AN INTRODUCTION TO THE SUPER-NORMAL-IRREDUCIBLE-IRRATIONAL NUMBERS AND THE THIRD ORDER OF LOGIC AXIOM.

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Abstract. We define the super-normal-irreducible irrationals with the help of the n-irreducible sequents (see my previous articles) built from irrationals and instead of taking the integer part of the irrational (or it inverse), we add a super-normal-irreducible formula which give the position of the first digit breaking some super-normal number definition. From 79 irreducible-irrational numbers, we deduce from the second order logic axiom that they are all super-normal numbers. Moreover, with some random digits, the probability that the super-normal-irreducible formula holds for the 79 ones is about 9.2×10^{-10} and we take in account that some irreducible-irrational numbers are only some different functions of the same irreducible-irrational number. From this large coincidence, we introduce the third of order logic of axiom which states that every irreducible-irrational numbers are super-normal numbers as well. From that new third of order of logic axiom, we deduce the none-existence of an exotic 4-sphere. Finally, we conclude with some considerations about the finite number of n-irreducible sequents.

We define the super-normal-irreducible irrational numbers from an irreductible sequent with the formula $\tilde{\phi}_a$ and the hypotheses $\tilde{\Gamma}_a$ which is valid only for a unique irrational number a and its subsequents are valid only with $a = 0$ or $a = 1$ or a is negative. We add the following formula and hypotheses to convert it into a n-irreducible sequent $\mathcal{S}$ which is relevant for the super-normality of the irreducible-irrational number a:

$$\Gamma, \Gamma_{Peano}, \vdash \phi_{[\underbrace{f_s(...f_s(}_{n\text{ times}} c_0 \underbrace{)...)}_{n\text{ times}} /x]} \wedge \neg\exists y \exists z \left(\phi_{[y/x]} \wedge \phi_{[z/x]} \wedge \neg y = z\right)$$

$$\Gamma \equiv \tilde{\Gamma}_a \ \wedge \forall n \forall b \forall d \exists k_0$$

$$a\,(b,0) = a \wedge$$

$$d\,(b,n)\, b^{k_0-n} < a\,(b,n) < (d\,(n)+1)\, b^{k_0-n} \ \wedge$$

$$a\,(b,n+1) = a\,(b,n) - d\,(b,n)\, b^{k_0-n} \ \wedge$$

$$N_{digit}\,(b,0,d) = 0 \ \wedge$$

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$$d\left(b,n\right)=d\rightarrow N_{digit}\left(b,n,d\right)=N_{digit}\left(b,n-1,d\right)+1\ \wedge$$

$$\neg d\left(b,n\right)=d\rightarrow N_{digit}\left(b,n,d\right)=N_{digit}\left(b,n-1,d\right)\ \wedge$$

$$\exists\delta\forall\delta' Abs\left(\delta'\right)<\delta\rightarrow Abs\left(Cexp\left(x'+\delta'\right)-Cexp\left(x'\right)-\delta' Cexp\left(x'\right)\right)<Abs\left(\delta'\epsilon\right)\ \wedge$$

$$n=1\rightarrow Cexp\left(n\right)=n\ \wedge$$

$$n\in\mathbb{N}\rightarrow\left(f\left(n\right)\in\mathbb{N}\wedge\neg f\left(n\right)<Cexp\left(n\right)\wedge f\left(n\right)<Cexp\left(n\right)+1\right)\ \wedge$$

$$\phi\equiv\tilde{\phi}_a\wedge\forall x'\forall\epsilon\forall d$$

$$\left(x'<x\wedge\neg x'<0\wedge d<x'\wedge\neg d<0\right)\rightarrow\left(x'\right)^{\epsilon} Abs\left(1-x'^{\frac{N_{digit}f\left(x',f\left(x'\right),d\right)}{f\left(x'\right)}}\right)<1\ \wedge$$

$$\neg x^{\epsilon} Abs\left(1-x^{\frac{N_{digit}\left(x,f\left(x\right),d\right)}{f\left(x\right)}}\right)<1$$

where $Cexp(x)=Exp(x-1)$.

That n-irreducible sequent $\mathcal{S}$ give the smallest integer x such that the distance between the ratio of digits d over the first $Ceiling\left(Exp\left(x-1\right)\right)$ digits in base x and $1/x$ is smaller than $1/x^{1+\epsilon}$ for all strictly positive ϵ.

If we can check numerically that $x>N_Z$, from the second order logic axiom, we can deduce that there is no integer x such the super-normality of a is violated.

From the first 79 irreducible-irrational numbers known (most of them are famous mathematical constants), we found that all of them are super-normal up to $x=5$. Since the number of digits grow up exponentially with x, the meaning of the results ar not far from $x=+\infty$. The probability to find the same result with random digits and by taking in account that some irreducible-irrational numbers are only different functions of the same irreducible-irrational number is 9.2×10^{-10}.

Therefore, we postulate at third order of logic that every irreducible-irrational number is a super-normal-irreducible-irrational number.

We have considered the following 79 mathematical constants:

The Conway constant and its inverse:
1.303577269034296391257099112152551890730702504659404875754861390628550
0.767119850701915359540715997135284064594703512060941496039823831701833

The Feigenbaum constant and its inverse:
4.669201609102990671853203820466201617258185577475768632745651343004134
0.214169377062326492478934818893161783413809015659045443500181457191667

The Niven constant :
0.7052111401

The three Khinchin constants:
2.685452001065306445309714835481795693820382293994462953051152345557218
1.74540566
0.78853056591150896106027632345455466647274966822328164975515640230178

The Gibbs constants with unit jump:
$-1/2 + \pi/2 \ \int_0^\pi sin(x)/x \ dx$
$+1/2 + \pi/2 \ \int_0^\pi sin(x)/x \ dx$

The Laplace constant and its inverse:
0.74884655311308928346715056409562822525263616664765469205128189325078962574
1.33538706406916188846116862523247357548584662065863300298670528152253847401

The Golomb-Dickman constant and its inverse:
0.62432998854355087099293638310083724
1.60171707005908755336789757833219300864266450324786224313214921769356604558

The Artin constant and its inverse:
0.37395581361920228805472805434641641511162924860615
2.67411272557002150896041183044548803750239862839769228621522584609442347765

The Landau-Ramanujan inverse square constant:
$(a \ Log\left(x\right) N\left(x\right) N\left(x\right) = Log\left(x\right) \left(N\left(x\right)\right)^2 / b^2 = x \times x)$
$(\tilde{a} \ \sqrt{Log\left(x\right)} N\left(x\right) = \sqrt{Log\left(x\right)} N\left(x\right) / b = x)$
1.712217963443786953812066012296122346962156854776346981721036801209944169
1.308517467764105529600931476113571198780190163845213947667124446332082976

Euler–Mascheroni constant:
γ

The Levy constant :
$e^{\pi^2/12/ln(2)}$

The alternating harmonic series constant:
$ln(2)$

The Universal Parabolic constant:
$ln\left(1 + \sqrt{2}\right) + \sqrt{2}$
The half square constants:
$\sqrt{2}, 1/\sqrt{2}$

The equilateral triangle constants:
$\sqrt{3}, 2\sqrt{3}, \sqrt{3}/2, 1/\sqrt{3}, 1/2/\sqrt{3}, 2/\sqrt{3}$

The Markov constant:
$\sqrt{5}$

The gold number:
$1/2 + \sqrt{5}/2$

The exponential constant:
e

The minimum value of x^x:
$1/e$

The minimum of x^x:
$Exp\left(1/e\right)$

The maximum of $1/x^x$:
$Exp\left(-1/e\right)$

The exponential geometric series:
$e/(e-1)$

The negative exponential integral:
$\int_{-1}^{0} Exp(x)dx = (e-1)/e$

The right angle tangent logarithmic spiral:
$a, Exp(a), 1/Exp(a), 2a, Exp(2a) 1/Exp(2a)$
0.27441063190284810044017506211094801781885840256703474410204202357688193718860

The integral of x^x:
$\int_0^1 x^x dx$

The Gamma integral 1:
$\int_0^1 1/\Gamma\left(x\right) dx$

The Gamma integral 2:
$\int_0^1 1/\Gamma\left(x+1\right) dx$

The root of LogIntegral:
1.45136923488338105028396848589202744949303228364801586309300

The π constant and it inverse:
$\pi, 1/\pi$

The constant from Fourrier transformations:
$2\pi, 1/(2\pi), \sqrt{2\pi}, 1/\sqrt{2\pi}$

The imaginary root of $Exp\left(z\right)=I$
$\pi/2$

The normalized radius of monotone volumes of n-spheres:
$2/\pi$

The half square and equilateral triangle angles:
$\pi/2,\pi/4,\pi/3,\pi/6$

The Cycloide surface constant:
$3\pi,1/\sqrt{3\pi}$

The Basel problem constant:
$\pi^2/6$

The maximal volume of a n-sphere:
$V\left(4,1\right)$
$\left(V\left(4,1\right)\right)^{-1/5}$

The maximal surface of a n-sphere:
$S\left(6,1\right)$
$\left(S\left(6,1\right)\right)^{-1/6}$

The maximal volume of a none Exotic n-sphere:
$V\left(6,1/2\right),V\left(6,1/\pi\right),V\left(6,1/(2\pi)\right)$
$\left(V\left(6,1/2\right)\right)^{-1/7},\left(V\left(6,1/\pi\right)\right)^{-1/7},\left(V\left(6,1/(2\pi)\right)\right)^{-1/7}$

The maximal volume of an exotic n-sphere:
$V\left(7,1/2\right)$
$\left(V\left(7,1/2\right)\right)^{-1/8}$

The harmonic series times Cosinus:
0.042019
0.2602714

The harmonic series times Cosinus:
1.0707983
0.2853977

Where both number are calculated with radian angle and angle defined with unit diameter: $\tilde{\theta}=\theta/2$.

Where the volume and the surface of the n-spheres are defined as:

$V\left(n,r\right)=r^{n+1}\pi^{n/2+1/2}/\Gamma\left(n/2+3/2\right)$
$S\left(n,r\right)=r^{n}2\pi^{n/2+1/2}/\Gamma\left(n/2+1/2\right)$

By defining a unit radius ($r = 1$) or a unit diameter ($2\ r = 1$) or a unit maximal geodesic length ($\pi\ r = 1$) or unit circumference ($2\pi\ r = 1$), we can calculate the surface and the volume of the n-spheres.

By defining a unit surface or a unit volume, we can calculate the radius, the diameter, the maximal geodesic length or the circumference of the n-spheres.

With a unit diameter or a unit maximal geodesic length or a unit circumference, the volume and the surface of the n-sphere are decreasing.

With a unit diameter, the volume of the 0-sphere is 1.

We have checked that the volume with a unit diameter or the diameter with a unit volume of a 4-sphere is not a super-normal-irrational number. Therefore, it does not exist an exotic 4-sphere and the corresponding maximal volume with a unit diameter of an exotic sphere is the 7-sphere. Finally, to prove the the none-existence of an exotic 4-sphere without the third order of logic axiom is likely impossible.

We have also checked that those numbers are not super-normal-irreducible-irrational numbers:

$$e^2, 1/e^4, 11e, 1/(3e), 5\pi, 1/(8\pi), \pi^3, 1/\pi^8, 1/ln\left(3\right), ln\left(6\right), 1/\sqrt{5}, \sqrt{13}, 1/e^{\pi}, 1/\pi^e, e^e$$

Therefore, they can not be made with irreducible sequents.

To conclude this article, we calculate some bound about the total number of n-irreducible sequents. First, we consider a sequence of words (written with l different letters) where each elements of the sequence can not be equal to another element by removing some letters. We focus on a specific symbol for each element of the sequence and the waves formed by it above the background of other letters. If each successive element is strictly larger than the previous one, we have the following maximal number of element:

$$N_{increasing}\left(l, n_1, ..., n_l\right) = \prod_{l'=1}^{l} \sum_{k=0}^{n_{l'}} k! \binom{n_{l'}+k}{k}$$

where $n_{l'}$ is the number of symbols l' inside the initial word of the sequence.

If the element have the same length after the initial element, the maximal number of elements is:

$$N_{equal}\left(n, m, l\right) = \sum_{k=0}^{n} \left(m - k\right)^{l-1} \binom{m}{k}$$

where n is the length of the initial word.

The maximal number of n-irreducible sequent made with one word is therefore:

$$N_{equal}\left(382, 10^5, 23\right)$$

where we have supposed that the theory of everything is the N_Z-irreducible sequent made with two words of a total maximal length of 10^5 about. The formula inside sequents are written with 23 different symbols and 382 symbols are required to write the smallest n-irreducible sequent $x = 1 + 1 + 0$ with some of the Peano hypotheses.

We remark that the total number of n-irreducible sequents is much smaller than the number N_Z.

Finally, the irreducible-irrational numbers can not be smaller in absolute value than $1/N_Z$ since we can write a n-irreducible sequent with the following formula ϕ:

$$\phi \equiv \phi_a \wedge y \times a = 1 \wedge x < y + 1 \wedge y < x$$

References

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