# The role of the Foundation of General Relativity in Proton Radius Puzzle

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## ABSTRACT

Initially, we proceeded by presenting a synthesis of the Foundation of Albert Einstein's 1916 Theory of General Relativity, necessary for the development of a theory in accordance with the Foundation of the Theory of General Relativity, in order to demonstrate that the radius of the proton in Muonic-Hydrogen is smaller than that of Hydrogen by an amount equal to 4%, of the radius of the proton in the Hydrogen.

## INTRODUCTION: The synthesis of " Foundation of General Relativity"

In The Foundation of the General Theory of Relativity by Albert Einstein, one come to suppose that the general laws of motion, must be such that the mechanical behavior of physical system of bodies $S_1, S_2 ,..., S_n, S_{n+1}, ...,$ is partly conditioned, in quiet essential respect, by distant masses which we have not included in this system. Hence, if $\mathrm{R}_1, \mathrm{R}_2, ..., \mathrm{R}_n, \mathrm{R}_{n+1}, ...,$ are space in any kind of motion relatively to one another in which the bodies $S_1, S_2, ..., S_n, S_{n+1}, ...,$ are respectively at rest, there is none which we may look upon as privileged a priori, without falling into the epistemological defect, through the introduction of a merely factitious cause and not a thing that can be observed, as this would contradict Newton's mechanics, for which, if the laws of mechanics apply to the space $\mathrm{R}_n$ with respect to which the body $\mathrm{S}_n$ is at rest, they will not apply to the space $\mathrm{R}_{n+1}$ with respect to wich the body $\mathrm{S}_{n+1}$ is at rest. To suppose this, Albert Einstein used the Mach's principle according to which the inertia of bodies is a property induced by distant masses.

It follows that the laws of physics can applied to systems of reference in any kind of motion.
We assume that to a translational and uniformly accelerated relative motion of a reference system K' with respect to a Galilean reference system K, due to the fact that an observer at rest with respect to K' cannot conclude that he is actually on a reference system accelerated, we can give the following equivalent interpretation:
The space in question is influenced by a gravitational field (the notion of field was introduced by Faraday and Maxwell to indicate the set of values that a given physical quantity assumes in space) which generates the accelerated motion of the bodies with respect to K'.
In fact, just as the gravitational force field, enjoys the property of imparting the same acceleration to all bodies, so relatively to K', the acceleration which will have a mass sufficiently distant from the other masses, will be indipendent of the physical state and material nature of the mass (material point free to move).
In the following example, we introduce, in a space which is free of gravitational fields, a Galilean reference $K_O$(x,y,z,t) and a system of coordinates $K_O^{'}$(x',y',z',t') in uniform rotation relatively to $K_O$, with the origins of both systems, as well as their axes z and z', permanently coincide.
Since both systems can be chosen for the description of physical phenomena, we arrive at the following result: The differences between the spatial coordinates, can no longer be measured through a sample of length chosen as the unit of measurement unlike what happens in the theory of special relativity, in which two selected material points of a stationary rigid body there always corresponds a distance of quite definite lenght, moreover the rate of a clock depends upon where the clock may be.
For this reason to expose the General Theory of Relativity, infinitely small ( local ) coordinate systems will be chosen, limited by an infinitely small region of space.
From the thought expounded by Albert Einstein it follows that, the laws of nature be covariant in a general way to any substitution.
We thus arrived at the generalization of the theory of relativity, the development of which takes place by means of tensors.

We can verify that through the principle of special relativity and the postulate of the constant of the speed of light in vacuum, we obtain the trasformation laws of Lorentz, which for each pair of infinitely close events in the continuous space-time (in terms of geometry, for each pair of points in the four-dimensional universe of Minkowsky) leave the interval unchanged $ds^2 = -dx_1^2 - dx_2^2 - dx_3^2 + dx_4^2$ in all space $R_1, R_2, ..., R_n, R_{n+1}, ...,$ ( the unit of time has been chosen in such way that the speed of light is equal to 1).

In the general case, limiting to an infinitely small four-dimensional region, the desplacement from an inertial system K to a non-inertial ( arbitrary ) system K', change the expression of the invariant $ds^2$ between two infinitely close event:

$$ds^2 = -dx_1^2 - dx_2^2 - dx_3^2 + dx_4^2 \;\rightarrow\; ds^2 = \sum g_{\mu\nu} dx^\mu dx^\nu = g_{\mu\nu} dx^\mu dx^\nu$$

where we have suppressed the sign $\Sigma$, using the simplified notation introduced by Albert Einstein (if an index occurs twice in one term of an expression, it is always to be summed unless the contrary is expressly stated). The $g_{\mu\nu}$ components vary as a function of space and time and must be considered from physical point of view, as the quantities that describe the gravitational field, in relation to the chosen reference system, and moreover, simultaneously determine the metric proprerties of the four-dimensional space. The equation of the law of motion of a material point relative to K, describe a four-dimensional straight line, i.e a geodetic and will also be the equations of motion of the material point with respect to K': $\dfrac{d^2x^\tau}{ds^2} = \Gamma^\tau_{\mu\nu} dx^\mu dx^\nu$ with $\Gamma^\tau_{\mu\nu} = \{^{\ \tau}_{\mu\ \nu}\}$ 1)

since the geodetic is defined independently of the reference system exactly as the invariant $ds^2$.

$\Gamma^\tau_{\mu\nu}$ are the components of the gravitational field and vanish only when the point moves uniformly in a straight line: $-\Gamma^\tau_{\mu\nu} = \{^{\ \tau}_{\mu\ \nu}\} = g^{\sigma\tau}\{\mu\nu,\sigma\}$ with $\{\mu\nu,\sigma\} = \dfrac{1}{2}\left(\dfrac{\partial g_{\mu\sigma}}{\partial x^\nu} + \dfrac{\partial g_{\nu\sigma}}{\partial x^\mu} - \dfrac{\partial g_{\mu\nu}}{\partial x^\sigma}\right).$

Putting ourselves in the condition of first approximation in which the $g_{\mu\nu}$ are such as to make all small values of the $\Gamma^\tau_{\mu\nu}$ components ( at least those of the first order, neglecting the quantities of the second order and subsequent ), and limiting the speed of material point, such that is small, compared to the speed of light, we can consider only the terms in which $\mu = \nu = 4$, because the $\dfrac{dx^i}{ds}$ for $i = 1,2,3,$ will be very small; consequently $\dfrac{dx^4}{ds}$ will be close to 1, so from 1) we get:

$$\frac{d^2x^\tau}{ds^2} = \Gamma^\tau_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\tau}{ds} = \Gamma^\tau_{44} = \frac{d^2x^\tau}{dt^2} \text{ with } ds = dx^4 = dt.$$

Furthermore supposing that the motion of the matter, generating the field is slow in comparison to the speed of light (quasi-static gravitational field), the derivations with respect to time will be negligible with respect to the spatial derivations so that 1) gives us in first approximation the equation of the motion of the material point according to Newton's theory:

$$\frac{d^2x^\tau}{dt^2} = -\frac{1}{2}\frac{\partial g_{44}}{\partial x^\tau} \text{ with } \tau = 1,2,3 \qquad 2)$$

in which $\frac{g_{44}}{2}$ play the part of the gravitational potential.

By advantageously choosing of the coordinates so that $\sqrt{-g} = 1$, Albert Einstein come to the following equations of the gravitational field in the absence of matter, which conform to the momentum and the energy theorems:

$$\frac{\partial \Gamma^\alpha_{\mu\nu}}{\partial x^\alpha} + \Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha} = 0 \text{ with } \sqrt{-g} = 1 \qquad 3)$$

Multiplying 2) by $g^{\nu\sigma}$ gives the following equation:

$$g^{\nu\sigma}\frac{\partial \Gamma^\alpha_{\mu\nu}}{\partial x^\alpha} + g^{\nu\sigma}\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha} = [\frac{\partial}{\partial x^\alpha}(g^{\sigma\beta}\Gamma^\alpha_{\mu\beta}) + \chi(\mathbf{t}^\sigma_\mu - \frac{1}{2}\delta^\sigma_\mu \mathbf{t}) - g^{\nu\sigma}\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha}] + g^{\nu\sigma}\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha} = 0 \text{ with } \sqrt{-g} = 1 \qquad 4)$$

The quantities $\mathbf{t}^\sigma_\mu$ with $\mathbf{t}^\alpha_\alpha = \mathbf{t}$, appearing in 3), are to be considered as the energy components of the gravitational field only.

Since the theory of relativity has led to the conclusion that inert mass is energy, then this energy adding to the energy component of the gravitational field into the field equations of gravitation; if we consider the solar system, the total mass and therefore its total gravitating action, will depend on the total energy of the system, and therefore on the ponderable energy together with the gravitational energy.

In addition Albert Einstein highlights that the energy of the gravitational field shallact gravitatively in the same way as any other kind of energy and this leads us to write the equations of the gravitational field in general form, adding to the component $\mathbf{t}^\sigma_\mu$ of the energy of the gravitational field only, the components of the energy of the matter $\mathrm{T}^\sigma_\mu$, with $T^\alpha_\alpha = T$; so from 3) we obtain the following general equations of the gravitational field in mixed form:

$$g^{\nu\sigma}\frac{\partial \Gamma^\alpha_{\mu\nu}}{\partial x^\alpha} + g^{\nu\sigma}\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha} = \{\frac{\partial}{\partial x^\alpha}(g^{\sigma\beta}\Gamma^\alpha_{\mu\beta}) + \chi[(\mathbf{t}^\sigma_\mu + T^\sigma_\mu) - \frac{1}{2}\delta^\sigma_\mu(\mathbf{t} + T)] - g^{\nu\sigma}\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha}\} + g^{\nu\sigma}\Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha} = 0 \qquad 5)$$

with $\sqrt{-g} = 1$.

The 4) are traceable to the following general equation of the gravitational field in the symmetrical covariant form:

$$\frac{\partial\Gamma^{\alpha}_{\mu\nu}}{\partial x^{\alpha}}+\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha}=-\chi(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T) \quad \text{with } \sqrt{-g}=1 \qquad 6)$$

in which, $T_{\mu\nu}=g_{\mu\nu}p+\rho g_{\mu\alpha}\frac{dx^{\alpha}}{ds}g_{\nu\beta}\frac{dx^{\beta}}{ds}$ is the covariant energy tensor of a perfect fluid, where P and $\rho$ denotes the pressure and density of fluid respectively.

Since the energy tensor of matter is defined almost exclusively by the density of matter $\rho$, than $T_{\mu\nu}=\rho g_{\mu\alpha}\frac{dx^{\alpha}}{ds}g_{\nu\beta}\frac{dx^{\beta}}{ds}$, and for the order of approximation which we are, it cames down to the $T_{44}=\rho=\mathrm{T}$; then from 5) we get:

$$\nabla g_{44}=\chi\rho \qquad 7)$$

which represents the equation corresponding to the Poisson equation for the Newtonian gravitational field.

From 2) and 7), we obtained the following system of equations equivalent to Newton's law of gravitation:

$$\begin{cases} d^2x^{\tau}/dt^2=-(1/2)(\partial g_{44}/\partial x^{\tau}) \quad \text{with } \tau=1,2,3 \\ \nabla g_{44}=\chi\rho \end{cases}$$

and by 2) and 7) the expression for the gravitational potential becomes

$$-\frac{\chi}{8\pi}\int_V\frac{\rho d\tau}{r} \qquad 8)$$

while Newton's theory, with the unit of time which we have chosen, given

$$-\frac{G}{c^2}\int_V\frac{\rho d\tau}{r} \qquad 9)$$

Subsequently, we derive the gravitational potential for a field producing by a point of mass at the origin of coordinates, and we obtain to the first approximation, the radially symmetrical solution:

$$\begin{cases} g_{\rho\sigma}=\delta_{\rho\sigma}-a\frac{x_{\rho}x_{\sigma}}{\sigma^3} & \rho,\sigma=1,2,3 \\ g_{\rho 4}=g_{4\rho}=0 & \rho=1,2,3 \\ g_{44}=1-\frac{a}{r} & \end{cases} \qquad 10)$$

with $\delta_{\rho\sigma}=0$ for $\rho\neq\sigma$, $\delta_{\rho\sigma}=1$ for $\rho=\sigma$, and $r=\sqrt{x_1^2+x_2^2+x_3^2}$

Comparing 8) with 9) we get: $\chi = \dfrac{8\pi G}{c^2}$ 11)

while for 8), 10) and 11) we get: $a = \dfrac{\chi M}{4\pi}$ 12)

which we rewrite $\dfrac{a}{r} = \dfrac{\chi M}{4\pi r}$ 12')

and for 11) and 12') we get: $\dfrac{a}{r} = \dfrac{8\pi GM}{4\pi c^2 r}$ 12")

If we remove time from the equation 12") we get: $4\pi c^2 = \dfrac{8\pi J_G M}{c}$ with $|J_G| = |G|$ and $r = c$ 13)

It is easy to verify that the unit measuring-rod appear a little shortened in relation to the system of coordinates, by the presence of the gravitational field, if the rod is laid along a radius: $dx^1 = 1 - \dfrac{a}{2r}$ while the gravitational field of the point of mass has no influence on the lenght of a rod when it is placed along the tangential direction.

As a result, Euclidean geometry is no longer valid, and the ratio of the circumference to the diameter is less than $\pi$ !

The following two examples were suggested by Albert Einstein to highlight the fact that the development of Relativity implied a non-Euclidean geometry.

1) In a space-time region without gravitational fields let us consider a circumference lying in the plane XY of a Galilean reference $K_O$(x,y,z,t) with centre in the origin and radius r. Then, if we measure the radius r of the circumference, through a measuring-rod at rest with respect to a system $K'_O$ that we consider in uniform rotary motion with respect to $K_O$(with $Z \equiv Z'$ and $O \equiv O'$) the ratio between circumference and diameter, for the effect due to the theory of special relativity, result to be grater than $\pi$.
2) If the space-time territory in which the circumference lies, is under the sway of a gravitational field, the unit measuring-rod undergoes the shortening foreseen by the Theory of General Relativity and the ratio between circumference and diameter will be less than $\pi$. This is true all the time that we wish to take one and the same measuring-rod, independently of its place and orientation, as a realization of the same interval. In other words, given n unit measuring-rod all of the same lenght in contrast to the case where we had a space-time region without gravitational field

we will need, a number of measuring-rod greater than r, to arrive at a lenght equal to r, for the simple fact that the gravitational field with its presence shortens the measuring-rods.

Let's proceed by considering an alternative case to points 1) and 2):

3) In a space-time we introduce two system of reference K and K' ( with $Z \equiv Z'$ and $O \equiv O'$ at any moment) such that, each system of reference with respect to an observer at rest relatively to the other system of reference, rotate wich costant angular velocity and we consider a mass m at distance r from O, at rest with respect to the reference K.
Then relative to K', the motion of this mass m will appear to be in uniform rotation around the origin of K', and at a distance r from the origin of K'.
It follows that an observer placed in the origin of K' is unable to conclude that he is an a system of reference in uniform rotation, and then given the following equivalent interpretation:
The space-time territory in question, is under the sway of a gravitational field, of such intensity as to generate the uniform rotation of the mass m, around the origin of K', and at a distance r from the origin of K'.
For observer placed in the origin of K', not being able to conclude that the system of reference is in uniform rotation (due to the relativity of the motion), the ratio between circumference and diameter must be equal to $\pi$, and therefore, in the case of equivalent interpretation, it can only equal to $\pi$.
Since due to the sway of the gravitational field, the distance r of the mass m from the origin of K', will be shortened of $\Delta$r, we deduce that also the orbit of the mass m will no longer be equal to $2\pi$r but to $2\pi$(r - $\Delta$r), as the ratio between circumference and diameter, in the equivalent interpretation must remain unchanged.

## THE ROLE OF THE FOUNDATION OF GENERAL RELATIVITY IN THE PROTON RADIUS PUZZLE

Being from the point of view of first approximation, the $g_{\mu\nu}$ differ from value $g_{11} = g_{22} = g_{33} = -1$ and $g_{44} = 1$ for small quantities compared to 1; then must be $a \ll r$ and therefore $\frac{1}{\sqrt{1+a/r}} \simeq 1 - \frac{a}{2r}$, which as has been have shown in the Theory of General Relativity, has the following geometrical meaning: "The unit measuring-rod, when is laid along the radius, at a distance r from the point of mass that generating the gravitational field, appear shortened by an amount equal to $\frac{a}{2r}$, in relation to the system of coordinates K". This also applied to r = 1, i.e $a \ll 1$ and so 1- $\frac{a}{2r} \simeq$ 1-$\frac{a}{2}$.

Therefore, if for a measuring-rod of lenght r, laid along the radius, we can write r$\left(1\text{-}\frac{a}{2r}\right) = r - r\frac{a}{2r}$ ( the measuring-rod will be shortened by an amount equal to $r\left(r\frac{a}{2r}\right)$ ), then a spherical surface of radius $r - r\frac{a}{2r}$, from point of view of first approximation ( neglecting the quantities of the second order and subsequent), having shown in the point 3) that the ratio of the circumference to the diameter is equal to $\pi$, we get: $4\pi(r - r\frac{a}{2r})^2 = 4\pi(r^2\text{-}2r^2\frac{a}{2r} + \frac{r^2a^2}{4r^2}) \simeq 4\pi r^2\text{-}\ 4\pi r^2\frac{a}{r}$; i.e the spherical surface $4\pi r^2$ will be decrease by an amount equal to $4\pi r^2\frac{a}{r}$, and we can note that the value of r put to denominator, is always equal to the radius of spherical surface to which be subtracted the term $4\pi r^2\frac{a}{r}$.

Therefore $\frac{8\pi \mathrm{J}_G\mathrm{M}}{c} = 4\pi c^2\frac{a}{c}$, i.e the 13) gives us the value of that surface that must be subtracted to the spherical surface of r = c, which around the point of mass M that generated the gravitational field in that space-time region, to which the spherical surface, of r = c, belongs.

The equation $4\pi\frac{c^2}{x^2}\frac{(x^2a)}{(c/x)} = 8\pi\frac{\mathrm{J}_G\mathrm{M}}{c/x}$ 13')

is equivalent to 13), but to be in agreement with 12), in the 13') we must necessarily replace the mass M with mass $x^2$M.

So, in accordance with the Theory of General Relativity we get the following equivalent equation

$$4\pi\frac{c^2}{x^2}\frac{(x^2a)}{(c/x)} = 8\pi\frac{\mathrm{J}_G(x^2\mathrm{M})}{x^2(c/x)} \qquad 13")$$

Through the equation 13") we get the surface that must be subtracted to the spherical surface of radius r =$\frac{c}{x}$.

For $a = \frac{c}{x^3}$ we get: $4\pi \frac{c^2}{x^2} - 4\pi \frac{c^2}{x^2} \frac{(x^2 a)}{(c/x)} = \frac{1}{x^2}\left[4\pi c^2 - 4\pi c^2 \left(\frac{x^2 \frac{c}{x^3}}{c/x}\right)\right] = 0;$

i.e for $x^2 a = \frac{c}{x}$ the spherical surface of radius $\frac{c}{x}$ becomes null, and therefore, the volume contained from the spherical surface will come to missing from the spherical volume of that sphere with radius $\frac{c}{x} \leq r \leq +\infty$.

The volume vanished is a consequence of shortened suffered to unit measurement-rod in a gravitational field when it is placed along the radial direction.

From 13) we obtain: $4\pi \frac{c^2}{8\pi} a = \mathrm{J}_G \mathrm{M}$ 14)

Also the 14) is equivalent to 13) and we note that to the second member of the 14), we can get the physical sense of a volume that is take up from a mass M in a medium of density $\mathrm{J}_G$, with the mass M beloging to this medium; this volume, if we follow the same reasoning as in the case of equation 13"), must be subtracted by spherical volume of radius r = $\frac{c}{8\pi}$, as the 14) is equivalent to 13).

It is evident by the 14) that the volume to subtract is superiorly bounded from the spherical volume $\mathrm{J}_{MAX}$ of radius r = $\frac{c}{8\pi}$.

Therefore, leaving the $\mathrm{J}_G$ constant, we can vary the mass M until that max value $\mathrm{M}_{MAX}$, necessary to take up the spherical volume of radius r = $\frac{c}{8\pi}$.

As the mass M belongs to medium of volumetric density $\mathrm{J}_G$ , the mass of medium will need to be limited and equal to $\mathrm{M}_{MAX}$ and accordingly also its volume will need to be limited by a spherical surface of radius r = $\frac{c}{8\pi}$.

In this proof $\mathrm{J}_G$ have the dimension of volumetric density and as such we treated it, also we have replaced the point of mass M with a body of mass M, extensive in the space.

Starting from the Theory of General Relativity we derived a mathematical model, which allows us to idealize, empty space as a medium endowed with volumetric density. This does not mean that phsical space possesses a volumetric density, as this volumetric density, could only be the result of the mathematical model that we have built.

This does not mean, that the phsical space truly possesses a volumetric density, instead of being empty.

We apply this mathematical model to muonic hydrogen.

Although the proton consists of 3 quarks, we have considered it as a particle of uniform volumetric density because Albert Einstein pointed out that the mass is energy and energy is mass.

Then must be $M_P = M_{MAX}$ ($M_P$ is the proton mass), $V_{MAX}$ while is that volume within by spherical surface limited from the radius $r_P$ of the proton; then we get $J_G = \frac{(4/3)\pi r_P^3}{M_P} = J_P$.

For a muon bound to a proton, since the wave function of a particle, orbiting around the proton, penetrates in that region of space occupied by the proton, it as been calculated, that the probability of the muon to be inside the proton, is about 8 milion times greater than that of electron.

This mean that all the times the muon is inside the proton, we can apply our mathematical model and then $J_P M_\mu$ ($M_\mu$ is the muon mass that we treat as if it were equivalent to a material point), gives us the value of that volume that must be subtracted from the spherical volume of a sphere of radius $r_P$.

## CONCLUSION

We have shown that a particle (considered as a material point), penetrates inside another particle extended in space, causes in the latter a reduction of its spherical volume and so of its radius.

The theory having been built through the Foundations of the Theory of General Relativity, describes a phiysical phenomenon, which in an intrinsic way, was already foreseen by the Foundation of the Theory of General Relativity.

The reduced volume $V'_P$ of the proton, due to the penetration of the muon into it, is given by following equation:

$$V'_P = \frac{4}{3}\pi(r'_P)^3 = V_P - J_P M_\mu = \frac{4}{3}\pi(r_P)^3 - \frac{\frac{4}{3}\pi(r_P)^3}{M_P} M_\mu \qquad 15)$$

If we know the values $r_P$, $M_P$ and $M_\mu$, we get it

$$r'_P = r_P\left(\sqrt[3]{1 - \frac{M_\mu}{M_P}}\right) \tag{16)}$$

while, if we know the values $r'_P$, $M_P$ and $M_\mu$, we get it

$$r_P = \frac{r'_P}{\sqrt[3]{1 - \frac{M_\mu}{M_P}}} \tag{17)}$$

$M_\mu = 1.88353\times10^{-28}$ Kg, is the mass of muon (elementary particle similar to the electron)

$M_P = 1.67262\times10^{-27}$ Kg, is the mass of proton

$r'_P = 0.84087(39)$ fm, is the recent determination of the proton radius using the measurement of the Lamb shift in the muonic hydrogen atom

Through these values $(r'_P, M_P, M_\mu)$ and the formula 17), we calculate the radius of the proton

$$r_P = \frac{r'_P}{\sqrt[3]{1 - \frac{M_\mu}{M_P}}} = \frac{0.84087(39)\ \text{fm}}{\sqrt[3]{1 - \frac{1.88353\times10^{-28}\,\text{Kg}}{1.67262\times10^{-27}\,\text{Kg}}}} = \frac{0.84087(39)\ \text{fm}}{0.96096} = 0.87503(39)\ \text{fm}$$

Since the effect on the radius of a proton, caused by a particle with very small mass as electron are negligible, then $r_P = 0.87503(39)$fm would be the radius of the proton when an electron is orbiting around it.

We have obtained a value of $r_P$ in agreement with the 2014 value, equal to $r_P = 0.8751(61)$ fm, recommended by the Committee on Data for Science and Technology (CODATA 2014) and furthermore the result is in agreement with the Standard Model.

If experimentally the radius of muonic-hydrogen is equal to the radius of hydrogen, then from the point of view of theory development, the outcome of this experiment, would be a pivotal result not in agreement with the Foundations of Theory of General Relativity, as it would clash with those Foundations .

## References

1) Die Grundlage der allgemeinen Relativitätstheorie

Albert Einstein

2) Proton structure from the measurement of 2S-2P transition frequencies of muonic hydrogen

Aldo Antognini et al.