty < x_k < \infty$ - and an operator that is the sum - over k - of j operators of the form that equation (54) shows. The number K is positive and is common to all j uses of equation (54).

$$-\frac{\partial^2}{\partial(x_k)^2} + K \cdot (x_k)^2 \tag{54}$$

For $j \geq 2$, modeling related to the harmonic oscillator can feature partial differential equations, a radial coordinate, and $j-1$ angular coordinates. We use the symbol r to denote the radial coordinate. The domain for r can be $0 \leq r < \infty$ or $0 < r < \infty$. The question of whether a solution normalizes does not depend on whether the solution has a value for $r = 0$. Our work uses the domain $0 < r < \infty$.

For $j \geq 2$, mathematics associates the group $SU(j)$ with a symmetry that associates with a j-dimensional isotropic harmonic oscillator. (See reference [2].)

We use a symbol of the form g_{group} to denote the number of generators for a group. For $j \geq 2$, equation (55) pertains.

$$g_{SU(j)} = j^2 - 1 \tag{55}$$

For $j \geq 2$, one can split the overall operator into pieces. (See equation (54).) Equation (56) associates with a split into two pieces.

$$j = j_1 + j_2 \tag{56}$$

Equation (57) echoes mathematics and some extant modeling. Here, each of the positive integers j_1 and j_2 is at least two. The symbol $\supset$ denotes the notion that each group to the right of the symbol is a subgroup of the group to the left of the symbol.

$$SU(j_1 + j_2) \supset SU(j_1) \times SU(j_2) \times U(1) \tag{57}$$

We associate with the mathematics that equations (60), (61), (62), and (63) show the group $U(1)$. Equation (58) pertains.

$$g_{U(1)} = 1 \tag{58}$$

We associate the notation $U(1)_b$ with the mathematics that equations (11) and (12) show. We posit that applications of equation (57) pertain for which one replaces the $U(1)$ (in equation (57)) with $U(1)_b$. We posit that equation (59) pertains.

$$g_{U(1)_b} = 1 \tag{59}$$

2.2. Modeling regarding objects and their properties

We develop bases for modeling objects and their properties. We show a catalog that organizes some properties of objects.

2.2.1. Types of modeling

We discuss types of modeling that this essay features.

This essay uses the notation Φ to denote so-called families of elementary particles. This essay uses the notation $\Sigma\Phi$ to denote so-called subfamilies of elementary particles. The two-element term G family includes the photon - which associates with 2G - and the would-be graviton - which associates with 4G. Here, Φ=G.

Table 6 discusses some types of modeling that this essay deploys. The table features aspects that types of modeling produce. The table notes associations between some types of modeling and ALG modeling or PDE modeling. The letter a in the second column of the table associates with the letter a in equation (1).

Extant modeling KIN models tend to have roots in the principle of stationary action. Proposed modeling GFC, ENT, FIP, and UNI models have roots in double-entry arithmetic.

Table 7 discusses relationships between ENT, GFC, FIP, UNI, and KIN modeling. The table alludes to the evolution - regarding proposed modeling - of each of ENT, GFC, FIP, and UNI modeling from roots in extant modeling KIN modeling.

2.2.2. Photons - KIN modeling

Extant modeling models photons via two harmonic oscillators. For modeling a photon, one chooses two spatial axes. Each axis is perpendicular to the direction in which the photon moves. The two axes are perpendicular to each other. Extant modeling might label the two axes with, respectively, the symbols x and y. Each harmonic oscillator models a number of excitations that people attribute to the photon mode that people pair with the relevant axis. Equations (60), (61), and (62) show a number - n - of excitations and the ladder operators. Equation (63) shows the extant modeling range for the integer n.

$$|n> \tag{60}$$

$$a^{+}|n>= (1+n)^{1/2}|n+1> \tag{61}$$

$$a^{-}|n>= n^{1/2}|n-1> \tag{62}$$

$$n \geq 0 \tag{63}$$

Extant modeling associates the word mode with each of the two axes. One mode associates with the x axis. One mode associates with the y axis. Extant modeling associates the two-word term transverse polarization with each of the two modes.

Extant modeling has bases in notions of three spatial dimensions. Proposed modeling suggests considering, regarding photons, modeling that includes a third harmonic oscillator. Considering this third oscillator provides a step toward proposed modeling. This essay de-emphasizes the notion of adding the third oscillator to extant modeling.

The third oscillator associates with the direction of motion. Modeling might label the axis associating with the direction of motion with the symbol z. Extant modeling states that photons have zero mass. Extant modeling states that longitudinal polarization does not pertain for photons. Proposed modeling suggests extending each of equations (60), (61), and (62) to pertain for the domain that equation (64) shows. Regarding the z oscillator, equation (65) shows that this extension is compatible with zero longitudinal polarization. Longitudinal polarization does not excite. Proposed modeling suggests that equation (63) - and not equation (64) - continues to pertain regarding modes of transverse polarization.

$$n \geq -1 \tag{64}$$

$$a^{+}|-1>= (1+(-1))^{1/2}|0>= 0|0> \tag{65}$$

Equation (66) pertains regarding our conceptual extension - of extant modeling for photons - to include three spatial harmonic oscillators. The notation $\{\cdots\}$ denotes a set. The expression $KSAj$ parses as follows. The symbol K denotes kinematics modeling. (Elsewhere, we discuss notions of other modeling. See, for example, table 6.) The symbol S stands for the word spatial. (Elsewhere, we discuss notions of T and temporal. See, for example, discussion related to equation (71).) The symbol A stands for the word aspects. For example, one can read SA as denoting the two-word phrase spatial aspects. The

Table 6: Some types of modeling

Modeling	a	Notes
GFC	G	• GFC denotes the two-element phrase G-family components. • GFC modeling outputs the following. ◦ Characteristics - of gravity - that explain known eras in the rate of expansion of the universe and that suggest earlier eras. ◦ Notions that support the notion of isomers and that (generally) explain the ratio of dark matter density of the universe to ordinary matter density of the universe. ◦ Aspects that support the notion that proposed modeling explains other observed ratios of dark matter to ordinary matter. ◦ The list of elementary bosons that proposed modeling suggests that nature includes. (See, also, ENT modeling.) ◦ The list of elementary fermions that proposed modeling suggests that nature includes. (See, also, ENT modeling.) • GFC modeling associates with ALG modeling. • GFC modeling does not use harmonic oscillator ladder operators.
ENT	E	• ENT denotes the word entity. • ENT modeling outputs the following. ◦ The list of elementary particles that proposed modeling suggests that nature includes. (See, also, GFC modeling.) ◦ Insight regarding circumstances in which elementary particles excite and regarding the extent to which elementary particles excite. • ENT modeling associates with ALG modeling. • ENT modeling uses harmonic oscillator ladder operators.
FIP	F	• FIP denotes the four-word phrase fields, interactions, and particles. • FIP modeling outputs the following. ◦ Models for fields, particles, and interaction vertices. ◦ Insight regarding the handedness of elementary fermions and the handedness of some elementary bosons. ◦ Insight regarding generations of elementary fermions. ◦ Insight regarding bounds on the spins that elementary particles have. • FIP modeling associates with PDE modeling. • FIP modeling suggests possible use of harmonic oscillator ladder operators to describe aspects associating with transitions from modeling for fields to modeling for interaction vertices and elementary particles. (See discussion related to equations (100) and (101).)
KIN	K	• KIN denotes the word kinematics. • KIN modeling outputs the following. ◦ Models that echo - and provide perspective about - aspects of extant modeling kinematics models. ◦ Models that complement extant modeling kinematics models. • Some KIN modeling associates with ALG modeling. Some KIN modeling associates with PDE modeling. • Some KIN modeling uses harmonic oscillator ladder operators.
UNI	U	• UNI denotes the word united. • UNI modeling outputs the following. ◦ Models that catalog and interrelate properties that people say that objects have. ◦ Modeling that unites aspects of extant modeling and aspects of proposed modeling. ◦ Modeling that unites aspects of GFC, ENT, FIP, and KIN modeling. • UNI modeling has bases in ALG modeling and in PDE modeling. • UNI modeling tends not to use harmonic oscillator ladder operators.

Table 7: Relationships between proposed modeling ENT, GFC, FIP, UNI, and KIN modeling and extant modeling KIN modeling

Modeling	a	Notes
ENT	E	• ENT modeling uses - as a basis for itself - a proposed modeling interpretation of extant modeling KIN modeling regarding spin states for elementary bosons.
GFC	G	• GFC modeling uses - as a basis for itself - hypothetical combinations of spin states for elementary bosons. As such, GFC modeling has bases in ENT modeling and in KIN modeling.
FIP	F	• FIP modeling has bases in a PDE equation that extant modeling KIN modeling uses regarding multidimensional harmonic oscillators. Extant modeling KIN modeling uses the equation as if it is linear in energy. FIP modeling uses the equation as if it is quadratic in energy.
UNI	U	• UNI modeling uses - as a basis for itself - aspects of ENT modeling, aspects of GFC modeling, and aspects of FIP modeling. As such, UNI modeling has bases in ENT modeling, in GFC modeling, in FIP modeling, and in KIN modeling.
KIN	K	• FIP modeling provides an example of the suggesting - by proposed modeling - of KIN models that supplement extant modeling KIN modeling.

symbol j varies over the range of applicable oscillators. The symbol $\{KSA\}$ denotes a set of relevant $KSAj$ oscillators. Equation (67) pertains for mode x. The construct $@_k$ denotes a value k that does not change. (For example, equation (68) pertains.) Equation (69) pertains for mode y.

$$\{KSA\} = \{KSAz, KSAx, KSAy\} \tag{66}$$

$$n_{KSAz} = -1,\ n_{KSAx} = n,\ n_{KSAy} = @_0 \tag{67}$$

$$@_0 = 0 \tag{68}$$

$$n_{KSAz} = -1,\ n_{KSAx} = @_0,\ n_{KSAy} = n \tag{69}$$

For each of the two modes, equation (70) pertains. The symbol $\equiv$ denotes the notion of definition. The leftmost equality defines the symbol A_{KSA}.

$$A_{KSA} \equiv \sum_{\{KSAj\}} (n_{KSAj} + (1/2)) = n_{KSAz} + n_{KSAx} + n_{KSAy} + (3/2) = n + (1/2) \tag{70}$$

Extant modeling has bases in notions of one temporal dimension. Proposed modeling suggests including an oscillator that associates with the temporal dimension. Proposed modeling suggests that, for each of the two modes, equations (71), (72), and (73) pertain. Here, the symbol T stands for the word temporal. The symbol t denotes the one temporal coordinate.

$$\{KTA\} = \{KTAt\} \tag{71}$$

$$n_{KTAt} = n \tag{72}$$

$$A_{KTA} \equiv \sum_{\{KTAj\}} (n_{KTAj} + (1/2)) = n_{KTAt} + (1/2) = n + (1/2) \tag{73}$$

Equation (74) pertains for each photon mode.

$$A_{KTA} - A_{KSA} = 0 \tag{74}$$

We use the two-element term double-entry arithmetic to describe the equality that equation (75) shows. Adding a unit to one of A_{KTA} and A_{KSA} requires adding a unit to the other quantity.

$$A_{KA} \equiv A_{KTA} - A_{KSA} = 0 \tag{75}$$

Extant modeling includes two-mode photon models for which one mode features left circular polarization and the other mode features right circular polarization. Extant modeling circular polarization models are invariant with respect to choices of transverse axes. Compared to linear polarization models, circular polarization models are more invariant with respect to choice of observer. For models for a photon in a vacuum, all observers would agree on the number of excitations for left circular polarization and on the number of excitations for right circular polarization.

We convert kinematics notions above to pertain for circular polarization modes. From a perspective of equations underlying models, we use the substitutions that equation (76) shows. An expression of the form $a \leftarrow b$ denotes the six-element phrase b takes the place of a. The oscillator $KSA0$ associates with longitudinal polarization. We adopt the convention that an oscillator KSA(odd number) features left circular polarization. Oscillator $KSA1$ features left circular polarization. Oscillator $KSA2$ features right circular polarization.

$$KSAz \leftarrow KSA0,\ KSAx \leftarrow KSA1,\ KSAy \leftarrow KSA2 \tag{76}$$

2.2.3. Photons and gravitons - ENT modeling

We discuss aspects of ENT modeling for the photon.

Equations (77), (78), (79), (80), (81) and (82) pertain. Symbols of the form $ETAj$ denote oscillators that pair with the two-word term temporal aspects. However, space-time coordinates do not underlie ENT modeling. Symbols of the form $ESAj$ denote oscillators that pair with the two-word term spatial aspects. $ESA1$ pairs with left circular polarization. $ESA2$ pairs with right circular polarization. The two-word term longitudinal polarization pairs with $ESA0$. Equation (82) exemplifies double-entry arithmetic.

$$\{ETA\} = \{ETA0\} \tag{77}$$

$$n_{ETA0} = n \tag{78}$$

$$\{ESA\} = \{ESA0, ESA1, ESA2\} \tag{79}$$

$$n_{ESA0} = -1,\ n_{ESA1} = n,\ n_{ESA2} = @_0 \tag{80}$$

$$n_{ESA0} = -1,\ n_{ESA1} = @_0,\ n_{ESA2} = n \tag{81}$$

$$A_{EA} \equiv A_{ETA} - A_{ESA} = 0 \tag{82}$$

ENT modeling for the photon has similarities to KIN modeling for photons. (Compare equation (76) and discussion related to equation (82).) We anticipate ENT modeling for the Higgs boson. Longitudinal polarization pertains. Circular polarization does not pertain. For the Higgs boson, $\{ESA\} = \{ESA0\}$ pertains. We anticipate ENT modeling for the Z and W bosons. For each of the photon and the set of weak interaction bosons, $\{ESA\} = \{ESA0, ESA1, ESA2\}$ pertains.

Equation (83) defines the symbol Σ. Here, S is the spin - in the sense of the extant physics KIN modeling expression $S(S+1)\hbar^2$ that relates to (the square of) angular momentum. Σ is a nonnegative integer.

$$\Sigma \equiv 2S \tag{83}$$

We discuss ENT modeling for some elementary particles that are not the photon.

For some elementary particles, the number of ENT modeling spatial oscillators does not equal three. For the elementary particles discussed just above, equation (84) pertains. The symbol $|$ denotes the two-word phrase such that. (Elsewhere, we show that equation (84) does not pertain for ENT modeling for some elementary particles. See discussion - that follows equation (105) - regarding elementary fermions.)

$$\Sigma = 2S = \max(j | n_{ESAj} = 0) \tag{84}$$

We anticipate that - in ENT modeling and for integer $j \geq 1$ - the oscillator $ESA(2j-1)$ associates with $\Sigma = 2j$ left circular polarization. The oscillator $ESA(2j)$ associates with $\Sigma = 2j$ right circular polarization. For example, $ESA3$ and $ESA4$ associate with $\Sigma = 4$, $S = 2$, and the would-be graviton.

Table 8: An ENT representation for photon ground states

$ETA4$	$ETA3$	$ETA2$	$ETA1$	$ETA0$	$ESA0$	$ESA1$	$ESA2$	$ESA3$	$ESA4$	$\Sigma\Phi$
				0	-1	0	0			2G

Table 9: A basis for ENT representations for G-family ground states (with LCP denoting left circular polarization; and with RCP denoting right circular polarization)

ΣG	$ETA0$	$ESA0$	$ESA1$	$ESA2$	$ESA3$	$ESA4$	$ESA\cdots$
2G	0	-1	$\Sigma = 2$:LCP	$\Sigma = 2$:RCP	-	-	$\cdots$
4G	0	-1	-	-	$\Sigma = 4$:LCP	$\Sigma = 4$:RCP	$\cdots$
$\cdots$	0	-1	-	-	-	-	$\cdots$

Regarding ENT modeling, some aspects of this essay tend to emphasize ground states and de-emphasize excited states. Such work in this essay tends to feature harmonic oscillator states that pair with the numbers 0 and -1. Such work tends not necessarily to state explicitly distinctions between $@_k$ and k.

Table 8 shows an ENT representation for photon ground states.

We assume that table 9 pertains for G-family ground states. The word graviton associates with 4G.

We note aspects of ENT modeling that pertain for more than just the photon.

Equation (82) exhibits an invariance with respect to a choice between KIN modeling that is quadratic in energy and KIN modeling that is linear in energy. Regarding a photon, the KIN expression $0 = E^2 - (pc)^2$ is quadratic in energy. The symbol E denotes energy. The symbol p denotes the magnitude of momentum. The symbol c denotes the speed of light. One can consider that an ENT raising operator associates with adding one unit of each of the two relevant items - E^2 and $(pc)^2$ - that have the dimensions of the square of energy. For an object with mass m and modeling based on the equation $E^2 = (mc^2)^2 + (pc)^2$ from special relativity, one can consider that an ENT raising operator associates with adding one unit of each of the three relevant items - E^2, $(mc^2)^2$, and p^2c^2. The Klein-Gordon equation provides an example of KIN modeling - for other than just photons - that can be quadratic in energy. Regarding a photon, the KIN expression $0 = E - pc$ is linear in energy. One can consider that an ENT raising operator associates with adding one unit of each of the two relevant items - E and pc - that have the dimensions of energy. Each of the Dirac equation and the Schrodinger equation provides an example of KIN modeling - for other than just photons - that is linear in energy.

Either one of A_{ETA} and A_{ESA} can pair with the extant modeling KIN modeling notion of a photon ground state energy that associates with the expression $0+(1/2)$ and with the number one-half. (See, for example, equation (73).) People interpret extant modeling KIN models as exhibiting notions of nonzero energy of the vacuum. Proposed modeling suggests - via equations such as equation (75) - modeling that might obviate needs to consider nonzero energy of the vacuum.

2.2.4. Photons, gravitons, and other long-range force carriers - GFC modeling

We explore aspects regarding G-family forces and regarding so-called components of G-family forces.

In extant modeling KIN modeling, an excitation of a photon carries information through which people infer aspects of an event that includes the excitation. For example, people measure the energy of a photon and might use that information to infer information about an atomic transition that excited the photon.

In proposed modeling ENT modeling, excitations of a photon carry similar information. We anticipate that GFC modeling points to encoded information to which extant modeling KIN modeling does not point. The additional encoded information features the isomer or isomers that associate with the creation of the photon. (See table 15 and table 19c.)

We consider the left circular polarization mode of 2G.

We consider an excitation that models conceptually as combining an excitation of the left circular mode of 4G and the right circular mode of 2G. (This essay de-emphasizes the possible relevance of an actual object that combines a graviton and a photon. Our discussion of ENT modeling does not include such an object.) The combination yields a left circular polarization $\Sigma = 2$ (or, spin one) excitation. The combination associates with 2G.

Equation (85) provides notation that we use for such combinations. The symbol ΣG denotes a subfamily of the G-family. The symbol Γ denotes a set of positive even integers. We use the symbol λ to denote an element of Γ. Each value of λ associates with the oscillator pair $GSA(\lambda - 1)$-and-$GSA\lambda$. (For alluding to oscillators, we also allow the value $\lambda = 0$. Use of $\lambda = 0$ associates with one oscillator

Table 10: A basis for GFC representations for G-family components (with LCP denoting left circular polarization; and with RCP denoting right circular polarization)

$GTA\cdots$	$GTA0$	$GSA0$	$GSA1$	$GSA2$	$GSA3$	$GSA4$	$GSA\cdots$
$\cdots$	0	-1	$\lambda=2$:LCP	$\lambda=2$:RCP	$\lambda=4$:LCP	$\lambda=4$:RCP	$\cdots$

Table 11: G-family solutions that may be relevant and for which $\lambda \leq 8$

Other	GFC monopole	GFC dipole	GFC quadrupole	GFC octupole
0G$\emptyset$	2G2	ΣG24	ΣG246	ΣG2468
	4G4	ΣG26	ΣG248	
	6G6	ΣG28	ΣG268	
	8G8	ΣG46	ΣG468	
		ΣG48		
		ΣG68		

and not with a pair of oscillators. Regarding Γ, $\lambda = 0$ is never an element of Γ.) For the above example of subtracting spin one from spin two, the notation $\Gamma = 24$ pertains and equation (86) pertains.

$$\Sigma \mathrm{G} \Gamma \tag{85}$$

$$\Sigma = |-2+4| = 2 \tag{86}$$

Table 10 echoes table 9. Table 9 pertains for ENT modeling. Table 10 pertains for GFC modeling. We explore solutions for which equation (87) shows allowed values of λ.

$$2,\ 4,\ 6,\ 8 \tag{87}$$

Table 11 points to possibly relevant solutions for which the limit $\lambda \leq 8$ pertains. (The word solution pertains regarding harmonic oscillator mathematics and double-entry arithmetic. Here, a solution solves - or, satisfies - the equation $A_{GA} \equiv A_{GTA} - A_{GSA} = 0$. We anticipate that some solutions have relevance to models regarding G-family physics. We use the word component - as in component of a ΣG field or of a G-family force - regarding physics applications of solutions that are relevant to G-family physics. We anticipate that some solutions have relevance regarding modeling for aspects of physics other than G-family aspects. For example, see table 31.) The labels GFC monopole through GFC octupole pertain regarding GFC modeling. The label GFC monopole pairs with the existence of one mathematical solution for each item in the column labeled GFC monopole. The label GFC dipole pairs with the existence of two mathematical solutions for each item in the column labeled GFC dipole. For example, for $\Gamma = 24$, each one of the solutions 2G24 and 6G24 pertains. The symbol 6G24 pairs with the expression $\Sigma = |+2+4| = 6$. The label GFC quadrupole pairs with the existence of four mathematical solutions for each item in the column labeled GFC quadrupole. G-family physics does not include phenomena that might associate with the symbol 0G. For each of two GFC quadrupole items, the one 0GΓ mathematical solution is not relevant to G-family physics. For example, the solution 0G246, which pairs with $|-2-4+6|$, is not relevant to G-family physics. The label GFC octupole pairs with the existence of eight mathematical solutions for the one item in the column labeled GFC octupole. The solution 0G2468 is not relevant to G-family physics. The table notes a conceptually possible 0G$\emptyset$ solution. The symbol $\emptyset$ denotes the empty set.

Each G-family solution that this essay considers associates with one - and only one - of equation (88), equation (89), and equation (90). We use the symbol $\Sigma\gamma$ to refer to the set of G-family solutions ΣGΓ for which Σ appears in the list Γ. (See equation (88).) Here, the notation $\{a|b\}$ denotes the ten-element phrase the set of all a such that conditions b pertain. The symbol $\in$ denotes the four-word phrase is a member of (or, the four-word phrase is an element of). For example, 2G24 is a member of 2γ. Regarding the symbol $\Sigma\gamma'$, the symbol $\Rightarrow$ denotes the word implies. (See equation (89).) For example, 2G68 is a member of $2\gamma'$. We use the symbol $0\gamma'$ to refer to the set of G-family solutions for which a sum, similar to the sum that equation (86) shows, is zero. (See equation (90).) For example, 0G246 is a member of $0\gamma'$.

$$\Sigma\gamma \equiv \{\Sigma \mathrm{G}\Gamma | \Sigma \in \Gamma\} \tag{88}$$

Table 12: $\Sigma\gamma$ solutions for which both $\Sigma \leq 8$ and, for each $\lambda \in \Gamma$, $\lambda \leq 8$

Σ	GFC monopole	GFC dipole	GFC quadrupole	GFC octupole
2	2G2	2G24	2G248	
4	4G4	4G48	4G246	4G2468a, 4G2468b
6	6G6		6G468	
8	8G8			8G2468a, 8G2468b

Table 13: KIN modeling interpretations pairing with $\Sigma\gamma$ force components for which $\Sigma \leq 4$ and, for each $\lambda \in \Gamma$, $\lambda \leq 8$

Components	Property of an object (assuming that modeling pertains for zero translational motion)
2G2	Charge.
2G24	Magnetic dipole moment.
2G248	Magnetic dipole moment for which the direction of the axis (pairing with the dipole moment) changes over time. (Adjustment regarding 2G24. KIN spatial dipole. KIN RSDF r^{-3}.)
4G4	Mass.
4G48	Adjustment regarding 4G, to the extent that the object rotates. KIN spatial dipole. KIN RSDF r^{-3}.
4G246	Adjustment regarding 4G, to the extent that the object has a quadrupole moment of mass. KIN spatial quadruple. KIN RSDF r^{-4}.
4G2468a, 4G2468b	Adjustments regarding 4G, to the extents that quadrupole moments of mass rotate. KIN spatial octupole. KIN RSDF r^{-5}.

$$\Sigma\gamma' \equiv \{\Sigma \mathrm{G}\Gamma | \Gamma \neq \emptyset, \lambda \in \Gamma \Rightarrow \lambda \neq \Sigma\} \tag{89}$$

$$0\gamma' \equiv \{0\mathrm{G}\Gamma\} \tag{90}$$

We use the symbol $\gamma\lambda$ to refer to the set of G-family solutions ΣGΓ for which λ appears in the list Γ and Σ does not appear in the list Γ. (See equation (91).) The symbol $\notin$ denotes the five-word phrase is not a member of. For example, 6G24 is a member of $\gamma 2$ and of $\gamma 4$. 6G24 is also a member of $6\gamma'$.

$$\gamma\lambda \equiv \{\Sigma \mathrm{G}\Gamma | \lambda \in \Gamma, \Sigma \notin \Gamma\} \tag{91}$$

Table 12 lists G-family solutions ΣGΓ for which both $\Sigma \leq 8$ and, for each $\lambda \in \Gamma$, $\lambda \leq 8$. The expressions $|-2+4-6+8|$ and $|-2-4-6+8|$ show that two solutions comport with the notion of 4G2468. We use the letters a and b to distinguish between the two solutions. We use each of the letters x and y to refer to either one of the solutions or to both solutions. The expressions $|+2+4-6+8|$ and $|-2-4+6+8|$ show that two solutions comport with the notion of 8G2468.

Work leading to table 11 does not depend on choosing a kinematics model. Examples of kinematics models include Newtonian physics and general relativity.

We posit that the words monopole through octupole pair, for extant modeling KIN Newtonian modeling, with force laws. RSDF abbreviates the five-word term radial spatial dependence of force. The notion of RSDF pertains regarding KIN modeling. (The notion of RSDF does not directly pertain regarding GFC modeling.) Extant modeling pairs the word monopole with a potential energy that varies as r^{-1} and with the RSDF of r^{-2}. Here, r denotes an extant modeling KIN radial coordinate and the distance from the center of the one relevant object. Here, we de-emphasize angular aspects of forces. A series that starts with monopole continues. For example, extant modeling pairs the word dipole with a potential energy that varies as r^{-2} and with the RSDF of r^{-3}. (Perhaps, see table 13.)

Table 13 notes some aspects related to table 12. The table discusses measurable properties for an object that measures as not moving. In table 13, we use the notion that - for 2γ - $8 \in \Gamma$ does not necessarily associate with a factor - regarding RSDF - of r^{-1}. (See table 14.) In table 13, we posit that - for 4γ - $8 \in \Gamma$ associates with a factor - regarding RSDF - of r^{-1}. (See table 14.)

Table 14 posits some associations between GFC solutions and extant modeling KIN models.

Table 14: Some associations between GFC solutions and extant modeling KIN models

Aspect
• For a $\Sigma G\Gamma$ solution that associates with $\Sigma\gamma$, the strength of interactions scales with a property that associates with the $\lambda = \Sigma$ item in the list Γ. Other items in the list Γ associate with extant modeling KIN geometric factors and do not necessarily associate directly with interaction strengths.
• For 2γ, we posit that one can consider that the presence in Γ of $\lambda = 8$ pairs with a KIN factor of $(ct)^{-1}$ and not with a KIN factor of r^{-1}. (Here, c denotes the speed of light and t denotes the temporal coordinate. Perhaps, consider the notion that - at least regarding propagation of light in a vacuum - $r^{-1} = (ct)^{-1}$.)
• For 4γ, we posit that one can consider that the presence in Γ of $\lambda = 8$ pairs with a KIN factor of r^{-1}. (See table 21.)

Table 15: PRι_IISP modeling and isomers of span-one particles

Note
• The two-word phrase span-one particles denotes all elementary particles except G-family elementary particles. The set $\{\Sigma\Phi \vert \Phi \neq \mathrm{G}\}$ of subfamilies associates with all span-one particles.
• Proposed modeling includes so-called PRι_IISP modeling, with ι_I being one of the integers one, six, and 36. The models address aspects of astrophysics and aspects of cosmology. The two letters PR denote the term physics-relevant. The three letters ISP denote the four-word term isomers of span-one particles (or, the five-word term isomers of span-one elementary particles). The integer ι_I denotes a number of so-called isomers of the set of all span-one particles.
• In this respect, PR1ISP modeling associates with extant modeling.
• Proposed modeling suggests that PR6ISP models explain more astrophysics data and more cosmology data than do PR1ISP models. For example, PR6ISP modeling explains some observed ratios of dark matter to ordinary matter.
• PR36ISP models might explain more data than do PR6ISP models. In particular, PR36ISP models offer a new possible explanation for the dark energy density of the universe.

2.2.5. Isomers of elementary particles and of components of long-range forces

We discuss the notion of isomer.

Table 15 defines the two-element term span-one particles and notes some aspects regarding the proposed modeling notion of isomers of span-one particles. (This proposed modeling notion of isomers does not necessarily parallel the nuclear physics notion - same numbers of protons and neutrons, but different energy states - of isomers. This proposed modeling notion of isomers does not necessarily parallel the chemistry notion - same numbers of various atoms, but different spatial arrangements - of molecular isomers.)

Table 16 discusses notions and terminology pertaining to isomers.

Table 17 Previews aspects - of this essay - that discuss PR36ISP modeling and PR6ISP modeling.

2.2.6. Objects and observed properties - UNI modeling

We discuss aspects regarding UNI modeling, GFC solutions, and isomers.

We posit that UNI modeling has a basis in the values of λ to which equation (92) alludes. (See table 18.)

$$0, 2, 4, \ldots, 14, 16 \tag{92}$$

For $\lambda \geq 10$, this essay uses $[\![\lambda]\!]$ to denote elements of Γ.

Table 18 posits associations between properties relevant to objects and values of $USA\lambda$. (Here, we extrapolate - to UNI modeling - from GFC modeling.) The following sentences discuss choices regarding λ for relevant aspects. The possibility that the series two, four, eight, and 16 pertains - regarding key properties - tends to support the placement - in table 18 - of $\lambda = 16$ for momentum. The notion that a 2G solution should associate with magnetic fields that moving charges produce suggests that the relevant 2G solution is 2G$[\![14]\!]$ $[\![16]\!]$. Based on the notion of that 2G solution, we associate isomers of 2G components with $\lambda = 14$. Based on the notions that $\vert -2+4-6+8\vert$ (which associates with 4G2468) associates with 4G and that $\vert +2-4+6+8\vert$ equals 12, we associate isomers of 4G components with $\lambda = 12$. (Each one of four, eight, and 16 is not available.) We posit that $\lambda = 6$ associates with freeable energy (generally)

Table 16: Notions and terminology pertaining to isomers

Note
• For PR36ISP modeling, we designate individual isomers via symbols of the form I($i_2; i_4$). Each of i_2 and i_4 is an integer from the domain 0, 1, ..., 5. We associate ordinary matter with I(0;0). (All known ordinary matter associates with I(0;0). Proposed modeling suggests that some I(0;0) stuff measures as - some - dark matter.) We posit that the five isomers I(1;0) through I(5;0) associate with extant modeling notions of dark matter. We use notation of the form I(1,...,5;0) to denote - collectively - those five isomers.
• We associate the two-word term isomer zero with I(0;0). We associate the two-word term isomer one with I(1;0). ... We associate the two-word term isomer five with I(5;0).
• Each isomer of span-one particles associates with an instance of 2G2 and with an instance of 2G24.
• PR6ISP modeling includes the six isomers for which - collectively - we use the notation I(0,...,5;0). PR6ISP modeling does not include the other 30 PR36ISP isomers.
• PR1ISP modeling includes the one isomer I(0;0). PR1ISP modeling does not include the other 35 PR36ISP isomers.
• PR36ISP modeling posits six so-called isomers of the 4G4 component of 4G (or, gravity). We posit a notion of isomers of 4G. We number isomers of 4G via i_4. For each i_4, the isomer of 4G4 intermediates gravitational interactions between - and only between - stuff associating with I(0,...,5;i_4). For each i_4, the isomer of 4G intermediates gravitational interactions between - and only between - stuff associating with I(0,...,5;i_4).
• Regarding PR36ISP modeling, we say that the span of (an isomer of) 4G4 is six (as in six isomers). We say that the span of (an isomer of) 4G is six (as in six isomers).
• PR36ISP modeling posits six so-called isomers of the 2G248 component of 2G (or, electromagnetism). We posit a notion of isomers of 2G. We number isomers of 2G via i_2. For each i_2, the isomer of 2G248 intermediates electromagnetic interactions between - and only between - stuff associating with I(i_2;0,...,5). For each i_2, the isomer of 2G intermediates electromagnetic interactions between - and only between - stuff associating with I(i_2;0,...,5).
• Regarding PR36ISP modeling, we say that the span of (an isomer of) 2G248 is six (as in six isomers). We say that the span of (an isomer of) 2G is six (as in six isomers).
• Regarding PR36ISP modeling, we use the three-word term doubly dark matter to denote the 30 isomers that associate with the symbols I($i_2; i_4$) for which $i_4 \geq 1$. Electromagnetic and gravitational interactions between ordinary matter (or between I(0,0)) and doubly dark matter feature span-6 and span-2 electromagnetic (or, 2G) interactions with the five doubly dark matter isomers I(0;1,2,3,4,5). Electromagnetic and gravitational interactions between ordinary matter plus dark matter (or between I(0,1,2,3,4,5;0)) and doubly dark matter feature span-6 and span-2 electromagnetic (or, 2G) interactions with the 30 doubly dark matter isomers I(0,1,2,3,4,5;1,2,3,4,5).
• Regarding PR36ISP modeling, we posit the possibility that the 30 doubly dark matter isomers associate with dark energy density of the universe. We use the three-word term dark energy stuff to denote the stuff that would associate with the possible 30 doubly dark matter isomers and with dark energy density of the universe.
• Regarding PR6ISP modeling, the span of the one isomer of 4G4 is six. The span of the one isomer of 4G is six.
• Regarding PR6ISP modeling, one might posit a choice. Which one of six and one associates with the span of 2G248? Proposed modeling suggests that the choice of six for 2G248 (and, with that choice, the selection of a span of two for 2G68) associates with an explanation regarding an observation that detected twice as much - compared to the amount that people expected - depletion of cosmic microwave background radiation. (See discussion related to equation (190).) We assume that a span of six pertains for 2G248.
• Regarding PR36ISP modeling and PR6ISP modeling, we associate the three-word term isomer of mass with the notion of an isomer of 4G. The span of one instance of 4G4 is six.
• Regarding PR36ISP modeling, PR6ISP modeling, and PR1ISP modeling, we associate the three-word term isomer of charge with the notion of a single isomer I($i_2; i_4$). (The notion of an isomer of charge associates with a set of span-one elementary particles. The set includes elementary particles having negative charge, elementary particles having zero charge, and elementary particles having positive charge.) The span of one instance of 2G2 is one.

Table 17: Notions pertaining to isomers, PR36ISP modeling, and PR6ISP modeling

Note
• We think that PR36ISP modeling might explain a set of data that is larger than (and includes all of) the set of data the PR6ISP modeling seems to explain.
• This essay assumes - generally - that discussing PR36ISP modeling is as informative as and easier than discussing PR6ISP modeling.
• Based on numbering that this essay uses regarding isomers that associate only with dark matter, for each of PR36ISP modeling and PR6ISP modeling, the following notions pertain. Stuff associating with I(3;0) evolves similarly to stuff (which is mostly ordinary matter) associating with I(0;0). Stuff associating with each of I(1;0), I(2;0), I(4;0), and I(5;0) evolves into cold dark matter.
• Regarding PR6ISP modeling, this essay nominally assumes that the three isomers of components - of 2G - that have spans of two intermediate interactions, respectively, associating with the following three pairs of isomers I(0,3;0), I(1,4;0), and I(2,5;0). For example, one isomer of span-two components intermediates interactions within and between I(0;0) and I(3;0).
• Regarding PR6ISP modeling, any different pairings regarding span-two 2G components (possibly, but not certainly) might not adequately accurately explain the depletion result to which table 16 alludes.

and therefore with generations (for elementary fermions). We posit that the only remaining slot ($\lambda = 10$) that associates with two oscillators associates with total angular momentum. We posit the association that the table shows regarding the one-oscillator slot ($\lambda = 0$). The column with the two-word label scalar example and the column with the two-word label trio example allude to relevant examples. (For various items, we attempt to use widely used symbols. For example, q associates with charge. E associates with energy. P associates with momentum. J associates with total angular momentum.) The column with the two-element label six-fold aspect suggests relevance of - for each row for which $\lambda \geq 2$ - a count of six somethings. (See table 18c.) The two rightmost columns allude to relevant examples. The symbol k_B denotes the Boltzmann constant. The symbol T denotes temperature. Regarding $\lambda = 6$, table 18b shows two parallel branches. One branch pertains for elementary fermions. The other branch can pertain for a variety of objects.

Elsewhere, we show a table that complements table 18 and alludes to the property of color charge. (See table 22.)

We discuss notions regarding modeling that considers freeable energy. (See table 18.)

Modeling has flexibility regarding setting a zero point regarding E_F. For example, modeling regarding a hydrogen atom need not (but might) consider that the rest energies of the nucleon and of the electron associate with freeable energy.

This essay does not fully explore the notion that people might want to consider that useful modeling can associate with notions of zero vacuum energy.

2.2.7. Spans and isomers - GFC modeling

We continue discussion regarding GFC modeling and isomers.

Table 19 shows GFC representations for the G-family solutions for which - for each $\lambda \in \Gamma$ - $\lambda \leq 8$. The solutions associate with symmetries pertaining to ENT modeling and ground states. In table 19, the rightmost seven columns comport with double-entry arithmetic. (See table 19b.) Table 19c discusses the notion of span. (Regarding information in the column - in table 19a - regarding span, see discussion regarding equation (93) and equation (94).)

We discuss spans for components of G-family forces. We develop the second column - Span (for $\iota_I > 1$) - in table 19a.

For any one value of ι_I (as in PRι_IISP), equation (93) pertains for each span-one particle, for each component of G-family force, and for each hadron-like particle. For example, for PR6ISP modeling, for the electron, the number of isomers is six and the span of each isomer is one. (The electron does not pair directly with a G-family GFC solution. However, perhaps note discussion that develops table 29 and perhaps note table 32.) For PR6ISP modeling, for the 4G4 component of 4G, the number of isomers is one and the span of each isomer is six. (Gravity intermediates interactions between the six isomers of span-one particles.)

$$(\text{number of isomers}) \times (\text{span of one isomer}) = \iota_I \tag{93}$$

Table 18: Associations between properties relevant to objects and values of $USA\lambda$

(a) Aspects for $USA\lambda = 0, 2, 4, 8,$ or 16

Property	λ	Scalar example	Trio example	Six-fold aspect	Related property (that some KIN Newtonian models use)	Constant that associates with a property
Object type	0	n_{USA0}	(See table 18c.)	-	-	-
Charge	2	q^2	q_-, q_0, q_+ (See table 18c.)	2×3	Charge q	$\lvert q_e \rvert$
Energy	4	E^2	$3 \times (1/2)k_BT$	2×3	Mass m	k_B
Intrinsic angular momentum	8	$S(S+1)\hbar^2$	s_x, s_y, s_z	2×3	Angular velocity ω	$\hbar$
Momentum	16	P^2	p_x, p_y, p_z	2×3	Velocity v	c

(b) Aspects for $USA\lambda = 6, 10, 12,$ or 14

Property	λ	Scalar example	Trio example	Six-fold aspect	Related property (that some KIN Newtonian models use)	Constant that associates with a property
Generation (elementary fermion)	6	-	Three generations	$3 \rightarrow 2$	-	-
Freeable energy (other)	6	$(E_F)^2$ (See table 18c.)	$3 \times (1/2)k_BT$	2×3	$3 \times (1/2)k_BT$	k_B
Total angular momentum	10	$J(J+1)\hbar^2$	j_x, j_y, j_z	2×3	Angular velocity ω	$\hbar$
Isomers (with respect to 4G components) - I(i_2;0,...,5) with one non-changing value of i_2.	12	Up to 6 isomers	$3 = 6/2$	6	-	-
Isomers (with respect to 2G components) - I(0,...,5;i_4) with one non-changing value of i_4.	14	Up to 6 isomers	$3 = 6/2$	6	-	-

(c) Notes

Aspect	Note
Six-fold	• 2×3 denotes the notion that interactions with other objects can add or subtract regarding one of the (trio example) aspects of property. • $3 \rightarrow 2$ denotes a transition from one of three states to one of the two other states. • 6 denotes six isomers (with respect to components of a specific ΣG).
$\lambda = 0$	We associate one value (out of zero and minus one) of n_{USA0} with elementary fermions. The concept of generations pertains. The other value of n_{USA0} associates with all other objects. The concept of generations does not pertain. This notion of duality extends to a notion of trio. The notion of other objects divides into objects that model (via $n_{ESA0} = 0$ in ENT modeling) as having nonzero mass and objects that model (via $n_{ESA0} = -1$ in ENT modeling) as having zero mass. (In ENT modeling for elementary fermions, $n_{ESA0} = 0$ associates with nonzero charge and $n_{ESA0} = -1$ associates with zero charge.)
$\lambda = 2$	Modeling for an interaction might associate with, in effect, transmission of a unit of non-zero charge (for example, via a W boson) or transmission of a unit of zero charge (for example, via a Z boson).
$\lambda = 6$	We associate the symbol E_F with a notion of freeable energy. Models need to comport with $E_F > 0$.

Table 19: GFC information regarding G-family solutions for which, for each $\lambda \in \Gamma$, $\lambda \leq 8$

(a) $\Sigma\Phi\Gamma$, GTA symmetries, and other aspects (with NR denoting not relevant)

$\Sigma\Phi\Gamma$	Span (for $\iota_I > 1$)	GTA $SU(\_)$ symmetry	$GTA0$	$GSA0$	$GSA1$ and $GSA2$	$GSA3$ and $GSA4$	$GSA5$ and $GSA6$	$GSA7$ and $GSA8$
0G∅	NR	NR	-1	-1				
2G2	1	None	0	-1	$\pi_{0,@_0}$			
4G4	6	$SU(3)$	0	-1	A0+	$\pi_{0,@_0}$		
ΣG24	1	None	0	-2	$\pi_{0,@_0}$	$\pi_{0,@_0}$		
6G6	2	$SU(5)$	0	-1	A0+	A0+	$\pi_{0,@_0}$	
ΣG26	6	$SU(3)$	0	-2	$\pi_{0,@_0}$	A0+	$\pi_{0,@_0}$	
ΣG46	6	$SU(3)$	0	-2	A0+	$\pi_{0,@_0}$	$\pi_{0,@_0}$	
ΣG246	1	None	0	-3	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$	
8G8	1	$SU(7)$	0	-1	A0+	A0+	A0+	$\pi_{0,@_0}$
ΣG28	2	$SU(5)$	0	-2	$\pi_{0,@_0}$	A0+	A0+	$\pi_{0,@_0}$
ΣG48	2	$SU(5)$	0	-2	A0+	$\pi_{0,@_0}$	A0+	$\pi_{0,@_0}$
ΣG68	2	$SU(5)$	0	-2	A0+	A0+	$\pi_{0,@_0}$	$\pi_{0,@_0}$
ΣG248	6	$SU(3)$	0	-3	$\pi_{0,@_0}$	$\pi_{0,@_0}$	A0+	$\pi_{0,@_0}$
ΣG268	6	$SU(3)$	0	-3	$\pi_{0,@_0}$	A0+	$\pi_{0,@_0}$	$\pi_{0,@_0}$
ΣG468	6	$SU(3)$	0	-3	A0+	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$
ΣG2468	1	None	0	-4	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$

(b) Notes regarding notation that table 19a uses and regarding GTA symmetries

Note
• The symbol A0+ pertains for an oscillator pair for which, for each of the two oscillators, the symbol $@_0$ pertains.
• The symbol $\pi_{0,@_0}$ associates with the notion that either $n_{GSA(odd)} = 0$ and $n_{GSA(even)} = @_0$ pertains or $n_{GSA(odd)} = @_0$ and $n_{GSA(even)} = 0$ pertains. For example, equation (86) and 2G24 associate with $n_{GSA1} = @_0$ and $n_{GSA2} = 0$ and $n_{GSA3} = 0$ and $n_{GSA4} = @_0$. Here, the two values of zero anti-align with respect to odd and even. In contrast, 6G24 associates with $n_{GSA1} = 0$ and $n_{GSA2} = @_0$ and $n_{GSA3} = 0$ and $n_{GSA4} = @_0$. Here, the two values of zero align with respect to odd and even.
• For each row for which table 19a shows a GTA $SU(\_)$ symmetry of none, oscillator $GTA0$ suffices regarding double-entry arithmetic.
• For each row for which table 19a shows a GTA symmetry of $SU(j)$, double-entry arithmetic suggests adding $j-1$ GTA oscillators. For each added $GTAk$ oscillator, the value of n_{GTAk} is zero. The result satisfies double-entry arithmetic. The $SU(j)$ symmetry pairs with mathematics for an isotropic harmonic oscillator that features j component harmonic oscillators. Here, the set of component oscillators includes $GTA0$.

(c) Notes regarding G-family excitations, regarding information that associates with specific $\Sigma G\Gamma$, and regarding the notion of span

Note
• An excitation of a ΣG field does not (directly) encode information about a relevant $\Sigma G\Gamma$.
• For PRι_IISP modeling for which $\iota_I > 1$, the word span denotes the isomers among which a specific instance of a specific $\Sigma G\Gamma$ intermediates interactions.
• For PRι_IISP modeling for which $\iota_I > 1$, this essay tends (when not discussing specific isomers of span-one particles) to use the word span to denote the number of isomers among which a specific instance of a specific $\Sigma G\Gamma$ intermediates interactions. (See, for example, table 19a.)
• For PRι_IISP modeling for which $\iota_I > 1$, an excitation of a ΣG field encodes information that specifies relevant isomers of particles. The number of relevant isomers associates with the Γ of the relevant $\Sigma G\Gamma$. The word span denotes that number of relevant isomers.
• For PRι_IISP modeling for which $\iota_I > 1$, a de-excitation of a ΣG field must associate with an isomer in the list of isomers that associates with the relevant excitation.
• For PR1ISP modeling, there is one isomer of span-one particles and the span is always one.

We start from the span of six that we posit for 4G4. We consider GTA symmetries for G-family solutions. (See table 19a.) We aim to develop numbers that belong in the table 19a column that has the label span (for $\iota_I \geq 6$). The number of generators of each of $SU(3)$, $SU(5)$, and $SU(7)$ divides evenly the integer 48, which is the number of generators of $SU(7)$. Regarding 4G4, we posit that the expression $6 = g_{SU(7)}/g_{SU(3)}$ provides the span. We generalize. We assert that, for each G-family solution for which a GTA symmetry of $SU(j)$ pertains, equation (94) provides the span. We assume that we can generalize from the assumption that the span of 2G2 is one. (Ordinary matter photons do not interact - or, at least, do not interact much - with dark matter.) For each G-family solution with no GTA $SU(\cdots)$ symmetry, the span is one. (Here, we consider that the 0G$\emptyset$ solution is not relevant.) We anticipate that some G-family solutions - for which some λ exceed eight - have relevance and that equation (94) does not pertain. (See discussion related to equation (172).)

$$g_{SU(7)}/g_{SU(j)} \tag{94}$$

Equation (95) shows notation for denoting the span, s, for an elementary particle or for a component of a long-range force.

$$\Sigma(\mathrm{s})\Phi \quad \text{or} \quad \Sigma(\mathrm{s})\Phi\Gamma \tag{95}$$

We explore - regarding GFC modeling - extending the range of λ from the range that equation (87) shows to the range that equation (96) shows.

$$2,\ 4,\ 6,\ 8,\ 16 \tag{96}$$

We consider solutions for which $[\![16]\!]$ is a member of Γ and each one of the other members of λ - of Γ is either two, four, six, or eight. In other words, $\Gamma = \{[\![16]\!]\} \cup \Gamma'$ for some Γ' for which the members comport with equation (87). The equality $g_{SU(17)}/g_{SU(7)} = 288/48 = 6$ pertains. For PR36ISP modeling, we posit that equation (97) pertains. In other words, the span that associates with such a Γ is six times the span that associates with the associated Γ'.

$$\mathrm{s}_{\Gamma} = 6\mathrm{s}_{\Gamma'} \tag{97}$$

Table 20 points to some G-family solutions that one might extrapolate from aspects that underlie table 19.

We discuss notions regarding some aspects of table 20.

We associate the 4G2468$[\![16]\!]$ solution with an attractive component - of 4G - that might dominate early in the evolution of the universe. (See table 21. See discussion related to equation (172).) Regarding 6G46$[\![16]\!]$, discussion related to equation (172) suggests a role early in the evolution of the universe. The 4G246$[\![16]\!]$ solution might associate with an attractive KIN octupole component of 4G. The corresponding force might participate regarding ending the inflationary epoch. (See discussion related to equation (175).)

We pair some 0GΓ solutions with some elementary bosons. (See table 35.)

This essay de-emphasizes the possible physics relevance of some possible extrapolations.

Solution 10G$[\![10]\!]$ provides an example. Per equation (158), a strength factor of four pertains regarding 2G2 and a strength factor of three pertains regarding 4G4. We assume that a strength factor of two pertains regarding 6G6. We assume that a strength factor of one pertains regarding 8G8. We assume that a strength factor of zero pertains regarding 10G$[\![10]\!]$. A lack of physics relevance for 10G$[\![10]\!]$ seems to comport with table 18b.

Regarding other items in table 20, we posit that, for ΣGΓ solutions for which Σ is not zero or six, the combination of a presence of $\lambda = 16$ and an absence of $\lambda = 6$ associates with a lack of relevance to G-family physics. (Possibly, a possibly implied notion of a lack of relevance of - freeable - energy that can convert - via motion - to momentum pertains. Perhaps, note that each one of $\lambda = 16$ and $\lambda = 6$ pertains for 4G2468$[\![16]\!]$, 6G46$[\![16]\!]$, and 4G246$[\![16]\!]$.)

2.2.8. Gravity - GFC modeling and UNI modeling

We discuss gravitational properties of objects. For example, we explore aspects related to components of 4G. (See, for example, 4G4, 4G48, and so forth in table 13.)

We explore PR1ISP modeling.

We discuss adjustments - to the strength of 4G4 - to which table 13 alludes. Data about the rate of expansion of the universe seems to support some of the adjustments. (See table 47.) Modeling regarding the masses of some elementary bosons might echo some of the adjustments. (See discussion regarding equation (109).)

Table 20: Some G-family solutions that one might extrapolate from aspects that underlie table 19

Solutions that associate with table 19 and with the limits $\Gamma \neq \emptyset$ and $\lambda \leq 8$	Other solution, assuming the limits $\Gamma \neq \emptyset$ and $\lambda \leq 16$	Possibilities, regarding the other solution
4G4, 4G48, 4G246, 4G2468x	4G2468⟦16⟧	Might have a PR36ISP span of six. Might associate with a dominant force component for an era two eras before inflation.
6G6	6G46⟦16⟧	Might have a PR36ISP span of 36. Might associate with a significant effect during an era two eras before inflation.
4G4, 4G246	4G246⟦16⟧	Might have a PR36ISP span of six. Might associate with a significant force component around the time of inflation.
0G246, 0G2468	0G2468⟦16⟧	Might associate with the 0I elementary boson.
0G268	0G268⟦16⟧	Might associate with the 2J elementary boson.
2G2, 4G4, 6G6, 8G8	10G⟦10⟧	Seemingly not relevant. The strength of 10G⟦10⟧ would be zero.
2G2, 2G24, 2G248	2G248⟦16⟧	Might have a PR36ISP span of 36. Possibly not necessarily relevant. $6 \notin \Gamma$.
4G4, 4G48	4G48⟦16⟧	Might have a PR36ISP span of 12. Possibly not necessarily relevant. $6 \notin \Gamma$.
8G8	8G8⟦16⟧	Might have a PR36ISP span of six. Possibly not necessarily relevant. $6 \notin \Gamma$.

Table 21 discusses some aspects regarding the strength of gravitation and some components of 4γ plus 2γ. (The table does not discuss 6G46⟦16⟧, which is a component of 6γ.)

Proposed modeling suggests that the results that table 21 shows pertain for KIN Newtonian modeling. We posit that these results are compatible with extant modeling KIN general relativity modeling.

Table 21 uses the three-word term active gravitational energy. In extant modeling, the three-word term active gravitational mass refers to a mass that associates with the gravitational field that an object generates. The three-word term passive gravitational mass refers to a mass that associates with reactions of an object to externally generated gravitational fields. The two-word term inertial mass associates with modeling that links accelerations and forces. (Discussion related to equation (170) includes a possible notion of mass that does not necessarily associate with active gravitational mass, passive gravitational mass, or inertial mass. Perhaps, see also discussion related to equation (168).)

We explore PR6ISP modeling. (Similar results pertain for PR36ISP modeling.)

We consider some related thought experiments.

We consider three cases regarding a non-rotating, spherically symmetric ordinary matter star. Each case involves the idealization of a small, non-rotating, spherically symmetric planet. (The thought experiments do not mention the passive gravitational masses of the planets.) In the first case, the planet includes only ordinary matter. In the second case, the planet includes only the isomer (other than isomer zero) for which 4(2)G48 intermediates repulsion regarding ordinary matter. In the third case, the planet includes only one of the other four isomers. In each case, the planet starts at the same point (relative to the star) and with the same velocity. The orbits of the three planets are identical. (One might say the following. With respect to 4(6)G4, the planets behave identically.)

We vary the three original cases. We assume that the star rotates. Based on 4(2)G48, the following notions pertain. For each of case one and case two, the orbit of the planet changes. Across case one and case two, the orbits are identical. For case three, the orbit matches the orbit pertaining to the cases in which the star does not rotate.

We vary the three original cases. We assume that the star is not spherically symmetric. We assume that the star does not rotate. Based on 4(1)G246, the following notions pertain, relative to the cases in which the star is spherically symmetric and does not rotate. For case one, the orbit of the planet changes. For each of cases two and three, the orbit of the planet does not change.

The thought experiments point to possible difficulties regarding the notion of geodesic motion and regarding the preciseness of modeling based on general relativity.

Table 21: Aspects regarding the strength of gravitation and some components of 4γ plus 2γ

Component and aspect
• 4G48: We consider a thought experiment in which a first object has a spherically symmetric distribution of matter and has no angular momentum. A second object has the same spherically symmetric distribution of the same matter and has some angular momentum. The second object uses more (than does the first object) freeable energy to maintain its shape. (Without use of that energy, the second object would bulge near its equator and flatten near its poles.) A lesser amount of freeable energy associates with a lesser amount of active gravitational energy. (See discussion regarding table 18. Also, perhaps, note a parallel to equation (109).) The first object does not exhibit a 4G48 component of active gravitational rest energy. The second object exhibits a 4G48 component of active gravitational rest energy. 4G48 associates with a repulsive component that detracts from attraction that associates with 4G.
• 4G246: We consider a thought experiment in which a first object has a spherically symmetric distribution of matter and has no angular momentum. A second object has a non-spherically symmetric distribution of the same matter and has no angular momentum. The second object has more (than does the first object) freeable energy. (The second object would - during a transition to having the shape of the first object - lose freeable energy. A greater amount of freeable energy associates with a greater amount of active gravitational energy. See discussion regarding table 18.) The first object does not exhibit a 4G246 component of active gravitational rest energy. The second object exhibits a 4G246 component of active gravitational rest energy. 4G246 associates with an attractive component that augments attraction that associates with 4G.
• 4G246⟦16⟧: We consider a thought experiment in which a first object has a distribution of matter and does not exhibit changes over time. A second object has the same distribution of the same matter and exhibits changes over time. The second object has more (compared to the first object) freeable energy. (The second object would - during a transition to having the characteristics of the first object - lose freeable energy. A greater amount of freeable energy associates with a greater amount of active gravitational energy. See discussion regarding table 18.) The first object does not exhibit a 4G246⟦16⟧ component of active gravitational rest energy. The second object exhibits a 4G246⟦16⟧ component of active gravitational rest energy. 4G246⟦16⟧ associates with an attractive component that augments attraction that associates with 4G.
• 4G2468a and 4G2468b: We consider a thought experiment in which a first object has a non-spherically symmetric distribution of matter and has no angular momentum. A second object has the same non-spherically symmetric distribution of the same matter and has some angular momentum. The second object uses more (than does the first object) freeable energy to maintain its shape. A lesser amount of freeable energy associates with a lesser amount of active gravitational energy. (See discussion regarding table 18.) 4G2468a and 4G2468b associate with repulsive components that detract from attraction that associates with 4G.
• 4G2468⟦16⟧: We consider a thought experiment in which a first object has a distribution of matter, perhaps has some angular momentum, and does not change over time. A second object has the same distribution of the same matter, has the same angular momentum, and exhibits changes over time. The second object has more (compared to the first object) freeable energy. (The second object would - during a transition to having the characteristics of the first object - lose freeable energy. A greater amount of freeable energy associates with a greater amount of active gravitational energy. See discussion regarding table 18.) The first object does not exhibit a 4G2468⟦16⟧ component of active gravitational rest energy. The second object exhibits a 4G2468⟦16⟧ component of active gravitational rest energy. 4G2468⟦16⟧ associates with an attractive component that augments attraction that associates with 4G.
• 2G2: We consider a thought experiment in which a first object has a spherically symmetric distribution of matter and has no charge. A second object has the same spherically symmetric distribution of the same matter and has some net charge. The second object uses more (than does the first object) freeable energy to maintain its net charge. (Without use of that energy, the charge would repel itself and the object would bulge outward.) A lesser amount of freeable energy associates with a lesser amount of active gravitational energy. (Perhaps, note a parallel to equation (109). Perhaps, also, consider solutions - to the Einstein field equations - regarding a spherically symmetric non-rotating charged object.) Net charge associates with a repulsive component that detracts from attraction that associates with 4G.

We explore notions regarding the applicability of general relativity.

We explore thought experiments re the bending of paths of light.

We consider the bending of the path of light via the gravity associated with the sun. The sun associates with the isomer that includes ordinary matter. The light associates with the isomer that includes ordinary matter. Extant modeling based on general relativity and an appropriate stress-energy tensor works. From a standpoint of proposed modeling, PR1ISP models suffice. We think that extant modeling general relativity and proposed modeling PR1ISP models are mutually compatible.

We consider the bending of the path of light via gravity associated with a galaxy cluster. First, we assume that the galaxy cluster contains equal amounts of the relevant six isomers (one mostly ordinary matter and five exclusively dark matter), that we can ignore rotation, and that we can ignore deviations from spherical symmetry. The stress-energy tensor would associate with equal contributions from each of the six isomers. The light associates with one isomer. Seemingly, modeling based on general relativity works. Next, we relax one or more of the assumptions regarding rotation and spherical symmetry. Regarding allowing just rotation, 4(2)G48 pertains. Two - and not six - isomers impact the trajectories of light. We assume that each isomer contributes to rotation similarly to each other isomer. Modeling via general relativity over-estimates effects of 4(2)G48 by a factor of three. Regarding allowing just an irregular (and not rotating) distribution of stuff (and assuming that each of the six isomers distributes in a manner similar to that of the other isomers), general relativity over-estimates effects - that associate with 4(1)246 - by a factor of six.

We explore an association between UNI modeling and extant modeling KIN modeling based on general relativity.

We explore the notion that double-entry arithmetic suggests a UNI modeling temporal complement to associations between object properties of values of $USA\lambda$. (See table 18.) Presumably, 17 temporal oscillators pertain. We use the symbol λ_T to index one such oscillator and eight pairs of oscillators.

Table 22 posits a temporal parallel to the spatial table 18. Aspects of table 22a associate with fractional-charge elementary fermions and with color charge. Aspects of table 22a associate directly with isomers. Beyond those aspects, the table alludes to 10 oscillators ($UTA7$ through $UTA16$) that might associate with gravitation.

Table 23 discusses combinations of spans for components of gravitational forces and matching sets of isomers.

2.2.9. Elementary particles: fields, particles, and handedness - FIP modeling

We discuss aspects of FIP modeling regarding elementary particles. Aspects include conjecture that led to our developing FIP modeling, PDE modeling that points to model-centric relationships between fields and particles, a suggested limit on the spins of some types of elementary particles, and modeling pertaining to handedness of elementary particles.

We discuss conjecture that leads to our exploring FIP modeling.

The term - in equation (21) - that includes a factor of r^{-2} might associate with a spatial dependence that associates with the electromagnetic force and, hence, with photons. (A similar association might pertain for the gravitational force and, hence, with gravitons.) The association might be based on the square of potential energy. The term - in equation (20) - that includes a factor of r^2 might associate with a spatial dependence that associates with the strong force and, hence, with gluons. The association might be based on the square of potential energy.

We conjecture that equation (20) associates with an operator and that solutions to equation (20) associate with elementary particles other than G-family and U-family elementary particles.

We define the two-element term FIP-solution particles to denote all elementary particles - other than G-family elementary particles and U-family elementary particles - that proposed modeling FIP modeling matches directly or suggests indirectly. We think that FIP-solution particles include - at least - all elementary particles other than G-family and U-family elementary particles. This essay does not explore the notions that some G-family elementary particles or U-family elementary particles might be FIP-solution particles.

Table 24 lists some notions that pertain for some applications of PDE modeling. (Regarding the symbol D^*_{PSA}, see equation (49). Regarding the symbol D^*_{PTA}, see equation (50).)

We explore bounds regarding the FIP-solution particles that proposed modeling suggests.

Table 25 lists aspects that proposed modeling posits to associate with modeling for FIP-solution elementary particles. (See table 24.) Table 25 limits the range of relevant subfamilies. The table does not specify the number of subfamilies that nature embraces or the number of elementary particles within each subfamily.

Table 22: Associations between phenomena and values of $UTA\lambda_T$

(a) Aspects for $UTA\lambda_T = 0, 2, 4, 6, \ldots,$ or 16

Property	λ_T	Scalar example	Trio example	Six-fold aspect	Related property (that some KIN Newtonian models use)	Constant that associates with a property
Object type	0	n_{UTA0}	(See table 22b.)	-	-	-
Isomers (with respect to 2G components) - I(0,...,5;i_4) with one non-changing value of i_4.	2	Up to 6 isomers	$3 = 6/2$	6	-	-
Isomers (with respect to 4G components) - I(i_2;0,...,5) with one non-changing value of i_2.	4	Up to 6 isomers	$3 = 6/2$	6	-	-
Color charge	6	clear	red, blue, green	$3 \to 2$	-	-
Aspects of gravitation	8 - 16	-	-	See table 22b.)	Mass m	-

(b) Notes

Aspect	Note
$\lambda_T = 0$	We associate one value (out of zero and minus one) of n_{UTA0} with modeling that associates objects with entanglement with other objects. (For ENT modeling, $n_{ETA0} = -1$ pertains.) The other value of n_{UTA0} associates with modeling that associates with no entanglement with other objects. (For ENT modeling, $n_{ETA0} = 0$ pertains.) This notion of duality extends to a notion of a trio. The notion of models as entangled divides into two facets of modeling. One facet associates with entanglement between gluons and elementary fermions for which $n_{ETA0} = -1$. One facet associates with other modeling that associates with entanglement.
$\lambda_T \geq 8$	• The following remarks pertain regarding a combination of PR1ISP modeling and extant modeling KIN models based on general relativity. The related 10 oscillators might associate with contributions - by the object - to the 10 independent components of a stress-energy tensor. • The following remarks pertain regarding PR36ISP modeling or PR6ISP modeling. The related 10 oscillators might associate with 10 combinations of spans for components of gravitational forces and matching sets of isomers. (See table 23.)

Table 23: Combinations of spans for components of gravitational forces and matching sets of isomers, assuming PR36ISP modeling or PR6ISP modeling

Notion
• Across G-family force components that have a span of one, each instance of the set of force components associates with one isomer. Overall, there are six isomers. Six pairings of a set of span-one G-family force components and a matching set of isomers pertain. For example, regarding PR36ISP modeling and a fixed value of i_4, the six pairings associate respectively with I(0;i_4), I(1;i_4), I(2;i_4), I(3;i_4), I(4;i_4), and I(5;i_4). • Across G-family force components that have a span of two, each instance of the set of force components associates with two isomers. Overall, there are six isomers. Three pairings of a set of span-two G-family force components and a matching set of isomers pertain. For example, regarding PR36ISP modeling and a fixed value of i_4, the three pairings associate respectively with I(0,3;i_4), I(1,4;i_4), and I(2,5;i_4). • Across G-family force components that have a span of six, each (or, the one) instance of the set of force components associates with six isomers. Overall, there are six isomers. One pairing of a set of span-six G-family force components and a matching set of isomers pertains. For example, regarding PR36ISP modeling and a fixed value of i_4, the one pairing associates with I(0,1,2,3,4,5;i_4). • The sum of six, three, and one is ten.

Table 24: Some notions that pertain for some applications of FIP modeling

Notion
• The symbol S denotes spin divided by $\hbar$. The symbol $\hbar$ denotes the reduced Planck's constant. • For some solutions - which comport with equation (51) - to equation (25), $D_{PSA} \neq D^*_{PSA}$. • Solutions for which $\nu_{PSA} = -1/2$ can associate with notions of fields for FIP-solution fermions. • Solutions for which $\nu_{PSA} = -1$ can associate with notions of fields for FIP-solution bosons. • Solutions for which $\nu_{PSA} = -3/2$ can associate with notions of particles for FIP-solution fermions. • PTA aspects of PDE solutions are radial with respect to t, the PTA analog to the PSA radial coordinate r. • For some PDE solutions, $D_{PTA} \neq D^*_{PTA}$.

Table 25: Aspects that proposed modeling posits to associate with modeling for FIP-solution elementary particles

Aspect
• Each FIP-solution elementary fermion associates with a subfamily for which a $\nu_{PSA} = -1/2$ solution exists. The solution associates with the notion of fields for the elementary particles in the subfamily. The solution associates with the notion of volume-like. • Each FIP-solution elementary fermion associates with a subfamily for which a $\nu_{PSA} = -3/2$ solution exists. The solution associates with the notion of particles for the elementary particles in the subfamily. The solution associates with the notion of point-like. • Each FIP-solution elementary boson associates with a subfamily for which a $\nu_{PSA} = -1$ solution exists. The solution associates with the notion of fields for the elementary particles in the subfamily. The solution associates with the notion of volume-like. • Each FIP-solution elementary boson associates with a subfamily for which a $\nu_{PSA} = -1$ solution exists for each of three oscillator pairs. The trio of solutions associates with the notion of particles for the elementary particles in the subfamily. The solutions associate with the notion of point-like. • For each such solution, the relevant $\Omega_{...}$ is nonnegative, the relevant $\sigma_{...}$ is plus one, the relevant $2S_{...}$ is a nonnegative integer, and the relevant $D_{...}$ is a positive integer.

The order of rows in table 5b associates with non-decreasing values of Ω_{PSA}. A value of spin S associates with the value of Ω_{PSA}. Proposed modeling posits that each FIP-solution elementary particle associates with a field. Proposed modeling posits that D_{PSA} must be a positive integer. No larger values of S comport with equation (98). (For example, for fermion fields, $S = 3/2$ would associate with $\Omega_{PSA} = 15/4$ and with a negative value, -5, for D_{PSA}.) Equation (99) associates with a limit that pertains regarding FIP-solution particles. (See table 25. Also, our assumptions regarding the existence of FIP-solution particles include excluding solutions for which $\sigma_{PSA} = -1$. See table 5d. If we included solutions for which $\sigma_{PSA} = -1$, table 5d indicates a possibility for indefinitely large values of S.) We do not expect that nature embraces FIP-solution particles with spins other than zero, one-half, and one.

$$S \geq 0 \text{ and } D \geq 1 \tag{98}$$

$$0 \leq S \leq 1 \tag{99}$$

We explore modeling regarding the FIP-solution particles that proposed modeling suggests. This exploration pertains within the bounds that equations (98) and (99) imply.

Tables 5b and 5c show solutions that associate with fields for all relevant elementary particle cases. (Fields for FIP-solution elementary bosons associate with $\nu_{PSA} = -1 = \nu_{PTA}$. Relevant rows in the tables associate with $2S = 0$ and with $2S = 2$. Fields for FIP-solution elementary fermions associate with $\nu_{PSA} = -1/2 = \nu_{PTA}$. Relevant rows in the tables associate with $2S = 1$.) Tables 5b and 5c show solutions that associate with particles for all relevant elementary fermion cases. (Particles for FIP-solution elementary fermions associate with $\nu_{PSA} = -3/2 = \nu_{PTA}$. Relevant rows in the tables associate with $2S = 1$.) The tables do not discuss particles for relevant elementary boson cases.

Table 5b includes a column with label $D^*_{PSA} + 2\nu_{PSA}$. We use the symbol D' to denote D^*_{PSA}. Table 5c includes a column with label $3 + 2\nu_{PTA}$. We use the symbol D' to denote the three. These two columns comport with the notion that a relevant $D' + 2\nu_{PXA}$ should be positive for fields, which should associate with the notion of volume-like. These two columns comport with the notion that a relevant $D' + 2\nu_{PXA}$ should be zero for particles, which should associate with the notion of point-like. For each of tables 5b and 5c, $D' = 3$.

We pursue discussion based on relevance of the three PTA oscillators $PTA0$, $PTA1$, and $PTA2$ and three PSA oscillators $PSA0$, $PSA1$, and $PSA2$. (Compare with equation (54).)

In general, use of equation (54) allows separation of terms into clusters. Equation (54) is a sum of D_{PXA} terms. Each one of the D_{PXA} terms appears in exactly one cluster. For $D_{PXA} = 1$, there is one term (which associates with the $PXA0$ oscillator) and one cluster (which contains the one term). For $D_{PXA} = 3$, we use two clusters. One cluster associates with the $PXA0$ oscillator. One cluster associates with the $PXA1$-and-$PXA2$ oscillator pair. In these and similar cases, we apply - for each two-oscillator cluster - an analog to equations (20) and (21).

Here, specifically, $D_{PTA} = D_{PSA} = D' = 3$.

We anticipate aspects regarding modeling - for fields and particles - for FIP-solution bosons and FIP-solution fermions.

For each of fields for FIP-solution bosons and fields for FIP-solution fermions, modeling points to the notion that, for relevant choices of sets of oscillators and of D, equation (100) pertains. For fields for FIP-solution bosons, $\nu_{PSA} = -1$. For fields for FIP-solution fermions, $\nu_{PSA} = -1/2$. The notion of volume-like associates with equation (100).

$$D + 2\nu_{PSA} = 1 \tag{100}$$

For each of particles for FIP-solution bosons and particles for FIP-solution fermions, modeling points to the notion that, for relevant choices of sets of oscillators and of D, equation (101) pertains. For particles for FIP-solution bosons, $\nu_{PSA} = -1$. For particles for FIP-solution fermions, $\nu_{PSA} = -3/2$. The notion of point-like associates with equation (101).

$$D + 2\nu_{PSA} = 0 \tag{101}$$

This essay does not further explore the notion that modeling based on ladder operators might associate with transitions - between field states and particle states - that notions related to equations (100) and (101) suggest. (See table 6.)

We discuss modeling for fields for FIP-solution fermions. The expression $S = 1/2$ pertains.

Regarding modeling for fields for FIP-solution fermions, the $D_{PSA}+2\nu_{PSA}$ column in table 5b shows a value of two. The $3+2\nu_{PTA}$ column in table 5c shows a value of two. Seemingly, equation (100) might not pertain.

We focus on aspects that associate with fields that associate with fermion subfamilies 1Φ.

Regarding fields for elementary fermions, modeling can feature an effective $D_{\dagger}=2$ instead of $D'=3$. (Each elementary fermion associates with one - not two - values for handedness. For example, each known matter elementary fermion associates with left handedness and not with right handedness. Each known antimatter elementary fermion associates with right handedness and not with left handedness. A reduction from $D'=3$ dimensions to $D_{\dagger}=2$ associates, in effect, with the lack - for each particle - of a second handedness. Perhaps, note discussion - regarding photon modes - related to table 9. Also, perhaps, note table 3.) For $D_{\dagger}=2$, $D_{\dagger}+2\nu_{PSA}=D_{\dagger}+2\nu_{PTA}=1$. The notions of volume-like and field still pertain. Equation (100) pertains.

We focus on aspects that associate with fields that associate with individual elementary particles (or, individual generations) within fermion subfamilies 1Φ. In so doing, we shift our attention to aspects that are somewhat separate from aspects associating with $D_{\dagger}=2$.

From $D_1=D'=3$, proposed modeling applies the transformation that associates with equation (36). (Perhaps note that, in equation (36), $j=2$ and that, regarding discussion here, $j\nu_{PSA}$ is an integer.) The result $D_2=(2\cdot 3)-2=4$ pertains. We bring together aspects associating with $D_{\dagger}=2$ and aspects associating with $D_2=4$. The result $D_2-D_{\dagger}=4-2=2$ pertains. In effect, the transformation - from D_1 to D_2 adds - compared to models for which $D_{\dagger}=2$ pertains - two PTA oscillators and two PSA oscillators. Equation (100) pertains for each of $D=D_{\dagger}=2$ and $D=D_2-D_{\dagger}=2$. We associate the additional pair of PSA oscillators with a breakable $SU(2)$ symmetry and with three generations. We associate the additional pair of PTA oscillators with - for isolated interactions - conservation of fermion generation. (Perhaps, see table 53.)

We discuss modeling for particles for FIP-solution fermions.

Table 5b shows $D_{PSA}=3$ and $D_{PSA}+2\nu_{PSA}=0$. Table 5c shows $D_{PTA}=3$ and $D_{PTA}+2\nu_{PTA}=0$. Equation (101) pertains. We can reuse results that pertain for fields for FIP-solution fermions.

We discuss modeling for fields for FIP-solution bosons.

Regarding modeling for fields for $S=0$ FIP-solution bosons, one can use results that tables 5b and 5c show. Here, $D_{PSA}=3$, $D_{PSA}+2\nu_{PSA}=1$, $D_{PTA}=3$, $D_{PTA}+2\nu_{PTA}=1$. Equation (100) pertains. Two PTA oscillators associate with - for isolated interactions - conservation of fermion generation. (Perhaps, see table 53.) Two PSA oscillators associate with a lack of spin and, thus, with no handedness.

Regarding modeling for fields for $S=1$ FIP-solution bosons, one can use the notion of mapping the $D_{PSA}=1$ solutions - that tables 5b and 5c show - into the three dimensions that associate with $D'=3$. (For each of PDE modeling and KIN modeling, the $D_{PSA}=1$ solution has or would have no dependence on angular coordinates.) The mapping obviates concerns - about normalization - that tables 5b and 5c flag based on the results that $D_{PSA}+2\nu_{PSA}=-1$ and $D_{PTA}+2\nu_{PTA}=-1$. After the mappings, each one of the PSA aspect and the PTA aspect normalizes and associates with equation (100). (After the mappings, $D_{PSA}+2\nu_{PSA}$ is one and $3+2\nu_{PTA}$ is one.) Two PTA oscillators associate with - for isolated interactions - conservation of fermion generation. One PSA oscillator associates with whether the bosons have nonzero mass or zero mass. For the case of nonzero mass, of the other two PSA oscillators, one oscillator associates with one handedness (that is, left handedness for ordinary matter W bosons) and one oscillator associates with no handedness (or, longitudinal polarization). For the case of zero mass, of the other two PSA oscillators, one oscillator associates with left circular polarization and one oscillator associates with right circular polarization.

We discuss modeling for particles for FIP-solution bosons.

For FIP-solution bosons, we expect that modeling regarding particles associates with the equations $D''=2$, $\nu''=-1$ and $D+2\nu''=0$. (See tables 25 and 5e.) We base this expectation on the notion that, for FIP-solution elementary fermions, modeling regarding particles associates with the expression $D_{PTA}+2\nu_{PTA}=0=D_{PSA}+2\nu_{PSA}$. (See equation (101) and tables 5b and 5c.)

Regarding modeling for particles for FIP-solution bosons, we start from the $D_{PTA}=D_{PSA}=D'=3$ models for fields. We use the clusters $PTA1$-and-$PTA2$, $PTA0$-and-$PSA0$, and $PSA1$-and-$PSA2$. For each cluster, we use the equations $D''=2$, $\nu''=-1$ and $D+2\nu''=0$.

Regarding modeling for particles for $S=1$ FIP-solution bosons, notions - such as three oscillator pairs - that pertain for fields for $S=1$ FIP-solution bosons continue to pertain.

Regarding modeling for particles for $S=0$ FIP-solution bosons, the following notions pertain. The perhaps seemingly oscillator pair $PTA1$-and-$PTA2$ associates with the notion of - for isolated interactions

Table 26: Interaction vertices for interactions involving only span-one particles and long-range forces (with ν denoting the effective ν)

Interaction	ν	Example	Note
0f+1b↔2f+0b	-1	A Z boson creates a matter-and-antimatter pair of fermions.	-
1f+1b↔1f+1b	$-3/2$	An electron and a W^+ boson produce a neutrino.	Modeling for one instance of 1b might associate with 0I or 2J.
1f+1b↔3f+0b	$-3/2$	A matter fermion and a boson produce two matter fermions and one antimatter fermion.	-
(3f+0b↔3f+0b)	$-3/2$	-	-
0f+nb↔0f+nb, for n≥2	$-n$	A Higgs boson creates two photons.	Modeling for one instance of 1b might associate with 0I or 2J.
(1f+nb↔1f+nb, for n≥2)	$-n-(1/2)$	-	Modeling for each one of some instances of 1b might associate with 0I or 2J.

- conservation of fermion generation. The perhaps seemingly extra oscillator pair $PSA1$-and-$PSA2$ associates with a lack of circular polarization.

2.2.10. Elementary particles: interaction vertices - FIP modeling

We explore notions that underlie possible supplementary proposed modeling models regarding interaction vertices. (Perhaps, see aspects, that mention $\nu_{PSA} < 0$, of table 24.)

This work generalizes from work above that, nominally, pertains for FIP-solution elementary particles. Equations (49) and (50) pertain regarding all elementary particles. We posit that results - regarding some roles for ν_{PSA}, ν_{PTA}, and ν'' - from that work extend to all elementary particles. (See, for example, table 5b.)

This work need not completely match extant modeling regarding interaction vertices. Extant modeling notions of interaction vertices reflect modeling that has bases in equations that are linear in energy (and in $\hbar$). Relevant proposed modeling has bases in equations that are quadratic in energy (and in $\hbar$). Because this work associates with supplementary proposed modeling, this work does not necessarily point to lacks of compatibility between core proposed modeling and core extant modeling.

Table 26 lists types of interaction vertices that one aspect of supplementary proposed modeling includes. Here, in the symbol nf, n denotes a number of elementary fermions. In the symbol nb, n denotes a number of elementary bosons. A symbol of the form a↔b denotes two cases, namely a→b and b→a. A symbol of the form a→b denotes the notion that the interaction de-excites each component of a by one unit and excites each component of b by one unit. (Note, for example, that de-excitation of a photon mode does not necessarily produce a ground state. Note, for example regarding the 1f+1b↔1f+1b row in table 26, that 1b can associate - at least mathematically - with the aye boson or with the jay boson.) For each type of interaction vertex, the effective ν is the sum, over incoming field solutions, of the relevant ν_{PSA} and is also the sum, over outgoing field solutions, of the relevant ν_{PSA}. In effect, the value of effective ν can associate with aspects of a product of solutions of the form that equation (23) shows. The case 3f+0b↔3f+0b pertains mathematically, but does not explicitly involve bosons. We are uncertain, in the current context, as to the possible relevance of 3f+0b↔3f+0b. In a broader context, 3f+0b↔3f+0b might point toward possibilities for extending work herein. Elsewhere, we use the case 1f+nb↔1f+nb, with n≥2, mathematically. (See, for example, equation (161).)

Proposed modeling suggests that the notion of 3f does not necessarily violate extant modeling notions of fermion statistics. Supplementary proposed modeling features aspects that might appear to aggregate extant modeling KIN modeling QFT (or, quantum field theory) interactions. (For one example, supplementary proposed modeling does not necessarily require notions of virtual particles. For this example, supplementary proposed modeling appears to aggregate multiple QFT Feynman diagrams. For another example, supplementary proposed modeling points toward modeling that replaces bosons with potentials.) Leaving aside the notion of aggregation of interactions, 3f can involve dissimilar elementary fermions. Dissimilarity can associate with differences regarding generations; matter and antimatter; and (if nothing else) types of span-one particle - neutrino, charged lepton, quark, or arc.

2.3. Elementary particles and dark matter

Table 27 previews elementary particles that proposed modeling suggests. Table 27 alludes to all known elementary particles and to elementary particles that proposed modeling suggests. Elsewhere, we depict some aspects regarding subfamilies.

Table 28 explores the following analogy. Elementary particle is to subfamily as atom is to chemical element.

Discussion related to table 40 provides details about proposed modeling regarding dark matter. Table 41 alludes to data - related to dark matter - that proposed modeling seems to explain. (For more details, see table 48.) Elsewhere, we depict some aspects regarding dark matter and ordinary matter.

2.3.1. Elementary particles

We show a method for matching known elementary particles and suggesting new elementary particles. We use the method. We suggest elementary particles that people have yet to find.

This work features ENT modeling. We discuss subfamilies of elementary particles. We discuss elementary particles.

Table 29 previews aspects of our work to match and suggest elementary particles. (The order of the rows in table 29a associates with discussion that develops the table. The order of the rows in table 27a associates with values of spin. The two orderings do not match each other.) In table 29a, the leftmost six columns show representations for subfamilies. Each representation satisfies double-entry arithmetic. The column with the one-element label ΣΦ shows the subfamily that pertains. (Regarding 1Q - or, quarks - the table devotes one row to each of the two magnitudes of charge.) Table 29b explains aspects of table 29a. Table 29b notes associations between $ETA\lambda$ in table 29a and $USA\lambda$ in table 18.

We review proposed modeling ENT models for the photon. We note an association between proposed modeling ENT models and the extant modeling elementary particle Standard Model.

Table 8 pertains. Proposed modeling suggests that aspects related to oscillator $ETA0$ might associate with the extant modeling Standard Model notion that a $U(1)$ internal symmetry pertains regarding the photon.

We discuss proposed modeling ENT models for the weak interaction bosons.

Each of the Z and W bosons has nonzero mass. Three spin states can pertain. Regarding KIN modeling, equation (102) pertains for ground states. The ENT equation (103) pertains for ground states. We extend work regarding 2G. We associate $ESA1$ with left circular polarization. We associate $ESA2$ with right circular polarization. We associate $ESA0$ with longitudinal polarization.

$$n_{KSA0} = 0,\ n_{KSA1} = 0,\ n_{KSA2} = 0 \tag{102}$$

$$n_{ESA0} = 0,\ n_{ESA1} = 0,\ n_{ESA2} = 0 \tag{103}$$

A combination of double-entry arithmetic and table 29b suggests that equation (104) pertains. We associate n_{ETA2} with the W$^+$ boson and with positive charge. We associate n_{ETA1} with the W$^-$ boson and with negative charge. We associate n_{ETA0} with the Z boson and with zero charge. Equation (105) pertains for ground states.

$$\{ETA\} = \{ETA2, ETA1, ETA0\} \tag{104}$$

$$n_{ETA0} = 0,\ n_{ETA1} = 0,\ n_{ETA2} = 0 \tag{105}$$

We discuss a thought experiment that associates with the extant modeling notion of an excitation of one W$^-$ boson during an isolated interaction that converts an electron into a neutrino. For such an interaction, extant modeling suggests that the generation associated with the neutrino equals the generation (which is generation one) associated with the electron. Proposed modeling suggests modeling in which - for the W$^-$ boson - the $ETA1$ oscillator excites by one unit and one of the three $ESAj$ oscillators excites by one unit. The two other ETA oscillators do not excite. We posit that the pair of two ETA oscillators that do not excite associates with - for the interaction (or, interaction vertex) - conservation of fermion generation. We note that table 30 pertains.

Table 27: Known and proposed elementary particles

(a) Known and proposed elementary particles (with SM denoting known or Standard Model; with PM denoting proposed or proposed modeling; with (Di) denoting the seven-word phrase if the particles model as Dirac fermions; with (Ma) denoting the seven-word phrase if the particles model as Majorana fermions; and with TBD denoting the three-word phrase to be determined)

Description	Sub-family	Spin	Can model as Free; or, always models as Entangled	Mass	Number of zero-charge particles (includes anti-particles)	Number of charged particles (includes anti-particles)	Number of modes	Status
Higgs boson	0H	0	Free	>0	1	0	-	SM
Aye	0I	0	Entangled	=0	1	0	-	PM
Quarks	1Q	1/2	Entangled	>0	0	12	-	SM
Charged leptons	1C	1/2	Free	>0	0	6	-	SM
Neutrinos	1N	1/2	Free	>0	6(Di) or 3(Ma)	0	-	SM
Arcs	1R	1/2	Entangled	>0	6(Di) or 3(Ma)	0	-	PM
Weak interaction bosons	2W	1	Free	>0	1	2	-	SM
Jay	2J	1	Entangled	=0	1	0	-	PM
Gluons	2U	1	Entangled	=0	8	0	-	SM
Photon	2G	1	Free	=0	-	-	2	SM
Graviton	4G	2	Free	=0	-	-	2	PM
TBD	6G	3	Free	=0	-	-	2	PM
TBD	8G	4	Free	=0	-	-	2	PM

(b) Notes regarding items designated as PM in table 27a

Item	Note
0I	Aye (or, inflaton) - would be a zero-mass analog to the Higgs boson; might have a role during the inflationary epoch
1R	Arcs - would be zero-charge fermions; would be analogs to quarks; might be components of (dark matter) hadron-like particles
2J	Jay - would be a zero-mass spin-one boson; might have a role before inflation; might associate with the Pauli exclusion force
4G	Graviton - would be a zero-mass spin-two boson; might associate with extant modeling notions regarding quantum gravity
6G	Name to be determined - would be a zero-mass spin-three boson; might associate with some aspects of observations which people interpret as implying that there are at least two distinct rest energies for neutrinos
8G	Name to be determined - would be a zero-mass spin-four boson; might associate with observations which people interpret as implying that there are at least two distinct rest energies for neutrinos

Table 28: An analogy regarding modeling for elementary particles and modeling for atoms (with PM denoting proposed modeling)

An elementary particle models as ...	An atom models as ... (with $((\ldots))$ denoting a PM suggestion regarding extant modeling)
• Associating with a subfamily	• Associating with a chemical element
• Associating with a specific PM isomer of span-one particles	• ((Associating with a specific PM isomer of span-one particles))
• Being - or not being - entangled	• Being - or not being - part of a molecule or other structure
• Having a specific charge	• Having a specific charge
• Having a specific mass	• Having a specific mass
• Having a specific spin state	• Having a specific spin state
-	• Associating with a specific nuclear isotope
-	• Associating with a specific (nuclear) isomer of the isotope
• (If it is a fermion,) having a specific generation	-

Table 30 summarizes aspects that we posit - regarding modeling for elementary bosons - that associate with possible changes - during an interaction - in property value for an elementary fermion.

We discuss a thought experiment that associates with extant modeling notions of CP violation within a hadron. Extant modeling considers the production of two virtual W bosons. Proposed modeling suggests that the exciting once each of a W$^+$ and a W$^-$ associates with modeling that leaves - among ETA oscillators that do not excite - just one ETA oscillator - the $ETA0$ oscillator. A lack of conservation of fermion generation can pertain. Table 30 pertains.

Proposed modeling suggests that aspects related to oscillators $ETA2$, $ETA1$, and $ETA0$ might associate with the extant modeling Standard Model notion that an $SU(2)\times U(1)$ symmetry pertains regarding the weak interaction bosons. From the ground state and for any j such that $j \in \{2,1,0\}$, proposed modeling associates - with an excitement of n_{ETAj} - a $U(1)$ symmetry with oscillator $ETAj$. An $SU(2)$ symmetry associates with the ground states for the other two $ETAk$ oscillators. (See table 29.)

We discuss proposed modeling ENT models for the 0H subfamily (and, hence, for the Higgs boson).

Proposed modeling interpretation of extant modeling for the Higgs boson associates with the set $\{KSA\}$ having one member - $KSA0$. Longitudinal polarization and nonzero mass pertain. Circular polarization does not pertain.

Proposed modeling ENT models use the notion that excitation associates with the oscillator pair $ETA0$-and-$ESA0$. For a ground state, $n_{ETA0} = n_{ESA0} = 0$. For one excitation, $n_{ETA0} = n_{ESA0} = 1$.

Adding the notions that $n_{ETA2} = n_{ETA1} = n_{ESA1} = n_{ESA2} = -1$ comports with known phenomena and with double-entry arithmetic. (Also, the addition comports with aspects of FIP modeling. See discussion - related to equation (101) - of modeling for fields for $S = 0$ FIP-solution bosons.) The notion of $n_{ESA1} = n_{ESA2} = -1$ comports with spin zero. The notion of $n_{ETA2} = n_{ETA1} = -1$ comports with table 30. Conservation of elementary fermion generation pertains regarding interactions with elementary fermions.

Proposed modeling suggests - paralleling aspects for 2G - that a $U(1)$ symmetry pertains regarding the Higgs boson. We are uncertain as to the extent to which the symmetry that proposed modeling suggests might associate with a possible Standard Model internal symmetry.

We discuss proposed modeling ENT models for the aye (or, 0I) boson.

ENT modeling for the aye boson reflects aspects of ENT modeling for the Higgs boson. For the aye boson, $n_{ETA0} = -1$ and $n_{ESA0} = -1$ pertain for the ground state. The expression $n_{ESA0} = -1$ associates with zero mass. Excitation associating with n_{ETA0} can occur in entangled environments. Conservation of fermion generation pertains.

We assume that - for the aye boson - the notion of excitement associates essentially only with higher density (of energy) environments than does the notion of excitement for 2U elementary bosons. For 2U bosons, $n_{ETA0} = -1$. People observe effects of 2U bosons in hadrons.

This essay de-emphasizes the notion that modeling for the aye boson might, in effect, inherit a $U(1)$ symmetry from modeling for the Higgs boson.

We discuss proposed modeling ENT models for the jay (or, 2J) boson.

ENT modeling for the jay boson reflects aspects of ENT modeling for the Z and W bosons.

Table 29: Representations for elementary particle subfamilies

(a) Representations for subfamilies

n_{ETA7}-and-n_{ETA8}	n_{ETA5}-and-n_{ETA6}	n_{ETA1}-and-n_{ETA2}	n_{ETA0}	n_{ESA0}	Σ_S	$\Sigma\Phi$	ETA symmetry (bosons)
-	-	-	0	-1	0,0	2G	$U(1)$
-	-	0,0	0	0	0,0	2W	$SU(2) \times U(1)$
-	-	-	0	0	-	0H	$U(1)$
-	-	-	-1	-1	-	0I	-
0,0	-	-	-1	-1	0,0	2J	$SU(2)$
-	$-1, -1$	-	-1	-1	$-1, -1$	2U	$SU(3)$
-	-	$\pi_{0,-1}$	0	0	$\pi_{0,-1}$	1C	- -
-	-	$-1, -1$	0	-1	$\pi_{0,-1}$	1N	- -
-	0,0	$\pi_{0,-1}$	-1	0	$\pi_{0,-1}$	$1\mathrm{Q}^{\lvert 2/3 \rvert}$	- -
-	0,0	$\pi_{0,-1}$	-1	0	$\pi_{0,-1}$	$1\mathrm{Q}^{\lvert 1/3 \rvert}$	- -
-	0,0	$-1, -1$	-1	-1	$\pi_{0,-1}$	1R	- -
-	-	-	0	-1	0,0	4G	$U(1)$
-	-	-	0	-1	0,0	6G	$U(1)$
-	-	-	0	-1	0,0	8G	$U(1)$

(b) Notes

Note
• We make the following associations. (The choices do not seem to cause undo loss of generality. The choices reflect aspects of UNI modeling.) ○ We associate $ETA7$ and $ETA8$ with spin. (Perhaps, compare with table 18.) ○ We associate $ETA5$, $ETA6$, and $ETA0$ with color charge. (Perhaps, compare with table 22.) ○ We associate $ETA1$ and $ETA2$ with charge. (Perhaps, compare with table 18.) ○ We associate - for elementary fermion particles (but not necessarily for elementary particle subfamilies) - $ETA11$, $ETA12$, $ESA5$, and $ESA6$ with aspects regarding fermion generation. The relationships $n_{ETA11} = n_{ETA12} = n_{ESA5} = n_{ESA6} = -1$ pertain. (Perhaps, see table 18b and table 32.) • Regarding n_{ETA0}, the value 0 associates with the notion that modeling can associate with the notion of a free particle and the value -1 associates with the notion that modeling associates with the notion of entanglement. • Regarding elementary bosons and n_{ESA0}, the value 0 associates with nonzero (positive) mass and the value -1 associates with zero mass. • Regarding elementary fermions and n_{ESA0}, the value 0 associates with nonzero charge and the value -1 associates with zero charge. • Regarding Σ_S, the following aspects pertain. ○ For subfamilies that are not part of the G family, the choice n_{ESA1}-and-n_{ESA2} pertains regarding spin. The choice echoes table 8. ○ For subfamilies of the G family and Σ_S, choices that echo table 9 pertain. For example, for 4G, the choice n_{ESA3}-and-n_{ESA4} pertains. • The symbol $\pi_{0,-1}$ points to two physics relevant possibilities. For one possibility, $n_{EbA(odd)} = 0$ and $n_{EbA(even)} = -1$. For the other possibility, $n_{EbA(odd)} = -1$ and $n_{EbA(even)} = 0$. • Regarding ETA boson symmetries, the following notions pertain. ○ For 2G, 2W, and 2U, proposed modeling suggests that these symmetries might associate with Standard Model internal symmetries. ○ For 0H, we are uncertain as to the extent to which the symmetry that proposed modeling suggests might associate with a possible Standard Model internal symmetry. ○ For 2J, 4G, 6G, and 8G, proposed modeling suggests that - to the extent that people add these particles to the Standard Model - these symmetries might associate with Standard Model internal symmetries.

Table 30: ETA-related aspects - regarding modeling for elementary bosons - that associate with possible changes - during an interaction - in property value for an elementary fermion

Aspect
• For ENT modeling $ETA\lambda$ aspects regarding an elementary boson, the following notions pertain.
◦ Two cases have relevance.
◦ In one case, a pair of oscillators (with each one of the two values of $n_{ETA...}$ equal to its ground-state value and to the other value of $n_{ETA...}$) pertains. Here, an interaction (or, interaction vertex) associates with no change in the value of a specific elementary fermion property.
◦ In one case, just one oscillator (with its value of $n_{ETA...}$ equal to its ground-state value and not equal to the other values of $n_{ETA...}$) pertains. Here, an interaction (or, interaction vertex) can associate with a change in the value of a specific elementary fermion property.

The following notions associate with modeling for the ground state of the jay boson. The expression $n_{ESA0} = -1$ associates with zero mass. The expression $n_{ETA0} = -1$ associates with the notion that the jay boson models as entangled. We posit that - as for the Z and W bosons and for the photon - the expressions $n_{ESA1} = 0$ and $n_{ESA2} = 0$ pertain. We posit that the jay boson associates with the Pauli exclusion force. The Pauli exclusion force differentiates between the case of two fermions with the same spin state and the case of two fermions with differing spins states. We invoke double-entry arithmetic. We posit that the expressions $n_{ETA8} = 0$ and $n_{ETA7} = 0$ pertain.

Oscillators $ESA1$ (left circular polarization) and $ESA2$ (right circular polarization) can excite.

Discussion just above suggests the possibility of one jay boson with two modes. (Compare with the representation, in table 8, for the photon.) For this case, oscillator $ESA0$ does not excite. Discussion regarding the 0I boson might suggest that modeling for the jay boson might embrace the notion that oscillator $ESA0$ can excite. Again, paralleling notions regarding the 0I boson, jay boson $ESA0$ excitations might pertain essentially only regarding circumstances that feature higher energy density than energy densities that associate with hadrons. For this case, there would be one 2J particle with three spin states.

We associate the symbol $2\mathrm{J}_1$ with left circular polarization. We associate the symbol $2\mathrm{J}_2$ with right circular polarization. The symbol $2\mathrm{J}_0$ associates with the possibility of nonzero longitudinal polarization.

Proposed modeling suggests that each of $2\mathrm{J}_1$ and $2\mathrm{J}_2$ associates with a force that repels - from each other - two fermions that are - in general - adequately similar and that - specifically - would associate with the same angular momentum state. Each of $2\mathrm{J}_1$ and $2\mathrm{J}_2$ attempts, in effect, to catalyze an interaction that would leave the two fermions in states such that the angular momentum states of the two fermions differ from each other. (Possibly, people have discovered effects of jay bosons. See discussion regarding equation (164).) For such an interaction, one of n_{ETA8} and n_{ETA7} changes. The other one of n_{ETA8} and n_{ETA7} has a value of zero. Also, $n_{ETA0} = -1$. The case - that table 30 shows - regarding a change of a property being possible pertains. The property is angular momentum (or, angular momentum state).

Proposed modeling suggests - paralleling aspects for 2W - that an $SU(2)$ symmetry pertains regarding the jay boson.

We discuss proposed modeling ENT models for gluons (or, 2U bosons).

The following notions associate with modeling for the ground state of gluons. The expression $n_{ESA0} = -1$ associates with zero mass. The expressions $n_{ESA1} = -1$ and $n_{ESA2} = -1$ pertain. We invoke double-entry arithmetic. The expressions $n_{ETA5} = -1$, $n_{ETA6} = -1$, and $n_{ETA0} = -1$ pertain. For each j, $ETAj$ associates with a color charge.

Based on the notion of entangled environment, oscillators $ESA1$ (left circular polarization) and $ESA2$ (right circular polarization) can excite.

An interaction that preserves fermion color charge associates with one $ETAj$ oscillator and not the other two $ETAk$ oscillators. (Extant modeling associates two of the eight gluons with no change in fermion color charge.) Table 30 pertains. An interaction that changes fermion color charge associates with two $ETAj$ oscillators and not the other one $ETAk$ oscillator. (Extant modeling associates six of the eight gluons with change in fermion color charge.) Table 30 pertains.

Aspects related to oscillators $ETA6$, $ETA5$, and $ETA0$ associate with an $SU(3)$ symmetry. We suggest that this symmetry might associate with the extant modeling Standard Model notion that an $SU(3)$ internal symmetry pertains regarding gluons.

We discuss proposed modeling ENT models for elementary fermions.

ENT modeling for elementary fermions reflects ENT modeling for the weak interaction bosons.

We discuss aspects regarding ENT modeling regarding 1Φ subfamilies.

Proposed modeling associates nonzero charge with $n_{ESA0} = 0$. Proposed modeling associates negative charge with $n_{ESA0} = 0$, $n_{ETA2} = -1$, and $n_{ETA1} = 0$. Proposed modeling associates positive charge with $n_{ESA0} = 0$, $n_{ETA2} = 0$, and $n_{ETA1} = -1$. Proposed modeling associates zero charge with $n_{ESA0} = -1$, $n_{ETA2} = -1$, and $n_{ETA1} = -1$.

Regarding one of the two possible spin states, $n_{ESA1} = 0$, and $n_{ESA2} = -1$. (For this spin state, equation (84) might seem to pertain explicitly.) Regarding the other one of the two possible spin states, $n_{ESA1} = -1$, and $n_{ESA2} = 0$. (For this spin state, a notion similar to equation (84) might seem to pertain implicitly.)

We discuss aspects regarding ENT modeling regarding 1Φ elementary particles.

Discussion related to equation (101) suggests that modeling for elementary fermion fields and particles involves - compared to modeling for elementary fermion subfamilies - two additional harmonic oscillators.

We posit that ENT modeling for elementary fermions includes oscillators $ETA11$, $ETA12$, $ESA5$, and $ESA6$. (Perhaps, see table 29b.) We posit that $n_{ETA11} = n_{ETA12} = n_{ESA5} = n_{ESA6} = -1$ pertains. Including these oscillators comports with double-entry arithmetic.

For each spin state, one of n_{ESA1} and n_{ESA2} is minus one. Proposed modeling posits that a breakable $SU(2)$ symmetry associates with that instance of minus one and with the minus one that associates with the relevant one of n_{ESA5} and n_{ESA6}. Here, $ESA5$ is relevant regarding $ESA1$. $ESA6$ is relevant regarding $ESA2$. The group has three generators. ENT modeling associates these notions with three generations of elementary fermions. (Perhaps, contrast with table 29a. Table 29a pertains regarding elementary fermion subfamilies.)

For an elementary fermion, at least one of n_{ETA1} and n_{ETA2} is minus one. Minus one associates with n_{ETA11} and with n_{ETA12}. Here, $ETA11$ is relevant regarding $ETA1$. $ETA12$ is relevant regarding $ETA2$. Table 30 pertains. For interactions with elementary bosons, conservation of fermion generation pertains to the extent that such conservation pertains regarding the relevant elementary bosons.

We discuss proposed modeling ENT models for charged leptons.

The three generations associate - respectively - with the electron, muon, and tauon. A swap featuring $n_{ETA2} \leftrightarrow n_{ETA1}$ leads to modeling for the three respective antiparticles.

We discuss proposed modeling ENT models for neutrinos.

ENT modeling for neutrinos reflects ENT modeling for charged leptons. Neutrinos have zero charge. The expression $n_{ESA0} = n_{ETA2} = n_{ETA1} = -1$ associates with zero-charge. This essay does not recommend extents to which neutrinos model as Dirac fermions and as Majorana fermions. The case of Dirac fermions associates with - if one counts both matter particles and antimatter particles - six neutrinos. The case of Majorana fermions associates with three neutrinos, with each neutrino being its own antiparticle.

We discuss proposed modeling ENT models for quarks.

Compared to modeling for charged leptons, modeling for quarks changes n_{ETA0} from zero (which associates with the notion that a lepton can model as not entangled) to minus one (which associates with the notion that quarks model as entangled). Based on double-entry arithmetic, we add (compared to models for charged leptons) an oscillator pair. (See the n_{ETA5}-and-n_{ETA6} column in table 29a.) We set each of the corresponding two new n_{ETAj} to zero. Proposed modeling associates the new oscillator pair with an $SU(2)$ symmetry and three generators. The three generators associate with three color charges. These notions associate with quarks for which the magnitude of charge is two-thirds of the charge of a positron. The same notions associate with quarks for which the magnitude of charge is one-third of the charge of a positron. For each magnitude of charge, swapping n_{ETA1} and n_{ETA2} associates with changing the sign of charge.

We discuss proposed modeling ENT models for arcs.

ENT models for arcs reflect ENT models for quarks. Arcs have zero charge. The expression $n_{ESA0} = -1$ associates with zero charge. The expression $n_{ETA2} = n_{ETA1} = -1$ associates with zero charge. The result satisfies double-entry arithmetic. This essay does not recommend extents to which arcs model as Dirac fermions and as Majorana fermions. The case of Dirac fermions associates with - if one counts both matter particles and antimatter particles - six arcs. The case of Majorana fermions associates with three arcs, with each arc being its own antiparticle.

We discuss proposed modeling ENT models for G-family elementary particles.

An interaction between a G-family elementary particle and an object might - in effect - measure a property of the object. For an interaction that does not change the object, the interaction does not change the property of the object. (Regarding an interaction that ionizes an atom, modeling generally associates with not leaving the atom intact.) We consider aspects of table 9, table 12, and table 18.

Proposed modeling suggests that 2G associates with extant modeling classical physics notions of electromagnetism. Proposed modeling suggests that 2G associates with extant modeling quantum physics

Table 31: A possibly complete list of non-G-family elementary bosons

Bosons	0GΓ	Note: For the first item in the previous column, zero = ...	n_{ETA0}
0H (or, Higgs)	0G2468	$\lvert +2-4-6+8 \rvert$	0
0I	0G2468⟦16⟧	$\lvert +2-4-6-8+16 \rvert$	−1
2W: one of Z or W	0G268 or 0G246	$\lvert -2-6+8 \rvert$	0
2W: the other of Z or W	0G246 or 0G268	$\lvert -2-4+6 \rvert$	0
2J	0G268⟦16⟧	$\lvert -2-6-8+16 \rvert$	−1
2U	0G∅	$\lvert \sum_{\emptyset} \rvert$	−1

notions of the photon. 2G associates with conservation of elementary fermion charge. For each 2GΓ that associates with 2γ, the notion of $6 = \lambda \in \Gamma$ does not pertain. We are not aware of any evidence that photons associate with other than conservation of elementary fermion mass and conservation of elementary fermion generation.

Regarding 4G, 6G, and 8G, we associate Σ_S with $ESA(\Sigma-1)$-and-$ESA\Sigma$. (See table 29.) 4G associates with conservation of elementary fermion mass. 4G does not necessarily associate with conservation of elementary fermion generation. If at least two generations of neutrinos share one value of mass, 4G can catalyze neutrino oscillations. 6G associates with conservation of elementary fermion generation. 8G associates with conservation of elementary fermion spin. 8G does not necessarily associate with conservation of elementary fermion mass or conservation of elementary fermion generation. 8G can catalyze neutrino oscillations. 8G might help explain extant modeling notions that suggest differences between squares of neutrino masses.

We discuss the possible completeness of the list of elementary particles to which table 29a alludes.

Table 31 suggests that each one of some non-G-family elementary bosons associates with a ΣG solution for which $\Sigma = 0$. (This essay does not fully address the topic of which one of 0G268 and 0G246 associates with the Z boson. The other one of 0G268 and 0G246 associates with the W boson. Patterns in table 35a suggest that 0G268 associates with the Z boson.) In table 31, the presence of $\lambda = 16$ associates with $n_{ETA0} = -1$. Except regarding 2U (or, gluons), the absence of $\lambda = 16$ associates with $n_{ETA0} = 0$. To the extent that each non-G-family elementary boson associates with a ΣG solution for which $\Sigma = 0$, the list of non-G-family elementary bosons to which table 29a alludes might be complete. (Mathematically, for $\Sigma > 8$, $\Sigma = 14$ is the least value of Σ for which seemingly relevant 0G solutions exist.) The list of elementary fermions to which table 29a alludes might also be complete. (See discussion related to table 32.) Proposed modeling points to ΣG solutions for which $\Sigma \geq 10$. (See table 19a.) These solutions seem not to associate directly with properties that associate with $\lambda \geq 10$. (See table 18.) Also, the strength of a hypothetical 10G⟦10⟧ might be zero. (See table 20.) This essay de-emphasizes - but does not entirely dismiss - the notion that people might want to associate some ΣG solutions for which $\Sigma \geq 10$ pertains with the notion of elementary particles. (For a use - regarding masses of elementary bosons - of information in table 31, see table 35.)

Work that includes equation (99) suggests that - aside from the G family and the U family, $S \leq 1$ pertains for elementary particles.

We discuss one other possible limit that might have bases in proposed modeling. Regarding possible nonzero mass elementary bosons, aspects of table 35 and table 54 would combine to restrict would-be nonzero mass elementary bosons to have $S = 2$ and $Q = 0$. (Otherwise, the squares of the masses would be less than zero. Table 54 defines Q, which associates with charge.) The square of the masses of the would-be elementary bosons would be 1/17 times the square of the mass of the Higgs boson. (See equation (111).)

Table 32 speculates regarding a possible analog - to table 31 for elementary bosons - for elementary fermions. Here, we suggest relevance - regarding ENT modeling - for $\lambda = 12$. (This use - for ENT modeling - of $\lambda = 12$ does not necessarily conflict with UNI modeling USA use of $\lambda = 12$. Perhaps see table 18.) Possibly, $\lambda = 12$ associates - in ENT modeling - with three generations or with a notion of six, as in three generations times two possible values of handedness. In table 32, the presence of $\lambda = 16$ associates with $n_{ETA0} = -1$. Except for 2U (or, gluons), the absence of $\lambda = 16$ associates with $n_{ETA0} = 0$. We associate the symbol N/R with the two-word phrase not relevant. (This essay de-emphasizes the possibility that N/R might associate with a notion of new neutrinos and thereby - contrary to a pattern regarding $\lambda = 16$ and n_{ETA0} - with $n_{ETA0} = 0$.)

Table 33 summarizes information regarding spans for span-one particles, for hadron-like particles,

Table 32: A possibly complete list of elementary fermions (with N/R denoting not relevant)

Fermions	0GΓ	Note: For the first item in the previous column, zero = ...	n_{ETA0}
1Q - one charge	0G2468⟦12⟧⟦16⟧	$\lvert +2-4+6-8-12+16\rvert$	-1
1Q - the other charge	0G2468⟦12⟧⟦16⟧	$\lvert -2-4-6+8-12+16\rvert$	-1
One of 1C or 1N	0G246⟦12⟧ or 0G268⟦12⟧	$\lvert -2-4-6+12\rvert$	0
The other of 1N or 1C	0G268⟦12⟧ or 0G246⟦12⟧	$\lvert +2-6-8+12\rvert$	0
One of 1R or N/R	0G268⟦12⟧⟦16⟧ or 0G246⟦12⟧⟦16⟧	$\lvert -2+6-8-12+16\rvert$	-1
The other of N/R or 1R	0G246⟦12⟧⟦16⟧ or 0G268⟦12⟧⟦16⟧	$\lvert -2+4-6-12+16\rvert$	-1

and for some components of long-range forces. The table separates, based on a proposed modeling view, elementary particle Standard Model aspects from aspects that the elementary particle Standard Model does not embrace. The symbol 1Q⊗2U associates with known and possible hadrons. (See discussion regarding equation (166).) The symbol 1R⊗2U associates with possible hadron-like particles. (See discussion regarding equation (167).) Regarding the PR6ISP case, the pairings of isomers that isomers of 2G68 span might not equal the pairings of isomers that isomers of 4G48 span. The symbols †4G and †2G associate with this possible mismatch regarding pairings. Table 33c summarizes some concepts relevant to tables 33a and 33b. Discussion immediately below seems to support notions - in table 33c - regarding 2(2)GΓ.

The following proposed modeling notions seem to suggest that - for PR6ISP modeling - the isomer pairings I(0,3;0), I(1,4;0), and I(2,5;0) pertain for each 4(2)GΓ solution and for each 2(2)GΓ solution. (See table 33c.) Isomer I(3;0) - and not the other four dark matter isomers - echoes isomer I(0;0) relationships between masses of charged leptons and generation numbers for charged leptons. (See discussion related to table 44.) Of isomers I(1;0) through I(5;0), possibly only isomer I(3;0) has enough hydrogen atom like entities to explain data about some depletion of CMB (or, cosmic microwave background radiation). (See discussion related to table 46 and see discussion related to equation (190).) Possibly, case A - not case B - pertains regarding galaxy evolution. (See discussion related to table 52.) Nevertheless, this essay does not ignore - at least regarding galaxy evolution - other possibilities regarding pairings of isomers.

We discuss concepts regarding the 2(2)G68 solution and regarding interactions between dark matter and ordinary matter. Here, we assume that PR6ISP modeling comports with nature.

Elsewhere, we posit that 2(2)G68 associates with some electromagnetic (or, $\Sigma = 2$) interactions with atoms and other objects. (See discussion regarding table 19.) We posit that those interactions include hyperfine interactions.

Each of 2(1)G2 and 2(1)G24 associates with some electromagnetic (or, $\Sigma = 2$) interactions with atoms and other objects that include both baryons and leptons.

Unlike for the cases of electromagnetic interactions that associate with 2(1)G2 and 2(1)G24, 2G produced by ordinary matter objects interacts with non-ordinary-matter dark matter objects (for the case in which PR6ISP pertains to nature) via 2(2)G68. (For PR36ISP, the interactions are with doubly dark matter objects.) Unlike for the cases of electromagnetic interactions that associate with 2(1)G2 and 2(1)G24, 2G produced by some dark matter objects (for the case in which PR6ISP pertains to nature) interacts with ordinary matter via 2(2)G68. (For PR36ISP, the objects associate with doubly dark matter.)

We discuss other aspects that associate with table 13 and table 19.

Table 19 does not point to a G-family solution that would associate with an interaction with nonzero magnetic monopole moment. To the extent that proposed modeling adequately comports with nature, proposed modeling ENT modeling seems to suggest that nature does not exhibit magnetic monopole elementary particles.

Table 19 does not point to a G-family solution that would associate with a nonzero electric dipole moment for an object that does not feature - within the object - non-uniformity of charge. To the extent that an elementary particle models - with respect to KIN modeling - as having zero size, proposed modeling ENT modeling seems to suggest that the particle has zero electric dipole moment.

2.3.2. Properties of elementary bosons

We discuss the masses of elementary bosons.

We suggest that equation (106) comports with data. (For data, see reference [3].) The most accurately known of the masses is the mass of the Z boson. We use the nominal mass of the Z boson as a base

Table 33: Particles and solutions that associate with one isomer and particles and solutions that might associate with more than one isomer

(a) Particles

Standard Model entities	Possible entities	PRι_IISP span
0H	0I	1
1C	-	1
1N	-	1
1Q	1R	1
2W	-	1
-	2J	1
2U	-	1
2G	-	(See table 33b.)
-	4G	(See table 33b.)
-	6G	(See table 33b.)
-	8G	(See table 33b.)
1Q⊗2U	1R⊗2U	1

(b) Selected G-family components (with symbols of the form (†_) denoting aspects that table 33c discusses)

G-family component	PR1ISP span	PR6ISP span	PR36ISP span	RSDF	$\Sigma \in \Gamma$
2G2	1	1	1	r^{-2}	Yes
2G24	1	1	1	r^{-3}	Yes
2G248	1	6	6	r^{-3}	Yes
2G68	1	2 (†2G)	2	(†2G68)	No
2G⟦14⟧⟦16⟧	1	1	1	r^{-3}	No
4G4	1	6	6	r^{-2}	Yes
4G48	1	2 (†4G)	2	r^{-3}	Yes
4G246	1	1	1	r^{-4}	Yes
4G246⟦16⟧	1	6	6	r^{-5}	Yes
4G2468a	1	1	1	r^{-5}	Yes
4G2468b	1	1	1	r^{-5}	Yes
4G2468⟦16⟧	1	6	6	r^{-6}	Yes
6G6	1	2	2	r^{-2}	Yes
6G46⟦16⟧	1	6	36 (†36)	r^{-4}	Yes
6G468	1	6	6	r^{-3}	Yes
8G8	1	1	1	r^{-2}	Yes
8G2468a	1	1	1	r^{-4}	Yes
8G2468b	1	1	1	r^{-4}	Yes

(c) Notes regarding spans

Note
• (†4G): For PR6ISP modeling, the following notions pertain. Three instances of 4(2)GΓ pertain. One instance of 4(2)GΓ intermediates interactions throughout, but not beyond, I(0,3;0). One instance of 4(2)GΓ intermediates interactions throughout, but not beyond, I(1,4;0). One instance of 4(2)GΓ intermediates interactions throughout, but not beyond, I(2,5;0).
• (†2G): For PR6ISP modeling, the following notions pertain. Three instances of 2(2)GΓ pertain. One instance of 2(2)GΓ intermediates interactions throughout, but not beyond, I(0,3;0). One instance of 2(2)GΓ intermediates interactions throughout, but not beyond, I(1,4;0). One instance of 2(2)GΓ intermediates interactions throughout, but not beyond, I(2,5;0).
• (†2G68): This essay does not propose an RSDF regarding 2G68.
• (†36): See table 20 and equation (97).

Table 34: Rest energies for the Higgs, Z, and W bosons

Name	$\Sigma\Phi$	S	Experimental mc^2 (GeV)	Calculated mc^2 (GeV)	Difference (standard deviations)
Higgs boson	0H	0	125.10 ± 0.014	125.325	≈ 1.6
Z	2W	1	91.1876 ± 0.0021	91.1876	-
W	2W	1	80.379 ± 0.012	80.420	≈ 3.4

for calculations. Regarding the Higgs and W bosons, the larger deviation from equation (106) associates with the 9 : 7 ratio. Equation (106) suggests a W boson mass that is about 3.4 standard deviations high with respect to the measured mass of the W boson.

$$(m_{\text{Higgs boson}})^2 : (m_{\text{Z}})^2 : (m_{\text{W}})^2 \;::\; 17 : 9 : 7 \tag{106}$$

Table 34 provides numbers that associate with equation (106). (For data, see reference [3].)

Discussion regarding table 11 alludes to 0GΓ solutions. Within the constraints of $\Gamma \neq \emptyset$ and $\lambda \leq 8$, there are three 0GΓ solutions - 0G2468, 0G246, and 0G268. Removing the constraint of $\Gamma \neq \emptyset$ admits the 0G$\emptyset$ solution. For each of the four solutions, we define j_λ to be the number of λ elements in Γ.

We use the notation and the expression that equation (107) shows. (This essay does not explore the extent to which Z_{UTA8} associates with $UTA8$ through $UTA16$.)

$$Z_{UTA8} = (j_\lambda)^2 + 1 \tag{107}$$

We establish - for each of the values of λ of two, four, six, and eight - the notation $Z_{USA\lambda}$. (See table 18.) Charge associates with Z_{USA2}. active gravitational energy associates with Z_{USA4}. Freeable energy associates with Z_{USA6}. Spin associates with Z_{USA8}. We assume that Z_{USA2} is zero for zero-charge elementary bosons and is two for nonzero charge elementary bosons that have magnitudes of charges that equal the magnitude of the charge of the electron. (Perhaps, see discussion regarding table 54.) We assume that Z_{USA6} is zero for all elementary bosons. We assume that Z_{USA8} is zero for zero-spin elementary bosons and is one for spin-one elementary bosons. (Perhaps, see discussion regarding table 54.) We posit that equation (108) pertains for the 0H, 2W, and 2J bosons. We explore the notion that equation (109) shows. (The rightmost relationship follows from equation (108).)

$$Z_{UTA8} \approx Z_{USA2} + Z_{USA4} + Z_{USA6} + Z_{USA8} \tag{108}$$

$$m^2 \propto Z_{USA4} \approx Z_{UTA8} - Z_{USA2} - Z_{USA6} - Z_{USA8} \tag{109}$$

Table 35 shows modeling that interrelates all elementary bosons to which table 33a alludes. (Perhaps, compare with table 31.) Each row of table 35a uses equation (109). The three rows for which $n_{ETA0} = 0$ associate with equation (106). Each G-family boson has indirect representation in table 35a via a corresponding $Z_{USA\Sigma}$ and direct representation in table 35c. The ordering of the columns - in table 35a - associating with $USA\Sigma$ aspects associates with the ordering of terms in equation (109). The one 0I boson represents a zero-mass association with the one 0H boson. (Perhaps, see table 20.) The one 2J boson represents a zero-mass association with the two weak interaction bosons. Table 35c explores a conjecture regarding G-family bosons and Z_{USA8}. (Perhaps, see table 54.) Here, equation (110) would pertain.

$$Z_{UTA8} = Z_{USA8} = S^2 = (\Sigma/2)^2. \tag{110}$$

Table 35 associates with a notion that G-family solutions might point to all elementary bosons and, thus perhaps, to the notion that table 27 points to all elementary particles. (Note discussion - following on from equation (104) - that seemingly relates - at least indirectly - all elementary fermions to weak interaction bosons.)

Equation (111) shows the rest energy that would associate with a square of mass that is 1/17 times the square of an approximate mass of the Higgs boson. (Perhaps see remarks related to table 31. Perhaps, see equation (106).)

$$3.040 \times 10^4 \text{GeV} \tag{111}$$

Table 35: Some relationships among all elementary bosons to which table 33a alludes

(a) Relationships between non-G-family elementary bosons and GFC items for which $\Sigma = 0$

0GΓ	j_λ (for $[\![16]\!] \notin \Gamma$)	j_λ (for $[\![16]\!] \in \Gamma$)	Z_{USA4}	Z_{UTA8}	Z_{USA2}	Z_{USA6}	Z_{USA8}	Bosons	n_{ETA0}
0G2468	4	-	17	17	0	0	0	0H (or, Higgs)	0
0G268 or 0G246	3	-	9	10	0	0	1	2W: Z	0
0G246 or 0G268	3	-	7	10	2	0	1	2W: W	0
0G∅	0	-	0	1	0	0	1	2U	-1
0G2468$[\![16]\!]$	-	i	0	0	0	0	0	0I	-1
0G268$[\![16]\!]$	-	0	0	1	0	0	1	2J	-1

(b) Notes regarding table 35a

Note
• In table 35a, i denotes a square root of minus one. • For $[\![16]\!] \notin \Gamma$, the integer j_λ denotes the number of integers λ that appear in the Γ that associates with 0GΓ. • Except regarding the column with the label Z_{USA4}, each integer in the columns labeled with an expression of the form $Z_{...}$ satisfies - for some k in the set $\{i, 0, 1, 2, 3, \text{or } 4\}$ - the expression $k^2 + 1$. ∘ Perhaps, compare with - in table 5e - the column labeled $D + 2\nu''$. ∘ For example, the value $Z_{UTA8} = 17$ in table 35a associates with the value $D + 2\nu'' = 17$ in table 5e. For this example, $j_\lambda = 4$ and $S'' = 4$. ∘ For another example, the presence of i in the set of relevant values of k associates with the $\sigma'' = +1$ row in table 5e.

(c) Possible relationships regarding modeling for G-family bosons

$S = \Sigma/2$	Z_{USA4}	Z_{UTA8}	Z_{USA2}	Z_{USA6}	Z_{USA8}	Bosons	n_{ETA0}
1	0	1	0	0	1	2G	0
2	0	4	0	0	4	4G	0
3	0	9	0	0	9	6G	0
4	0	16	0	0	16	8G	0

Table 36: Approximate rest energies (in MeV) for quarks and charged leptons (with the symbol q denoting charge)

M''	Legend	M'=3, $q=-1\cdot\lvert q_e\rvert$	M'=2, $q=+(2/3)\cdot\lvert q_e\rvert$	M'=1, $q=-(1/3)\cdot\lvert q_e\rvert$
0	name	electron	up	down
0	data	$(0.511 \text{ to } 0.511)\times 10^0$	$(1.8 \text{ to } 2.7)\times 10^0$	$(4.4 \text{ to } 5.2)\times 10^0$
0	calculation	$m_e c^2 \approx 0.511\times 10^0$	$m_u c^2 \approx 2.2\times 10^0$	$m_d c^2 \approx 4.8\times 10^0$
1	name		charm	strange
1	data		$(1.24 \text{ to } 1.30)\times 10^3$	$(0.92 \text{ to } 1.04)\times 10^2$
1	calculation		$m_c c^2 \approx 1.263\times 10^3$	$m_s c^2 \approx 0.938\times 10^2$
2	name	muon	top	bottom
2	data	$(1.06 \text{ to } 1.06)\times 10^2$	$(1.56 \text{ to } 1.74)\times 10^5$	$(4.15 \text{ to } 4.22)\times 10^3$
2	calculation	$m_\mu c^2 \approx 1.06\times 10^2$	$m_t c^2 \approx 1.72\times 10^5$	$m_b c^2 \approx 4.18\times 10^3$
3	name	tauon		
3	data	$(1.777 \text{ to } 1.777)\times 10^3$		
3	calculation	$m_\tau c^2 \approx 1.777\times 10^3$		

2.3.3. Properties of elementary fermions

We discuss formulas that - based on the accuracy of measured quantities - predict a tauon mass that is consistent with and would be more accurate than the measured tauon mass.

Equation (112) shows an experimental result for the tauon mass, m_τ. (See reference [3].)

$$m_{\tau,\ \text{experimental}} \approx 1776.86 \pm 0.12\ \text{MeV}/c^2 \tag{112}$$

Equation (113) defines the symbol β'. Equation (114) defines β. Here, m denotes mass, e denotes electron, q denotes charge, ε_0 denotes the vacuum permittivity, and G_N denotes the gravitational constant. Equation (115) possibly pertains. Equation (115) predicts a tauon mass, which equation (116) shows. (For relevant data, see reference [3].) Eight standard deviations fit within one experimental standard deviation of the nominal experimental result. Equation (117) shows an approximate value of β that we calculate, using data that reference [3] shows, via equation (114). (For perspective regarding equations (113), (114), and (115), see discussion related to equation (156).)

$$\beta' = m_\tau/m_e \tag{113}$$

$$(4/3)\times\beta^{12} = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \tag{114}$$

$$\beta' = \beta \tag{115}$$

$$m_{\tau,\ \text{calculated}} \approx 1776.8400 \pm 0.0115\ \text{MeV}/c^2 \tag{116}$$

$$\beta \approx 3477.1891 \pm 0.0226 \tag{117}$$

We discuss formulas that - based on the accuracy of measured quantities - fit the masses of the six quarks and three charged leptons.

Table 36 shows, regarding the rest energies of quarks and charged leptons, data that people report and numbers that we calculate via equation (120). Below, we discuss the table and the data before we discuss the equation and the calculations. Equation (120) results from fitting data. (Equation (120) provides - for elementary fermions - a somewhat analog to equation (109) for elementary bosons. For elementary fermions, a notion of $\log(m/m_{ref})$ - and not a notion of m^2 - pertains. The choice of a positive value of m_{ref} can be arbitrary. Equation (120) associates with $m_{ref}=m_e$. This essay does not show modeling that would generate equation (120).)

The data in table 36 reflect information from reference [3]. For each particle other than the top quark, reference [3] provides one estimate. For the top quark, reference [3] provides estimates associating with each of three bases. For each quark, table 36 shows a data range that runs from one standard deviation below the minimum nominal value that reference [3] shows to one standard deviation above the maximum nominal value that reference [3] shows. Each standard deviation associates with the reported standard deviation that associates with the nominal value. For charged leptons (that is, for $M' = 3$), the table does not completely specify accuracy regarding ranges.

The following concepts pertain regarding developing equation (120). Use of modular arithmetic in equation (122) anticipates uses of equation (120) that pertain to neutrino masses and that pertain regarding inferences about dark matter. (Regarding equation (122), we take the liberty to define and use the notion that $3/2 \bmod 3 \equiv 3/2$.) The notion of $M'' = 3/2$ associates with modeling. (No elementary particle associates with $M'' = 3/2$.) Regarding equations (124) and (125), uses of $M' = 0$ anticipate uses of equation (120) that pertain to arc masses. Equation (118) produces a meaningful value for $m(1, 3)$. (No known or suggested elementary particle associates with $M'' = 1$ and $M' = 3$. However, work that leads to and includes table 37 suggests that the mass of each of two generations of arcs is $m(1, 3)$.) For each $0 \leq M'' \leq 2$, equation (119) produces a meaningful value of $m(M'', 3/2)$. (No charged elementary particle associates with $M' = 3/2$. The notion of $M' = 3/2$ associates with the average of $M' = 2$ and $M' = 1$ and associates with equation (119). Aspects of equations (120), (124), and (125) associate with the concept that $m(M'', 3/2)$ values have meaning. The concepts of $M' = 3/2$ and $m(M'', 3/2)$ are useful mathematically, though the concepts are not necessarily directly relevant to charged elementary particles.) Within each cluster of rows - in table 36 - for which $M'' \neq 3$, the fine-structure constant plays a role regarding linking the masses that pertain for that cluster of rows. (Aspects of equation (120) comport with this role.) Regarding equations (126), (127), and (128), we choose values that fit data. Regarding each charged lepton, our calculations fit data to more significant figures than the numbers in table 36 show.

$$m(1, 3)m(2, 3) = m(0, 3)m(3, 3) \tag{118}$$

$$(m(M'', 3/2))^2 = m(M'', 2)m(M'', 1) \tag{119}$$

The following concepts pertain regarding developing and using equation (120). We use equation (114) to calculate β. Equation (120) calculates the same value of m_τ that equation (116) calculates.

Equation (120) shows a formula that approximately fits the masses of the six quarks and three charged leptons. The formula includes two integer variables and seven parameters. One integer variable, M'', associates somewhat with generation. For the electron and each of the six quarks, the generation equals $M''+1$. For each of the muon and the tauon, the generation equals M''. The other integer variable, M', associates with magnitude of charge. The seven parameters can be m_e, m_μ (or, the mass of a muon), β, α, $d'(0)$, $d'(1)$, and $d'(2)$. The symbol α denotes the fine-structure constant. (See equation (121).) Here, $d'(k)$ pertains regarding generation-$(k+1)$ quarks. For each generation, the number $d'(k)$ associates with the extent to which the two relevant quark masses do not equal the geometric mean of the two quark masses. (See equation (119).) Regarding charged leptons, $M' = 3$, the term $g(M')$ is zero, and the factor - in equation (120) - that includes the fine-structure constant is one. (See equation (124).)

$$m(M'', M') = m_e \times (\beta^{1/3})^{M''+(j''_{M''})d''} \times (\alpha^{-1/4})^{g(M')\cdot(1+M'')+j'_{M'}d'(M'')} \tag{120}$$

$$\alpha = ((q_e)^2/(4\pi\varepsilon_0))/(\hbar c) \tag{121}$$

$$j''_{M''} = 0, +1, 0, -1 \text{ for, respectively, } M'' \bmod 3 = 0, 1, 3/2, 2; \text{ with } 3/2 \bmod 3 \equiv 3/2 \tag{122}$$

$$d'' = (2 - (\log(m_\mu/m_e)/\log(\beta^{1/3}))) \approx 3.840679 \times 10^{-2} \tag{123}$$

$$g(M') = 0, 3/2, 3/2, 3/2, 3/2, \text{ for, respectively, } M' = 3, 2, 3/2, 1, 0 \tag{124}$$

$$j'_{M'} = 0, -1, 0, +1, +3 \text{ for, respectively, } M' = 3, 2, 3/2, 1, 0 \tag{125}$$

$$d'(0) \sim 0.318 \tag{126}$$

$$d'(1) \sim -1.057 \tag{127}$$

$$d'(2) \sim -1.5091 \tag{128}$$

$$m(1,3) \approx 8.59341\text{MeV}/c^2 \tag{129}$$

We explore possibly useful variations and extensions regarding uses of equation (120).

Equations (130), (131), and (132) characterize a possible approach to re-estimating rest energies for the six quarks.

$$d'(0) \approx 0.264835 \tag{130}$$

$$d'(1) = -1 \tag{131}$$

$$d'(2) = -3/2 \tag{132}$$

The calculations yield new calculated rest energies for the six quarks. (See table 37.) Of the six quarks, the rest energies that one calculates via equation (120) differ from measured values (that reference [3] provides) by more than 1.2 units of estimated error for, at most, $m(1,2)$ (or, the charm quark) and $m(2,2)$ (or, the top quark). (Our calculations use the estimated errors - regarding experimental data - that reference [3] provides.) For the charm quark, the calculated number differs from the experimental number by about 4.6 units of estimated error. For the top quark, the largest (of the three differences associating with the three experimental interpretations) difference would be about 4.0 units of estimated error and one other difference would be about 0.6 units of estimated error.

To the extent that table 37 comports with nature, various straightforward equations interrelate the masses of elementary fermions. Equation (133) provides an example.

$$(m_s)^2 m_\mu = m_e m_\tau m_c \tag{133}$$

Equation (134) points to possibilities for estimating rest energies for arcs and neutrinos. Equations (135) and (136) would pertain.

$$m(M'',0) = m(M'',1) \cdot (m(M'',1)/m(M'',2)) \tag{134}$$

$$m(0,0) \approx m(1,0) = m(1,3) \tag{135}$$

$$m(2,0) = m(2,3) \tag{136}$$

To the extent that $m(0,0)$, $m(1,0)$, and $m(2,0)$ associate with masses of arc particles, approximate rest energies (in MeV) for arcs are 8.593 for generation one, 8.593 for generation two, and 105.66 for generation three.

We consider the possible extension that has bases in equations (137) and (138).

$$m(-1,3) = (\beta')^{-1} m(2,3) \tag{137}$$

$$d'(-1) = 0 \tag{138}$$

Equation (139) pertains.

$$m(-1,M')c^2 \approx 3.0386 \times 10^{-2}\ \text{MeV, for } M' = 3, 2, 3/2, 1, \text{ and } 0 \tag{139}$$

We discuss possible rest energies for neutrinos.

Equation (140) provides extant modeling limits for the sum, across three generations, of neutrino masses. (The limits have bases in interpretations of astrophysics data. See reference [3].) The integer j is an index for designating types of neutrinos.

$$0.06\text{eV}/c^2 \lesssim \sum_{j=1}^{3} m_j \lesssim 0.12\text{eV}/c^2 \tag{140}$$

Table 37: Suggested rest energies for some elementary fermions

Particles	Approximate rest energy	Note
Tauon	1776.8400 ± 0.0115 MeV	The error reflects the measured error re G_N
Up quark	2.335 MeV	
Down quark	4.479 MeV	
Charm quark	1.178×10^3 MeV	
Strange quark	1.006×10^2 MeV	
Top quark	1.695×10^5 MeV	
Bottom quark	4.232×10^3 MeV	
Arcs - generation one	8.593 MeV	
Arcs - generation two	8.593 MeV	
Arcs - generation three	1.0566×10^2 MeV	Equals the muon rest energy
Neutrinos (each of at least two mass eigenstates)	3.4475×10^{-2} eV	
Neutrinos (no more than one mass eigenstate)	4.1629×10^{-6} eV	Might instead equal 4.4305×10^{-4} eV

Extending work that produces equation (139) produces equations (141), (142), and (143). (Here, equation (138) extends to the notion that $d'(-4) = 0$ pertains. Here, $m(-4, 3) = (\beta')^{-1}m(-1, 3)$ pertains. Compare with equation (137). We assume that $d'(-5) = 0$ and $d'(-6) = 0$ pertain.)

$$m(-6, 0)c^2 = m(-6, 3/2)c^2 \approx 4.1629 \times 10^{-6} \text{ eV} \tag{141}$$

$$m(-5, 0)c^2 = m(-5, 3/2)c^2 \approx 4.4305 \times 10^{-4} \text{ eV} \tag{142}$$

$$m(-4, 0)c^2 = m(-4, 3/2)c^2 \approx 3.4475 \times 10^{-2} \text{ eV} \tag{143}$$

We posit that equation (143) provides the rest energies for either just two neutrinos or for all three neutrinos. Either case can comport with equation (140).

The case for which the rest energies of just two neutrinos associate with equation (143) might comport with the extant modeling notion that at least two neutrino masses are unequal. (Extant modeling suggests that indirect observations imply at least two neutrino masses differ from each other. See, for example, reference [3].) Either of equations (141) and (142) might pertain for the other neutrino. (Perhaps, note that no other lepton associates with $1 \equiv M''$ (mod 3). Here, the symbol $\equiv$ denotes the three-word phrase is congruent with. The lack of such a congruence for other leptons might suggest that equation (142) does not yield a neutrino mass.)

The case for which the rest energies of all three neutrinos associate with equation (143) might comport with data. Gravity catalyzes neutrino oscillations. (See discussion related to table 29.) Extant modeling interpretations of data suggest that the squares of masses of neutrinos might differ from each other. Proposed modeling suggests that such inferred differences regarding squares of masses might associate with effects of neutrino interactions with (at least) 8G. Differences - between 4G and associated conservation of elementary fermion mass and 6G and associated conservation of elementary fermion generation - might echo extant modeling KIN notions that, for neutrinos, mass eigenstates differ from generation eigenstates.

Table 37 lists approximate rest energies that proposed modeling suggests for some elementary fermions. (Some results regarding quarks differ from results that table 36 shows. Equations (130), (131), and (132) lead to results that table 37 shows for quarks.)

We discuss the topic of anomalous magnetic dipole moments for charged leptons.

We note an aspect of seeming synergy between table 13 and table 18b. The components of 2G that table 13 lists do not refer to a value of six for λ. Modeling seems compatible with the notion that - with respect to nominal aspects of electromagnetism - the three charged leptons exhibit identical characteristics.

We discuss the possibility that proposed modeling can produce useful results regarding the topic of anomalous magnetic dipole moments for charged leptons. (This essay de-emphasizes discussing the extent to which the 2G248 solution might associate with anomalous magnetic dipole moments for elementary particles. Perhaps, note table 13.)

Equations (144), (145), and (146) show extant modeling KIN interpretations of results of experiments regarding anomalous magnetic dipole moments. (See reference [3].) The subscripts e, μ, and τ denote, respectively, electron, muon, and tauon. The symbol a associates with anomalous magnetic dipole moment.)

$$a_e \approx 0.00115965218091 \tag{144}$$

$$a_\mu \approx 0.0011659209 \tag{145}$$

$$-0.052 < a_\tau < +0.013 \tag{146}$$

Extant modeling provides means, associating with Feynman diagrams, to calculate an anomalous magnetic dipole moment for each of, at least, the electron and the muon. The extant modeling Standard Model suggests computations whereby the anomalous magnetic dipole moment for a charged lepton is a sum of terms. The first term is $\alpha/(2\pi)$. The second term is proportional to α^2. The third term is proportional to α^3. The exponent associated with α associates with a number of virtual photons.

Regarding the tauon, equation (147) shows a result associating with a first-order Standard Model (or, extant modeling) calculation. (See reference [4].)

$$a_{\tau,\mathrm{SM}} \approx +1.177 \times 10^{-3} \tag{147}$$

Proposed modeling suggests that notions of anomalous electromagnetic moments associate with $\gamma 2$ solutions. Electromagnetic dipole solutions associate with $\gamma 2$ solutions for which RSDF is r^{-3}. The following remarks pertain for other than the 2G24 solution, which associates with the extant modeling nominal magnetic moment result of $g \approx 2$. (2G24 associates with 2γ and not with $\gamma 2$.) Relevant G-family solutions (for which $\lambda \leq 8$) might be 4G26, 6G24, 6G28, 8G26, and (if we allow $\Sigma \geq 10$) 10G28. Solutions 6G28 and 10G28 might not have relevance, because $8 \in \Gamma$ might associate with $(ct)^{-1}$ and might not necessarily associate with r^{-1}. (See table 14.) Regarding anomalous electromagnetic dipole moments, we assume that 4G26, 6G24, and 8G26 pertain.

For each of solutions 4G26 and 8G26, $4 \notin \Gamma$. Solutions 4G26 and 8G26 might associate with results that do not vary with charged lepton rest mass. For solution 6G24, $4 \in \Gamma$. Solution 6G24 might associate with a result that varies with charged lepton rest mass.

We explore modeling for which equation (148) pertains. Here, the subscript cl can be any one of e, μ, and τ. The symbol $a_{4\mathrm{G}26^*}$ associates with the notion of combining effects of 4G26 and 8G26. We explore the notion that t_{cl} might be one of $(\log(m_{\mathrm{cl}}/m_e))^2$, $(M'')^2$, and $(\text{generation})^2$. For each of the three possibilities regarding t_{cl}, $(a_\tau - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ is more than -0.003 and less than -0.0006. For t_{cl} being $(\log(m_{\mathrm{cl}}/m_e))^2$, $(a_\tau - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ is approximately -0.00228.

$$a_{\mathrm{cl}} \approx a_{4\mathrm{G}26^*} + a_{6\mathrm{G}24} t_{\mathrm{cl}} \tag{148}$$

Proposed modeling might provide modeling relevant to anomalous magnetic dipole moments for charged leptons.

Also, people report the possibility that extant modeling misestimates the anomalous magnetic dipole moment of the muon. (See reference [5].) Perhaps, aspects of proposed modeling point to possibilities for more accurate estimates. Possibilities might associate with the notion of components of $\gamma 2$ or with the notion of 2J bosons.

2.3.4. Strengths of long-range forces

We explore concepts that might associate with the extant modeling notion that the strength of gravity is much less than the strength of electromagnetism.

We explore modeling for interactions that involve a charged elementary fermion, such as an electron, that models as not entangled.

We assume that we can work within aspects of proposed modeling that de-emphasize translational motion and multicomponent objects. We assume that conservation of angular momentum pertains.

We associate the symbol 1F with that fermion. We explore interactions that model as if the number of incoming elementary bosons equals the number of outgoing elementary bosons. (Perhaps, see table 26.) Equation (149) shows an interaction in which the fermion absorbs a photon. Conservation of angular momentum pertains. The spin of the fermion flips. Trying to replace, in equation (149), 2G with 4G does not work. The angular momentum associated with the fermion can change by no more than one unit.

The interaction would not conserve angular momentum. Equation (150) can pertain. One can consider that the 2J particle in equation (150) associates with $2J_1$ or $2J_2$. (See table 38.)

$$1\text{F} + 2\text{G} \rightarrow 1\text{F} + 0\text{I} \tag{149}$$

$$1\text{F} + 4\text{G} \rightarrow 1\text{F} + 2\text{J} \tag{150}$$

The notion that $1\text{F} + 4\text{G} \rightarrow 1\text{F} + 0\text{I}$ does not pertain might associate with extant modeling notions that the strength of gravity is much less than the strength of electromagnetism.

We explore the strengths - for the monopole components of interactions between pairs of identical charged leptons - of electromagnetism and gravity. We use KIN Newtonian modeling.

For each of the three charged leptons, equation (151) characterizes the strength of the 2G2 component of electromagnetism. Here, r denotes the distance between the two particles. Here, F denotes the strength of the force. The equation associates with a magnitude of the force. The interaction is repulsive. Equation (152) shows notation regarding the masses of charged leptons. (See discussion related to table 36.) Here, the three in $m(M'', 3)$ associates with charged leptons. (Compare with equation (120), which pertains to the masses of quarks and charged leptons.) Equation (153) repeats equation (113). Equation (154) shows results that reflect data. (We used data that reference [3] shows.) Equation (155) provides a 4G4 analog to the 2G2 equation (151). The symbol G_N denotes the gravitational constant. The equation associates with a magnitude of the force. Here, the interaction is attractive.

$$r^2F = (q_e)^2/(4\pi\varepsilon_0) \tag{151}$$

$$m(M'', 3) = m_x, \text{ for the pairs } M'' = 0, x = e;\ M'' = 2, x = \mu;\text{ and } M'' = 3, x = \tau \tag{152}$$

$$\beta' = m_\tau/m_e \tag{153}$$

$$m(M'', 3) = y_{M''}(\beta')^{M''/3}m_e, \text{with } y_0 = y_3 = 1 \text{ and } y_2 \approx 0.9009 \tag{154}$$

$$r^2F = G_N(m(M'', 3))^2 \tag{155}$$

We pursue the concept that a value of M'' can point to a relationship between the strength of electromagnetism and the strength of gravity. Based on the definitions just above, equation (156) pertains within experimental errors regarding relevant data. (Reference [3] provides the data.) Here, in essence, the equation $y_{18} = y_0 = 1$ pertains. Equation (156) echoes equation (114).

$$((q_e)^2/(4\pi\varepsilon_0))/4 = (G_N(m(18, 3))^2)/3, \text{ with } m(18, 3) = (\beta')^6m_e \tag{156}$$

The following notes pertain. Equation (156) links the ratio of the masses of two elementary fermions to a ratio of the strengths of two G-family force components. Equation (156) links the strength of 2G2 interactions to the strength of 4G4 interactions. Equation (157) associates the fine-structure constant, α, with a function of the tauon mass and the electron mass. (Regarding the fine-structure constant, see equation (121).) Equation (158) recasts equation (114) to feature, in effect, the magnitudes of three interactions, with each one of the interactions involving two similar particles. (For example, $G_N(m_\tau)^2$ associates with a gravitational interaction between two tauons.) Equation (159) shows a ratio that pertains for interactions between two electrons.

$$\alpha = ((q_e)^2/(4\pi\varepsilon_0\hbar c)) = (4/3) \times (m_\tau/m_e)^{12}G_N(m_e)^2/(\hbar c) \tag{157}$$

$$(4/3)((G_N(m_\tau)^2)/(G_N(m_e)^2))^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \tag{158}$$

$$(((q_e)^2/(4\pi\varepsilon_0))/4)\ /\ ((G_N(m_e)^2)/3) \approx 3.124 \times 10^{42} \tag{159}$$

We explore a possible relationship between the strength of electromagnetism associating with G-family monopole interactions with charge and the strength of electromagnetism associating with G-family dipole interactions with nominal magnetic dipole moment.

Equation (160) provides one definition of the fine-structure constant. (Compare with equation (121), which provides a more common definition.) In equation (160), $(q_e)^2/(4\pi\varepsilon_0 c)$ associates with the strength of 2G2.

$$\alpha = ((q_e/\hbar)^2/(4\pi\varepsilon_0 c)) \cdot \hbar \tag{160}$$

Equation (160) provides a link between the strength of 2G2 and the strength of 2G24. The equation includes the term $(q_e/\hbar)^2$. The Josephson constant K_{J} equals $2q_e/h$ (or, $q_e/(2\pi\hbar)$). Extant modeling considers that magnetic flux is always an integer multiple of $h/(2q_e)$.

We explore a concept regarding extant modeling notions that associate with relationships between the strengths of the electromagnetic, weak, and strong interactions.

We use the symbol ΣB to denote an elementary boson having a spin of Σ/2. The expression 1F+2B→1F+0B can pertain for each of the following cases - 2B associates with 2G, 2B associates with 2W, and 2B associates with 2U. (Per table 26, 0B can associate - at least mathematically - with 0I.) This notion might associate with extant modeling notions that associate with relationships between the strengths of the electromagnetic, weak, and strong interactions.

We explore the relative strengths of interactions regarding G-family bosons with spins of at least two.

Equations (161) and (162) parallel equation (150). Compared to equation (150), equation (161) requires dissipation (from the incoming G-family boson) of one more unit (of magnitude $\hbar$) of spin. Compared to equation (161), equation (162) requires dissipation (from the incoming G-family boson) of one more unit (of magnitude $\hbar$) of spin.

$$1\mathrm{F} + 6\mathrm{G} + 0\mathrm{I} \rightarrow 1\mathrm{F} + 2\mathrm{J} + 2\mathrm{J} \tag{161}$$

$$1\mathrm{F} + 8\mathrm{G} + 0\mathrm{I} + 0\mathrm{I} \rightarrow 1\mathrm{F} + 2\mathrm{J} + 2\mathrm{J} + 2\mathrm{J} \tag{162}$$

We explore the notion that a strength scaling relationship might pertain regarding G-family components ΣGΓ that share a value of Γ. For two such ΣGΓ, Σ_1GΓ and Σ_2GΓ, equation (163) pertains.

$$|\Sigma_2 - \Sigma_1|/4 \text{ is an integer} \tag{163}$$

We interpret equation (160) as suggesting that a factor of α might pertain regarding modeling the absorbing of a unit of spin. For a step from equation (150) to equation (162), two factors of α would pertain.

2.3.5. Interactions involving the jay boson

We note one observational result that might associate with effects associating with the jay boson.

Reference [6] reports a possible discrepancy between the observed energy associating with one type of fine-structure transition in positronium and a prediction based on extant modeling. (Perhaps, see also reference [7].) Equation (164) states a transition frequency. The observed value of transition frequency associates with the energy that associates with the transition. Equation (165) associates with extant modeling. The observed energy might exceed the predicted energy. Reference [6] characterizes the transition via the expression $2^3S_1 \rightarrow 2^3P_0$.

$$18501.02 \pm 0.61 \text{ MHz} \tag{164}$$

$$18498.25 \pm 0.08 \text{ MHz} \tag{165}$$

We explore the topic of interactions and effects associating with the jay boson.

Table 38 discusses aspects regarding physics, interactions, and modeling involving the jay (or, 2J) boson. (Regarding Pauli crystals, see reference [8] and reference [9].)

Table 39 shows some possible reactions involving pairs of jay bosons. The leftmost column describes the pair of incoming jay bosons. We discuss, as an example, the case of incoming $2\mathrm{J}_1$+$2\mathrm{J}_2$. The incoming particles associate with units of spin that have opposite circular polarizations. In effect, the circular polarizations sum to zero circular polarization. The outgoing pair 0I+0I is possible. The outgoing pair 2G+0I is not possible. The outgoing circular polarizations would sum to plus one or minus one.

Table 38: Aspects regarding the 2J boson

(a) Aspects - associating with observations and modeling - that might associate with the 2J boson

Aspect
• Interactions - between identical fermions - that associate with extant modeling notions of a Pauli exclusion force. (A pair of such identical fermions can be, for example, two hadrons in an atomic nucleus or two elementary particles. In extant modeling, the notion of identical might involve rest energy, charge, generation, and - for example, in an atom - spin orientation and orbital state. Aspects such as spin orientation and orbital state associate with extant modeling KIN aspects. Proposed modeling would suggest - regarding the notion of identical - including a number that associates with isomer. This inclusion would add to the list that associates with extant modeling.)
• Forces associating with some energy levels of positronium atoms. (See discussion related to equation (164).)
• Patterns that Pauli crystals exhibit.
• Some interaction vertices that involve an incoming spin-one-half elementary fermion, an incoming or outgoing ΣG for which $\Sigma \geq 4$, and an outgoing spin-one-half elementary fermion. (See discussion related to equation (150). For this example, a 2J boson absorbs, in effect, one unit of spin that associates originally with an incoming boson. The unit associates with $\hbar$.)
• Some interaction vertices that involve no fermions. (See discussion related to equation (174). For this example, two incoming 2J bosons associate with, in effect, two units of spin that associate with an outgoing component of a graviton. Each unit of spin associates with $\hbar$.)

(b) Suggested aspects regarding the 2J boson

Aspect
• The Pauli exclusion force (in extant modeling) associates with (in proposed modeling) a repulsive force based on $2J_1$ and $2J_2$. The proposed modeling 2J force, in effect, tries to flip the spin of a fermion.
• The positronium energy shift might involve the notion that the two fermions - an electron and a positron - have identical properties (including the spin orientations), except for the signs of the charges. We posit that an energy level shift (regarding at least one of the two positronium states) associates with, in effect, aspects of $2J_1$ and $2J_2$. Here, at least with respect to extant modeling based on the Dirac equation, a notion associating with charge exchange (between the electron and positron) might be appropriate.
• We posit that the 2J boson associates with some interaction vertices that involve an incoming spin-one fermion, an incoming or outgoing ΣG for which $\Sigma \geq 4$, and an outgoing spin-one fermion. (See, for example, equation (150).)
• We posit that the 2J boson can associate with some interaction vertices that involve no fermions. (See, for example, discussion related to equation (174).)

Table 39: Some possible reactions involving pairs of jay bosons

Incoming particles	Allowed outgoing particles	Precluded outgoing particles
$2J_1+2J_1$ or $2J_2+2J_2$	4G+0I	2G+0I
$2J_1+2J_2$	0I+0I	2G+0I
$2J_0+2J_0$	0I+0I	2G+0I

Table 40: Perspective regarding PR6ISP modeling

PR6ISP modeling ...
• Explains observed dark matter to ordinary matter ratios of five-plus to one, four to one, zero-plus to one, and one to zero-plus.
• Associates with a six-fold aspect to which table 18 alludes.
• Echoes the notion that ENT modeling intertwines 2G-related aspects and 4G-related aspects in ways that extant modeling does not. (See, for example, equation (120).)
• Echoes the exponent of six that equation (156) discusses.
• Echoes the six ranges that equation (168) and table 44 feature.

2.3.6. Dark matter particles

We discuss one type of dark matter.

We discuss the symbols that equations (166) and (167) show. The symbol 1Q⊗2U denotes a particle that includes (regarding non-virtual particles) just quarks and gluons. The word hadron pertains for the particle. The one-element term hadron-like pertains for the particle. Examples of 1Q⊗2U particles include protons, neutrons, and pions. The symbol 1R⊗2U denotes a particle that includes just arcs and gluons. The one-element term hadron-like pertains for the particle. The particle does not include (non-virtual) quarks.

$$1\mathrm{Q} \otimes 2\mathrm{U} \tag{166}$$

$$1\mathrm{R} \otimes 2\mathrm{U} \tag{167}$$

A 1R⊗2U hadron-like particle contains no (non-virtual) charged particles. The 1R⊗2U hadron-like particles do not interact with 2γ. Isomer I(0;0) 1R⊗2U hadron-like particles measure as being dark matter. (Perhaps, see table 15 and table 16.)

Within the perspective of PR1ISP modeling, we know of no notions that would provide bases to explain observed ratios of dark matter effects to ordinary matter effects, including the ratio of five-plus to one for densities of the universe.

We explore the notion that some five-plus to one ratios reflect something fundamental in nature. We associate some results from this exploration with PR6ISP modeling. (See table 15, table 16, and table 19c.)

The notion of isomers I(0;0) through I(5;0) associates with a six-fold aspect. (See table 18.)

GFC modeling interrelates interactions with charge and the 2G2 component of the 2G force. We posit that nature includes six isomers of charge. GFC modeling interrelates interactions with nominal magnetic dipole moment and the 2G24 component of the 2G force. We posit that each isomer of charge associates with one isomer of nominal magnetic dipole moment. We posit that each of six pairings of one isomer of charge and one isomer of nominal magnetic moment associates with its own isomer of all span-one particles. Isomer I(0;0) measures mostly as ordinary matter. (I(0;0) 1R ⊗ 2U hadron-like particles measure as dark matter. Hence, we use the word mostly.) We posit that each of the other five isomers - I(1;0) through I(5;0) - of charge, nominal magnetic dipole moment, and related span-one particles measures as dark matter. (PR1ISP modeling does not include these five isomers.) Each of the six isomers associates with its own 2U particles (or, gluons). We posit that one isomer of 4G4 interacts with each one of the one (mostly) ordinary matter isomer and five dark matter isomers.

We posit that the next two sentences pertain. The six-isomer notion explains the five that pertains regarding five-plus to one ratios of amounts of dark matter to ordinary matter. The existence of 1R⊗2U hadron-like particles explains the plus that pertains regarding five-plus to one ratios of amounts of dark matter to ordinary matter. Such five-plus to one ratios pertain regarding densities of the universe and regarding the compositions of some (perhaps, most) galaxy clusters.

Table 40 provides perspective regarding PR6ISP modeling.

Regarding each one of the six isomers that associate with PR6ISP models, we suggest that each combination - that table 36 shows - of magnitude of charge and magnitude of mass pertains to a span-one fermion that associates with the isomer. For example, each isomer includes a charged lepton for which the magnitude of charge equals the magnitude of the charge of the ordinary matter electron and for which the rest energy equals the rest energy of the electron. However, regarding charged leptons, the combination of mass and generation number does not necessarily match across isomers. (See table 44.) For example,

Table 41: Cumulative features of various types of modeling (with NR denoting not relevant)

Modeling	ι_I	New descriptions and new explanations	New subtleties
Extant modeling	NR	• (Baseline)	-
PR1ISP	1	• New elementary particles • One type of dark matter • Possible eras early in the development of the universe	• Internal symmetries • Known eras regarding the rate of expansion of the universe
PR6ISP	6	• More types of dark matter • Ratios of dark matter effects to ordinary matter effects • Objects, smaller than galaxies, that feature dark matter	• Galaxy formation and evolution • Eras regarding the rate of expansion of the universe • Spans • Ranges of applicability of some extant modeling kinematics models
PR36ISP	36	• Possible dark energy stuff	• Dark energy density of the universe • Spans

Table 42: Relationships regarding PR1ISP, PR6ISP, and G-family forces

Aspect
• Absent the notion that some components of G-family forces have spans of more than one, PR6ISP would associate with six non-interacting sub-universes.
• In PR6ISP models, each sub-universe consists of an isomer of PR1ISP. The six isomers of PR1ISP might exhibit differing matches between generation of charged lepton and mass of charged lepton. (See discussion related to table 44.)
• In PR6ISP models, the main interactions between PR1ISP-like isomers associate with gravity (or, 4G). Some other interactions between PR1ISP-like isomers associate with a KIN dipole component (or, 2G248 - which associates with the notion of GFC quadrupole) of electromagnetism (or, 2G).

for isomer I(1;0), the generation three charged lepton may have the same mass as the ordinary matter electron. (See table 36.) The ordinary matter electron has a generation number of one.

We preview features of each of PR1ISP modeling, PR6ISP modeling, and PR36ISP modeling.

Table 41 discusses cumulative features of various types of modeling. Generally, each row augments the rows above that row. (Table 19c discusses the symbol ι_I.) Regarding extant modeling, the symbol NR denotes the concept that the notion of isomers is not relevant. We think that PR6ISP modeling provides useful insight about nature. We think that the notions of isomers and PR6ISP modeling point to limitations regarding the ranges of applicability of some extant modeling kinematics models. (For example, discussion related to table 21 suggests limits regarding the applicability of general relativity.) We think that PR36ISP modeling might provide a new description for phenomena that measure as dark energy density of the universe.

Table 42 shows relationships regarding PR1ISP, PR6ISP, and G-family forces.

2.3.7. Isomers of quarks and charged leptons

We consider PR6ISP modeling.

Table 43 lists aspects that seem to associate with each other regarding the one isomer that associates with ordinary matter (and some dark matter) and the five isomers that associate with (most) dark matter.

We explore modeling that associates each of the six relevant isomers with a range of M''. (Regarding M'', perhaps see discussion related to equation (120).) In equation (168), the integer n numbers the isomers. The symbol $\leftrightarrow$ associates with the two-word phrase associates with. The notation I(n;0) pertains. The ordinary matter isomer associates with $n = 0$.

$$\text{isomer } n \;\leftrightarrow\; 3n \leq M'' \leq 3n+3, \text{ for } 0 \leq n \leq 5 \tag{168}$$

Table 44 shows, for each value of n, relationships between quark generation and charged lepton aspects. For each n, the order for quarks is generation one, generation two, and then generation three.

Regarding table 44, we de-emphasize the following notions. Dark matter lepton active gravitational masses might associate with $m(M'', 3)$ and $M'' > 3$. (However, numbers that associate with $m(M'', 3)$

Table 43: Aspects that seem to associate with each other regarding the one isomer that associates with ordinary matter (and some dark matter) and the five isomers that associate with (most) dark matter

Aspect
• The exponent of six in equation (156) associates with the notion of six isomers, one of which associates with ordinary matter and five of which associate with (most) dark matter.
• The number, six, of isomers associates with a six-fold aspect and a possible six-fold symmetry. (See table 18.)
• The would-be six-fold symmetry breaks - across the six isomers - based on aspects that associate with relationships between - for charged leptons - active gravitational mass and generation.

Table 44: Relationships between quark generation and charged lepton aspects

M''	n	Quark n	Quark generation	Lepton n (for n even)	Lepton aspect (for even n)	Lepton n (for n odd)	Lepton aspect (for odd n)
0	0	0	1	0	1	-	-
1	0	0	2	-	-	-	-
2	0	0	3	0	2	-	-
3	0 or 1	1	1	0	3	1	3
4	1	1	2	-	-	-	-
5	1	1	3	-	-	1	1
6	1 or 2	2	1	2	2	1	2
7	2	2	2	-	-	-	-
8	2	2	3	2	3	-	-
9	2 or 3	3	1	2	1	3	1
10	3	3	2	-	-	-	-
11	3	3	3	-	-	3	2
12	3 or 4	4	1	4	3	3	3
13	4	4	2	-	-	-	-
14	4	4	3	4	1	-	-
15	4 or 5	5	1	4	2	5	2
16	5	5	2	-	-	-	-
17	5	5	3	-	-	5	3
18	5	-	-	-	-	5	1

and $M'' > 3$ might have physics relevance. Perhaps, note discussions - regarding notions of mass - related to table 21 and related to equation (168).) Results that associate with $M'' < 0$ might be useful for estimating magnitudes of ordinary matter 2G interactions with dark matter analogs to ordinary matter charged leptons.

Table 44 has roots in models that associate with the relative strengths of 2G2 and 4G4. We posit that, for each item (in table 44) that associates with a particle, equation (169) provides the active gravitational mass. Here, the notions of $n = 0$ and $m_{grav}(M'', M')$ associate with work that associates with the isomer I(0;0) and equation (120). For example, for the dark matter lepton for which $n = 1$ and M''=3, the generation is three and the active gravitational mass equals the active gravitational mass of the ordinary matter electron.

$$m_{grav}(M'' + 3n, M') = m_{grav}(M'', M'), \text{ for } 0 \leq n \leq 5 \tag{169}$$

We speculate regarding the extent to which aspects of table 44 associate with origins for baryon asymmetry.

Aspects of extant modeling consider that early in the universe baryon symmetry likely pertained. Unverified extant modeling posits mechanisms that might have led to asymmetry. Some conjectured mechanisms would suggest asymmetries between matter elementary fermions and antimatter elementary fermions. One set of such elementary fermions might feature the neutrinos. (See reference [10].)

Observed baryon asymmetry associates with isomer I(0;0) (or, ordinary matter).

We think that some aspects of proposed modeling might shed light on baryon asymmetry. For example, a modeling centric notion of baryon symmetry might pertain regarding the combination of isomer I(0;0) and isomer I(3;0).

We consider a thought experiment. We consider that modeling for isomer I(3;0) quarks parallels modeling for isomer I(0;0) quarks. Per table 44, modeling for isomer I(3;0) leptons can differ from modeling for isomer I(0;0) leptons. One difference might associate with handedness, for example regarding neutrinos.

2.3.8. Right-handed W bosons and neutrinos

Reference [11] notes that the (extant modeling) Standard Model predicts that the fraction f_+ of W bosons - produced by decays of top quarks - that are right-handed is $f_+ = 3.6 \times 10^{-4}$. Reference [3] suggests that, with a confidence level of 90 percent, the rest energy of a W_R (or, right-handed W) would exceed 715 GeV. (Perhaps, note also, reference [12].)

Proposed modeling suggests that each of isomers I(0;0) through I(5;0) includes its own isomer of W bosons. The suggested active gravitational mass for dark matter W bosons is the same as the active gravitational mass for the ordinary matter W boson.

We suggest that leptons associating with each one of isomers I(0;0), I(2;0), and i(4;0) might associate with left-handedness and that leptons associating with isomers I(1;0), I(3;0), and i(5;0) might associate with right-handedness. (Note the pattern that table 44 exhibits regarding charged leptons.) We suggest that W bosons associating with isomers I(0;0), I(2;0), and i(4;0) might associate with left-handedness and that W bosons associating with isomers I(1;0), I(3;0), and i(5;0) might associate with right-handedness. Table 43 and equation (156) suggest that equation (170) pertains regarding measurements that feature aspects centric to ordinary matter and interactions intermediated by span-six aspects of 2G. (Note, for example, 2(6)G248 in table 33.) We know of no measurements that associate with interactions intermediated by 4G. To the extent that equation (170) has relevance to nature, one might use the four-word phrase not necessarily gravitational mass to describe $m_{W_R(\mathrm{I}(1;0)),\ \text{inferred not via 4G}}$. (Perhaps, compare with discussions - regarding notions of mass - related to table 21 and related to table 44.)

$$m_{W_R(\mathrm{I}(1;0)),\ \text{inferred not via 4G}} c^2 = \beta m_W c^2 \approx 2.8 \times 10^5 \text{ GeV} \tag{170}$$

We consider a thought experiment. We consider a possibly relevant notion that would have bases in statistics related to inferable not necessarily gravitational masses. Perhaps equation (171) approximates fractions of non-longitudinal polarization W bosons observed via ordinary matter non-4G interactions. (For isomers other than I(0;0) and I(1;0), the $m_{W_R(\mathrm{I}(\ldots;0)),\ \text{inferred}\ \ldots} c^2$ would be larger than $m_{W_R(\mathrm{I}(1;0)),\ \text{inferred}\ \ldots} c^2$. Effects based on the existence of isomer I(3;0) W bosons and isomer I(5;0) W bosons would be small compared to effects associating with each of isomer I(0;0) W bosons and isomer I(1;0) W bosons.)

$$f_+ \sim e^{(\beta^{-1})} - 1 \approx \beta^{-1} \approx 2.9 \times 10^{-4} \tag{171}$$

Table 45: Opportunities for advances regarding cosmology

Opportunity
• Describe aspects of the universe that occurred before inflation.
• Identify - within a context that is broader than inflation - the inflaton elementary particle that extant modeling hypothesizes.
• Describe mechanisms underlying three eras in the rate of expansion of the universe.
• Explain the magnitude of the current increase in the rate of expansion of the universe.
• Describe bases leading to the ratio of dark matter density of the universe to ordinary matter density of the universe.

Equation (171) is not necessarily incompatible with the estimate - $f_+ = 3.6 \times 10^{-4}$ - based on the Standard Model.

Regarding neutrinos, similar notions might pertain. Proposed modeling suggests that neutrinos do not interact with 2G. Direct inferences of the presence of right-handed neutrinos might associate with interactions - mediated by 4G - between isomer I(1;0) neutrinos and isomer I(0;0). This essay de-emphasizes discussing the question of when people might have observations that would point to right-handed neutrinos.

2.4. Cosmology

Table 45 lists opportunities for advances regarding cosmology. Proposed modeling suggests advances regarding each opportunity.

2.4.1. An earlier of two eras that might occur before inflation

We discuss possibilities regarding times before the inflationary epoch.

We explore possibilities pertaining to an era before a later (but also before inflation) era that proposed modeling associates with prominence for the jay boson and the 4G2468x components of 4γ. (Regarding the later of the two eras before inflation, see discussion related to equation (174). Regarding the symbol 4G2468x, see discussion related to table 12.)

We assume that modeling associating with G-family solutions for which the RSDF is r^{-6} pertains. No solutions of the form ΣG2468⟦10⟧ comport with $\Sigma = 4$. One solution of the form ΣG2468⟦16⟧ comports with $\Sigma = 4$. (Here, $|-2-4-6-8+16|$ equals four. Perhaps, see table 20.) Regarding KIN Newtonian modeling, the RSDF (or, radial spatial dependence of force) would be r^{-6}. Table 21 notes that attraction (not repulsion) pertains. (Perhaps, also note that extrapolation based on aspects of table 47 might point to attraction.)

We consider interactions between two similar, neighboring, non-overlapping objects (or clumps of energy). Equation (172) suggests scaling for a 4G2468⟦16⟧ component of G-family force. Here, υ is a non-dimensional scaling factor that associates with linear size (or, a length) pertaining to each object and that associates with the distance between the centers of the objects, ρ is the relevant object property for the case for which $\upsilon = 1$, and r is the distance (for the case of $\upsilon = 1$) between the centers of the objects. The factor υ^3 provides for scaling for an object that has three spatial dimensions. The force would be independent of υ. That independence might suggest, from a standpoint of physics, that a 4G2468⟦16⟧ component of 4G would associate with concentrating matter or energy before the suggested era in which much of the matter in the universe consists of jay bosons.

$$(\upsilon^3\rho)^2/(\upsilon r)^6 \tag{172}$$

We assume that 4G provides the dominant phenomena that pertain early in this era. (For later eras, we identify a combination of stuff - or non-G-family phenomena - and dominant components of G-family forces.)

We assume that interactions of the form that equation (173) shows pertain. Here, we assume that the net circular polarization for before the interaction is zero.

$$4(6)\mathrm{G2468}[\![16]\!]{+}4(6)\mathrm{G2468}[\![16]\!] \rightarrow 2(1)\mathrm{J}_1{+}2(1)\mathrm{J}_2 \tag{173}$$

For each value of i_4, we assume that interactions - to which equation (173) alludes - populate roughly equally isomers I(0;i_4) through I(5;i_4).

For each value of i_4, interactions - to which equation (173) alludes - would occur independently of similar interactions that associate with other sets - I(0,1,2,3,4,5;$\neq i_4$) - of isomers.

We explore the PR36ISP modeling topic of the relative abundance of each of the six isomers of 4G.

One possibility is that some mechanism, such as a mechanism associated with 6(36)G46⟦16⟧, leads to sufficient transfers of energy to catalyze nearly similar formation across the six isomers of 4G. (See table 20. We assume that the span of 6G46⟦16⟧ does not extend beyond the relevant ι_I isomers.)

One possibility is that the isomers of 4G form relatively independently from each other.

We know of no data that would suggest a choice among such possibilities.

For PRι_IISP models for which ι_I exceeds one, we posit roughly equal creation of ι_I isomers of jay bosons.

We note one aspect regarding modeling.

This essay de-emphasizes possible associations - from the standpoint of modeling - between 4G2468⟦16⟧ and the cosmological constant.

2.4.2. The later of two eras that might occur before inflation

We explore the notion that, just before the inflationary epoch, the main component of the universe might have consisted of jay bosons.

Extant modeling seems to suggest that nature creates photons (or, 2G) primarily after the inflationary epoch. Regarding times just before inflation, we assume that the allowed reactions that table 39 shows pertain.

We assume that the particle density is sufficiently large that modeling can associate the production of 4G with the 4G2468x components of 4G.

Equation (174) describes a possible interaction.

$$2(1)\mathrm{J}_1 + 2(1)\mathrm{J}_1 \rightarrow 4(1)\mathrm{G2468x} + 0(1)\mathrm{I} \tag{174}$$

4G has a span of six. To the extent that ι_I exceeds one, isomers within each I(0,1,2,3,4,5;i_4) interact with each other during and after this period.

Table 39 suggests that interactions between pairs of jay bosons do not create photons. A lack of photons is compatible with extant modeling that suggests that significant presence of photons starts after inflation.

2.4.3. Inflation

We discuss possibilities regarding the inflationary epoch.

Extant modeling suggests that an inflationary epoch might have occurred. Extant modeling suggests that the epoch started around 10^{-36} seconds after the Big Bang. Extant modeling suggests that the epoch ended around 10^{-33} seconds to 10^{-32} seconds after the Big Bang. We are not certain as to the extent to which data confirms the occurrence of an inflationary epoch.

Extant modeling includes models that people claim would support notions of inflation. The models point to states of the universe, at and somewhat after the inflationary epoch, that would provide bases for evolution that would be consistent with observations about later phenomena and would be consistent with aspects of extant modeling. (Reference [13] summarizes aspects related to inflation, points to references regarding extant modeling, and discusses some extant modeling work.)

Reference [14] suggests the possibility that a repulsive aspect of gravity drove phenomena associated with the inflationary epoch. The reference suggests that the composition of the universe was nearly uniform spatially. The reference suggests the importance of a so-called inflaton field.

Proposed modeling suggests the possibility that, during the inflationary epoch, aye particles (or, 0I particles) provided a major non-long-range-force component of the universe. The aye particle matches extant modeling notions of a boson with zero spin. (See reference [13].) Extant modeling uses the word inflaton to name that boson. Proposed modeling suggests the possibility that the octupole components of 4γ provided the repulsive aspect of gravity. (Components 4G4268x associate with GFC octupole and with KIN octupole.) Those components interact with individual span-one particles and are repulsive. Equation (175) shows such an interaction. Here, x and y might be either of a and b.

$$0(1)\mathrm{I} + 4(1)\mathrm{G2468x} \rightarrow 0(1)\mathrm{I} + 4(1)\mathrm{G2468y} \tag{175}$$

Around the time of the inflationary epoch, octupole attraction associating with 4G246⟦16⟧ might play a role. (Perhaps, see table 21.)

Table 46: Ordinary matter, four cold dark matter isomers, and the one other dark matter isomer

Isomers n (as in I(n;0))	Aspect - regarding each isomer I(n;0)
0	Includes ordinary matter.
3	Evolves similarly to isomer I(0;0).
1, 2, 4, and 5	Evolves into cold dark matter.

2.4.4. Just after inflation

The end of the inflationary epoch might associate with a change, regarding effects of 4γ, from octupole repulsion being dominant to quadrupole attraction being dominant. The end of the inflationary epoch might also associate with a growth of spatial inhomogeneities regarding (at least) aye particles. The quadrupole component of 4γ might help catalyze some of the spatial inhomogeneities. The quadrupole component of 4γ might amplify some of the spatial inhomogeneities.

Proposed modeling suggests the possibility that, for some time just after the inflationary epoch, the aye particle might have been - within each isomer I(i_2;i_4) - a dominant non-long-range-force component. Interactions between aye particles would produce components of 2G forces. (See equation (176).) Interactions of 2G with itself produce matter-and-antimatter pairs of span-one fermions. Proposed modeling suggests the possibility that attraction based on the (quadrupole) 4G246 component of 4γ contributed to clumping.

$$0\mathrm{I} + 0\mathrm{I} \rightarrow 2\mathrm{G} + 2\mathrm{G} \tag{176}$$

2.4.5. Dissimilarities between isomers

We consider a thought experiment regarding isomer I(0;0) (or, the isomer that includes ordinary matter) and a so-called alt isomer. Here, the alt isomer is one of I(1;0), I(2;0), I(4;0), and I(5;0).

The stuff that associates with the alt isomer and the stuff that associates with isomer I(0;0) exhibit similarities with respect to phenomena involving quarks, gluons, and W-family bosons.

We consider a time at which the densities of stuff are high and the compositions of stuff associating with the isomers are essentially similar. Similar evolution would occur to the extent that one considers just quarks, gluons, and W-family bosons.

We consider three-quark baryons (real or virtual) that consist of generation three quarks. The charged baryons are more massive than the neutral (or, charge-neutral) baryons. (Consider the masses - per table 37 - of the constituent quarks.)

For the alt isomer, generation three leptons are less massive than the tauon that associates with isomer I(0;0) generation three. Interactions that produce generation three leptons (and produce or consume W bosons) facilitate - in the alt isomer compared to isomer I(0;0) - more transitions from all-generation-three charged baryons to all-generation-three neutral baryons.

Over time, in both isomers, generation three quarks and generation two quarks evolve, via interactions that entangle multiple W bosons, into generation one quarks.

We consider a time when the transitions to all-generation-one quarks have just completed. Densities of stuff have dropped. We consider all-generation-one baryons. The alt isomer contains more alt neutrons than isomer I(0;0) contains neutrons. The mass of the alt isomer generation one charged lepton exceeds the mass of the isomer I(0;0) generation one charged lepton (or, the mass of the electron). The (already more abundant, compared to isomer I(0;0)) alt isomer neutrons have difficulties (compared to isomer I(0;0) neutrons) decaying into charged baryons.

From then on, the alt isomer has, compared to isomer I(0;0), more neutrons and fewer protons. The alt isomer has, compared to isomer I(0;0), fewer charged leptons. The alt isomer has, compared to isomer I(0;0), fewer charged leptons with masses equal to the mass of the isomer I(0;0) electron.

Even to the extent that stuff associating with the alt isomer forms some stars, the alt isomer becomes cold dark matter consisting mainly of alt neutrons and alt hydrogen-like atoms. Also, the collection of - mostly old - alt isomer photons cools.

We consider isomer I(0;0) and isomer I(3;0).

Presumably, similar evolution pertains regarding isomer I(0;0) and isomer I(3;0). For example, isomer I(3;0) stuff forms stars in numbers similar to isomer I(0;0) numbers.

Table 46 pertains.

Table 47: Aspects regarding three eras associating with the expansion of the universe

Aspect	Era: Inflation	Era: Next billions of years	Era: Most recent billions of years
Observed changes in the rate	?	Decrease	Increase
Extant modeling KIN model-based changes in the rate	Increase	Decrease	Increase
Proposed modeling ENT model-based changes in the rate	Increase	Decrease	Increase
Drivers, as suggested by ENT modeling and GFC modeling (4G components that dominate between largest objects)	4G2468a, 4G2468b	4G246	4G48
KIN RSDF for the 4G components	r^{-5}	r^{-4}	r^{-3}
Proposed modeling interpretation of KIN modeling for the net force associating with the components	Repulsive	Attractive	Repulsive

2.4.6. Filaments and baryon acoustic oscillations

Proposed modeling is compatible with the extant modeling notion that ordinary matter baryon acoustic oscillations contributed to the formation of filaments.

Regarding models for which ι_I (as in PRι_IISP) exceeds one, each of the five dark matter isomers has its own baryon-like particles and its own 2(1)G physics. Proposed modeling suggests, for models for which ι_I exceeds one, that dark matter baryon-like acoustic oscillations occurred in the early universe. Proposed modeling suggests that dark matter baryon-like acoustic oscillations contributed (along with ordinary matter baryon acoustic oscillations) to the formation of filaments.

2.4.7. The rate of expansion of the universe

Table 47 posits concepts regarding three eras in the rate of expansion of the universe. (Regarding observations that associate with the eras that associate with decrease and recent increase, see references [15], [16], [17], and [18].) We know of no observations that pertain directly to the era of inflation. Extant modeling suggests the existence of an era of inflation.

Table 47 suggests associations between repulsion and 4G48. Table 47 suggests associations between attraction and 4G246. We suggest these associations, based on data.

Work elsewhere in this essay reinforces the notions that 4G246 associates with attraction and that 4G2468a, 4G2468b, and 4G48 associate with repulsion. (See table 21.)

Two thought experiments provide notions that lead to table 47.

We consider one thought experiment. We consider two similar neighboring clumps of stuff. We assume that the clumps are moving away from each other. We assume that the clumps will continue to move away from each other. We assume that, initially, interactions associating with RSDF $r^{-(n+1)}$ dominate regarding interactions between the two clumps. We assume that the two clumps interact via interactions associating with RSDF r^{-n}. We assume that no other forces have adequate relevance. We assume that the distance between the objects increases adequately. Eventually, the RSDF r^{-n} force dominates the RSDF $r^{-(n+1)}$ force.

We consider a similar thought experiment. We consider two similar neighboring clumps. We assume that these clumps are less interactive (for example, less massive) than the two clumps in the first thought experiment. Generally, dominance of the RSDF r^{-n} force over the RSDF $r^{-(n+1)}$ force occurs sooner for the two clumps in the second thought experiment than it does for the two clumps in the first thought experiment.

Interactions between galaxy-like clumps transit to 4G4 RSDF r^{-2} dominance quickly compared to the current age of the universe. Mutual attraction occurs. Interactions between adequately larger clumps can still exhibit 4G48 RSDF r^{-3} dominance. Mutual repulsion occurs.

We discuss modeling regarding recent increases in the rate of expansion.

People suggest that extant modeling underestimates recent increases in the rate of expansion. (See, for example, reference [19], reference [20], reference [21], and reference [22]. However, some people note possible objections to some notions of underestimates. See, for example, references [23] and [24].) People suggest phenomenological remedies regarding the modeling. (See, for example, reference [25].)

Proposed modeling suggests a basis for such underestimates.

We consider a thought experiment.

Here, we assume that people use models that associate with data about the rate of expansion during the era of decreases in that rate. We assume that the models have bases in equations of state and in general relativity.

Proposed modeling associates dominant effects - for the era of decreasing rate - with the span of one that associates with 4G246. Proposed modeling associates dominant effects for the recent era with the span of two that associates with 4G48.

Applying decreasing-rate era equations of state and general relativity to current era phenomena associates with underestimating a key factor - 4G48 repulsion - by, conceptually, a factor of two.

2.4.8. Dark matter density of the universe

Extant modeling discusses five partial densities of the universe. The symbol Ω_{c} denotes dark matter (or, cold dark matter) density of the universe. The symbol Ω_{b} denotes ordinary matter (or, baryonic matter) density of the universe. The symbol Ω_ν denotes neutrino density of the universe. The symbol Ω_γ denotes photon density of the universe. The symbol Ω_Λ denotes dark energy density of the universe. Each of the five densities associates with data. Equation (177) pertains regarding the total density of the universe, Ω.

$$\Omega = \Omega_{\mathrm{c}} + \Omega_{\mathrm{b}} + \Omega_\nu + \Omega_\gamma + \Omega_\Lambda \tag{177}$$

Reference [3] provides the data that equations (178), (179), (180), and (181) show.

$$\Omega_{\mathrm{c}} \approx 0.265 \pm 0.007 \tag{178}$$

$$\Omega_{\mathrm{b}} \approx 0.0493 \pm 0.0006 \tag{179}$$

$$\Omega_\nu \leq 0.003, \text{ also } \Omega_\nu \geq 0.0012 \tag{180}$$

$$\Omega_\gamma \approx 0.0000538 \pm 0.0000015 \tag{181}$$

In extant modeling, the symbol Ω_{c} associates with all dark matter. To the extent that proposed modeling PR6ISP modeling comports with nature, the symbol Ω_{c} associates with all of the three aspects - isomer I(0;0) 1R⊗2U hadron-like particles, the four dark matter isomers that we associate above with the word cold, and the one dark matter isomer I(3;0) that we do not necessarily associate above with the word cold - that proposed modeling associates with the term dark matter. (See table 46.)

Proposed modeling suggests considering - for each isomer I(j;0), with $0 \leq j \leq 5$ - equation (182). (Technically, the isomers share a fraction of Ω_γ, but the total Ω_γ is small.) The symbol $\Omega_{\mathrm{1R2U,j}}$ denotes the density of the universe that associates with the 1R⊗2U hadron-like particles that associate with isomer I(j;0). From here on, we de-emphasize the densities of neutrinos and the densities of photons. Equation (183) pertains. Even though isomers evolve differently with respect to quark-based hadrons, we assume that there is adequate similarity in evolution so that equation (184) pertains. Equations (185) and (186) pertain.

$$\Omega_j = \Omega_{\mathrm{b},j} + \Omega_{\mathrm{1R2U},j} + \Omega_{\nu,j} + \Omega_{\gamma,j} \tag{182}$$

$$\Omega_{\mathrm{b}} + \Omega_{\mathrm{c}} \approx \sum_{j=0}^{5} \Omega_j \tag{183}$$

$$\Omega_{\mathrm{1R2U},j} \approx \Omega_{\mathrm{1R2U},0}, \text{ for } 0 \leq j \leq 5 \tag{184}$$

$$\Omega_{\mathrm{b}} + \Omega_{\mathrm{c}} \approx \Omega_{\mathrm{b}} + \Omega_{\mathrm{1R2U},0} + 5(\Omega_{\mathrm{1R2U},0} + \Omega_{\mathrm{b}}) \tag{185}$$

$$\Omega_{\mathrm{1R2U},0} \approx (\Omega_{\mathrm{c}} - 5\Omega_{\mathrm{b}})/6 \tag{186}$$

Equation (187) estimates $\Omega_{\mathrm{1R2U},0}$ for the current state of the universe.

$$\Omega_{\mathrm{1R2U},0} \approx 0.0031 \tag{187}$$

Except possibly regarding dark energy density (or, Ω_Λ), proposed modeling suggests that ratios of the actual values of the various $\Omega_{...}$ in equation (177) remain constant for essentially the entire history of the universe. (This essay does not speculate - regarding this topic - regarding the very earliest times after the Big Bang. Regarding Ω_Λ, see discussion related to equation (189).) PR6ISP proposes no significant mechanisms for transferring - adequately after the Big Bang - stuff between ordinary matter and dark matter. (We assume that net transfers based on components - for which the spans are greater than one - of 2G are negligible.)

We discuss measurements via which people infer densities - of dark matter and ordinary matter - of the universe.

People use data from observations of CMB (or, cosmic microwave background radiation) to infer ratios - of dark matter density of the universe to ordinary matter density of the universe - to which equations (178), (179), (180), and (181) point. A ratio of five-plus to one might pertain for billions of years.

Regarding data based on CMB, measured ratios of dark matter density of the universe to ordinary matter density of the universe would not much change regarding times for which equation (188) pertains. That time range starts somewhat after 380,000 years after the Big Bang and continues through now. (Perhaps, see reference [26].)

$$\Omega_\gamma \ll \Omega_\mathrm{b} \text{ and } \Omega_\nu \ll \Omega_\mathrm{b} \tag{188}$$

2.4.9. Dark energy density of the universe

We explore possible explanations for nonzero dark energy density of the universe.

Equation (189) shows a ratio of presently inferred density of the universe of dark energy to presently inferred density of the universe of dark matter plus ordinary matter plus (ordinary matter) neutrinos plus (ordinary matter) photons. (Reference [3] provides the five items of data.) Inferences that reference [27] discusses might suggest that inferred dark energy density increases with time. Reference [26] suggests that an inferred dark energy density of essentially zero associates with times around 380,000 years after the Big Bang. We know of no inferences that would not comport with a somewhat steady increase - regarding the inferred ratio associating with equation (189) - from approximately zero over time since somewhat after the Big Bang.

$$\Omega_\Lambda/(\Omega_\mathrm{c} + \Omega_\mathrm{b} + \Omega_\nu + \Omega_\gamma) \approx 2.18 \tag{189}$$

Some aspects of extant modeling associate inferred dark energy densities of the universe with phenomena for which people use terms such as vacuum energy, vacuum fluctuations, or quintessence. Proposed modeling is not necessarily incompatible with such extant modeling. Nevertheless, we discuss possibilities for proposed modeling that might explain nonzero dark energy density.

For any one of PR1ISP modeling, PR6ISP modeling, and PR36ISP modeling, aspects related to the aye (or, 0I) boson or the jay (or, 2J) boson might lead to phenomena similar to effects that extant modeling associates with some terms such as vacuum energy, vacuum fluctuations, or quintessence. (See discussion related to equations (149) and (150). Perhaps, also note discussion related to equation (172).)

For PR6ISP modeling, proposed modeling includes the notion of 2(6)G248, whereas extended extant modeling might associate with the notion of 2(1)G248. The difference, in proposed modeling, between 2(6)G248 and 2(1)G248 might associate with nature's indirectly producing effects, regarding CMB, that people associate (via extant modeling) with some nonzero dark energy density. The difference associates with interactions between ordinary matter and dark matter.

PR36ISP modeling offers another possibility. (This possibility associates with a six-fold aspect that table 18 associates with the parameter i_4 - as in I(0,...,5;i_4).) We assume that the spans of 4(6)G4 and the other 4(>1)GΓ components are orthogonal to the spans of 2(6)G248 and the other 2(>1)GΓ components. The PR36ISP universe associates with six isomers of a PR6ISP sub-universe. (Perhaps, compare with table 42.) Each PR6ISP sub-universe includes its own isomer of 4(6)G4. We continue to associate ordinary matter (and some dark matter) with isomer I(0;0) and most dark matter with isomers I(1;0) through I(5;0). We use the three-word term doubly dark matter to associate with the 30 isomers that associate with the symbols I(0,...,5;i_4) for which $1 \leq i_4 \leq 5$. (See table 16.) Doubly dark matter isomers do not interact with ordinary matter via 4G. Dark matter isomers do not interact with ordinary matter via 2G. Differences between 2(>1)GΓ and 2(1)GΓ associate with interactions between ordinary matter plus dark matter and doubly dark matter. All interactions - mediated by 2G - that PR6ISP modeling would associate with interactions between ordinary matter and dark matter isomers become - for PR36ISP modeling - interactions between ordinary matter and doubly dark matter. Dark energy density might associate with stuff associating with the 30 doubly dark matter isomers. Modeling suggests

Table 48: Opportunities for advances regarding astrophysics

Opportunity
• Describe mechanisms leading to an observed amount of depletion - some of which has bases in hyperfine interactions with hydrogen atoms - of cosmic microwave background radiation.
• Hone scenarios associating with the formation of galaxies.
• Explain data - that extant modeling seems not to explain - about the following.
◦ Large clumps of ordinary matter gas and of dark matter.
◦ Ratios of dark matter to ordinary matter in galaxy clusters.
◦ Amounts of stuff that does and does not pass through - with mainly just gravitational interactions - collisions of galaxy clusters.
◦ Some aspects of interactions between galaxies.
◦ Ratios - within galaxies - of dark matter to ordinary matter.
◦ Dark matter effects within the Milky Way galaxy.
◦ High-mass neutron stars.

an upper bound of approximately five regarding a possible future value for the ratio that associates with equation (189).

2.5. Astrophysics

Table 48 lists opportunities for advances regarding astrophysics. Proposed modeling suggests advances regarding each opportunity.

We discuss ratios that proposed modeling might predict or explain.

Table 49 lists some approximate ratios - of effects of other than ordinary matter to effects of ordinary matter - that PR6ISP modeling or PR36ISP modeling might explain. (Regarding depletion of CMB, PR36ISP modeling suggests the possibility that an approximate ratio of doubly dark matter effects to ordinary matter effects pertains. Otherwise, each row in table 49 associates with either PR6ISP or PR36ISP and with ratios of dark matter effects to ordinary matter effects.) We designed PR6ISP modeling to explain the five-plus to one ratio that people observe regarding densities of the universe. Here, the five associates with dark matter isomers of known span-one elementary particles and the plus associates with hadron-like particles that do not interact with 2γ force components. Galaxy clusters seem to be sufficiently large to comport with similar ratios. (However, galaxy clusters that are remnants of collisions of galaxy clusters might be exceptions. See discussion related to table 50.) Discussion regarding 2(2)G68 associates with the approximately one to one ratio. (See discussion related to equation (190).) For PR6ISP, the depletion that does not associate directly with I(0;0) associates with I(3;0). For PR36ISP, the depletion that does not associate directly with I(0;0) associates with I(0;3). The following notions pertain regarding galaxies. DMA:OMA ratios of zero-plus to one, four to one, and one to zero-plus comport with roles of non-monopole components of gravity in scenarios regarding galaxy formation. (See discussion related to table 52.) This essay does not speculate regarding the feasibility of measuring $\lesssim$4:1 ratios regarding early galaxies or somewhat early galaxies. People infer - based on observations of recent objects - that ratios of 1:0^+ pertained for some early galaxies. This essay does not speculate regarding the feasibility of directly detecting early galaxies for which ratios of 1:0^+ would pertain. DMA:OMA ratios of zero-plus to one, four to one, and one to zero-plus comport with scenarios regarding some galaxies for which observations associate with times well after galaxy formation. (See other discussion related to table 52.)

Discussion below points to observations that might associate with each of the ratios that table 49 shows, other than - for early galaxies - $\lesssim$4:1. (Regarding $\lesssim$4:1 for early galaxies, perhaps see discussion related to table 52.)

2.5.1. CMB depletion via hyperfine interactions

People measure specific depletion of CMB and attribute some of that depletion to hyperfine interactions with (ordinary matter) hydrogen atoms. (See reference [28].) The amount of depletion is twice or somewhat more than twice the amount that people expected. At least one person speculates that the amount above expectations associates with effects of dark matter. (See reference [29].)

Proposed modeling suggests the following explanation. Solution 2(2)G68 (or, 2G68) might associate with hyperfine interactions. (Perhaps, note equation (190).) Solution 2G68 has a span of two. (See table 33b.) Half or somewhat less than half of the observed absorption associates with the ordinary matter isomer of hydrogen atoms. An approximately equal amount of the observed effect associates with

Table 49: Approximate ratios - that proposed modeling might explain - of other than ordinary matter effects to ordinary matter effects (with OOM denoting the four-word term other than ordinary matter and denoting one of DM and DDM; with DM denoting dark matter; with DDM denoting doubly dark matter; with OM denoting ordinary matter; with A denoting amount; and with OM CMB denoting cosmic microwave background radiation)

Approximate OOMA:OMA	Amounts	Relevant OOM might be DM	Relevant OOM might be DDM
5^+:1	Density of the universe	$\iota_I = 6$, $\iota_I = 36$	
5^+:1	Amount of stuff in some galaxy clusters	$\iota_I = 6$, $\iota_I = 36$	
1:1 or 1^+:1	Amount of a type of depletion of OM CMB, possibly via interactions with OOM atoms or known to be via interactions with OM atoms.	$\iota_I = 6$	$\iota_I = 36$
0^+:1	Amount of stuff in some early galaxies	$\iota_I = 6$, $\iota_I = 36$	
$\lesssim$4:1	Amount of stuff in some somewhat early galaxies	$\iota_I = 6$, $\iota_I = 36$	
1:0^+	Amount of stuff in some early galaxies	$\iota_I = 6$, $\iota_I = 36$	
0^+:1	Amount of stuff in some later galaxies	$\iota_I = 6$, $\iota_I = 36$	
$\approx$4:1	Amount of stuff in some later galaxies	$\iota_I = 6$, $\iota_I = 36$	
1:0^+	Amount of stuff in some later galaxies	$\iota_I = 6$, $\iota_I = 36$	

hydrogen-atom isomers that associate with one dark matter isomer (or, I(3;0)) for PR6ISP modeling or one doubly dark matter isomer (or, I(0;3)) for PR36ISP modeling.

$$2G68 \notin 2\gamma,\ \ 2G68 \notin \gamma 2 \tag{190}$$

To the extent that the absorption by ordinary matter is less than half of the total absorption, the following explanations might pertain regarding the difference between less than half and equal to half. One explanation associates with the notion that the evolution of the relevant non-ordinary-matter isomer might differ from the evolution of the ordinary matter isomer. The non-ordinary-matter isomer might have more hydrogen-atom-like objects than does the ordinary matter isomer. One explanation associates with 2GΓ solutions with spans of at least two. Each one of solutions 2(6)G46 and 2(6)G468 might pertain. For each one, the solution is not a member of 2γ and is not a member of $\gamma 2$. The number six appears in both the Γ for 2(6)G46 and the Γ for 2(6)G468. Solution 2(6)G46 associates with a KIN spatial dipole effect. Solution 2(6)G468 associates with a KIN spatial dipole effect (and with the notion of GFC quadrupole solution).

Proposed modeling might contribute to credibility for assumptions and calculations that led to the prediction for the amount of depletion that associates with ordinary matter hydrogen atoms. (Regarding the assumptions and calculations, see reference [30].)

2.5.2. Large clumps of ordinary matter gas and of dark matter

Reference [31] discusses observations that point to the notion that - on a large scale - clumping of matter - ordinary matter gas and dark matter - might be less than extant modeling models suggest. Observed phenomena have bases in gravitational lensing of light. The article alludes to a dozen observational studies and points to at least two papers - reference [32] and reference [33]. Clumps would be - to use wording from reference [31] - too thin. (Reference [31] suggests a result of too thin by about ten percent. This essay does not explore the topic of quantifying such thinness.) A distribution of galaxies would be - to use wording from reference [22] - too smooth. Reference [22] suggests a notion of ten percent more evenly spread than extant modeling predicts.

Proposed modeling suggests that such effects might associate with the notion that 4(2)G48 repels more stuff than would 4(1)G48. (See table 33 and table 21.) Early formation of clumps associates with 4(1)G246 attraction. Early clumps associate with single isomers. Effects of 4(2)G48 repulsion would dilute matter around early clumps more than would effects that extant modeling might associate with, in effect, 4(1)G48 repulsion.

2.5.3. Galaxy clusters - ratios of dark matter to ordinary matter

Regarding some galaxy clusters, people report inferred ratios of dark matter amounts to ordinary matter amounts.

References [34] and [35] report ratios of five-plus to one. The observations have bases in gravitational lensing. Reference [36] reports, for so-called massive galaxy clusters, a ratio of roughly 5.7 to one. (Perhaps, note reference [37].) The observations have bases in X-ray emissions.

Table 50: Aspects regarding a collision between two galaxy clusters (with the assumption that each of the two galaxy clusters has not undergone earlier collisions)

Aspect
• Up to essentially nearly all ordinary matter IGM (in each galaxy cluster) interacts with ordinary matter IGM (in the other galaxy cluster) and slows down. (The notion of up to essentially all associates with equally sized colliding galaxy clusters and with a head-on collision.)
• Much of the stuff associating with ordinary matter stars passes through with just gravitational interactions having significance.
• No more than somewhat less than 20 percent of dark matter significantly interacts non-gravitationally with dark matter and, based on non-gravitational interactions, slows down. (For each galaxy cluster, this dark matter associates with the IGM associating with isomer three.)
• At least 80 percent of dark matter passes through with just gravitational interactions having significance.
• Essentially all of the incoming 1R⊗2U passes through the collision with just gravitational interactions having significance.

Proposed modeling PR6ISP modeling and PR36ISP modeling are not incompatible with these galaxy cluster centric ratios.

Reference [38] suggests a formula that associates - across 64 galaxy clusters - dark matter mass, hot gas baryonic mass (or, essentially, ordinary matter mass), and two radii from the centers of each galaxy cluster. The reference suggests that the formula supports the notion of a relationship between dark matter and baryons. This essay de-emphasizes discussing the extent to which proposed modeling comports with this formula. Proposed modeling might suggest a relationship, based on proposed similarities between dark matter and ordinary matter.

2.5.4. Galaxy clusters - collisions

People use the two-word term Bullet Cluster to refer, specifically, to one of two galaxy clusters that collided and, generally, to the pair of galaxy clusters. The clusters are now moving away from each other. Extant modeling makes the following interpretations based on observations. For each of the two clusters, dark matter continues to move along trajectories generally consistent with just gravitational interactions during the collision. For each of the two clusters, (ordinary matter) stars move along trajectories generally consistent with just gravitational interactions during the collision. For each of the two clusters, (ordinary matter) gas somewhat generally moves along with the cluster, but generally lags behind the other two components (dark matter and stars). Regarding such gas, people use the acronym IGM and the two-word term intergalactic medium. Extant modeling suggests that the IGM component of each original cluster interacted electromagnetically with the IGM component of the other original cluster. Electromagnetic interactions led to slowing the motion of the gas.

If each of the six dark matter or ordinary matter isomers evolved similarly, there might be problems regarding explaining aspects of the Bullet Cluster. One might expect that, in each galaxy cluster, more (than the observed amount of) dark matter would lag. The lag would occur because of one-isomer 2G-mediated interactions within each of the five dark matter isomers. Possibly, for each dark matter isomer, there would not be enough star-related stuff to explain the amount of dark matter that is not lagging. Possibly, across the six (five dark matter and one ordinary matter) isomers, there would not be enough 1R⊗2U dark matter to significantly help regarding explaining the amount of dark matter that is not lagging.

We assume that four dark matter isomers associate with proposed modeling notions of cold dark matter and that one dark matter isomer exhibits behavior similar to behavior that ordinary matter exhibits. (See discussion related to table 44 and see table 46.)

Proposed modeling suggests that, for each of the two galaxy clusters, essentially all the stuff associating with isomers I(1;0), I(2;0), I(4;0), and I(5;0) would pass through the collision with just gravitational interactions having significance. For isomer I(3;0), incoming 1R⊗2U would pass through. For isomer I(0;0), incoming 1R⊗2U (which measures as dark matter) would pass through. Thus, at least 80 percent of the incoming dark matter would pass through the collision with just gravitational interactions having significance.

Table 50 lists aspects regarding a collision between two galaxy clusters. Here, we assume that each of the two galaxy clusters has not undergone earlier collisions.

We suggest that these proposed modeling notions might comport with various possible findings about

IGM after a collision such as the Bullet Cluster collision. The findings might point to variations regarding the fractions of IGM that, in effect, stay with (the cores of) outgoing galaxy clusters and the fractions of IGM that, in effect, (at least somewhat) detach from (the cores of) outgoing galaxy clusters.

We discuss possible aspects regarding an outgoing galaxy cluster.

Suppose that, before a collision, ordinary matter IGM comprised much of the ordinary matter in the galaxy cluster. Suppose that, because of the collision, the galaxy cluster has a significant net loss of ordinary matter IGM. After the collision, the galaxy cluster could have a (perhaps somewhat arbitrarily) large ratio of amount of dark matter to amount of ordinary matter.

To the extent that IGM detaches from galaxy clusters after the galaxy clusters collide, the detached IGM might form one or more objects. Some such objects might have roughly equal amounts of dark matter and ordinary matter. The dark matter would associate with isomer three.

2.5.5. Interactions between galaxies

Reference [39] reports measurements pertaining to external gravitational effects on components of individual galaxies. The article suggests that - compared to expected results based on notions that associate with the strong equivalence principle and with general relativity - observations point to unexpected effects - of neighboring galaxies - regarding galaxy rotation curves. The article suggests the possibility of associating the unexpected effects with the notion of an external field effect and possibly with aspects of MOND (or, Milgromian dynamics or modified Newtonian dynamics).

Proposed modeling provides the possibility that the unexpected results associate with differences in spans between 4G4 (for which the span is six) and (perhaps mainly just) 4G48 (for which the span is two) and (maybe also) other components of 4γ (for which the spans are one).

2.5.6. Galaxies - formation

We discuss scenarios regarding galaxy formation and evolution. We anticipate that such galaxy formation and evolution scenarios will explain galaxy centric data that table 49 shows.

Models for galaxy formation and evolution might take into account the following factors - one-isomer repulsion (which associates with the 4G2468a and 4G2468b solutions), one-isomer attraction (which associates with 4G246), two-isomer repulsion (which associates with 4G48), six-isomer attraction (which associates with 4G4), dissimilarities between isomers, the compositions of filaments and galaxy clusters, statistical variations in densities of stuff, and collisions between galaxies. Modeling might feature a notion of a multicomponent fluid with varying concentrations of gas-like or dust-like components and of objects (such as stars, black holes, galaxies, and galaxy clusters) for which formation associates significantly with six-isomer (or 4G4) attraction.

We focus on early-stage galaxy formation and evolution. For purposes of this discussion, we assume that we can de-emphasize collisions between galaxies. We suggest the two-word term untouched galaxy for a galaxy that does not collide, before and during the time relevant to observations, with other galaxies. We emphasize formation scenarios and evolution scenarios for untouched galaxies. (Reference [40] and reference [41] discuss data that pertains regarding a time range from about one billion years after the Big Bang to about 1.5 billion years after the Big Bang. Observations suggest that, out of a sample of more than 100 galaxies or galaxy-like rotating disks of material, about 15 percent of the objects might have been untouched.)

We assume that differences - in early evolution - regarding the various isomers do not lead, for the present discussion, to adequately significant differences - regarding 4G interactions and galaxy formation - between isomers. (This assumption might be adequately useful, even given our discussion regarding cold dark matter and our discussion regarding the Bullet Cluster. Regarding cold dark matter, see discussion related to table 46. Regarding the Bullet Cluster, see discussion related to table 50.)

We organize this discussion based on the isomer or isomers that originally clump based, respectively, on 4G246 attraction or on 4G246 attraction and 4G4 attraction. Each one of some galaxies associates with an original clump that associates with just one isomer. Multi-isomer original clumps are possible. Because of 4(2)G48 repulsion, an upper limit on the number of isomers that an original clump features might be three.

Table 51 discusses a scenario for the formation and evolution of a galaxy for which the original clump contains essentially just one isomer. Regarding this isomer, we use the word featured. We assume that stuff that will become the galaxy is always in somewhat proximity with itself. We assume that no collisions between would-be galaxies or between galaxies occur.

Table 51: A scenario for the formation and evolution of a galaxy for which the original clump contains essentially just one isomer (with the two-word phrase featured isomer associating with that one isomer)

Step
• Early on, stuff associating with each one of the six isomers expands, essentially independently from the stuff associating with other isomers, based on repulsion associating with 4(1)G2468a and 4(1)G2468b.
• Then, each isomer starts to clump, essentially independently from the other isomers, based on attraction associating with 4(1)G246.
• With respect to clumps associating with any one isomer, 4(2)G48 repels one other isomer and repels some stuff associating with the first-mentioned isomer.
• A galaxy forms based on a clump that contains mostly the featured isomer.
• The galaxy attracts and accrues, via 4(6)G4 attraction, stuff associating with the four isomers that the featured isomer does not repel (via 4(2)G48 repulsion). The galaxy can contain small amounts of stuff associating with the isomer that the featured isomer repels.

2.5.7. Galaxies - ratios of dark matter to ordinary matter

We continue to explore the realm of one-isomer clumps.

One of two cases pertains. For so-called case A, one isomer of 4(2)G48 spans (or connects) isomers zero and three. (Regarding numbering for isomers, see n in table 46. For example, the two-word term isomer three associates with I(3;0).) For so-called case B, one isomer of 4(2)G48 spans isomer zero and one isomer out of isomers one, two, four, and five. Discussion related to equation (190) suggests that case A pertains. The existence of many spiral galaxies might point to the notion that case A pertains. (Compare the rightmost column in table 52a and the rightmost column in table 52b.) However, here, we discuss both cases.

Table 52 pertains. (See table 49.) The following sentences illustrate the notion that some statements in table 52 are at least somewhat conceptual. We assume that local densities for the isomers are somewhat the same. We assume that the galaxy remains adequately untouched. For each row in the table, OM stars can form (and become visible) over time, whether or not significant OM star formation occurs early on. The notation DMA:OMA=1:0^+ denotes the notion that the ratio of OMA to DMA might be arbitrarily small. (Table 49 defines the three-letter terms DMA and OMA.) The notion of three or four DM isomers in a halo refers to the notion that one or zero (respectively) of the DM isomers in the halo is the featured isomer. We de-emphasize some aspects regarding 1R⊗2U hadron-like particles.

Table 52 reflects at least two assumptions. Each core clump features one isomer. Each galaxy does not collide with other galaxies. Yet, data of which we know and discussion below seem to indicate that ratios that table 52 features might pertain somewhat broadly. We think that galaxies that have core clumps that feature more than one isomer are more likely to appear as elliptical galaxies (and not as spiral galaxies) than are galaxies that have core clumps that feature only one isomer. Such likelihood can associate with starting as being elliptical. Such likelihood can associate with earlier transitions - via collisions - from spiral to elliptical.

We explore the extent to which the galaxy formation scenarios comport with observations.

Observations regarding stars and galaxies tend to have bases in ordinary matter isomer 2G phenomena (or, observable electromagnetism). (The previous sentence de-emphasizes some observations - regarding collisions between black holes or neutron stars - that have bases in 4G phenomena.) People report ratios of amounts of dark matter to amounts of ordinary matter.

We discuss observations associating with early in the era of galaxy formation. Table 49 comports with these results. We suggest that visible early galaxies associate with generalization of label-A0 or with generalization of label-B0. (See table 52.) Label-A3 or label-B3 evolves similarly to label-A0 or label-B0, but is not necessarily adequately visible early on.

- Reference [42] provides data about early-stage galaxies. (See, for example, figure 7 in reference [42]. The figure provides two graphs. Key concepts include redshift, stellar mass, peak halo mass, and a stellar - peak halo mass ratio.) Data associated with redshifts of at least seven suggests that some galaxies accrue, over time, dark matter, with the original fractions of dark matter being small. Use of reference [43] suggests that redshifts of at least seven pertain to times ending about 770 million years after the Big Bang.

- Reference [44] reports zero-plus to one ratios. The observations have bases in the velocities of stars within galaxies and associate with the three-word term galaxy rotation curves.

Table 52: Aspects regarding untouched galaxies that associate with original one-isomer clumps (with just one of cases A and B pertaining to all galaxies)

(a) Case A

Label	Featured isomer (n)	Early aspects regarding the galaxy	Possible later aspects regarding the galaxy
A0	0	Forms some ordinary matter stars early on. Starts at DMA:OMA=0^+:1.	Attracts cold dark matter over time. Can get to DMA:OMA≈4:1, with most DM in a halo. Might be a spiral galaxy.
A3	3	Forms some dark matter stars early on. Starts at DMA:OMA=1:0^+.	Attracts the four other DM isomers over time. Some OM stars can form over time. Can settle at DMA:OMA=1:0^+. The three-word term dark matter galaxy pertains.
AX	Any one of 1, 2, 4, and 5	Might form dark matter stars early on. Starts at DMA:OMA=1:0^+.	Attracts the OM isomer and three other isomers over time. OM stars can form over time. Can get to DMA:OMA≈4:1, with three or four DM isomers in a halo. Might become an elliptical galaxy.

(b) Case B

Label	Featured isomer (n)	Early aspects regarding the galaxy	Possible later aspects regarding the galaxy
B0	0	Forms some ordinary matter stars early on. Starts at DMA:OMA=0^+:1.	Attracts isomer three and three cold dark matter isomers over time. Can get to DMA:OMA≈4:1, with three DM isomers in a halo. Might appear to be an elliptical galaxy.
BP	The DM isomer that 4(2)G48 connects to the OM isomer	Might form dark matter stars early on. Starts at DMA:OMA=1:0^+.	Attracts the other DM isomers over time. OM stars can form over time. Can settle at DMA:OMA=1:0^+. The three-word term dark matter galaxy pertains.
B3	3	Forms some dark matter stars early on. Starts at DMA:OMA=1:0^+.	Attracts the OM isomer and three other DM isomers over time. OM stars can form over time. Can get to DMA:OMA≈4:1, with three DM isomers in a halo. Might appear to be an elliptical galaxy.
BY	Any one of the other three DM isomers	Might form dark matter stars early on. Starts at DMA:OMA=1:0^+.	Attracts the OM isomer and three other DM isomers over time. OM stars can form over time. Can get to DMA:OMA≈4:1, with three or four DM isomers in a halo. Might appear to be an elliptical galaxy.

We discuss observations associating with later times. Table 49 comports with these results.

- Reference [45] discusses some MED09 spiral - or, disk - galaxies. A redshift of approximately $z = 1.57$ pertains. (See reference [46].) The redshift associates with a time of 4.12 billion years after the Big Bang. (We used reference [43] to calculate the time.) Reference [45] reports ratios of amount of dark matter to amount of ordinary matter of approximately four to one. The observations have bases in gravitational lensing. We suggest that each label - other than label-A3 or label-BP - that table 52 shows might pertain. (We note, without further comment, that this example might associate with the notion that case A pertains to nature and that case B does not pertain to nature. This example features spiral galaxies. Label-A0 suggests an association with spiral galaxies. Each other label - pertaining to case A or to case B - either associates with dark matter galaxies or might suggest an association with - at least statistically - evolution into elliptical galaxies. See table 52.)
  - To the extent that such an MED09 galaxy models as being nearly untouched, proposed modeling offers the following possibility. The galaxy began based on a one isomer clump. The clump might have featured the ordinary matter isomer. The clump might have featured a dark matter isomer that does not repel ordinary matter. Over time, the galaxy accrued stuff associating with the isomers that the original clump did not repel. Accrual led to a DMA:OMA ratio of approximately four to one.
  - To the extent that such an MED09 galaxy models as not being untouched, proposed modeling offers the following possibility. One type of collision merges colliding galaxies. One type of collision features galaxies that separate after exchanging material. For either type of collision, incoming galaxies having approximately four times as much dark matter as ordinary matter might produce outgoing galaxies having approximately four times as much dark matter as ordinary matter.
- Reference [47] discusses the Dragonfly 44 galaxy. A redshift of $z = 0.023$ pertains. The redshift associates with a time of 13.45 billion years after the Big Bang. (We used reference [43] to calculate the time.) People discuss the notion that ordinary matter accounts for perhaps as little as one part in 10 thousand of the matter in the galaxy. (See reference [48].) The observations have bases in light emitted by visible stars. This case associates with the three-word term dark matter galaxy. We suggest that label-A3 or label-BP might pertain. (See table 52.)

We discuss observations that associate with both early times and later times. Table 49 comports with these results.

- References [49] and [50] discuss observations of ultrafaint dwarf galaxies (or, UFD) for which recent dark-matter-to-ordinary-matter ratios of about 1000 to one pertain. Reference [49] suggests that the notion of just small amounts of ordinary matter seems to pertain throughout the evolution of such galaxies. The observations seem compatible with either one of label-A3 and label-BP. (See table 52.) Proposed modeling notions of dark matter galaxies seem to associate with both early times and later times.

The following notions pertain regarding other data of which we know. Here, the ratios are ratios of dark matter amounts to ordinary matter amounts. Table 49 seems to comport with these results. (See table 52.)

- Reference [51] discusses six baryon-dominated ultra-diffuse galaxies that seem to lack dark matter, at least to the radii studied (regarding gas kinematics) via observations of light with a wavelength of 21 centimeters. These observations seem not to be incompatible with the early stages of label-A0 or label-B0.

- Reference [52] discusses 19 dwarf galaxies that lack having much dark matter, from their centers to beyond radii for which extant modeling suggests that dark matter should dominate. These observations measure r-band light that the galaxies emitted. These observations seem not to be incompatible with the early stages of label-A0 or label-B0.

- People report two disparate results regarding the galaxy NGC1052-DF2. Proposed modeling seems to be able to explain either ratio. Proposed modeling might not necessarily explain ratios that would lie between the two reported ratios.

- Reference [53] suggests a ratio of much less than one to one. The observation has bases in the velocities of stars - or, galaxy rotation curves. This observation seems not to be incompatible with the early stages of label-A0 or label-B0.
- Reference [54] suggests that at least 75 percent of the stuff within the half mass radius is dark matter. This ratio seems similar to ratios that reference [45] discusses regarding some MED09 galaxies. (See discussion above regarding MED09 galaxies.) We suggest that each label - other than label-A3 or label-BP - that table 52 shows can pertain.

- The galaxy NGC1052-DF4 might associate with a ratio of much less than one to one. (See reference [55].) The observation has bases in the velocities of stars - or, galaxy rotation curves. This observation seems not to be incompatible with the early stages of label-A0 or label-B0.
- The compact elliptical galaxy Markarian 1216 has an unexpectedly large amount of dark matter in its core and may have stopped accumulating each of ordinary matter and dark matter approximately 4 billion years after the Big Bang. (See reference [56].) Observations feature the X-ray brightness and temperature of hot gas. This galaxy might associate with an original clump that features three isomers. One isomer would be the ordinary matter isomer. Around the time that the galaxy stopped accruing material, there might have been - near the galaxy - essentially nothing left for the galaxy to attract via 4(6)G4.
- The galaxy XMM-2599 stopped producing visible stars by approximately 1.8 billion years after the Big Bang. (See reference [57].) People speculate regarding a so-called quenching mechanism. Proposed modeling suggests that phenomena similar to phenomena that might pertain regarding Markarian 1216 might pertain regarding XMM-2599.

People report other data. Table 49 and table 52 seem not to be incompatible with these results. We are uncertain as to the extents to which proposed modeling provides insight that extant modeling does not provide.

- One example features a rotating disk galaxy, for which observations pertain to the state of the galaxy about 1.5 billion years after the Big Bang. (See reference [58].) People deduce that the galaxy originally featured dark matter and that the galaxy attracted ordinary matter.
- One example features so-called massive early-type strong gravitation lens galaxies. (See reference [59].) Results suggest, for matter within one so-called effective radius, a minimum ratio of dark matter to dark matter plus ordinary matter of about 0.38. Assuming, for example, that measurements associating with material within larger radii would yield larger ratios, these observational results might support the notion that the galaxies accumulated dark matter over time.
- One example pertains to early stages of galaxies that are not visible at visible light wavelengths. (See reference [60].) Observations feature sub-millimeter wavelength light. We might assume that proposed modeling galaxy formation scenarios comport with such galaxies. We are not certain about the extent to which proposed modeling might provide insight regarding subtleties, such as regarding star formation rates, associating with this example.
- We are uncertain as to the extent to which proposed modeling might provide insight regarding possible inconsistencies - regarding numbers of observed early-stage galaxies and numbers of later stage galaxies - that associate with various observations and models. (For a discussion of some possible inconsistencies, see reference [61].)
- We are uncertain as to the extent to which proposed modeling might provide insight regarding the existence of two types - born and tidal - of ultra-diffuse galaxies. (See reference [62].)

Observations that we discuss above indicate that some galaxies do not exhibit dark matter halos. Proposed modeling that we discuss above comports with the notion that some galaxies do not exhibit dark matter halos.

2.5.8. Some components of galaxies

We discuss effects, within galaxies, that might associate with dark matter.

Reference [63] reports, based on a study of 11 galaxy clusters, more instances of more gravitational lensing - likely associating with clumps of dark matter that associate with individual galaxies - than extant modeling simulations predict. Reference [64] suggests that the number of instances - 13 - compares with an expected number of about one. We suggest the possibility that the clumps might be dark matter galaxies. (See, for example, table 52.) Perhaps some of the dark matter galaxies are dwarf dark matter galaxies. We suggest the possibility that galaxies with significant amounts of ordinary matter gravitationally captured (or at least attracted) such dark matter clumps.

People study globular cluster systems within ultra-diffuse galaxies. Regarding 85 globular cluster systems in ultra-diffuse galaxies in the Coma cluster of galaxies, reference [65] suggests that 65 percent of the ultra-diffuse galaxies are more massive than people might expect based on extant modeling relationships, for so-called normal galaxies, between stellar mass and halo mass. We are uncertain as to the extent to which proposed modeling might explain this result. For example, proposed modeling might suggest that phenomena related to isomers might play a role. (See, for example, table 52.) Higher-mass galaxies might tend to feature more dark matter isomers (or tend to feature more material that associates with such isomers) than do lower-mass galaxies.

Discussion related to table 52 is not incompatible with the notion that visible stars do not include much dark matter.

Discussion related to table 52 is not incompatible with the notion that some black holes that form based on the collapse of stars might originally associate with single isomers. Discussion above is not incompatible with the notion that supermassive black holes might contain material associating with more than one isomer. (Perhaps, note references [66] and [67].)

We suggest that proposed modeling might provide insight about other aspects regarding black holes. People suggest gaps in understanding about the formation of intermediate-mass and large-mass black holes. (Perhaps, note reference [68].) Proposed modeling suggests the possibility that the 4G(1)246 attractive component of G-family forces plays key roles in the early formation of some intermediate-mass and large-mass black holes.

Regarding the coalescing of two black holes, proposed modeling suggests that people might be able to estimate the extent to which 4(2)G48 repulsion pertains. Effects of 4(2)G48 repulsion would vary based on the amounts of various isomers that each black hole in a pair of colliding black holes features.

2.5.9. Dark matter effects within the Milky Way galaxy

People look for possible effects, within the Milky Way galaxy, that might associate with dark matter.

For one example, data regarding the stellar stream GD-1 suggests effects of an object of 10^6 to 10^8 solar masses. (See reference [69].) Researchers tried to identify and did not identify an ordinary matter object that might have caused the effects. The object might be a clump of dark matter. (See reference [70].) Proposed modeling offers the possibility that the object is an originally dark matter centric clump of stuff. Such a clump would likely associate with isomer I(3;0).

For other examples, people report inhomogeneities regarding Milky Way dark matter. (See references [70] and [71].) Researchers note that simulations suggest that such dark matter may have velocities similar to velocities of nearby ordinary matter stars. We suggest that these notions are not incompatible with proposed modeling notions of the existence of dark matter stars that would be similar to ordinary matter stars. Such dark matter stars would likely associate with isomer I(3;0).

2.5.10. High-mass stellar mass black holes

Observations associate with some so-called stellar mass black holes having more mass than extant modeling might suggest. (See reference [72].)

We suggest that some high-mass stellar mass black holes might result from mergers of two (or more) stellar mass black holes, with at least one merging black hole associating with an isomer that differs from the isomers pertaining to each other black hole that forms part of the merged object.

2.5.11. High-mass neutron stars

We discuss possible bases for and properties of high-mass neutron stars.

Observations associate with most known neutron star pairs having masses in the range that equation (191) shows and one neutron star pair having a mass of about 3.4 solar masses. (See references [73] and [74].) Here, M denotes the mass of a pair. The symbol $M_\odot$ denotes the mass of the sun. The 3.4 number results from the second detection via gravitational waves of a merger of two neutron stars. People assign the name GW190425 to that detection.

$$2.5M_\odot \lesssim M \lesssim 2.9M_\odot \tag{191}$$

People speculate - based on, at least, the GW190425 result - about needs for new modeling regarding neutron stars. (See references [73] and [75].)

The span of 4G4 is six.

We suggest that some high-mass neutron stars might result from mergers of two (or more) neutron stars, with at least one merging neutron star associating with an isomer that differs from the isomers pertaining to each other neutron star that forms part of the merged object. Reference [76] discusses a high-mass neutron star for which the magnetic field associates with two poles that do not diametrically oppose (with respect to the center of the star) each other. We suggest the possibility that this star resulted from a merger of two neutron stars. One of the original stars would associate with isomer I(0;0). The other original star might associate with isomer I(3;0). Reference [77] suggests that the same neutron star - J0740 - might have a size that is smaller than extant modeling and observations of other neutron stars might suggest. We suggest that a lack of Pauli exclusion force interactions (or, interactions mediated by jay bosons) between the two isomers might associate with the unexpectedly small size.

3. Results

This unit summarizes results that proposed modeling produces.

Figure 1 and figure 2 summarize results that proposed modeling produces.

3.1. Physics properties

Proposed modeling suggests perspective about modeling and about notions associating with the word object. For example, figure 9 points to a parameter that distinguishes the notion that an object always models as entangled from the notion that an object can model as not entangled.

Proposed modeling catalogs and interrelates some properties of objects. Figure 9 provides a catalog. Examples of extant modeling properties include charge, energy, angular momentum, and momentum. The property of isomer (of span-one elementary particles) arises from proposed modeling. Discussions related to table 18 and table 22 underlie the catalog.

Principles for organizing and uniting the properties come from proposed modeling models that feature components of long-range forces (or, G-family forces). (See, for example, table 13, table 15, and table 19.)

3.2. Elementary particles

Proposed modeling matches all known elementary particles. Proposed modeling suggests elementary particles that people have yet to find. Figure 3 summarizes some information about known and suggested elementary particles.

Particles associating with figure 3 might suffice - from the standpoint of elementary particles - to explain data that extant modeling does not yet explain and to predict data that extant modeling does not necessarily predict. Some of that data associates with the field of cosmology. Some of that data associates with the field of astrophysics. Some of that data associates with the field of elementary particles.

Table 27 alludes to all known elementary particles and to candidate elementary particles that proposed modeling suggests. Figure 4 shows outputs - from one modeling technique - that associate with known and suggested elementary particles. Table 29 shows representations that proposed modeling associates with elementary particles. Table 33 provides information about spans for components of G-family elementary particles.

Proposed modeling points to various associations among properties of elementary particles and strengths of interactions.

Figure 5 shows suggested rest energies for all elementary fermions other than the electron and muon (for which people have determined masses rather accurately).

3.3. Cosmology

Proposed modeling suggests advances associating with the opportunities that table 45 lists.

Figure 7 suggests eras - in the evolution of the universe - that might precede inflation. Figure 7 also suggests insight regarding mechanisms leading to eras regarding the rate of expansion of the universe. Figure 6 depicts information about the ratio of dark matter density of the universe to ordinary matter density of the universe.

3.4. Astrophysics

Proposed modeling suggests advances associating with the opportunities that table 48 lists and with data to which table 49 alludes.

Figure 6 depicts information about the ratio of dark matter density of the universe to ordinary matter density of the universe. Figure 8 notes seemingly prevalent ratios of dark matter to ordinary matter. This essay discusses aspects of galaxy formation and other phenomena that seem to lead to the seemingly prevalent ratios.

4. Discussion

This unit provides possibly useful perspective about some physics topics and about proposed modeling.

4.1. Possibilities regarding other subfamilies of elementary particles

We discuss a possibility for another subfamily of elementary fermions.

Nature might embrace a sibling subfamily to 1R. (See table 32. Perhaps, compare with table 29.) We use the symbol 1R$'$ to denote this possible sibling. One interpretation of the 1R' solution might be the following. To the extent that neutrinos model as Majorana fermions, the notion that both 1R solutions and 1R' solutions associate with nature associates with 1R particles modeling as Dirac fermions. This essay de-emphasizes the notion that 1R$'$ elementary particles might be distinct from 1R elementary particles.

We discuss a possibility for another subfamily of elementary bosons.

For the set - of λ - $\{2,4,6,8,10,12,14\}$, four 0G solutions pertain. These solutions might associate with $Z_{UTA8} = 7^2 + 1 = 50$ and with elementary bosons with rest energies of somewhat more than 200 GeV. (Compare with discussion related to table 35.) These solutions might associate - paralleling the 2W subfamily - with elementary bosons that have spin one. Two of the bosons might have no charge. One of the bosons might have a magnitude of charge equal to $|q_e|/3$. One of the bosons might have a magnitude of charge equal to $2|q_e|/3$. The bosons might have a role regarding catalyzing baryon asymmetry. Absent evidence for so-called leptoquarks, this essay de-emphasizes the notion of such elementary bosons.

4.2. Possibilities regarding dynamics within black holes

We discuss dynamics within black holes.

People might consider applying the notion of components of 4G to dynamics within black holes. For example, octupole repulsion might prevent some conditions that extant modeling might associate with the notion of a singularity.

Aside from aspects regarding 4(2)G48 near the edges of black holes, this essay de-emphasizes discussing dynamics within black holes.

4.3. Possible associations between UNI modeling and the group $SU(17)$

We explore the notion that modeling associating with the group $SU(17)$ associates with aspects of table 18. This work posits - regarding each of various aspects for which a notion of three degrees of freedom pertains - that the number three associates with the number of generators of the group $SU(2)$. The number three seems to be important. The notion of possible further physics relevance of $SU(2)$ may be of lesser importance. Notions of broken or breakable instances of $SU(2)$ may be relevant. We do not necessarily try to interpret further meaning regarding the would-be instances of $SU(2)$. (This essay de-emphasizes trying to connect this work to extant modeling notions of space-time symmetries and to extant modeling notions of internal symmetries.)

We consider the notion that the set $\{\lambda|\lambda = 0,2,4,6,\ldots,14,16\}$ associates with 17 components of an isotropic harmonic oscillator. Indices k - as in equation (54) - associating with the components are zero, one, two, three, four, five, …, 13, 14, 15, and 16.

We deploy equation (56) with $j = 17$, $j_1 = 15$, $j_2 = 2$, and with the assumption that j_2 associates with the pair consisting of oscillator 7 and oscillator 8. Per equation (55), an $SU(2)$ symmetry associates with the 7-and-8 pair. We posit that the three generators that associate with the 7-and-8 pair associate with the spin-related (or, intrinsic angular momentum related) trio that table 18 shows. We posit that the notion of three degrees of freedom has relevance. Per equation (57), we posit that one instance of $U(1)$ pertains. We posit that one of the two ladder operators associates with adding to the property that associates with a (or, any one) member of the trio. The other ladder operator associates with subtracting from the property.

Table 53: Aspects that we posit regarding the possibility that UNI modeling USA modeling based on $SU(17)$ pertains

Aspect
• For UNI modeling $USA\lambda$ aspects for which $\lambda \geq 2$, we consider modeling that associates with the viewpoint of an observer. • The following notions pertain regarding the first seven steps. ∘ Each one of seven steps produces an instance of $SU(2)$ symmetry (which associates with two USA oscillators) and an instance of $U(1)$. ∘ For cases for which one of table 18a and table 18b shows 2×3, the instance of $U(1)$ associates with the notion that changes regarding the value of a property (that associates with the three oscillators) seem - to an observer - to require interactions with other objects. For a change, a property of at least one other object changes. ∘ For cases for which table 18b directly shows 6, the $U(1)$ (that associates with equation (57)) associates with a factor of two (in the number six) based on two would-have-been ladder operators. An object cannot change isomer. • The first seven steps of the process leave an instance of $SU(3)$. • The following notions pertain regarding the eighth step. ∘ For the sub-case for which table 18b shows 2×3, the instance of $U(1)$ associates with the notion that changes regarding the value of a property (that associates with the three oscillators) seem - to an observer - to require interactions with other objects. For a change, a property of at least one other object changes. ∘ For the sub-case for which table 18b shows $3 \rightarrow 2$, there are six possible transitions from one elementary fermion generation to another elementary fermion generation. We posit that the two ladder operators that associate with the $U(1)$ associate - perhaps indirectly - with a factor of two in the number six.

We envision continuing a program that uses equation (57) with - at each step - $j_2 = 2$ and successively smaller values of j_1. We specify that the seventh step - which features $j_1 = 3$ - must leave, as the three remaining oscillators the 0-5-and-6 oscillator trio. We de-emphasize discussing steps that associate with values - of j_1 - of 15, 13, …, and five. The order of taking those steps is not relevant.

Table 53 summarizes aspects that we posit regarding the possibility that UNI modeling USA modeling based on $SU(17)$ pertains.

Possibly, similar notions pertain regarding UNI modeling UTA modeling and at least one of $SU(17)$ and $SU(7)$. (Perhaps, compare table 22a and table 18.)

This essay de-emphasizes speculating regarding the notion of an association between the number of relevant $U(1)$ and completeness (or other attributes) of a specification for an object.

4.4. Possible associations between proposed modeling and entropy

Possibly, modeling related to 6G associates with notions related to entropy.

For elementary fermions, possibly the notion of three generations associates - uniquely among elementary particles - with notions of an association between three states and entropy.

More generally, aspects related to entropy might associate with - or supplant - proposed modeling notions of freeable energy.

This essay does not further discuss entropy.

4.5. Possibilities regarding symmetries related to CPT symmetry

Aspects of ENT ETA modeling associate with symmetry regarding charge reversal (or, C symmetry). (See table 29.) Aspects of ENT ESA modeling associate with symmetry regarding two values of circular polarization and, hence, with some aspects regarding parity reversal (or, P symmetry). (See discussion related to equation (79).) ENT modeling seems not to fully associate with other aspects regarding (direction of motion and) parity reversal. ENT modeling seems not to fully associate with aspects regarding so-called time reversal (Or, T symmetry).

We think that, to the extent proposed modeling gains traction, people might want to explore notions of CPTI symmetries. Here, the letter I denotes the word isomer. A relevant notion that would associate with the two-word term isomer reversal might associate with pairs I(even integer;i_4) and I(odd integer;i_4), with the absolute value of the difference between the even integer and the odd integer being three.

Table 54: Possible relevance - regarding six-fold aspects - of notions, each associating with a two-dimensional aspect and a complementary three-dimensional aspect

Note
• Some PDE aspects of mathematics regarding isotropic harmonic oscillators associate with the expression $X(X+D-2)$. (See equation (25).) Here, D denotes a number of dimensions. For $D=2$, the expression evaluates to X^2. For $D=3$, the expression evaluates to $X(X+1)$.
• Table 18 alludes to various six-fold aspects. Table 22a alludes to at least one more six-fold aspect.
• Each of the next three items alludes to a possibly relevant six-fold aspect. For each item, one two-dimensional aspect seems to complement one three-dimensional aspect.
• A two-dimensional construct associates with the trio example regarding charge. (See table 18.) The construct associates with $q_-$and q_+, but not with q_0. A complementary three-dimensional construct associates with the notion of $Q(Q+1)$. Z_{USA2} might associate with $Q(Q+1)$. (See discussion related to table 35.)
• A three-dimensional construct associates with the trio example regarding energy. (See table 18.) The construct associates with $3 \times (1/2)k_BT$ and three degrees of freedom. A complementary two-dimensional construct associates with the notion of m^2. Z_{USA4} associates with m^2. (See discussion related to table 35.)
• A three-dimensional construct associates with the trio example regarding intrinsic angular momentum. (See table 18.) The construct associates with $S(S+1)$ and three degrees of freedom. A complementary two-dimensional construct associates with the notion of S^2. Z_{USA8} might associate with S^2. (See discussion related to table 35.)

4.6. *A basis for possible insight regarding physics properties*

We discuss a basis for possible insight regarding physics properties.

Table 54 speculates regarding a possible relationship between aspects of table 18 and equation (109).

We think that, to the extent proposed modeling gains traction, people might want to explore notions of such complementarities within six-fold aspects.

4.7. *Possible insight regarding kinematics models*

We think that, to the extent proposed modeling gains traction, people might want to explore possibilities for adding insight regarding extant modeling KIN models or developing new KIN models based on double-entry arithmetic and - for example - relationships (to which table 18 might point) between items - in the column labeled scalar example - in table 18.

5. Concluding remarks

Proposed modeling might provide impetus for people to tackle broad agendas that our work suggests. Proposed modeling might provide means to fulfill aspects of such agendas. Proposed modeling might fulfill aspects of such agendas.

Opportunities might exist to develop more sophisticated modeling than the modeling that we present. Such a new level of work might provide more insight than we provide.

Proposed modeling suggests applied mathematics techniques that might have uses other than uses that we make.

Proposed modeling might suggest - directly or indirectly - opportunities for observational research, experimental research, development of precision measuring techniques, development of data analysis techniques, numerical simulations, and theoretical research regarding elementary particle physics, astrophysics, and cosmology.

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