Quantization of Klein-Gordon Scalar Field in Cosmological Inertial Frame

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ABSTRACT

In the Cosmological Special Theory of Relativity, we quantized Klein-Gordon scalar field in Cosmological Special Theory of Relativity. We treat Lagrangian density and Hamiltonian in quantized Klein-Gordon scalar field.

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1. Introduction

Our article's aim is that we make quantization of Klein-Gordon scalar field in Cosmological Special Theory of Relativity (CSTR).

At first, space-time relations are in cosmological special theory of relativity (CSTR).[1]

$$ct = \gamma(ct' + \frac{v_0}{c}\Omega(t_0)x'), \quad x\Omega(t_0) = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t')$$

$$\begin{matrix}\Omega(t_0)y = \Omega(t_0)y', \\ \Omega(t_0)z = \Omega(t_0)z'\end{matrix}, \quad \gamma = 1/\sqrt{1-\frac{v_0^2}{c^2}\Omega^2(t_0)}, \quad t_0 \text{ is cosmological time} \tag{1}$$

Proper time is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2]$$

$$= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)[dx'^2 + dy'^2 + dz'^2], \quad t_0 \text{ is cosmological time} \tag{2}$$

Angular frequency-wave number relation is in CSTR.

$$\omega' = \gamma(\omega - v_0\Omega(t_0)k_1), \quad k_1' = \gamma(k_1 - \frac{v_0}{c^2}\Omega(t_0)\omega)$$

$$k_2' = k_2, k_3' = k_3, \gamma = 1/\sqrt{1-\frac{v_0^2}{c^2}\Omega^2(t_0)} \tag{3}$$

2. Quantization of Klein-Gordon Scalar Field in CSTR

Lagrangian density of Klein-Gordon scalar field in CSTR,

$$L = -\frac{1}{2}[-(\frac{1}{c}\frac{\partial\phi}{\partial t})^2\Omega(t_0) + \frac{1}{\Omega(t_0)}\vec{\nabla}\phi\cdot\vec{\nabla}\phi - \frac{m_0^2c^2}{\hbar^2}\phi^2] \tag{4}$$

Hence, Euler-Lagrange equation is in CSTR,

$$\partial_\mu[\frac{\partial L}{\partial(\partial_\mu\phi)}] - \frac{\partial L}{\partial\phi} = [\Omega(t_0)\frac{1}{c^2}(\frac{\partial}{\partial t})^2 - \frac{1}{\Omega(t_0)}\nabla^2 + \frac{m_0^2c^2}{\hbar^2}]\phi = 0) \tag{5}$$

Hamiltonian of Klein-Gordon scalar field is in CSTR,

$$H = \frac{1}{2}[(\frac{1}{c}\frac{\partial\phi}{\partial t})^2\Omega(t_0) + \frac{1}{\Omega(t_0)}\vec{\nabla}\phi\cdot\vec{\nabla}\phi + \frac{m_0^2c^2}{\hbar^2}\phi^2] \tag{6}$$

The Klein-Gordon scalar field is divided by positive frequency mode and negative frequency mode.

$$\phi(x) = \phi^{(+)}(x) + \phi^{(-)}(x) \tag{7}$$

The positive frequency mode is

$$\phi^{(+)}(x) = \int \frac{d^3k}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}} a(k) f_k(x) \tag{8}$$

The negative frequency mode is

$$\phi^{(-)}(x) = \int \frac{d^3k}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}} a^{(+)}(k) f_k(x) \tag{9}$$

In this time, $f_k(x)$ is

$$f_k(x) = \frac{1}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}} \exp[i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})] \tag{10}$$

In this time,

$$\frac{\omega_k}{c} = (k^2 + \frac{{m_0}^2 c^2}{\hbar^2})^{\frac{1}{2}} \tag{11}$$

Quantization of complex scalar field is in CSTR,

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} [a(k)\exp\{i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}]$$

$$+\int \frac{d^3k}{(2\pi)^2 2\omega_k} [b^+(k)\exp\{-i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}] \tag{12}$$

$$\phi^+(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} [b(k)\exp\{i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}]$$

$$+\int \frac{d^3k}{(2\pi)^2 2\omega_k} [a^+(k)\exp\{-i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}] \tag{13}$$

Hence, Hamiltonian H is in CSTR,

$$H = \int \frac{d^3k}{(2\pi)^3 2\omega_k} [a^+(k)a(x) + b^+(k)b(k)] \tag{14}$$

In this time,

$$[a(k), a^+(k')] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{k}')$$

$$[b(k), b^{+}(k')] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{k}') \quad (15)$$

3. Conclusion

We quantized Klein-Gordon scalar field in CSTR. We treat Lagranian density and Hamiltonian.

References

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