Vacuum Energy Levels

Niveaux d'Energie du Vide

مستويات الطاقة للفراغ

Alaya Kouki

alaya.kouki@ctf.com.tn

Abstract

We have unified fields (electromagnetic field & gravitation field) by the introduction of a new universal constant . We had show that vacuum has many levels of energy and in every level we can found an infinite of other levels. Space and time are not inert, they act and interact with corpuscles and waves.

Résumé

On a unifié les champs électromagnétiques et gravitationnel en un seul champs via l'introduction d'une nouvelle constante universelle. On a montré aussi que le vide possède une infinité de niveaux d'énergie. L'espace-temps n'est pas inerte et il interagit avec les corpuscules et les ondes.

ملخص

لقد قمنا بتوحيد المجال الكهرمغناطيسي والجاذبية في مجال واحد وذالك عبر إدخال ساكن كوني جديد. بينا أن الفراغ له عدة مستويات من الطاقة لا نهائية.الفضاء-زمان ليس عدما ولكن يتفاعل مع الجزيئات والموجات.

Keywords :

Mechanical impedance of vacuum, inertial time, inertial length, wave-corpuscle duality, unity-multiplicity duality, viscosity-dispersion duality, vacuum energy levels, dark energy, dark matter, Newton gauge, Maxwell charge, Maxwell force, scale invariance gauge, unification of fields.

1)Introduction:

From 1899 , Planck had established an absolute system of unities as follows[1]:

$$M_P = \sqrt{\frac{\hbar.c}{G}} = 2.18\ \ 10^{-8} kg \quad (1)\ ,$$

$$L_P = \sqrt{\frac{\hbar.G}{c^3}} = 1.6\ \ 10^{-35} m \qquad (2)\ ,$$

$$T_P = \sqrt{\frac{\hbar.G}{c^5}} = 5.39\ \ 10^{-44} s \qquad (3).$$

The Planck system signify that extension in space-temps is equivalent to energy. So the equation of motion of a corpuscle should be written in a full space-time which act on the corpuscle by a friction force in the opposite direction of the its speed. The equation of motion of such a corpuscle is:

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{f} - a.\boldsymbol{v} \quad (4)$$

Where **:** $\boldsymbol{p}$**:** the moment of the corpuscle;

f: all unknown forces which act on the corpuscle;

$-a.\boldsymbol{v}$: an universal friction force due to the energy of the space-time;

a : friction coefficient of the space-time;

The friction coefficient $''a''$of the space-time is declared as a new universal constant.Space-time is vacuum and vacuum is space-time.

Of course equation (4) is not invariant by transformations of space and time but we will see how to change our view in exchanging energy. Let’s take it as the first idea which comes to us as thinking in classical manner.

The MKS system (or cgs system) is a meshing of space-time where the unities of measure are as follows:

$M = 1\ kg$: the unit of mass;

$L = 1\ m$: the unit of length;

$T = 1\ s$: the unit of time.

But here there is any relation ship between those unities. When Michelson try to measure the speed of light in 1891 in order to detect any motion of the ''aether'' he was surprised that the speed of light is constant in any direction: the speed of light $''c''$ was declared an universal constant and it is independent from the choice of the referential of motion and from any corpuscle. With the constant $''G''$ the gravitational constant we can establish another relation ship between mass and length for example and with Planck constant $''\hbar''$ we can resolve a system of three equations with three parameters and so Planck get the solutions (1), (2) & (3): there is a general equivalence between mass, length and time. Space-time can't be ''inert'' and should act on corpuscles. Space-time can't be only a theatre of interactions between corpuscles but it participates and interact with them.

The Planck meshing of space-time should conduct to the minimum energy in the minimum volume of the space-time. If we calculate it i.e. we have a mass M_P in a volume L_P^3 we get an enormous value and anything can't move in this media: we conclude that Planck meshing of space-time is not the good choice, another system should replace it and so one of the constants which form the Planck system is a derived constant.

It is evident that we have particles which have a mass less than Planck mass (for example electrons). The constant which should be removed is the gravitational constant: gravitation strength is neglected in the subatomic particles.

We propose to do the meshing of the space-time with three constants $''\hbar''$, $''c''$&$''a''$. The result is:

$$M = \frac{1}{c}.\sqrt{\hbar.a} \quad (5) \quad ,$$

$$L = \sqrt{\frac{\hbar}{a}} \quad (6) \quad ,$$

$$T = \frac{1}{c}.\sqrt{\frac{\hbar}{a}} \quad (7) \quad .$$

We can expect that in the MKS system the constant $''a''$ should have a very low value: if we neglect it in the equation (4) we get the classical dynamic law.

2)Determination of the constant $''a''$:

The most evident experience to have an idea about the value of the constant $''a''$ is to determine the density of energy of vacuum in the Universe by observations and to identify it to the theoretical vacuum energy [2]. There is also others experiences which allow us to determine this constant such as the photo-electric experience and the black body radiation experience [3]. We will try in the following to determine this constant referring to recent cosmology observations.

2-1)Wave-corpuscle duality:

2-1-1)Lorentz transformations:

Let's have a corpuscle of a mass m in motion in an inertial referential $R(O,x,y,z,t)$. Let's have another inertial referential $R'(O',x',y',z',t')$ in motion with a speed V along the axis (O,x) and that origins are coincident in the beginning of motion . Axis (O,x)&(O',x') are co-linear.

The Lorentz transformations of space and time between the two referential are [4]:

$$x' = \frac{x - V.t}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (8)$$

$$t' = \frac{t - x.V/c^2}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (9) \qquad \text{or} \qquad c.t' = \frac{c.t - \frac{V}{c}.x}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (9\text{-bis})$$

For monochromatic plane waves the transformations of wave-vector and frequency are the same of space and time:

$$k' = \frac{k - \omega.V/c^2}{\sqrt{1-\frac{V^2}{c^2}}} \quad (10) \qquad \text{or} \qquad c.k' = \frac{c.k - \frac{V}{c}.\omega}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (10\text{-bis})$$

$$\omega' = \frac{\omega - k.V}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (11)$$

Where: k & k' are respectively the wave-vector in the referential R& R';

ω & ω' are respectively the frequency in the referential R & R'.

2-2-2)The principle of relativity:

The principle of relativity is that the equations of nature are invariant by Lorentz transformations i.e there is invariance of the action of the corpuscle and also there is invariance of phase for plane waves.

Every theory takes its validity by respecting the following steps:

1st step–The respect of the principle of least action i.e. that every physical phenomenon is described by a principle of action. The principle of conservation of energy and conservation of momentum comes from the least action principle. We should search an action that the equations of motion which comes from its minimisation describe the phenomenon in the laboratory and Nature. The action of a corpuscle is:

$$S_{corpuscle} = \int L\big(X, \dot{X}\big).dt \quad (12)$$

Where $L(X, \dot{X})$ its Lagrange function.

2nd step-The respect of the principle of locality i.e. the phenomenon that happen in a region of space and time affect directly only their nearest environment. If we act on a system in the position (X, t) in space-time, the only direct effect is on the nearest infinitely close neighbourhood. How to guaranty that a theory respect the principle of locality?: it is done by the principle of least action.

Let's take the action of a corpuscle as:

$$S_{corpuscle} = \int L\big(X, \dot{X}, t\big).dt \qquad (13)$$

To guaranty the locality the Lagrange function in equation (13) should depend only of the spatial coordinates of the system i.e. for a corpuscle it should depend only from its position $X(t)$ and its first derivative $\dot{X}(t)$. The neighbouring points does not intervene only via its time derivative i.e. at the limit when $\Delta t \to zero$ by $[X(t + \Delta t) - X(t)]/\Delta t$.

In other terms in a referential, the effect is detected at a distance x only after a time t and that $\frac{dx}{dt}$ has a finite value $v(x, t)$ which is less or equal to the speed of information c.

3rd step-The respect of the principle of relativity i.e. the equations of motion should be invariants by Lorentz transformations. The equations of motion should be invariants in inertials referentials i.e. the same equations in different referentials.

4th step-The respect of the gauge invariance i.e. a change on the system which does not affect the action or the equations of motion: it is called a symmetry. Let's take an example in the classical fundamental law of dynamics:

$$\boldsymbol{F} = m.\frac{d^2\boldsymbol{X}}{dt^2} \quad (14)$$

This equation remain the same if we translate the origin of coordinates of a fix value or we rotate the axles of coordinates of a fixed angles. Which is conserved in classical dynamics is the total energy of the corpuscle: When there is a symmetry there is something which is conserved.

2-2-2-1)The principle of least action:

2-2-2-1-1)The equations of Euler-Lagrange:

A corpuscle which have the generalised coordinates $\{q_i, i = 1,2,3\}$ follow a trajectory developed in time and which have the equation [5]:

$$q = q_i(t), \;\; i = 1,2,3 \qquad (15)$$

Here referring to the referential R we have $(q_1 = x, q_2 = y, q_3 = z)$.

The components of generalised speed are defined as:

$$\dot{q_i} = \frac{dq_i(t)}{dt} \;, i = 1,2,3 \qquad (16)$$

The action S associated to the corpuscle is defined as:

$$S = \int L(q_i, \dot{q}_i, t).dt \qquad (17)$$

Where L is a function of q_i, $\dot{q_i}$ and possible of t.

The quantity S is extreme for the real trajectory of the corpuscle. We have:

$$dS = 0 \qquad (18)$$

If we put that the q_i are independents from each else and that the variation of the function $L(q_i, \dot{q}_i, t)$is happened at constant time we get the Euler-Lagrange equations as follows:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0 \qquad (19)$$

The solutions of equations (19) define the trajectory of the corpuscle.

The quantity L is linked to the energy of the corpuscle an it is called almost the kinetic potential. It is the difference of kinetic energy and the potential energy of the corpuscle in the case that the forces which act on the corpuscle are derived from a potential i.e. they are conservatives forces.

In case that those forces are non conservatives the Euler-Lagrange are [5]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \qquad (20)$$

Where Q_i :generalized forces.

The importance of Lagrange equations is that instead we treat with vectors quantity such as forces and accelerations in the classical dynamics, we have scalar quantities where appear only positions and speeds .

2-2-2-1-2) The Hamilton equations:

We define the generalised moments as:

$$p_i = \frac{\partial L}{\partial \dot{q}_i} \qquad (21)$$

We define the Hamilton function as:

$$H = \sum \dot{q}_i \,.\, p_i - L \qquad (22)$$

From that the q_i&p_i are independents and that $L(q_i, \dot{q}_i, t)$ is independent from time we get the equations of Hamilton as the following:

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad (23)$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \qquad (24)$$

$$\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t} \qquad (25)$$

The Hamiltonian of a corpuscle is the total energy of the corpuscle i.e. the sum of its kinetic energy and its potential energy. The advantage of the equations of Hamilton is that they are equations of the first order but Lagrange equations are of the second order. Also with the equations of Hamilton we deal only with positions and moments, the notion of inertia doesn't appear explicitly.

2-2-2-2)Equation of motion:

Let's suppose that the corpuscle is in rest in the referential R'. The action S' of the corpuscle in this referential is as:

$$dS' = L'.dt' \qquad (26)$$

Where L' is a constant that we search. In principle this constant is the kinetic potential of the corpuscle in the referential R'. It is different from the conception of the classical mechanics which consider the mass is ''inert'' and has no function only to resist to the variation to the speed of the corpuscle.

The principle of relativity requires that:

$$dS' = dS \qquad (27)$$

So we get:

$$L.dt = L'.dt' \qquad (28)$$

The position of the corpuscle in the referential R is $x = V.t$ and so from equation (9) we get:

$$dt' = dt.\sqrt{1 - \frac{V^2}{c^2}} \qquad (29)$$

Replace (29) in (28) we get :

$$L = L'.\sqrt{1 - \frac{V^2}{c^2}} \qquad (30)$$

For weak speeds we should found the expression of the kinetic energy in the classical mechanics. So when $V \ll c$ we get from equation (30):

$$L \approx L' - \frac{1}{2}.L'.\frac{V^2}{c^2} \qquad (31)$$

It is evident from equation (31) that:

$$L' = -m.c^2 \qquad (32)$$

And so from (30) and (31) we have :

$$L = -m.c^2.\sqrt{1 - \frac{V^2}{c^2}} \qquad (33)$$

We generalise the equation (33) for every speed of the corpuscle :

$$L = -m.c^2.\sqrt{1 - \frac{v^2}{c^2}} \qquad (34)$$

And that from (34) the moment of the corpuscle is:

$$\boldsymbol{p} = \frac{\partial L}{\partial \boldsymbol{v}} = \frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (35)$$

We define the inertia of the corpuscle as the ratio of its moment to its speed:

$$\xi = \frac{p}{v} = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (36)$$

The energy of the corpuscle is equal to its Hamiltonian in (22) i.e.:

$$E = H = \boldsymbol{p}.\boldsymbol{v} - L = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \xi.c^2 \quad (37)$$

If this corpuscle is in motion in the referential R it signify that it is under a friction force due to vacuum which is the extension of the space-time. Remainder that space-time according to Planck system of unities is like a ‘’foam’’ of energy. The same idea of continuous media for vacuum (or space-time) is presented by Lev Landau &E.Lifchitz in their theory of fields [6]:

to determine the position of a corpuscle we should have a referential filled with an infinite number of bodies everywhere in space and it behave like a ‘’medium’’. Every body had its own clock which it function in an arbitrary manner. This system of bodies is the referential of the theory of the gravitation i.e. the theory of general relativity as presented by Landau & E.Lifchitz. If we choose an arbitrary referential in the theory of general relativity than the laws of Nature should be available in any system of coordinates: we will conclude a similar conclusion in our actual development. So we summarise that we can predict that there is a gravitational interaction field due to our corpuscle and the strength of this field can determine the coupling constant $''G''$. Yes constant $''G''$ is an universal constant and is also predictable.

The action of the ‘’foam’’ on the corpuscle is a friction force which act in the opposite direction of motion. This force is a serial coefficients of the exponents of the speed of the corpuscle. We take only the first exponent i.e. the friction force is as:

$$\boldsymbol{f} = -a.\boldsymbol{v} \qquad (38)$$

This friction is of course independent from the choice of the corpuscle i.e. it is universal. The coefficient of friction $''a''$ is declared as a new universal constant.

We associate to the corpuscle an inertial time as:

$$\xi = a.\tau \qquad (39)$$

Its inertial time in rest is as :

$$m = a.\tau_0 \qquad (40)$$

Idem for inertial length as :

$$l = c.\tau \qquad (41)$$

And inertial length in rest as :

$$l_0 = c.\tau_0 = \frac{m.c}{a} \qquad (42)$$

We have always this relation ship for inertial time :

$$\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (43)$$

It is very easy to verify that equation (43) is invariant by Lorentz transformations i.e. we have in referential R' the inertial time of the corpuscle is:

$$\tau' = \frac{\tau_0}{\sqrt{1-\frac{v'^2}{c^2}}} \qquad (44)$$

Where :

$$v' = \frac{dx'}{dt'} \qquad (45)$$

Proof:

We can consider that the speed of the corpuscle is constant between the instant t and $t + dt$ in the referential R which corresponds to the instants t' and $t' + dt'$ in the referential $'$. From equation (9) we get:

$$dt' = \frac{1-v.V/c^2}{\sqrt{1-\frac{V^2}{c^2}}}.dt \quad (46)$$

If we indicate by τ' the inertial time of the corpuscle in the referential R' and by τ its inertial time in the referential R than from (46) we deduce that:

$$\tau' = \frac{1-v.V/c^2}{\sqrt{1-\frac{V^2}{c^2}}}.\tau \quad (47)$$

Replace (43) the expression of τ in (47) than we have:

$$\tau' = \frac{\tau_0}{\sqrt{1-\frac{(v-V)^2}{(1-\frac{v.V}{c^2})^2.c^2}}} \quad (48)$$

Let's determinate the speed v' of the corpuscle. From (8) and (9) we have:

$$v' = \frac{dx'}{dt'} = \frac{v-V}{1-\frac{v.V}{c^2}} \quad (49)$$

Replace (49) in (48) we get:

$$\tau' = \frac{\tau_0}{\sqrt{1-\frac{v'^2}{c^2}}} \qquad (50)$$

And that's CQFD.

The constancy of the speed $''c''$ implies the constancy of the energy and momentum in inertial referential as the following:

$$E^2 - p^2.c^2 = m^2.c^4 \quad (51)$$

And also the constancy of the pseudo-module [7] :

$$ds^2 = c^2.dt^2 - dx^2 - dy^2 - dz^2 \quad (52)$$

The moment can be written as the following:

$$\boldsymbol{p} = \xi.\boldsymbol{v} = \frac{E}{c^2}.\boldsymbol{v} \qquad (53)$$

It is evident that a corpuscle with a speed c has a moment $\frac{E}{c}$ according to (53).

The Hamiltonian is as:

$$H = a.\tau.c^2 \qquad (54)$$

If we add another dimension to the referential R which is the inertial position $''c.\tau''$ of the corpuscle , the speed of the corpuscle along this dimension is from (23):

$$\frac{\partial H}{\partial(a.\tau.c)} = c \qquad (55)$$

Where $''a.\tau.c''$ is the moment of the corpuscle along its inertial dimension.

The friction force along this dimension is from (24) as:

$$\frac{-\partial H}{\partial(c.\tau)} = -a.c \qquad (56)$$

It is like that the corpuscle had a speed of $''c''$ along its inertial dimension and there is always a friction force equal to $''-a.c''$ which act on along this dimension.

It is evident that the equation of motion of the corpuscle in the three dimensional space is as (thinking in classical manner) :

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{f} - a.\boldsymbol{v} \qquad (57) \qquad \text{or} \qquad \frac{d}{dt}(\boldsymbol{p} + a.\boldsymbol{X}) = \boldsymbol{f} \qquad (57\text{-bis})$$

Where $\boldsymbol{f}$ **:** all unknown forces which act on the corpuscle ;

$-a.\boldsymbol{v}$: an universal friction force which act always in the opposite side of the direction of motion due to vacuum which the same the space-time.

$\boldsymbol{X}$**:** the position of the corpuscle.

But equation (57) is non relativist invariance we can't take it and we will see how to replace it to get the kinetic energy of the corpuscle as in classical mechanics.

Let's remark that when we write the position $\boldsymbol{X}$of the corpuscle in fourth space dimensions as:

$$\boldsymbol{X} = (c.\tau, x, y, z) \qquad (58)$$

The speed of the corpuscle in fourth dimensions is :

$$\boldsymbol{V} = (c.\frac{d\tau}{dt}, \dot{x}, \dot{y}, \dot{z}) \quad (59)$$

This speed should coincide to $''c''$ along the inertial dimension when the energy of the corpuscle is varying. So we have:

$$\frac{d\tau}{dt} = 1 \qquad (60)$$:when the energy of the corpuscle is varying;

$$d\tau = 0 \qquad (61)$$: when the energy of the corpuscle is constant (i.e. its three dimensional speed is constant in module) .

So the speed in fourth dimensions is :

$$\boldsymbol{V}=(c,\dot{x}, \dot{y}, \dot{z}) = (c, \boldsymbol{v}) \qquad (62)$$

The moment in fourth dimensions is :

$$\boldsymbol{P} = a.\tau.\boldsymbol{V} = (a.\tau.c, \boldsymbol{p}) = m.c.\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{v}{c.\sqrt{1-\frac{v^2}{c^2}}}\right) = m.c.u^i = p^i \qquad (63)$$

Where u^i is the quadri-dimensional speed in the forth dimensions space-time (ct, x, y, z).

$$u^i = \frac{dx^i}{ds} \quad (64)$$

With $x^0 = c.t, x^1 = x, x^2 = y, x^3 = z$ (65)

$$ds = c.dt.\sqrt{1 - \frac{v^2}{c^2}}$$ (66):deduced

from (52).

And:

$$p^i = m.c.u^i = (\frac{E}{c}, \boldsymbol{p}) \quad (67)$$

The square of the vector x^i is the following pseudo-module:

$$(x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 \quad (68)$$

This square doesn't change for every fixed rotations of the fourth coordinates system where the Lorentz transformations are a particular case (We follow the same analysis done by Lev Landau & E.Lifchitz in their book "theory of fields").

In general we call a quadri-vector A^i the fourth quantities A^0, A^1, A^2, A^3 when in fourth transformations of coordinates system they are transformed as the x^i. In Lorentz transformations we have:

$$A^{0\prime} = \frac{A^0 - \frac{V}{c}.A^1}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A^{1\prime} = \frac{A^1 - \frac{V}{c}.A^0}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A^{2\prime} = A^2 \ , \quad A^{3\prime} = A^3 \qquad (69)$$

Here V is the three dimensional speed of R' $(O', c.t', x', y', z')$.

The square of every quadri-vector is the pseudo-scalar as defined in (52).

In order to simplify writing equations, we introduce another kind of quadri-vector as the following:

$$A_0 = A^0, \quad A_1 = -A^1, \quad A_2 = -A^2, \qquad A_3 = -A^3 \qquad (70)$$

The square of the quadric-vector is now as the following:

$$A^iA_i = A^0A_0 + A^1A_1 + A^2A_2 + A^3A_3 \qquad (71)$$

The quantities A^i are called the contra-variant corposants and the quantities A_i are called the covariant corposants of the quadri-vector.

The scalar product of two quadric-vectors is as the following:

$$A^iB_i = A^0B_0 + A^1B_1 + A^2B_2 + A^3B_3 = A_iB^i \qquad (72)$$

The product A^iB_i is an invariant scalar for every fixed rotations of coordinates system.

So for a corpuscle its four speed is :

$$u_0 = u^0 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad ,$$

$$u_1 = -u^1 = \frac{-\dot{x}}{c.\sqrt{1-\frac{v^2}{c^2}}} ,$$

$$u_2 = -u^2 = \frac{-\dot{y}}{c.\sqrt{1-\frac{v^2}{c^2}}} \quad ,$$

$$u_3 = -u^3 = \frac{-\dot{z}}{c.\sqrt{1-\frac{v^2}{c^2}}} \qquad (73)$$

We remark that:

$$ds^2 = dx_i dx^i \qquad (74)$$

So we get that:

$$u^iu_i = 1 \qquad , \ p^ip_i = m^2.c^2 \qquad (75)$$

The fourth acceleration of the corpuscle is defined as:

$$w^i = \frac{du^i}{ds} = \frac{d^2x^i}{ds^2} \qquad (76)$$

Deriving (75) we get:

$$u_iw^i = u^iw_i = 0 \qquad (77)$$

We define the quadri- force vector as :

$$g_i = \frac{dp_i}{ds} = m.c.\frac{du_i}{ds} \qquad (78)$$

Its elements verify the identity:

$$g_i u^i = 0 \qquad (79)$$:deduced from (77)

We have:

$$g^i = \frac{dp^i}{ds} = m.c.\frac{du^i}{ds} = \left(\frac{\boldsymbol{F.v}}{c^2.\sqrt{1-\frac{v^2}{c^2}}}, \frac{\boldsymbol{F}}{c.\sqrt{1-\frac{v^2}{c^2}}}\right) \qquad (80)$$

With:

$$\boldsymbol{F} = \frac{\boldsymbol{dp}}{dt} = \frac{d}{dt}\left(\frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) \qquad (81)$$:the ordinary three-dimensional force

Lets develop the expression of g_i :

$$g_0 = g^0 = \frac{1}{c^2.\sqrt{1-\frac{v^2}{c^2}}}.\frac{dE}{dt} \quad \text{with} \quad E = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad \& \quad \boldsymbol{F}.\boldsymbol{v} = \frac{dE}{dt}$$

$$g_1 = -g^1 = \frac{-1}{c.\sqrt{1-\frac{v^2}{c^2}}}.\frac{d}{dt}\left(\frac{m.\dot{x}}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$g_2 = -g^2 = \frac{-1}{c.\sqrt{1-\frac{v^2}{c^2}}}.\frac{d}{dt}\left(\frac{m.\dot{y}}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$g_3 = -g^3 = \frac{-1}{c.\sqrt{1-\frac{v^2}{c^2}}}.\frac{d}{dt}\left(\frac{m.\dot{z}}{\sqrt{1-\frac{v^2}{c^2}}}\right) \qquad (82)$$

Replace (73) & (82) in (82) we get:

$$0 = g_i u^i = \frac{1}{c^2.(1-\frac{v^2}{c^2})}.\frac{dE}{dt} - \frac{\dot{x}}{c^2.\left(1-\frac{v^2}{c^2}\right)}.\frac{d}{dt}\left(\frac{m.\dot{x}}{\sqrt{1-\frac{v^2}{c^2}}}\right) - \frac{\dot{y}}{c^2.\left(1-\frac{v^2}{c^2}\right)}.\frac{d}{dt}\left(\frac{m.\dot{y}}{\sqrt{1-\frac{v^2}{c^2}}}\right) -$$

$$\frac{\dot{z}}{c^2.\left(1-\frac{v^2}{c^2}\right)}.\frac{d}{dt}\left(\frac{m.\dot{z}}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$\frac{dE}{dt} - \dot{x}.\frac{d}{dt}\left(\frac{E}{c^2}.\dot{x}\right) - \dot{y}.\frac{d}{dt}\left(\frac{E}{c^2}.\dot{y}\right) - \dot{z}.\frac{d}{dt}\left(\frac{E}{c^2}.\dot{z}\right) = 0 \qquad (83)$$

Suppose that $v \neq c$ than (83) becomes:

$$\frac{dE}{dt} - \frac{(\dot{x}^2+\dot{y}^2+\dot{z}^2)}{c^2}.\frac{dE}{dt} - \frac{\dot{x}}{c^2}.E.\frac{d\dot{x}}{dt} - \frac{\dot{y}}{c^2}.E.\frac{d\dot{y}}{dt} - \frac{\dot{z}}{c^2}.E.\frac{d\dot{z}}{dt} = 0 \quad (84)$$

So:

$$\frac{dE}{dt}\left(1 - \frac{v^2}{c^2}\right) - \frac{E}{2.c^2}.\frac{dv^2}{dt} = 0 \qquad (85)$$

Let’s do the following balance in equation (85):

*If $v = constant$ than we get from (85) that $0 = 0$ (nothing) but we are in contradiction to our hypothesis that the energy is varying so we should exclude this case.

*The energy is varying so we have $\frac{dE}{dt} = \frac{d}{d\tau}(a.c^2.\tau) = a.c^2$ than we get from (85)&(43):

$$a.c^2.\left(1 - \frac{v^2}{c^2}\right) - \frac{m}{2.\sqrt{1-\frac{v^2}{c^2}}}.\frac{dv^2}{dt} = a.c^2.\left(1 - \frac{v^2}{c^2}\right) - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.\frac{d\varepsilon}{dt} = 0 \qquad (86)$$

With:

$$\varepsilon = \frac{1}{2}.m.v^2 \qquad (87)$$

And :

$$dt = \frac{1}{a.c^2}.\frac{d\varepsilon}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (88)$$

So we get from (86), (87) & (88) that:

$$1-\frac{v^2}{c^2}=1 \qquad (89)$$

It comes that the corpuscle should be in rest $(v=0)$ but we have exclude the constant speed. From (86) we have:

$$\frac{d\varepsilon}{dt}=a.c^2.\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} \qquad (90)$$

To resolve the dilemma of equation (90) that the energy is varying and the speed of the corpuscle is approximately constant is that the speed of the corpuscle vary slowly close to the speed of light $''c''$:it is the balance which we search. I think that this dilemma is similar to the dilemma of wave-corpuscle duality: how to have a corpuscle present in a position $(t,\boldsymbol{x})$ and the same time it is a plane wave present everywhere, the solution was that the speed of the corpuscle should be identified to the group speed of a packet of plane waves.

Another solution is that the speed of the corpuscle is varying slowly nearly zero to be approximately conform to equation (89). In this case we have:

$$\varepsilon\approx\frac{1}{2}.m.v^2-\frac{3}{4}.m.\frac{v^4}{c^2} \qquad (91)$$

Let's remark that when the corpuscle has a speed closely to $''c''$ it signify that a corpuscle is like light and it can have a wave behaviour. It signify also that it should be maintained in motion by an external force equal approximately to $''a.c''$.

Let's define the proper time $''\zeta''$of the corpuscle as the following:

$$d\zeta=\frac{ds}{c}=dt.\sqrt{1-\frac{v^2}{c^2}} \qquad (92)$$

The time $''\zeta''$ is the time indicated by a clock moving with the corpuscle at the same speed: it is like it is attached to the corpuscle. Between two positions A & B is space-time we have:

$$\zeta_B-\zeta_A=\frac{1}{c}.\int_{t_A}^{t_B}ds=\int_{t_A}^{t_B}\sqrt{1-\frac{v^2}{c^2}}.dt \qquad (93)$$

We remark that the proper time is always less the time of the referential of motion :we conclude that a mobile clock function slowly than a fixed clock.

The laws of Nature are invariants in inertial referentials. The referential of the fixed clock is an inertial referential but the referential of the mobile clock is not an inertial referential. If the motion of the corpuscle is happened approximately in constant speed than its *universe line* is a straight line parallel to the axle of time. A *universe line* of a corpuscle is its trajectory in four-dimensional space-time constructed by the *universe points* from which the corpuscle passes . A *universe point* is the three dimensional space coordinates x, y, z and the time t when the corpuscle passes.

The interval of time of any clock is of course $\frac{1}{c}.\int ds$ among its universe line (Suppose that this clock is attached to a corpuscle). The universe line of a fixed clock in an inertial referential is a straight line parallel to the axle of time. In another hand we have that the fixed clock indicate always a time interval superior than the interval time indicated by the mobile clock. It comes that the integral $\int ds$ between two universe points presents its maximum value if those points are linked by a universe straight line. We suppose that those points and lines which links them are that the elementary intervals ds along those lines are *times genre.*

An interval is a *time genre* when the module of the line which link two universe points $(c.t_1, x_1, y_1, z_1)$ and $(c.t_2, x_2, y_2, z_2)$ is positive.

This module is the following square:

$$s_{12}^2 = c^2.t_{12}^2 - l_{12}^2 \qquad (94)$$

With : $t_{12} = t_2 - t_1$ and $l_{12}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

If the square (94) is negative than the interval is a *space genre.*

Two events had a causal relation only if the interval which separate them is time genre. This result comes from that any interaction can't spread with a speed more than the light speed. The notions of "before'' and " after" had an absolute sense.

In our case the corpuscle can't be in motion in constant speed because its energy is varying continuously. Than to be conform with the image of constant speed the motion of the corpuscle should be in curved trajectory to get the same length interval as in straight motion in constant speed. An important conclusion that the motion of free massive corpuscle never can't be in straight line: it is always curved. This phenomenon is called universal gravitation: the vacuum curve the motion of every massive corpuscle.

The same analysis can be done referring to the proper time of the corpuscle. From (92) we have:

$$\int ds = \int c.d\zeta = \int \sqrt{1-\frac{v^2}{c^2}}c.dt < \int c.dt \quad (95)$$

If the speed is augmenting continuously than from (95) we get an interval less than the case of constant speed and to be in coherence with the image of constant speed than an observer attached to the corpuscle see the light in curved motion.

To be coherent from the beginning, let's search the action of the corpuscle in the quadric-dimensional space-time. The first idea which comes is that this action is proportional to the integral of the interval ds. We should at first verify that $ds' = ds$ to be conform with (27). We can take it also as a demonstration of equation (27) by following the same path of Lev Landau & E.Lifchitz in their book "Theory of fields".

In the referential R' we have also from equation (94):

$$s'^2_{12} = c^2.t'^2_{12} - l'^2_{12} \quad (96)$$

With : $t'_{12} = t'_2 - t'_1$ and $l'^2_{12} = (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2$

For light which move with constant speed c in the two inertial referentials R & R' we have of course that:

$$s'^2_{12} = s^2_{12} = 0 \quad (97)$$

We can consider that equation (94) or equation (52) are the intervals between two points in the quadric-dimensional space-time (ct, x, y, z) but with a special form geometry: The Minkowski geometry.

From (97) it is evident that if $ds = 0$ in an inertial referential that $ds' = 0$ in any other inertial referential. In another hand ds & ds' are infinitely little in the same order that we can consider ds^2 & ds'^2 are mutually proportional:

$$ds^2 = \gamma . ds'^2 \qquad (98)$$

The factor of proportionality γ is only a function of the absolute relative speed of the two referentials and is not a function of space coordinates and time coordinate otherwise the points of space-time will not be equivalents as implies the homogeneity of space-time. This factor is also independent from the sens of the relative speed because this makes default of the isotropy of space-time .

Let's have three inertial referentials R, R_1 & R_2 and let's take V_1 & V_2 the relatives speeds of respectively R_1 & R_2 in the referential R .So we have:

$$ds^2 = \gamma(V_1). ds_1^2 \quad \& \quad ds^2 = \gamma(V_2). ds_2^2 \qquad (99)$$

We have also:

$$ds_1^2 = \gamma(V_{12}). ds_2^2 \qquad (100)$$

Where V_{12} the absolute speed of K_2 referring to K_1. So from (99) & (100) we get:

$$\frac{\gamma(V_2)}{\gamma(V_1)} = \gamma(V_{12}) \qquad (101)$$

But V_{12} depends not only of the modules of $\boldsymbol{V_1}$& $\boldsymbol{V_2}$ but also of the angle between the two vectors. As this angle doesn't exist in the first member of equation (101), so equation (101) can't be verified if only the function $\gamma(V)$ is equal to one. So we have:

$$ds^2 = ds'^2 \qquad (102)$$

We return now to search the action of the corpuscle in four dimensions. For a free corpuscle i.e. a corpuscle which is not under any force, the action is an integral of a scalar. The unique convenient scalar is the interval ds .So the action should be as:

$$S = \alpha . \int_A^B ds \qquad (103)$$

With : α a constant which characterise the corpuscle.

The integral $\int_A^B ds$ has its great value along a universe straight line. Along a curved line this integral will be more great. For the principle of least action of a mechanical system we can define an integral S called action which presents a minimum for real motion i.e. its variations δS are equal to zero. It comes that the constant α should be negative.

We have:

$$\delta S = \alpha . \int_A^B \delta . ds = 0 \qquad (104)$$

Or from (74) we have:

$ds = \frac{dx_i dx^i}{ds}$ so $\delta ds = \frac{dx_i \delta dx^i}{ds} = u_i d\delta x^i$

Than :

$$\delta S = \alpha . \int_A^B u_i d\delta x^i = \alpha . \int_A^B [d(u_i \delta x^i) - \delta x^i du_i] \qquad (105)$$

So:

$$\delta S = \alpha . [u_i \delta x^i]_A^B - \alpha . \int_A^B \delta x^i \frac{du_i}{ds} ds \qquad (106)$$

As in classical mechanics to find the equations of motions of a corpuscle we should compare many trajectories of the corpuscle which all pass from two given points i.e. satisfying the limit conditions that $(\delta x^i)_A = (\delta x^i)_B = 0$. The real trajectory of the corpuscle is deduced from the condition that $\delta S = 0$. Than we conclude from (106) that $\frac{du_i}{ds} = 0$ expressing the constancy of the speed of a free corpuscle in the quadri-dimensional form of coordinates.

To determinate the variation of the action as a function of coordinates , we put that the point A is given as $(\delta x^i)_A = 0$ and that the point B is any point of space-time which satisfy the equation of motion i.e. its belong to the real trajectory of the corpuscle. In consequence the integral in (106) is equal to zero and by putting that $(\delta x^i)_B = \delta x^i$ it comes :

$$\delta S = -\alpha . u_i . \delta x^i \qquad (107)$$

Referring to classical mechanics the partials derivations $\frac{\partial S}{\partial x}, \frac{\partial S}{\partial y}, \frac{\partial S}{\partial z}$ are the components of the moment vector of the corpuscle and $-\frac{\partial S}{\partial t}$ is its energy. So it comes that the covariant components of the four moment of the corpuscle is:

$$p_i = -\frac{\partial S}{\partial x^i} = -\alpha . u_i = (\frac{E}{c}, -\boldsymbol{p}) \qquad (108)$$

So we deduce from (108) and (73) that:

$$\alpha = -m.c \qquad (109)$$

The transformations of energy and moment are as follows:

$$E' = \frac{E-\boldsymbol{p}.\boldsymbol{V}}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (110)$$

$$\boldsymbol{p}' = \frac{\boldsymbol{p}-\frac{E}{c^2}.\boldsymbol{V}}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (111)$$

Equations (110) ,(111) and (75) are what implies the relativist invariance of equation of motion deduced from the principle of least action in four dimensions .

To find the equation of motion it comes from (75) that:

$$p^i p_i = \frac{\partial S}{\partial x_i}\frac{\partial S}{\partial x^i} = \frac{1}{c^2}(\frac{\partial S}{\partial t})^2 - (\frac{\partial S}{\partial x})^2 - (\frac{\partial S}{\partial y})^2 - (\frac{\partial S}{\partial z})^2 = m^2.c^2 \qquad (112)$$

Equation (112) is the Hamilton-Jacobi relativist equation of motion of the corpuscle. .

We have obtain the equation of motion (112) with the hypothesis that the quadric-dimensional speed of the corpuscle $\frac{du_i}{ds} = 0$. But we had seen that a massive corpuscle can't never move in a straight line with constant speed otherwise it should be like light or it is in rest. Equation of motion is not the good equation, it is only a step to get the good equation.

2-2-2-3)Theory of fields:

Let's develop equation (57) and see what does it mean:

$$\boldsymbol{f} = \frac{d^2(a.\tau.\boldsymbol{x})}{d\tau^2} = a.\tau.\frac{d^2\boldsymbol{x}}{d\tau^2} + 2.a.\boldsymbol{v} = a.\tau.\frac{d\boldsymbol{v}}{d\tau} + 2.a.\boldsymbol{v} = \frac{d^2(\xi.\boldsymbol{x})}{dt^2} = \frac{d\boldsymbol{p}}{dt} - a.\boldsymbol{v} \qquad (113)$$

Let's note that equation (113) is independent from the choice of the origin of the referential.

Let's develop the conventional definition of the force:

$$\frac{d\boldsymbol{p}}{dt} = \frac{d(a.\tau.\boldsymbol{v})}{dt} = a.\boldsymbol{v} + a.\tau.\frac{d\boldsymbol{v}}{dt} = \frac{d}{dt}\left(\frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}.\frac{d\boldsymbol{v}}{dt} + \frac{m.\boldsymbol{v}}{2.(1-\frac{v^2}{c^2})^{3/2}}.\frac{dv^2}{c^2.dt} \quad (114)$$

It comes that:

$$a.\boldsymbol{v} = \frac{m.\boldsymbol{v}}{2.(1-\frac{v^2}{c^2})^{3/2}}.\frac{dv^2}{c^2.dt} \quad (115)$$

So:

$$a = \frac{m}{2.(1-\frac{v^2}{c^2})^{3/2}}.\frac{dv^2}{c^2.d\tau} \quad (116)$$

We have:

$$d\tau = d\left(\frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{\tau_0}{2.(1-\frac{v^2}{c^2})^{3/2}}.\frac{dv^2}{c^2} \quad (117)$$

Replace (117) in (116) we have:

$$a = \frac{m}{\tau_0} \quad (118)$$

We get nothing special.

Let's define the following force as:

$$\boldsymbol{G} = \frac{d\boldsymbol{P}}{ds} = \left(\frac{1}{c^2.\sqrt{1-\frac{v^2}{c^2}}}.\frac{d}{dt}\left(\frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}\right), \frac{1}{c.\sqrt{1-\frac{v^2}{c^2}}}.\frac{d\boldsymbol{p}}{dt}\right) = \left(\frac{\frac{dW}{dt}}{c^2.\sqrt{1-\frac{v^2}{c^2}}}, \frac{\boldsymbol{F}}{c.\sqrt{1-\frac{v^2}{c^2}}}\right) \quad (119)$$

With that:

$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$ (120): the classical definition of the force.

$$\frac{dW}{dt} = \frac{d}{dt}\left(\frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{dH}{dt} \quad (121)$$

If we adopt the same definition in classical mechanics that W is the work of the classical force $\boldsymbol{F}$ than we have:

$$\boldsymbol{F}.\boldsymbol{v} = \frac{dW}{dt} = \frac{dE}{dt} \qquad (122)$$

We have also this relation:

$$\boldsymbol{p} = \frac{E}{c^2}.\boldsymbol{v} \qquad (123)$$

We have also the following invariant as a consequence of the invariance of the speed of light:

$$\frac{E^2}{c^2} - p^2 = m^2.c^2 \qquad (124)$$

So from (124) and (123) we get:

$$v^2 = c^2 - \frac{m^2.c^6}{E^2} \qquad (125)$$

From (119) we have:

$$\boldsymbol{f}.\boldsymbol{v} = \boldsymbol{F}.\boldsymbol{v} - a.v^2 = \frac{dE}{dt} - a.c^2 - \frac{a.m^2.c^6}{E^2} = \frac{dE}{dt} - a.c^2.\left(2 - \frac{v^2}{c^2}\right) = \frac{dE}{dt} - a.v^2$$
(126)

It comes that:

$$v = c \qquad (127)$$

The speed of the corpuscle is equal to $''c''$ only referring to its proper time but the referential which is attached to the corpuscle is not an inertial referential. The only way to get out from this contradiction is to accept that the corpuscle can have a wave behaviour like light in a curved line motion. Along its inertial time coordinate the corpuscle is like light and had the speed $''c''$. It is like maintained in motion with a force:

$$f = a.c \qquad (128)$$

The work of this force along the inertial length of the corpuscle is:

$$\Delta E_{kinetic} = f.\Delta l \qquad (129)$$

With :

$$\Delta l = l - l_0 = c(\tau - \tau_0) \approx \frac{1}{2}.\frac{\tau_0.v^2}{c} \quad \text{if } v \ll c \ (130)$$

Of course the force $f = a.c$ respect relativist invariance. This force ''maintain'' the corpuscle in motion with a speed c along its inertial coordinate. In the four dimensions space-time the motion of the corpuscle is in a curved universe line but we can consider this motion is constant between the time t and $t + dt$ and the corpuscle is maintained in motion by the force $\boldsymbol{f} = a.\boldsymbol{v}$ just in this laps of time. At the same time the corpuscle can have a waving behaviour.

Let's suppose that the corpuscle is in constant speed. The characteristics of a plane wave is its frequency ω and wave-vector **.** We can form a four-vector k^i as the following:

$$k^0 = \frac{\omega}{c}\ ,\ k^1 = k_x\ \ ,k^2 = k_y\ \ ,k^3 = k_z \qquad (131)$$

Of course the four components should have the same dimension.

If the corpuscle can have a waving behaviour necessary there is a relation ship between its four dimension moment and its four dimension wave-vector:

$$p^i = \beta.k^i \qquad (132)$$

Where : β: a new universal constant .

So we have:

$$\frac{E}{c} = \beta.\frac{\omega}{c} \qquad (133)$$

$$\boldsymbol{p} = \beta.\boldsymbol{k} \qquad (144)$$

It comes that from equation (51) :

$$(\beta.\frac{\omega}{c})^2 - (\beta.k)^2 = (m.c)^2 \qquad (145)$$

In other terms equation (145) becomes:

$$\frac{\omega^2}{c^2} - k^2 = (\frac{m.c}{\beta})^2 \qquad (146)$$

The wave-vector associated to the corpuscle is not a linear function of the frequency i.e. the medium in which the corpuscle move is a dispersive medium. A dispersive medium for waves correspond for the corpuscle to a viscous medium : there is friction in space-time and this doesn't surprise us.

The group speed of the wave is as defined :

$$v_g = \frac{d\omega}{dk} = \frac{d\omega}{dE}.\frac{dE}{dk} = v \qquad (147)$$

The phase speed is as defined:

$$v_f = \frac{\omega}{k} = \frac{\beta}{p}.c.\sqrt{\frac{p^2}{\beta^2} + \frac{m^2.c^2}{\beta^2}} = c.\sqrt{1 + \frac{m^2.c^2}{p^2}} = c.\sqrt{1 + \frac{c^2}{v^2}.(1 - \frac{v^2}{c^2})} = \frac{c^2}{v} \quad (148)$$

The corpuscle is like a packet of waves which are reinforced around its position and

annihilate themselves above. This condition requires that:

$$\Delta k_x.\Delta x \geq 1 \quad ,\Delta k_y.\Delta y \geq 1 \quad ,\Delta k_z.\Delta z \geq 1 \qquad (149)$$

$$\Delta\omega.\Delta t \geq 1 \qquad (150)$$

Where Δk: the uncertainty about the wave-vector;

ΔX: the uncertainty about the position of the corpuscle;

$\Delta\omega$: the uncertainty about the frequency (i.e. about the energy of the corpuscle)

Δt: the uncertainty about the time.

The equation of motion is:

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{f} - a.\boldsymbol{v} \qquad (151)$$

Which is only valid locally in the position $\boldsymbol{X} \pm \Delta\boldsymbol{X}$ at the time $\boldsymbol{t} \pm \Delta\boldsymbol{t}$ .So this equation can't be the solution to found the real trajectory of the corpuscle. The only way to found the trajectory of the corpuscle is to determine its action i.e. to apply the principle of least action in a curved space-time and respecting the principle of uncertainty (149) &(150).

2-2-2-3-1)Motion of a charged corpuscle in an electromagnetic field:

In the interaction of a charged corpuscle with an electromagnetic field we consider only its electric charge e which can be positive, negative or equal to zero and we neglect its spin the intrinsic momentum of the corpuscle.

The electromagnetic field is characterised by the quadric-potential A_i which its components are a function of coordinates and time. The action of the corpuscle is the sum of the action (103) for a free corpuscle and the action of the electromagnetic field:

$$S = \int_A^B \left(-mc.ds - \gamma.\frac{e}{c}A_i dx^i\right) \qquad (152)$$

The factor $\frac{1}{c}$ is chosen for commodity.

The coefficient γ is a conversion factor since there is not any known relation ship between the potential vector or the electric charge with the MKS system or cgs system. The only fact which we know is that gravitational field is acting in great scale and electromagnetic field is acting in microscopic scale.

The time component of the quadric-potential is the *scalar potential* of the field and it is noted $A^0 = \varphi$ and the three space components of the field are the *vector potential* $\boldsymbol{A}$ of the field.

We have:

$$A^i = (\varphi, \boldsymbol{A}) \qquad (153)$$

We can write the integral (152) as:

$$S = \int_A^B \left(-mc.ds + \gamma.\frac{e}{c}.\boldsymbol{A}.d\boldsymbol{X} - \gamma e.\varphi dt\right) \qquad (154)$$

Introduce the speed $\boldsymbol{v} = \frac{d\boldsymbol{X}}{dt}$ of the corpuscle , equation (154) becomes:

$$S = \int_{t_1}^{t_2} \left(-m.c^2.\sqrt{1-\frac{v^2}{c^2}} + \gamma\frac{e}{c}.\boldsymbol{A}.\boldsymbol{v} - \gamma e.\varphi\right).dt \qquad (155)$$

So the Lagrange function of the corpuscle is:

$$L = -m.c^2.\sqrt{1-\frac{v^2}{c^2}} + \gamma\frac{e}{c}.\boldsymbol{A}.\boldsymbol{v} - \gamma e.\varphi \qquad (157)$$

The equation (157) is different from equation (35) for a free corpuscle by the term $\gamma\frac{e}{c}.\boldsymbol{A}.\boldsymbol{v} - \gamma e.\varphi$ which describe the interaction of the corpuscle with the field.

The derivative $\frac{\partial L}{\partial \boldsymbol{v}}$ is the generalised momentum of the corpuscle noted as **.** We found that:

$$\boldsymbol{P} = \frac{m.v}{\sqrt{1-\frac{v^2}{c^2}}} + \gamma\frac{e}{c}.\boldsymbol{A} = \boldsymbol{p} + \gamma\frac{e}{c}.\boldsymbol{A} \qquad (158)$$

Where $\boldsymbol{p} = \frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}$ is the ordinary momentum of the corpuscle.

The Hamilton function of the corpuscle in the field is:

$$H = \boldsymbol{v}.\frac{\partial L}{\partial \boldsymbol{v}} - L = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} + \gamma e.\varphi \qquad (159)$$

The Hamilton function should be written as a function of generalised momentum and as a function of speed. From (158) & (159) we have:

$$(\frac{H-\gamma e.\varphi}{c})^2 - (\boldsymbol{P} - \gamma\frac{e}{c}.\boldsymbol{A})^2 = m^2.c^2 \qquad (160)$$

Than we have:

$$H = \sqrt{m^2.c^4 + c^2.(\boldsymbol{P} - \gamma\frac{e}{c}.\boldsymbol{A})^2} + \gamma e.\varphi \qquad (161)$$

Let's write the Hamilton-Jacobi equation for a corpuscle placed in an electro-magnetic field. This equation is obtained by replacing in the Hamilton function the generalised momentum by $\frac{\partial S}{\partial \boldsymbol{X}}$ and H by $-\frac{\partial S}{\partial t}$. We get from (160):

$$(grad\ S - \gamma\frac{e}{c}.\boldsymbol{A})^2 - \frac{1}{c^2}.(\frac{\partial S}{\partial t} + \gamma e.\varphi)^2 + m^2.c^2 = 0 \qquad (162)$$

2-2-3-2)Locally equation of motion of an electric charge in a field:

We suppose that the electric charge is small and can't affect the electromagnetic field . We get the equation of motion by varying the action so we can use the Lagrange equations (19) as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \boldsymbol{v}}\right) - \frac{\partial L}{\partial \boldsymbol{X}} = \mathbf{0} \qquad (163)$$

We neglect any non conservative force in equation (163). The Lagrange function L is given by equation (157).

The derivative $\frac{\partial L}{\partial \boldsymbol{v}}$ is given by equation (158). Also we have:

$$\frac{\partial L}{\partial \boldsymbol{X}} \equiv \nabla L = \gamma \frac{e}{c}.grad(\boldsymbol{A}.\boldsymbol{v}) - \gamma e.grad(\varphi) \qquad (164)$$

As we know in mathematics that for any two vectors $\boldsymbol{A}$ & $\boldsymbol{v}$ we have:

$$grad(\boldsymbol{A}.\boldsymbol{v}) = (\boldsymbol{A}\nabla).\boldsymbol{v} + (\boldsymbol{v}\nabla).\boldsymbol{A} + \boldsymbol{v} \times rot\boldsymbol{A} + \boldsymbol{A} \times rot\boldsymbol{v} \qquad (165)$$

Locally the speed $\boldsymbol{v}$ of the corpuscle is approximately constant so we take it constant . We have:

$$\frac{\partial L}{\partial \boldsymbol{X}} = \gamma \frac{e}{c}(\boldsymbol{v}\nabla).\boldsymbol{A} + \gamma \frac{e}{c}.\boldsymbol{v} \times rot\boldsymbol{A} - \gamma e.grad(\varphi) \qquad (166)$$

So locally the Lagrange equations are as the following:

$$\frac{d}{dt}\left(\boldsymbol{p} + \gamma \frac{e}{c}\boldsymbol{A}\right) = \gamma \frac{e}{c}(\boldsymbol{v}\nabla).\boldsymbol{A} + \gamma \frac{e}{c}.\boldsymbol{v} \times rot\boldsymbol{A} - \gamma e.grad(\varphi) \qquad (167)$$

Or the total differential $\frac{d\boldsymbol{A}}{dt}.dt$ include two terms the local variation $\frac{\partial \boldsymbol{A}}{\partial t}.dt$ of the potential vector as a function of time in a given point of space and its variation when we translate of a distance dX to another point. This second term is equal to $(d\boldsymbol{X}\nabla)\boldsymbol{A}$. So we have:

$$\frac{d\boldsymbol{A}}{dt} = \frac{\partial \boldsymbol{A}}{\partial t} + (\boldsymbol{v}\nabla)\boldsymbol{A} \qquad (168)$$

Introduce equation (168) in equation (167) we get:

$$\frac{d\boldsymbol{p}}{dt} = -\gamma \frac{e}{c}.\frac{\partial \boldsymbol{A}}{\partial t} - \gamma e.grad(\varphi) + \gamma \frac{e}{c}.\boldsymbol{v} \times rot\boldsymbol{A} \qquad (169)$$

Equation (169) is the locally equation of motion of a charged corpuscle in an electromagnetic field. In the first member of this equation we found the derivative of the momentum of the corpuscle to time so the second member of this equation represent the force exerted by the electromagnetic field on the electric charge. This force is composed of two terms. The first term is independent from the speed of the corpuscle but the second term it depends and it is proportional to the module of the speed and perpendicular to it.

The first term is called the *intensity of electric field* , it is noted **E** and is equal for one unit charge:

$$\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{A}}{\partial t} - \gamma.grad(\varphi) \qquad (170)$$

The factor after the speed $\boldsymbol{v}$ is the second term of the force applied on a unit charge and it is called *magnetic field vector* which is noted $\boldsymbol{B}$ and so we have:

$$\boldsymbol{B} = \frac{\gamma}{c}.rot\boldsymbol{A} \qquad (171)$$

So the locally equation of motion of an electric charge in an electromagnetic field is:

$$\frac{d\boldsymbol{p}}{dt} = e.\boldsymbol{E} + e.\boldsymbol{v} \times \boldsymbol{B} \qquad (172)$$

Let's establish the variation of the total energy of the corpuscle locally:

$$\frac{dE_{total}}{dt} = \boldsymbol{v}.\frac{d\boldsymbol{p}}{dt} = e\boldsymbol{E}.\boldsymbol{v} \qquad (173)$$

We remark that the work furnished to the corpuscle is only due to electric field, the magnetic field doesn't do any work for electric charges in motion.

Mechanical motion are locally invariants to the change of the direction of time which we can deduce it from $\frac{d\boldsymbol{p}}{dt} = \frac{d}{dt}\left(\frac{m.\frac{dX}{dt}}{\sqrt{1-\frac{\left(\frac{dX}{dt}\right)^2}{c^2}}}\right)$ in other terms the motion in inverse direction of a mechanical system is possible and product the same affects as in the first direction.

For electromagnetic field let's remark that when we do the following substitutions $t \to -t$, $\boldsymbol{E} \to \boldsymbol{E}$, $\boldsymbol{B} \to -\boldsymbol{B}$ the equation (172) doesn't change but regarding to equations (170) & (171) the scalar potential doesn't change and the potential vector change its sign: $\varphi \to \varphi$,

$\boldsymbol{A} \rightarrow -\boldsymbol{A}$. So when a certain local motion is possible in the electromagnetic field, the motion in the inverse direction is also possible with the condition to inverse the direction of magnetic field.

2-2-3-3)Gauge invariance:

Let's remark that when we add a constant to potential vector and/or another constant to the scalar potential in the equations (171) and (170) than the electric field and the magnetic field doesn't change: the question is that are the potentials of the field determined in only one manner.

The electromagnetic field is characterised by its action on the charges in which they move and in the equation (172) there is only the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ so we can conclude that two fields are physically the same only if they are characterised by the same vectors $\boldsymbol{E}$ & $\boldsymbol{B}$.

For a given potentials $\boldsymbol{A}$ & φ we can determine the field by equations (170) & (171) but as we had seen to a unique and the same field can correspond many different potentials . In the general case lets add to the components of the potential A_k the quantity $-\frac{\partial f}{\partial x^k}$ where f is an arbitrary function of coordinates and time. We get the new potential:

$$A'_k = A_k - \frac{\partial f}{\partial x^k} \qquad (174)$$

This substitution engender in the integral of the action (152) a supplement term which is a total differential:

$$\gamma \frac{e}{c} . \frac{\partial f}{\partial x^k} . dx^k = d(\gamma \frac{e}{c} . f) \qquad (175)$$

Which doesn't affect the equations of motion .

If we introduce instead the quadric-dimensional potential another vector potential and another scalar potential and instead the coordinates x^i the coordinates ct, x , y, z we can write the fourth equalities (174) as:

$$\boldsymbol{A}' = \boldsymbol{A} + grad(f) \quad , \ \varphi' = \varphi - \frac{1}{c} . \frac{\partial f}{\partial t} \qquad (176)$$

It is very easy to verify that the electric field and the magnetic field given by equations (170) & (171) don't vary if we replace $\boldsymbol{A}$ & φ by the potentials $\boldsymbol{A}'$& φ' given by equations (176). So the transformation of potentials (174) doesn't affect the field. So the potentials are not defined in a unique manner, the vector potential is defined with a gradient function nearly and the scalar potential is also defined to time derivative of the same function nearly.

Only the values invariants referring to the transformations of potentials (176) have a physical signification. So all the equations should be invariants referring to this transformation: this invariance is called *gauge invariance.*

2-2-2-4)Unification of fields:

Let's take the locally equation of motion (151). It can be written as:

$$\frac{d}{dt}(\boldsymbol{p} + a.\boldsymbol{X}) = \boldsymbol{f} \qquad (177)$$

Where:

$\boldsymbol{p}$: the momentum of the corpuscle;

$\boldsymbol{X}$: the position of the corpuscle.

We can write the equation (177) as the following:

$$\frac{d\boldsymbol{P}}{dt} = \boldsymbol{f} \qquad (178)$$

With :

$$\boldsymbol{P} = \boldsymbol{p} + a.\boldsymbol{X} \qquad (179)$$

Equation (179) is like equation (158) but here the corpuscle is not charged. The only potential which can curve the motion of the corpuscle is the field of gravitation.

Let's define the following generalised momentum as the following:

$$\boldsymbol{P} = \boldsymbol{p} + \mu.\boldsymbol{U} \qquad (180)$$

With:

$\boldsymbol{p}$: the momentum of the corpuscle;

$\mu = \gamma.\frac{e}{c}$ (181) :if we are dealing with a charged corpuscle

$\mu = a$ (182): if we are dealing with a non charged corpuscle

$\boldsymbol{U} = \boldsymbol{A}$ (183): if we are dealing with a charged corpuscle in motion in an electromagnetic field;

$\boldsymbol{U} = \boldsymbol{X}$ (184): if we are dealing with a non charged corpuscle in motion in a gravitational field.

So we can write the Hamilton-Jacobi equation of motion for any corpuscle charged or not charged in any field by going back to the equation (162) as:

$$(grad\ S - \mu.\boldsymbol{U})^2 - \frac{1}{c^2}.\left(\frac{\partial S}{\partial t} + c.\mu.\varphi\right)^2 + m^2.c^2 = 0 \quad (185)$$

Let's take a non charged corpuscle. The force of first specie which is applied on the corpuscle as referring to equation (170) is:

$$\boldsymbol{G} = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{X}}{\partial t} - \gamma.grad(\varphi) = -grad(\varphi') \quad (186)$$

With :

$\frac{\partial \boldsymbol{X}}{\partial t} = \boldsymbol{0}$: we accept that this force vary slowly with time or independent from time.

$\varphi' = \gamma.\varphi$: is the gravitational field

We choose the coefficient γ as $\boldsymbol{G}$ becomes an acceleration. The corpuscle will be under a force of a first specie as in classical mechanics:

$$\boldsymbol{f} = m.\boldsymbol{G} \quad (187)$$

The force of the second specie is as referring to equation (171):

$$\boldsymbol{F} = \frac{\gamma}{c}.rot\boldsymbol{X} = \boldsymbol{0} \quad (188)$$

The total energy of the corpuscle as referring to equation (159) is:

$$E_{total} = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{a.c}{\gamma}.\varphi' \qquad (189)$$

If we have $v \ll c$ than the total energy is :

$$E_{total} \approx m.c^2 + \frac{1}{2}.m.v^2 + \varphi'' \quad (190)$$

With:

$$\varphi'' = \frac{a.c}{\gamma}.\varphi' \qquad (191)$$

2-2-2-4-1)Newton gauge:

Let's take a corpuscle in rest. Which gravitational field it create?.

From equation (186) we have:

$$div(\boldsymbol{G}) = -\nabla^2\varphi' = -\Delta\varphi' \qquad (192)$$

The *Newton gauge* is when we have:

$$div(\boldsymbol{G}) = -4.\pi.G.\rho \qquad (193)$$

With :

G: gravitational constant (*Newton constant*)

ρ: density of masses

The general solution for the equation (193) is as:

$$\varphi' = -k\int\frac{\rho dV}{R} \qquad (194)$$

Where :

$dV = dx.dy.dz$: volume element;

R: the distance between the corpuscle in the center and the volume element.

Of course we suppose that $\boldsymbol{G}$ has a spherical symmetry.

For low speed corpuscles the equation (194) determine the gravitational field for any masses distribution. For one corpuscle we have:

$$\varphi' = -\frac{G.m}{R} \qquad (195)$$

The force which is exerted on a corpuscle of a mass m' in this field is:

$$f = -m'.\frac{\partial\varphi\prime}{\partial R} = -\frac{G.m.m\prime}{R^2} \qquad (196)$$

We found the *second law of Newton.*

2-2-2-4-2)Coulomb gauge :

Let's have a charge in rest .Which electrical field create?

From equation (170)& (171) we have in general:

$$rot\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial}{\partial t}(rot\boldsymbol{A}) - \gamma.rotgrad(\varphi) = -\frac{\partial\boldsymbol{B}}{\partial t} \qquad (197)$$

$$div\boldsymbol{B} = \boldsymbol{0} \qquad (198)$$

Equations (197)and (198) are called the *first group of Maxwell equations without sources* .

In electrostatic the *Coulomb gauge* is as:

$$div\boldsymbol{A} = \nabla\boldsymbol{A} = \boldsymbol{0} \qquad (199)$$

So we have in electrostatic:

$$div\boldsymbol{E} = \nabla\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial\nabla\boldsymbol{A}}{\partial t} - \gamma.\nabla^2\varphi = -\gamma.\nabla^2\varphi = -\nabla^2\varphi' \qquad (200)$$

$$rot\boldsymbol{B} = \frac{\gamma}{c}\nabla(\nabla\boldsymbol{A}) - \nabla^2\boldsymbol{A} = -\nabla^2\boldsymbol{A} \qquad (201)$$

With:

$$\varphi' = \gamma.\varphi$$

Like the *Newton gauge* we suppose that the field $\boldsymbol{E}$ is spherical and radial and we impose that:

$$divE = 4.\pi.\sigma.\rho \qquad (202)$$

With:

σ: electrical constant (*Coulomb constant*) ;

ρ: density of charges;

Equation (202) is called *First Maxwell equation with source.*

The general solution of equation (202) is as:

$$\varphi' = \sigma.\int \frac{\rho dV}{R} \qquad (203)$$

Where :

$dV = dx.dy.dz$: volume element;

R: the distance between the charge in the center and the volume element.

For low speed charges the equation (203) determine the scalar potential for any charges distribution. For one charge we have:

$$\varphi' = \frac{\sigma.e}{R} \qquad (204)$$

The force which is exerted on a charge e' by the charge e is :

$$F = -e'.\frac{\partial\varphi'}{\partial R} = \frac{\sigma.e.e'}{R^2} \qquad (205)$$

So we found the *law of Coulomb.*

The electric field is as:

$$\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{A}}{\partial t} - grad(\varphi') = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{A}}{\partial t} + \frac{\sigma.e}{R^3}.\boldsymbol{R} \qquad (206)$$

If we add another condition which is that the electrical field vary slowly or constant in time , we get:

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{0} \qquad (207)$$

$$\boldsymbol{E} = \frac{\sigma.e}{R^3}.\boldsymbol{R} = \frac{\boldsymbol{F}}{e\prime} \qquad (208)$$

2-2-2-4-3)Continuity equation for masses:

The variation of density of masses as a function of time is:

$$\frac{\partial}{\partial t}\int \rho.dV \qquad (209)$$

The variation of quantity of masses per unit time depends on the quantity of masses getting out or in the element volume. The quantity of masses going in this volume is equal to $\rho\boldsymbol{v}.d\boldsymbol{f}$ where $\boldsymbol{v}$ is the speed of a corpuscle in the point of space where exist the element . The total mass going out the volume is as:

$$\oint \rho\boldsymbol{v}.d\boldsymbol{f} \qquad (210)$$

Where the integral (210) is extended to the total closed surface bordering the volume. So we have:

$$\frac{\partial}{\partial t}\int \rho.dV = -\oint \rho\boldsymbol{v}.d\boldsymbol{f} = -\oint \boldsymbol{j}.d\boldsymbol{f} \qquad (211)$$

The negative sign forward the second member of equation (211) is that the first member should be positive when the total mass in the volume augment . The element of surface $d\boldsymbol{f}$ is oriented to the exterior of the volume.

Apply Gauss theorem to the second member of equation (211) we get:

$$\oint \boldsymbol{j}.d\boldsymbol{f} = \int div\boldsymbol{j}\; dV \qquad (212)$$

Replace (212) in equation (211) we have:

$$\int (div\boldsymbol{j} + \frac{\partial\rho}{\partial t})\; dV = 0 \qquad (213)$$

The equation (213) is valid for any volume so we should have:

$$div\boldsymbol{j} + \frac{\partial\rho}{\partial t} = 0 \qquad (214)$$

Equation (214) is *the continuity equation*.

2-2-2-4-4)Quadric-vector current of mass or flux of mass:

For a corpuscle an element of its mass is as:

$$dm = \rho . dV \qquad (215)$$

Multiplying the two terms by dx^i we get:

$$dm . dx^i = \rho . dV . dx^i = \rho . dV . \frac{dx^i}{dt} . dt \qquad (216)$$

In the left dm is a scalar and dx^i is a quadric-vector so the product is a quadric-vector. In the right $dV . dt$ is a scalar so $\rho . \frac{dx^i}{dt}$ is a quadric-vector noted as j^i and called *quadric-vector of density of current of mass or flux of mass.* We have:

$$j^i = \rho . \frac{dx^i}{dt} \qquad (217)$$

The three space components of this quadric-vector define the three dimensional flux of mass:

$$\boldsymbol{j} = \rho . \boldsymbol{v} \qquad (218)$$

The time component of this quadric-vector is $\rho . c$ so we have:

$$j^i = (\rho . c, \boldsymbol{j}) \qquad (219)$$

The total mass in a volume V is the mass of the corpuscle so we have:

$$m = \int \rho . dV = \frac{1}{c} . \int j^0 dV \qquad (220)$$

From equation (214) we deduce that:

$$div\boldsymbol{j} + \frac{\partial \rho}{\partial t} = \frac{\partial j^1}{\partial x^1} + \frac{\partial j^2}{\partial x^2} + \frac{\partial j^3}{\partial x^3} + \frac{1}{c} . \frac{\partial (\rho . c)}{\partial t} = \frac{\partial j^i}{\partial x^i} = 0 \qquad (221)$$

For vacuum where absence of mass we have from (220):

$$m_0 = \rho_0 . \frac{4}{3} . \pi . R^3 \qquad (222)$$

With:

m_0 :the mass of vacuum contained in a sphere of radius R

ρ_0 : the density of vacuum

$\boldsymbol{j_0} = \mathbf{0}$:density of current of vacuum.

Normally we should take in consideration the action of vacuum on the corpuscle when establishing the equation of motion of the corpuscle referring to the principle of least action .

2-2-2-4-5)Applications in classic physics:

Let's have a classic corpuscle in motion nearly the border of the Universe. We draw a sphere tangent to its motion. The gravitational action on this corpuscle referring to (196)is:

$$F_0 = \frac{-G.m_0.m}{R^2} \qquad (223)$$

Replace m_0 by its expression in (222) so:

$$F_0 = -G.\frac{4}{3}.\rho_0.\pi.m.R \qquad (224)$$

The corpuscle interact with every point of space-time as like an harmonic oscillator.

The density of vacuum referring to equation (5) & (6) is:

$$\rho_0 = \frac{M}{L^3} = \frac{a^2}{\hbar.c} \qquad (225)$$

So:

$$F_0 = -\frac{4}{3}.\pi.G.\frac{a^2}{\hbar.c}.m.R \qquad (226)$$

For fine structure the electromagnetic force is the most important.

Let's have two charges e in interaction .From equation (205) we have:

$$F = \frac{\sigma.e^2}{R^2} \qquad (227)$$

To compare the generalized momentums of classic non charged corpuscle and a classic charged corpuscle , they differ by terms $a.\boldsymbol{X}$ and $\gamma.\frac{e}{c}.\boldsymbol{A}$. Let's choose the coefficient γ as a two coefficients $\gamma\varepsilon$ in order to get:

$$a = \gamma.\frac{e}{c} \qquad (228)$$

$$\boldsymbol{X} = \varepsilon.\boldsymbol{A} \qquad (229)$$

If we replace (228) in (226) we get for fine structure:

$$F = -\frac{4}{3}.\pi.G.\frac{\gamma^2.e^2}{\hbar.c^3}.m.R \qquad (230)$$

The same conclusion, in fine structure charges interact with others like harmonic oscillator.

In equation (228) , a & c are universal constants , γ is a conversion factor so we can deduce that there is an universal constant e_0 which has a dimension of electric charge. This constant is called *Maxwell constant* .

In fine structure the electrical force is so great compared to the gravitational force. For two charged corpuscles with the same mass and the same charge in absolute value we have:

$$\frac{\sigma.e^2}{R^2} \gg \frac{G.m^2}{R^2} \qquad (231)$$

So we get:

$$m \ll e.\sqrt{\frac{\sigma}{G}} \qquad (234)$$

The gravitational interaction is negligible if the mass of the corpuscles are under the following constant :

$$M_0 = e_0.\sqrt{\frac{\sigma}{G}} = \frac{a.c}{\gamma}.\sqrt{\frac{\sigma}{G}} \qquad (235)$$

The constant M_0 is called *Maxwell mass.*

The radius of interaction for microscopic sizes is as:

$$M_0 = \frac{4}{3}.\pi.R_0^3.\rho_0 \qquad (236)$$

So:

$$R_0 = \left(\frac{3}{4.\pi}.\frac{\hbar.c}{a^2}.M_0\right)^{\frac{1}{3}} \qquad (237)$$

The constant R_0 is called *Maxwell radius.*

We define also the *Maxwell force:*

$$f_0 = a.c \qquad (238)$$

Also the *Maxwell pressure:*

$$p_0 = \frac{f_0}{4.\pi.R_0^2} \qquad (239)$$

Maxwell period is as per definition:

$$T_0 = \frac{R_0}{c} \qquad (240)$$

With M_0, T_0, R_0 we can define a new system of unities which match well with the microscopic scale. For great scale we take the system of unities as defined by equations (5), (6) &(7).

For microscopic scale the motion of the corpuscle is like nearly the center of the Universe. The vacuum around the corpuscle create an attractive/repulsive force because of the dissymmetry of the position of the two half of the Universe referring to the position of the corpuscle: one half is always more near to the corpuscle than the other half referring to the center: this force maintain the orbital speed of the corpuscle as constant in module after a certain distance of the corpuscle from the center :we say that it is due to *Dark Matter*.

For great scale the motion of the corpuscle is like nearly the border of the Universe . The vacuum in the sphere tangent to the motion of the corpuscle create a repulsive/attractive force in the direction of the center. The corpuscle becomes accelerated in the direction of the position of the corpuscle- center of the Universe: we say that it is due *Dark Energy*.

This phenomenon in the Universe is called *scale invariance gauge*.[8]

Of course equation of continuity is also valid for charges: instead we speak about flux of masses, we say flux of charges…etc.

Negative charge correspond to negative pressure of vacuum in microscopic scale and vice versa.

2-2-2-4-7)Lorentz gauge:

The *second Maxwell equation with sources* is as the following[9]:

$$rot\boldsymbol{B} = \frac{4.\pi.\sigma}{c^2}.\boldsymbol{j} + \frac{1}{c^2}.\frac{\partial \boldsymbol{E}}{\partial t} \quad (241)$$

It is possible to get the scalar potential φ and the vector potential $\boldsymbol{A}$ not coupled.

From equation (170) &(171)we have:

$$div\boldsymbol{E} = \nabla.\boldsymbol{E} = -\gamma.\nabla^2\varphi - \gamma.\frac{e}{c}.\frac{\partial \nabla.\boldsymbol{A}}{\partial t} = 4.\pi.\sigma.\rho \quad (242)$$

$$\nabla \times \boldsymbol{B} = \frac{\gamma}{c}\nabla \times (\nabla \times \boldsymbol{A}) = \frac{\gamma}{c}\nabla(\nabla.\boldsymbol{A}) - \frac{\gamma}{c}\nabla^2\boldsymbol{A} = \frac{4.\pi.\sigma}{c^2}.\boldsymbol{j} + \frac{1}{c^2}.\frac{\partial \boldsymbol{E}}{\partial t} \quad (243)$$

With the condition (*Lorentz gauge*):

$$\frac{\gamma}{c^2}.\frac{\partial}{\partial t}\varphi + \gamma.\frac{e}{c}.\nabla.\boldsymbol{A} = 0 \quad (244)$$

We get:

$$(242) \rightarrow \frac{\gamma}{c^2}.\frac{\partial^2\varphi}{\partial t^2} - \gamma.\nabla^2\varphi - \frac{\partial}{\partial t}\left(\frac{\gamma}{c^2}.\frac{\partial\varphi}{\partial t} + \gamma.\frac{e}{c}\nabla\boldsymbol{A}\right) = 4.\pi.\sigma.\rho \quad (245)$$

$$(243) \rightarrow \frac{\gamma e}{c}.\nabla^2\boldsymbol{A} = -\frac{4.\pi.\sigma e}{c^2}.\boldsymbol{j} - \frac{e}{c^2}.\left(-\frac{\gamma}{c}.\frac{\partial^2\boldsymbol{A}}{\partial t^2} - \gamma\nabla\frac{\partial\varphi}{\partial t}\right) + \nabla\left(\frac{\gamma}{c^2}.\frac{\partial}{\partial t}\varphi\right) =$$
$$-\frac{4.\pi.\sigma e}{c^2}.\boldsymbol{j} + \frac{\boldsymbol{\gamma.e}}{\boldsymbol{c^3}}.\frac{\partial^2\boldsymbol{A}}{\partial t^2} + \nabla[\left(\frac{\gamma}{c^2} + \frac{\gamma.e}{c^2}\right)\frac{\partial\varphi}{\partial t}] \quad (246)$$

Let’s add another condition:

$$\frac{\partial\varphi}{\partial t} = 0 \quad (247)$$

(246) becomes:

$$\gamma.\nabla^2\boldsymbol{A} = -\frac{4.\pi.\sigma}{c}.\boldsymbol{j} + \frac{\gamma}{c^2}.\frac{\partial^2\boldsymbol{A}}{\partial t^2} \quad (248)$$

Finally we have:

$$(245) \rightarrow \frac{1}{c^2}.\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi = -\nabla^2\varphi = \frac{4.\pi.\sigma.\rho}{\gamma} \quad (249)$$

$$(246) \rightarrow \frac{\boldsymbol{1}}{\boldsymbol{c^2}}.\frac{\partial^2\boldsymbol{A}}{\partial t^2} - \nabla^2\boldsymbol{A} = \frac{\boldsymbol{1}}{\boldsymbol{c^2}}.\frac{\partial^2\boldsymbol{A}}{\partial t^2} = \frac{4.\pi.\sigma}{c.\gamma}.\boldsymbol{j} \quad (250)$$

If vacuum, equations (249) and (250) had solutions in microscopic scale & also in great scale.

2-2-2-4-6)Equation of motion of a charged corpuscle:

Equations (197) and (198) are the *two first Maxwell equations without sources.* Those equations don't characterize completely the electromagnetic field because if we want to determine for example $\frac{\partial \boldsymbol{B}}{\partial t}$ we haven't another equation for $\frac{\partial \boldsymbol{E}}{\partial t}$.

Equation (198) can be written as referring to Gauss theorem:

$$\int div\boldsymbol{B}\, dV = \oint \boldsymbol{B}.d\boldsymbol{f} = 0 = \Phi \qquad (251)$$

The integral of the second term in (251) is for all the surface bordering the volume on which done the integral of the first term.

The integral of a vector taken on a surface is called *flux of this vector* (Φ) through the surface. So the flux of an electromagnetic field through a closed surface is equal to zero.

Equation (197) can be written as referring to Stokes theorem:

$$\int rot\boldsymbol{E}.d\boldsymbol{f} = \oint \boldsymbol{E}.d\boldsymbol{l} = -\frac{\partial}{\partial t}\int \boldsymbol{B}.d\boldsymbol{f} = -\frac{\partial \Phi}{\partial t} = f.e.m \qquad (252)$$

The integral of a vector along a closed contour is called *circulation* of this vector along this contour. The circulation of electric field is called *force electromotive (f.e.m)* in the considered contour.

Let's write the equation of motion of a charged corpuscle in motion in en electromagnetic field using quadric-coordinates.

From equation (152) we have:

$$\delta S = \delta \int \left(-m.c.ds - \gamma.\frac{e}{c}.A_i dx^i\right) = 0 \qquad (253)$$

Or $ds = \sqrt{dx_i dx^i}$ so we get:

$$\delta S = -\int (m.c.\frac{dx_i d\delta x^i}{ds} + \gamma.\frac{e}{c}.A_i d\delta x^i + \gamma.\frac{e}{c}.\delta A_i dx^i)$$

$$= -\int \left(m.c.u_i d\delta x^i + \gamma.\frac{e}{c}.A_i d\delta x^i + \gamma.\frac{e}{c}.\delta A_i dx^i\right) = 0 \quad (254)$$

With:

$u_i = \frac{dx_i}{ds}$: the quadric-vector speed.

Integrate by party the first two terms in (254):

$$\int(m.c.u_i d\delta x^i + \gamma.\frac{e}{c}.A_i d\delta x^i) = \int(m.c.u_i + \gamma.\frac{e}{c}.A_i)d\delta x^i - \int(m.c.du_i + \gamma.\frac{e}{c}.dA_i)\delta x^i = 0 - \int(m.c.du_i + \gamma.\frac{e}{c}.dA_i)\delta x^i \quad (255)$$

Replace (255) in (254) we get:

$$\delta S = -\int \{(m.c.du_i + \gamma.\frac{e}{c}.dA_i)\delta x^i - \gamma.\frac{e}{c}.\delta A_i dx^i\}$$

$$= -\int\{(m.c.du_i + \gamma.\frac{e}{c}.\frac{\partial A_i}{\partial x^k}dx^k)\delta x^i - \gamma.\frac{e}{c}.\frac{\partial A_i}{\partial x^k}\delta x^k dx^i\} \quad (256)$$

Equation (256) because we have:

$dA_i = \frac{\partial A_i}{\partial x^k}dx^k$, $\delta A_i = \frac{\partial A_i}{\partial x^k}\delta x^k$

Replace in (256) $du_i = \frac{du_i}{ds}.ds$ and $dx^i = u^i ds$ and permit indices i & k in the third term (which doesn't change the result):

$$\delta S = -\int\{(m.c.\frac{du_i}{ds}ds\delta x^i + \gamma.\frac{e}{c}.\frac{\partial A_i}{\partial x^k}u^k ds\delta x^i - \gamma.\frac{e}{c}.\frac{\partial A_k}{\partial x^i}u^k ds\delta x^i = -\int\{m.c.\frac{du_i}{ds} - \gamma.\frac{e}{c}.(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k})u^k\}ds\delta x^i = 0 \quad (257)$$

The variations δx^i are arbitrary so we should have:

$$m.c.\frac{du_i}{ds} - \gamma.\frac{e}{c}.\left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}\right)u^k = 0 \quad (258)$$

Equation (248) is the equation of motion of the charge written in quadric coordinates.

Let's introduce the notation:

$$F_{ik} = \left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}\right) \qquad (259)$$

This anti-symmetric tensor is called *tensor of electromagnetic field.* The equation of motion of the corpuscle becomes as:

$$m.c.\frac{du_i}{ds} = \gamma.\frac{e}{c}.F^{ik}u_k \qquad (260)$$

With the notation $A_i = (\varphi, -\boldsymbol{A})$ we have from (259)&(170) &(171):

$$i = 0, k = 0 \rightarrow F_{00} = \frac{\partial A_0}{\partial x^0} - \frac{\partial A_0}{\partial x^0} = 0$$

$$i = 0, k = 1 \rightarrow F_{01} = \frac{\partial A_1}{\partial x^0} - \frac{\partial A_0}{\partial x^1} = -\frac{\partial A_x}{\partial (c.t)} - \frac{\partial \varphi}{\partial x} = \frac{1}{\gamma}.E_x$$

$$i = 0, k = 2 \rightarrow F_{02} = \frac{1}{\gamma}.E_y$$

$$i = 0, k = 3 \rightarrow F_{03} = \frac{1}{\gamma}.E_z$$

$$i = 1, k = 0 \rightarrow F_{10} = \frac{\partial A_0}{\partial x^1} - \frac{\partial A_1}{\partial x^0} = \frac{\partial \varphi}{\partial x} + \frac{\partial A_x}{\partial (c.t)} = \frac{-1}{\gamma}.E_x$$

$$i = 1, k = 1 \rightarrow F_{11} = 0$$

$$i = 1, k = 2 \rightarrow F_{12} = \frac{\partial A_2}{\partial x^1} - \frac{\partial A_1}{\partial x^2} = -\frac{\partial A_y}{\partial x} + \frac{\partial A_x}{\partial y} = -\frac{c}{\gamma}B_z$$

$$i = 1, k = 3 \rightarrow F_{13} = \frac{\partial A_3}{\partial x^1} - \frac{\partial A_1}{\partial x^3} = -\frac{\partial A_z}{\partial x} + \frac{\partial A_x}{\partial z} = \frac{c}{\gamma}B_y$$

$$i = 2, k = 3 \rightarrow F_{23} = \frac{\partial A_3}{\partial x^2} - \frac{\partial A_2}{\partial x^3} = -\frac{\partial A_z}{\partial y} + \frac{\partial A_y}{\partial z} = -\frac{c}{\gamma}B_x$$

……(etc)

So:

$$F_{ik} = \frac{1}{\gamma}.\begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -c.B_z & c.B_y \\ -E_y & c.B_z & 0 & -c.B_x \\ -E_z & -c.B_y & c.B_x & 0 \end{pmatrix} \qquad (261)$$

$$F^{ik} = \frac{1}{\gamma}.\begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -c.B_z & c.B_y \\ E_y & c.B_z & 0 & -c.B_x \\ E_z & -c.B_y & c.B_x & 0 \end{pmatrix} \qquad (262)$$

For space components $(i = 1,2,3)$ the equation (260) is exactly the vector equation of motion (172).

For time component $(i = 0)$ the equation (260) is exactly the work equation (173) which becomes from the locally equation of motion.

We can verify that only three equations from the four equations (260) are independents: we can multiply the two members of equation (260) by u^i and as we know that u^i and $\frac{du_i}{ds}$ are orthogonal and the second member is :

$$F^{ik}u_k u^i = F_{ik}u^k u^i = 0 \qquad (263)$$

From equation (255) we have:

$$\delta S = -(mcu_i + \gamma \frac{e}{c}.A_i)\delta x^i \qquad (264)$$

So we have:

$$-\frac{\partial S}{\partial x^i} = m.c.u_i + \gamma \frac{e}{c}.A_i = p_i + \gamma \frac{e}{c} A_i = P_i \qquad (265)$$

So:

$$P^i = \left(\frac{\frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} + \gamma.e.\varphi}{c}, \boldsymbol{p} + \gamma\frac{e}{c}.\boldsymbol{A} \right) \quad (266)$$

2-2-2-4-7)Lorentz transformations of the field:

2-2-2-4-7-1)Tensor algebra:

a)four-vector position of a universe point :

Let's have an inertial reference $R(O, ct, x, y, z)$ .A universe point $\boldsymbol{X}$ have the coordinates in this spae-time (*Minkovski vectorial space*) as the following [10]:

$$\boldsymbol{X} = \sum_i x^i.e_i \quad \text{(a-1)}$$

Where:

$\boldsymbol{e}_i$: base of the Minkovski vectorial space M.

$x^0 = c.t,\ x^1 = x\ ,x^2 = y\ ,x^3 = z$:*the contra-variant coordinates of the Universe point.*

$i = 0,1,2,3$:variable indices.

With *Einstein convention for repetitive indices* we write (a-1) as the following:

$$\boldsymbol{X} = x^i.e_i \quad \text{(a-2)}$$

We do implicitly summation if only the same indices is viewed *one time in the top and another time in the down.* For example T_{ii} represent a diagonal element of a tensor (a matrices) and not a summation. The trace of the matrices is T_i^i so the convention summation is applied for repetitive indices.

We call *free indices* an indices on which the summation rule is not applied and so it remains as it is in the final expression and we call *mute indices* an indices which is the subject of an implicitly summation and don't appear as it is in the final expression. For free indices we respect the *rule of "balance"*. In an equation the free indices which appears in the two members should corresponds one to one and appears in the same position (up or down).

We can associate for our space-time a scalar product , which is of course commutative. Let's have two four-vectors $\boldsymbol{X} = x^i.\boldsymbol{e_i}$ & $\boldsymbol{Y} = y^j.\boldsymbol{e_j}$, the scalar product is as follows:

$$\boldsymbol{X}.\boldsymbol{Y} = x^i y^j \boldsymbol{e_i}.\boldsymbol{e_j} \qquad \text{(a-3)}$$

We pose a table of numbers of two indices as:

$$g_{ij} = \boldsymbol{e_i}.\boldsymbol{e_j} \qquad \text{(a-4)}$$

The scalar product is as:

$$\boldsymbol{X}.\boldsymbol{Y} = g_{ij} x^i y^j \qquad \text{(a-5)}$$

We hope of course that the scalar product have an expression which compatible with the notion of interval in four dimension space-time. For this we should have $\boldsymbol{X}.\boldsymbol{X} = c^2t^2 - x^2 - y^2 - z^2$. We get a convenient scalar product is the g_{ij} which we call *metric tensor* is as follows:

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \text{(a-6)}$$

In this table i is the line indices and j is the column indices.

b)Covariant coordinates:

We pose:

$$y_i = g_{ij} y^j \qquad \text{(b-1)}$$

The repetitive indices (up and down) in the right member of (b-1) is j .We should made summation on all values of this indices. The indices i is a free indices which appears with the same name with the same position in the two members of the equation. We call *covariant coordinates* of the Universe point the components y_i.

With this notation we have $y_0 = y^0$ & $y_i = -y^i \ for \ i = 1,2,3$. The metric tensor permit to download or to upload the indices as an escalator with a general rule : the downloading or the uploading of a space indices changes its sign and the downloading and uploading of a time indices doesn't change the sign.

With those notations the scalar product of two four-vectors is as:

$$\boldsymbol{X}.\boldsymbol{Y} = x^i y_i \qquad \text{(b-2)}$$

And also:

$$\boldsymbol{X}.\boldsymbol{Y} = x_i y^i \quad with\; x_i = g_{ij}x^j \qquad \text{(b-3)}$$

And :

$$\boldsymbol{X}.\boldsymbol{e_i} = x^j \boldsymbol{e_j}.\boldsymbol{e_i} = g_{ji}x^j = x_i \qquad \text{(b-4)}$$

We can also write the inverse transformation which gives us the contra-covariant coordinates as a function of covariant coordinates by defining a new *table of numbers* g^{ij} as :

$$y^i = g^{ij}y_j \qquad \text{(b-5)}$$

We can write:

$$y^i = g^{ij}y_j = g^{ij}g_{jk}y^k = \delta^i_k y^k \qquad \text{(b-6)}$$

With:

$$g^{ij}g_{jk} = \delta^i_k \qquad \text{(b-7)}$$

With :

$\delta^i_k = 0\; if\; i \neq j\; and\; 1\; if\; i = j$:Kronecker symbols.

As matrices the g^{ij} is the inverse of the matrices g_{ij}. We have:

$$g^{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \text{(b-8)}$$

c)Duality:

For a space vector M we can define a *linear forms.* A linear form associate for every vector a real number (or complex). We note $\tilde{R}$ a linear form and $\tilde{R}(\boldsymbol{X})$ the real number associated to the vector $\boldsymbol{X}$. A linear form is a linear function of its vector argument. So we have such relations as $\tilde{R}(\boldsymbol{X}+\boldsymbol{Y})=\tilde{R}(\boldsymbol{X})+\tilde{R}(\boldsymbol{Y})$ etc…

We can define on the ensemble of linear forms an addition and a multiplication with a real scalar. This two operations confer to the ensemble of linear forms a structure of space vector: it is called the *dual* of our initial space vector M and noted as M^*.

Also if M have a finite dimension, its dual have the same dimension. Also if it is defined a scalar product in the space vector M we can define a bijection between the space and its dual. We associate to every vector $\boldsymbol{Y}$ a linear form $\tilde{Y}$ defined as $\tilde{Y}(\boldsymbol{X})=\boldsymbol{Y}.\boldsymbol{X}$ .A Universe point in four dimensions space-time can be considered as a vector or a linear form. In fact the two representations are the same one subject.

In the dual space we choose the base:

$$\tilde{\epsilon}^i(\boldsymbol{e}_j)=\delta_j^i \qquad \text{(c-1)}$$

We have :

$$\boldsymbol{e}_i.\boldsymbol{Y}=\boldsymbol{e}_i.e_j y^j=g_{ij}y^j=y_i \qquad \text{(c-2)}$$

$$\tilde{\epsilon}^i(\boldsymbol{Y})=\tilde{\epsilon}^i(y^j e_j)=y^j\tilde{\epsilon}^i(\boldsymbol{e_j})=y^j\delta_j^i=y^i\neq y_i \;\; for\; i=1,2,3. \qquad \text{(c-3)}$$

So we have in general:

$$\boldsymbol{e}_i.\boldsymbol{Y}=y_i \qquad \& \qquad \tilde{\epsilon}^i(\boldsymbol{Y})=y^i \qquad \text{(c-4)}$$

So we can form from a four-vector $\boldsymbol{Y}=y^j e_j$ a linear form $y_j\tilde{\epsilon}^j$. The action of this linear form on a vector $\boldsymbol{X}=x^i e_i$ is as $y_j\tilde{\epsilon}^j(x^i e_i)=y_j x^j=\boldsymbol{Y}.\boldsymbol{X}$. The form constructed coincide with the linear form $\tilde{Y}$ associated to the vector $\boldsymbol{Y}$. If the components contra-variants are the components of the four-vector , the covariant components are the components of the linear form associated to this vector on the dual base. As we can confound vector and linear form in one physical subject , the writing of contra-variants components and covariant components are different writing of the same quantity.

d)Change of referential, change of base:

We can write the contra-variant coordinates by Lorentz transformations as:

$$x'^i = \mathcal{L}^i{}_j x^j \qquad \text{(d-1)}$$

Where x^j are the contra-variant components in the referential R and x'^i are the contra-variant components of the Universe point in the referential R'. We associated the line indices i for the new referential and the column indices j for the old referential. In the Lorentz transformations the table $\mathcal{L}^i{}_j$ is the following matrices :

$$\mathcal{L}^i{}_j = \begin{pmatrix} \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} & \frac{-\frac{V}{c}}{\sqrt{1-\frac{V^2}{c^2}}} & 0 & 0 \\ \frac{-\frac{V}{c}}{\sqrt{1-\frac{V^2}{c^2}}} & \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \text{(d-2)}$$

The inverse transformation is as follows:

$$x^i = (\mathcal{L}^{-1})^i{}_j x'^j \qquad \text{(d-3)}$$

The inverse matrices $(\mathcal{L}^{-1})^i{}_j$ is obtained by change in the matrices $\mathcal{L}^i_j$, $V\ by - V$.

The transformations of covariant coordinates in the dual space are defined as:

$$x'_i = \mathcal{L}_i{}^j x_j \qquad \text{(d-4)}$$

Where $\mathcal{L}_i{}^j$ is a table of numbers. Note that $\mathcal{L}_i{}^j$ is different from $\mathcal{L}^j{}_i$.We can deduce the link between $\mathcal{L}^i_j$ *and* $\mathcal{L}^j_i$ by the invariance of the scalar product which is a consequence of the invariance of the interval. We have in this case:

$$x'^i y'_i = x^i y_i \quad with \quad x'^i = \mathcal{L}^i{}_j x^j \quad and \quad y'_i = \mathcal{L}_i{}^k y_k \qquad \text{(d-5)}$$

So:

$$\mathcal{L}^i{}_j x^j \mathcal{L}_i{}^k y_k = x^i y_i = x^j y_j = x^j y_k \delta^k_j \qquad \text{(d-6)}$$

The relation (d-6) should be verified for every couple of vectors so:

$$\mathcal{L}^i{}_j \mathcal{L}_i{}^k = \delta^k_j \qquad \text{(d-7)}$$

Let’s note that the left term in (d-7) is not a product of two matrices. It is a summation on two lines indices. In Lorentz transformations the matrices are symmetric and the matrices of transformations of covariant coordinates is the inverse matrices of transformations of contra-variant coordinates and we get this matrices by changing the speed V in contra-variant matrices by $-V$.

The link between the two transformations is:

$$\boldsymbol{X}.\boldsymbol{Y} = x^i g_{ij} y^j = x'^k g_{kl} y'^l \qquad \text{(d-8)}$$

The metric tensor which expressing an orthogonal base is the same in all bases and so it is an invariant by Lorentz transformation. From (d-8)and (d-5) we get:

$$x^i g_{ij} y^j = \mathcal{L}^k{}_j x^j g_{kl} \mathcal{L}^l{}_m y^m \qquad \text{(d-9)}$$

This relation (d-9) is always verified , so we deduce that:

$$g_{jm} = \mathcal{L}^k{}_j g_{kl} \mathcal{L}^l{}_m \qquad \text{(d-10)}$$

Multiply (d-10) by g^{nj} we get:

$$g^{nj} g_{jm} = \delta^n_m = g^{nj} \mathcal{L}^k{}_j g_{kl} \mathcal{L}^l{}_m \qquad \text{(d-11)}$$

So:

$$\left(g^{nj} g_{kl} \mathcal{L}^k{}_j\right) \mathcal{L}^l{}_m = \delta^n_m \qquad \text{(d-12)}$$

(d-7)→ $\left(g^{nj} g_{kl} \mathcal{L}^k{}_j\right) \mathcal{L}^l{}_m = \mathcal{L}^i_m \mathcal{L}^n_i = \mathcal{L}^l_m \mathcal{L}^n_l$ (d-13)

So:

$$\mathcal{L}_l{}^n = g^{nj} g_{kl} \mathcal{L}^k{}_j \qquad \text{(d-14)}$$

The inverse relation of (d-14) is:

$$\mathcal{L}^n{}_l = g_{il} g^{nj} \mathcal{L}_j{}^i \qquad \text{(d-15)}$$

Let’s remark that for the coordinates the change of a space coordinate indices change the sign and the change of the time coordinate indices doesn’t change the sign . In the passage from a transformation to another only changes in the sign the coefficients indexed space & time. The coefficients only space or only time are unchanged. This is what we observe in Lorentz transformation.

We can choose that acting on Lorentz transformations is only by the metric tensor and define new quantities as the following:

$$\mathcal{L}^{ij} = g^{jk}\mathcal{L}^{i}{}_{k} \qquad \text{(d-16)}$$

$$\mathcal{L}_{ij} = g_{ik}\mathcal{L}^{k}{}_{j} \qquad \text{(d-17)}$$

We have with those tensors:

$$x'^{i} = \mathcal{L}^{i}{}_{j}\, x^{j} = \mathcal{L}^{i}{}_{j}\, g^{jk}x_{k} = \mathcal{L}^{ik}x_{k} \qquad \text{(d-18)}$$

Where the line indices is assigned to the new referential and the column indices is assigned to the old referential.

Also we have:

$$x'_{i} = \mathcal{L}_{ik}x^{k} \qquad \text{(d-19)}$$

For the inverse change of the referential we can use the relation (d-7) without using the transformation $\mathcal{L}^{-1}$.We have:

$$\mathcal{L}^{i}{}_{m}\, x'_{i} = \mathcal{L}^{i}{}_{m}\, \mathcal{L}_{i}{}^{k}x_{k} = \delta^{k}_{m}x_{k} = x_{m} \qquad \text{(d-20)}$$

So:

$$x_{i} = \mathcal{L}^{k}{}_{i}x'_{k} \qquad \text{(d-21)}$$

This transformation is of course different from the direct transformation:

$$x'_{i} = \mathcal{L}_{i}{}^{k}x_{k} \qquad \text{(d-22)}$$

The indices relative to the new referential i and the other relative to the old referential k change the position (up/down) between the two expressions. In terms of matrices in case of Lorentz transformation it correspond to a change of sign in the components space & time and so taking the inverse of the matrices.

We can do the same for the contra-variant components of coordinates or any combination of mixture components as the following:

$$x'_i = \mathcal{L}_i{}^j x_j \qquad \text{(d-23)}$$

$$x'^i = \mathcal{L}^i{}_j x^j \qquad \text{(d-24)}$$

$$x_i = \mathcal{L}^j{}_i x'_j \qquad \text{(d-25)}$$

$$x^i = \mathcal{L}_j{}^i x'^j \qquad \text{(d-26)}$$

Those four combinations are obtained by respecting the balance rule by assigning the first indices to the new referential, assigning the second indices to the old referential and summing on the indices of the coordinate which to be transformed.

We finish this paragraph by examining the transformations of the base vectors of our space-time. We remark that:

$$y_i = \boldsymbol{Y}.e_i = \tilde{Y}(e_i) \ \& \ y'_i = \boldsymbol{Y}.e'_i = \tilde{Y}(e'_i) \qquad \text{(d-27)}$$

Where the e'_i are the transformations of the base vectors. We can write:

$$\boldsymbol{Y}.e'_i = \mathcal{L}_i{}^j \boldsymbol{Y}.e_j \qquad \text{(d-28)}$$

In other terms the law of vector transformations is the same law of the transformations of covariant coordinates which the inverse of the one of contra-variant components transformations.

For the dual base we have:

$$y'^i = \widetilde{e'^i}(\boldsymbol{Y}) \ \& \ y^i = \tilde{e}^i(\boldsymbol{Y}) \qquad \text{(d-29)}$$

So we deduce:

$$\widetilde{e'^i} = \mathcal{L}^i{}_j \tilde{e}^j \qquad \text{(d-30)}$$

The vectors of the dual base are transformed like the contra-variant components **.**

e)Tensors:

e-a)Contra-variant tensors :

The operation of tensors product permit to associate to a vector space M a space $M \otimes M$ more great. For every couple of vectors $\boldsymbol{X}$ & $\boldsymbol{Y}$ of M we associate a vector $\boldsymbol{X} \otimes \boldsymbol{Y}$ of $M \otimes M$.A base of $M \otimes M$ is formed of 16 tensor products obtained with the four-vectors base of M , $e_i \otimes e_j$. The components of $\boldsymbol{X} \otimes \boldsymbol{Y}$ in this base are the components of $\boldsymbol{X}$ & $\boldsymbol{Y}$ as the following:

$$\boldsymbol{X} \otimes \boldsymbol{Y} = x^i y^j e_i \otimes e_j \qquad \text{(e-a-1)}$$

The dimension of tensor product space is 16. We define a tensor of order 2 completely contra-variant T^{ij} which components are defined on the base $e_i \otimes e_j$. In a base change with Lorentz transformation the new components of the tensor are as:

$$T'^{ij} = \mathcal{L}^i{}_k \mathcal{L}^j{}_m T^{km} \qquad \text{(e-a-2)}$$

The inverse transformation is as:

$$T^{ij} = \mathcal{L}_k{}^i \mathcal{L}_m^{\ j} T'^{km} \qquad \text{(e-a-3)}$$

We can also consider that the tensor T^{ik} is the image of its dual M^* in M. The image $\boldsymbol{W}$ of a vector $\boldsymbol{V}$ is :

$$W^i = T^{ij} V_j \qquad \text{(e-a-4)}$$

Its transformation as a four vector is:

$$W'^i = \mathcal{L}^i{}_k W^k = \mathcal{L}^i{}_k T^{kj} V_k \qquad \text{(e-a-5)}$$

But:

$$V_k = \mathcal{L}^m{}_k V'_m \qquad \text{(e-a-6)}$$

So:

$$W'^i = \mathcal{L}^i{}_k \mathcal{L}^j{}_m T^{km} V'_j = T'^{ij} V'_j \qquad \text{(e-a-7)}$$

The operation of tensor product can be generalised for any number of terms. We can define the space $M^{\otimes k}$ tensor product of M , k manner its self. The elements of this space have a dimension 4^k are the tensors completely contra-variant of order k and their elements are written as $T^{ijkl....p}$.Those components are transformed as k Lorentz transformations.

e-b)Covariant tensors , Mix tensors :

Which was done for the space M can be done for its dual M^* .We can define tensors of an order 2 completely covariant where the components are written on the dual base product tensor $\tilde{e}^i \otimes \tilde{e}^j$ as T_{ij} . The Lorentz transformation of those quantities are:

$$T'_{ij} = \mathcal{L}_i{}^k \mathcal{L}_j{}^l T_{kl} \qquad \text{(e-b-1)}$$

We can do the tensor product of any number of dual space. We can also define subjects as the tensor product of the space M with its dual M^*.We obtain mix tensors of an order 2 (or more if we use many times M & $M*$) which the components are written as $T^i{}_j$ for $M \otimes M^*$ and $T_i{}^j$ for $M^* \otimes M$. The transformation rule of such mix tensor is :

$$T'^i{}_j = \mathcal{L}^i{}_l \mathcal{L}_j{}^k T^l{}_k \qquad \text{(e-b-2)}$$

And it can be generalised for every mix tensor of any order.

Covariant components and contra-variants components describe the same physical subject. The same thing is for tensors: a physical quantity represented as a tensor can be also written as a tensor completely contra-variant, completely covariant, or mix in arbitrary manner. Like for four-vectors the metric tensor g^{ij} or g_{ij} can be used to upload or download indices, so we can write:

$$T^{ij} = g^{ik} g^{jl} T_{kl} \qquad \text{(e-b-3)}$$

$$T^i{}_j = g^{ik} g_{jl} T_k{}^l \qquad \text{(e-b-4)}$$

$$T^i{}_j = g_{jl} T^{il} \qquad \text{(e-b-5)}$$

In terms of linear applications, all those forms are different manners to write the image $\boldsymbol{Y}$ of a four-vector :

$$Y^i = T^{ik} X_k = T^i{}_k X^k \qquad \text{(e-b-6)}$$

And:

$Y_i = T_{ik}X^k = T_i{}^k X_k$ (e-b-6)

e-c)Terminology:

A tensor of an order 2 is symmetric if:

$T^{ij} = T^{ji}$ (e-c-1)

We deduce immediately $T_{ij} = T_{ji}$ & $T^i{}_j = T_j{}^i$. So for a symmetric mix tensor we can write it as T_j^i without order of indices. Note that in this case it doesn't implies that T_j^i is the same T_i^j.

A tensor of an order2 is anti-symmetric if:

$T^{ij} = -T^{ji}$ (e-c-2)

A symmetric tensor can be written as:

$$T^{ij} = \begin{pmatrix} 0 & a_x & a_y & a_z \\ -a_x & 0 & -b_z & b_y \\ -a_y & b_z & 0 & -b_x \\ -a_z & -b_y & b_x & 0 \end{pmatrix} = (\boldsymbol{a}, \boldsymbol{b}) \qquad \text{(e-c-3)}$$

Where $\boldsymbol{a}$ is a vector and $\boldsymbol{b}$ is a pseudo-vector (which is transformed to its symmetric opposite in a base change included a space reflexion). The couple electric field/magnetic field obey to those conditions.

We call *trace* of a tensor of an order 2 the quantity $T^i{}_i = T_i{}^i$.

We call *contraction* of a tensor the expression like $T^i{}_i{}^j$. The contraction of a tensor order k is a tensor order $k-2$. The contraction of a tensor order 3 for example gives a tensor order 1 i.e. a four-vector. The trace is a contraction of a tensor order 2 and it gives a tensor order 0 i.e. a four-scalar.

Example for contraction of a tensor order3:

$$T'^i{}_i{}^j = \mathcal{L}^i{}_l \mathcal{L}_i{}^m \mathcal{L}^j{}_n T^l{}_m{}^n = \delta_l^m \mathcal{L}^j{}_n T^l{}_m{}^n = \mathcal{L}^j{}_n T^l{}_l{}^n \qquad \text{(e-c-4)}$$

So it is a four-vector.

As a tensor we have the metric tensor which is invariant by Lorentz transformation. It is a symmetric tensor. Its mix form $g^i_j = g^{ik} g_{kj} = \delta^i_j$. The Kronecker symbol is the mix form of the metric tensor. The relation between the contra-variant form and the covariant form $g^{ij} g_{jk} = \delta^i_k$ is only a simple downloading of indices.

Finally we define a tensor order 4 completely anti-symmetric (Levi-Civita tensor) ϵ^{ijkl}.By the 256 elements of this tensor only are not equal to zero whose indices correspond to one permutation of (0,1,2,3). If the permutation is pair the correspondent element is equal to +1. It is equal to -1 if the permutation is impair. So there is 24 elements of the tensor not equal to zero, 12 equal to +1 and 12 equal to -1. We have $\epsilon^{ijkl} = -\epsilon_{ijkl}$. Finally we have: $\epsilon^{ijkl}\epsilon_{ijkl} = -24$.

f)Derivation & vector analysis:

f-a)Derivation:

We can define for a four-vector which is a Universe point , the derivation by the contra-variant coordinate as:

$$\partial_i = \frac{\partial}{\partial x^i} \qquad \text{(f-a-1)}$$

For a scalar function its variation is :

$$df = \partial_i f(x^i).dx^i = \frac{\partial f}{\partial x^i}.dx^i \qquad \text{(f-a-2)}$$

df is a scalar, and dx^i is a contra-variant vector, ∂_i is a covariant *vector.* It is transformed as it is in a Lorentz transformation:

$$\partial'_i = \mathcal{L}_i^{\ j} \partial_i \qquad \text{(f-a-3)}$$

Where ∂' represents the derivatives according to the new contra-variant coordinates.

The derivative according to the covariant coordinates is:

$$\partial^i = g^{ij} \partial_j \qquad \text{(f-a-4)}$$

f-b)Vector analysis:

If f is a scalar function, $\partial_i f$ generalise the gradient and we have:

$$\partial_i f = (\frac{\partial f}{c\partial t}, \nabla f) \qquad \text{(f-b-1)}$$

And:

$$\partial^i f = (\frac{\partial f}{c\partial t}, -\nabla f) \qquad \text{(f-b-2)}$$

If we have a four-vector $A^i(x^j) = (a^0, \boldsymbol{a})$ its divergence is defined as:

$$\partial^i A_i = \partial_i A^i = \frac{\partial a^0}{c\partial t} + \nabla . \boldsymbol{a} \qquad \text{(f-b-3)}$$

The analogue of the rotational is a tensor of order 2 completely anti-symmetric :

$$\partial^i A^j - \partial^j A^i \qquad \text{(f-b-4)}$$

In its covariant form the rotational is:

$$\partial_i A_j - \partial_j A_i \qquad \text{(f-b-5)}$$

The *Laplace operator* of the space-time is the norm of the vector ∂^i :

$$\partial_i \partial^i = \frac{\partial^2}{c^2 \partial t^2} - \Delta = \Box \qquad \text{(f-b-6)}$$

Which is the dalembertian $\Box$.

f-c)Integration:

We define a volume integral of space-time for any types of quantity as :

$$\int d\Omega \qquad \text{(f-c-1)}$$

Where $d\Omega = cdtdxdydz$ the integral element in space-time.

A surface in space in three dimensions is a variety in three dimensions. We can define an integral on those surfaces (a flux) with the condition to define a four-vector element surface dS^i . A surface element is a little subject of three dimensions. It is defined by three four-vector dx^i, dy^i, dz^i . dS^i should be orthogonal to any vector of the element and its length should be a measure of the *volume* of the surface element. To define dS^i we form at first a tensor of an order 3, dS^{ijk} as:

$$dS^{ijk} = \begin{vmatrix} dx^i & dy^i & dz^i \\ dx^j & dy^j & dz^j \\ dx^k & dy^k & dz^k \end{vmatrix} \qquad \text{(f-c-2)}$$

The surface element is obtained by contracting this tensor with the tensor of order 4 completely anti-symmetric:

$$dS^i = -\frac{1}{6}\epsilon^{ijkl}\, dS_{jkl} \qquad \text{(f-c-3)}$$

We establish for surface integrals a theorem which generalise the Gauss theorem as:

$$\int_S A^i dS_i = \int_V \partial_i A^i d\Omega \qquad \text{(f-c-4)}$$

Where V is a volume in space-time and S is its surface border.

So the integral of the divergence extended to all the space is equal to the flux on the *sphere at infinite*. This is in general equal to zero for physical fields.

We can also define an integral on two dimensions varieties . The element of the integral is a tensor anti-symmetric of an order 2 madden on the vectors dx^i& dy^j delimitate the integral element:

$$df^{ij} = dx^i dy^j - dx^j dy^i \qquad \text{(f-c-5)}$$

Finally we can define a curvilinear integral on a universe line. The theorem of Stokes link the integral on a variety in two dimensions to the integral on its contour:

$$\int A_i dx^i = \int df^{ij}(\partial_i A_j - \partial_j A_i) \qquad \text{(f-c-6)}$$

2-2-2-5)Generalised equation of motion:

2-2-2-5-1) A corpuscle in a field:

Let’s have a system of many corpuscles in a free field. The total action of this system is as:

$$S = S_{free\ corpuscles} + S_{free\ \ fields} + S_{Interaction} \qquad (267)$$

We consider here charged corpuscles to facilitate writing the equation of motion. For non charged corpuscles do the conversion factors in unified field as defined in (181), (182), (183) & (184).

In a first step we consider only one charged corpuscle in interaction with a free field. In a second step we consider many charged corpuscles in interaction between each other and the free field.

The action of a free corpuscle is as:

$$S_{free\ corpuscles} = -mc \int ds \qquad (268)$$

The field can be represented by a unique potential four-vector as $A^i = (\varphi, \boldsymbol{A})$ and its action is as:

$$S_{Intercation} = -\int \gamma.\frac{e}{c} A_i dx^i = -q \int A_i dx^i \qquad (269)$$

With:

$q = \gamma.\frac{e}{c}$ is a constant which we call *charge* of the corpuscle.

2-2-2-5-2)Electromagnetic field tensor :

F_{ik} is per definition an anti-symmetric tensor of an order 2, the four-rotational of the potential $(\varphi, \boldsymbol{A})$. It depends only of six independent coordinates. The three space-time coordinates are the components of a space vector, and the three only space coordinates are the components of a pseudo-vector.

We can write the space-time components as the following:

$$F_{0i} = \partial_0 A_i - \partial_i A_0 = -\frac{\partial A_i}{c\partial t} - \frac{\partial \varphi}{\partial x^i} = \frac{E_i}{\gamma} \quad \text{for } i = 1,2,3 \qquad (270)$$

We pose :

$$\boldsymbol{E} = -\frac{\gamma}{c}.\frac{\partial \boldsymbol{A}}{\partial t} - \gamma.grad(\varphi) \qquad (271)$$

Which is called *electric field* the real space vector defined.

The space coordinates of the field tensor are:

$$F_{12} = -\frac{c.B_z}{\gamma} = -\frac{\partial A_y}{\partial x} + \frac{\partial A_x}{\partial y} \qquad (272)$$

$$F_{13} = \frac{c.B_y}{\gamma} = -\frac{\partial A_z}{\partial x} + \frac{\partial A_x}{\partial z} \qquad (273)$$

$$F_{23} = -\frac{c.B_x}{\gamma} = -\frac{\partial A_z}{\partial y} + \frac{\partial A_y}{\partial z} \qquad (274)$$

If we introduce the pseudo-vector called *magnetic field*:

$$\boldsymbol{B} = \frac{\gamma}{c}.rot\boldsymbol{A} \qquad (275)$$

The electromagnetic tensor describe well the Maxwell equations of electromagnetism.

We have:

$$F_{ik} = (\boldsymbol{E}, \boldsymbol{B}) \quad \& \quad F^{ik} = (-\boldsymbol{E}, \boldsymbol{B}) \qquad (276)$$

2-2-2-5-3) Change of referential for the field:

We have in an inertial referential:

$$F'^{ik}\left(x'^{i} = \mathcal{L}^{i}{}_{j}x^{j}\right) = \mathcal{L}^{i}{}_{l}\mathcal{L}^{k}{}_{m}F^{lm}(x^{j}) \qquad (277)$$

Where the quantities F'^{ik} are relative to the new referential $'$.

The transformations of fields are as the following:

$$E'_x = E_x \qquad (288)$$

$$E'_y = \frac{E_y - V.B_z}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (289)$$

$$E'_z = \frac{E_z + V.B_y}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (290)$$

$$B'_x = B_x \qquad (291)$$

$$B'_y = \frac{B_y + \frac{V}{c^2}.E_z}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (292)$$

$$B'_z = \frac{B_z - \frac{V}{c^2}.E_y}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (293)$$

For the inverse transformations change $V\ by - V$.

The transformations of potentials are as the following [11]:

$$A^0 = \frac{A'^0 + \frac{V}{c}.A'^1}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (294)$$

$$A^1 = \frac{A'^1 + \frac{V}{c}A'^0}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (295)$$

$$A^2 = A'^2 \qquad (296)$$

$$A^3 = A'^3 \qquad (297)$$

For covariant components of the potentials we have:

$$A_0 = \frac{A'_0 - \frac{V}{c}.A'_1}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A_1 = \frac{A'_1 - \frac{V}{c}.A'_0}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A_2 = A'_2 \quad , \quad A_3 = A'_3 \qquad (298)$$

With the four-vector $A^i = (\varphi, \boldsymbol{A})$ we have:

$$\varphi = \frac{\varphi' + \frac{V}{c}.A'_x}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A_x = \frac{A'_x + \frac{V}{c}.\varphi'}{\sqrt{1-\frac{V^2}{c^2}}} \quad , \quad A_y = A'_y \quad , \quad A_z = A'_z \qquad (298)$$

2-2-2-5-4) Invariants of the field:

There is two invariants which have physical interest. They are:

$$F_{ik}F^{ik} = inv. \qquad (299)$$

$$\epsilon^{iklm}F_{ik}F_{lm} = inv. \qquad (300)$$

This is due to the power of mathematics.

It comes that:

$$c^2B^2 - E^2 = inv. \qquad (301)$$

$$\boldsymbol{E}.\boldsymbol{B} = inv \qquad (302)$$

Another approach be described for the invariants of the field represented by anti-symmetric four-tensor .

Let's consider the complex vector:

$$\boldsymbol{F} = \boldsymbol{E} + ic\boldsymbol{B} \qquad (303)$$

The Lorentz transformation of this vector along the axle (O, x) according to (288)...(293) is as:

$$F_x = F'_x \quad , \quad F_y = F'_y ch\theta - iF'_z sh\theta = F'_y \cos(i\theta) - F'_z \sin(i\theta) \qquad (304)$$

$$F_z = F'_z \cos(i\theta) + F'_y \sin(i\theta) \qquad , \qquad th(\theta) = \frac{V}{c} \qquad (305)$$

The rotation of the vector $\boldsymbol{F}$ in the plan (O, x, t) of the four-dimensional space (it is the Lorentz transformation which we search here) is equivalent of a rotation of an imaginary angle in the plan (O, y, z) of the three dimensional space. The ensemble of all possible rotations in the four-dimensional space (included the simples rotations of axles x, y & z) is equivalent to the ensemble of all possible rotations of complexes angles in the three dimensional space (for the six rotation angles in the four-dimensional space correspond three complexes rotation angles of the three dimensional referential).

The unique invariant of the vector according to those rotations is its square $F^2 = E^2 - c^2B^2 + 2ic\boldsymbol{E}.\boldsymbol{B}$ .So the real quantities $E^2 - c^2B^2$ and $\boldsymbol{E}.\boldsymbol{B}$ are the unique invariants of the tensor F_{ik}.

2-2-2-5-5)First group of Maxwell equations:

Equation (259) signify the electromagnetic tensor is the rotational of the potential. In three dimension this propriety implies the nullity of its divergence. Let's establish this propriety in four dimensions. We have:

$$F_{ik} = \partial_i A_k - \partial_k A_i \qquad (306)$$

We deduce that:

$$\partial_j F_{ik} = \partial_j \partial_i A_k - \partial_j \partial_k A_i \qquad (307)$$

$$\partial_k F_{ji} = \partial_k \partial_j A_i - \partial_k \partial_i A_j \qquad (308)$$

$$\partial_i F_{kj} = \partial_i \partial_k A_j - \partial_i \partial_j A_k \qquad (309)$$

The sum of (307), (308) & (309) gives us:

$$\partial_j F_{ik} + \partial_k F_{ji} + \partial_i F_{kj} = 0 \qquad (310)$$

There is only four independents equations of (310) where $i \neq j \neq k$. Otherwise the components of (310) are equal to zero.

The first one is for indices 1,2,3:

$$\partial_1 F_{23} + \partial_3 F_{12} + \partial_2 F_{31} = 0 \qquad (311)$$

i.e.:

$$\nabla . \boldsymbol{B} = 0 \qquad (312)$$

For the other three equations we have also that:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad (313)$$

So we found the first pair of Maxwell equations (homogeny Maxwell equations)which are the existence of a scalar potential and a vector potential.

2-2-2-6)Fields as a function of sources:

We will establish the equations which links the field to its sources i.e. to the motion of charged corpuscles. In the following we suppose that is imposed the dynamics of corpuscles and we are interested only to the dynamics of the field. The dynamic variables are the values of potentials or fields in every space-time point.

2-2-2-6-1)Interaction field-current:

We consider an ensemble of punctual charged corpuscles whom motion is imposed and they are indexed with indices (α) .

Instead to pose that the charges are punctual we consider that the charge is repatriated in a continuous form . This allows as to define the *density of charge* ρ and to pose that ρdV is the charge contained in the volume dV.The density of charge is a function of coordinates and time. The integral of ρ represents for a given volume the charge contained in this volume. We shouldn't forget that the charges are punctual and that the density ρ is equal to zero everywhere except in the points where localised punctual charges; the integral $\int \rho dV$ should be equal to the sum of the charges contained in this volume. This permit us to represent the density ρ as :

$$\rho = \sum_{\alpha} e_{(\alpha)} \delta(\boldsymbol{r} - \boldsymbol{r}(\alpha)) \qquad (314)$$

Where :

$e_{(\alpha)}, \boldsymbol{r}_{(\alpha)}$ are respectively the charge and the position of the corpuscle α.

$\delta(x)$ is the function defined as $\delta(x) = 0$ for every $x \neq 0$, for $x = 0, \delta(0) = \infty$ but $\int_{-\infty}^{+\infty} \delta(x) dx = 1$.

If $f(x)$ is an arbitrary continuous function than:

$$\int_{-\infty}^{+\infty} f(x)\delta(x-a)dx = f(a)$$

In consequence we have $\int_{-\infty}^{+\infty} f(x)\delta(x)dx = f(0)$.

The limit of integration can be different than $\pm\infty$ and the domain of integration can be anyone but should contain the point where δ exist.

The signification of those equalities is that their members furnish the same result when used as a factors under the sign of integration:

$$\delta(-x) = \delta(x) , \delta(ax) = \frac{1}{|a|}\delta(x) \ .$$

And in general we have:

$$\delta[\varphi(x)] = \sum_i \frac{1}{|\varphi'(a_i)|} \delta(x - a_i)$$

Where the a_i are solution of $\varphi(x) = 0$ and $\varphi'(a_i)$ the derivative of $\varphi(x)$ at the point a_i.

For three dimensional space we can define a function $\delta(\boldsymbol{r})$ which equal to zero everywhere except in the origin the three-dimensional coordinates system and also its integral extended to the total space is equal to one. This function can be represented in the form of a product of $(x)\delta(y)\delta(z)$.

Per definition the charge of a corpuscle is an invariant i.e. it is independent from the choice of the referential. The density ρ is not an invariant but the product ρdV is an invariant.

Let's multiply by dx^i the two terms of the equality $= \rho dV$:

$$dedx^i = \rho dV dx^i = \rho dV dt \frac{dx^i}{dt} \quad (315)$$

In the left we found a four-vector (because de is a scalar and dx^i is a four-vector).In consequence we should found in the right a four-vector. As $dVdt$ is a scalar ,so $\rho \frac{dx^i}{dt}$ is a four-vector. This one noted j^i is called four-vector of *current density*:

$$j^i = \rho \frac{dx^i}{dt} \quad (316)$$

The three space components of this four-vector define the three-dimensional density of current:

$$\boldsymbol{j} = \rho \boldsymbol{v} \quad (317)$$

Where $\boldsymbol{v}$ is the speed of the considered charge. The time component of this four-vector is ρc . So we have:

$$j^i = (\rho c, \boldsymbol{j}) \quad (318)$$

The total charge contained in all space is equal to the integral $\int \rho dV$ extended for all space. We can represent this integral in a four-dimensional form:

$$\int \rho dV = \frac{1}{c} \int j^0 dV = \frac{1}{c} \int j^i \, dS_i \quad (319)$$

Where the integral is extended to a four-dimensional hyper-plan orthogonal to the axle x^0 .In a general manner the integral $\frac{1}{c}\int j^i dS_i$ extended to an arbitrary hyper-plan represents the sum of charges whom the universe lines cut this hyper-surface.

Instead of punctual charges e we introduce a continuous repartition of density ρ and so the action due to interaction charge-charge and charge-current is:

$$S_{interaction} = -\frac{1}{c}\int \rho\, A_i dx^i dV \qquad (320)$$

If we write it in this form:

$$S_{interaction} = -\frac{1}{c}\int \rho \frac{dx^i}{dt} A_i dV dt = -\frac{1}{c^2}\int A_i j^i\, d\Omega \qquad (321)$$

Where :

$$d\Omega = dVcdt = cdtdxdydz \qquad (322)\ \text{(four-volume)}$$

2-2-2-6-2)Interaction charge-field:

To find the form of the action $S_{free\ field}$ we refer to an important propriety of electromagnetic fields which is that the experience shows that the electromagnetic fields satisfy the *principle of superposition*: the field generated by a system of charges result only in a simple addition of fields due to every charge taken separately. In other terms the field vector resultant is equal to the sum of all vectors values in the point of every fields considered separately. Every solution of field equations is a field which can be realised in the Nature. According to the principle of superposition the sum of two fields should be a field which can exist in the Nature and should verify the equations of field.

It is known that the linear differentials equations had the propriety that the sum of their solutions is also a solution. In consequence the equations of the electromagnetic field should be linear differentials equations.

So in the action $S_{free\ field}$ we should have a quadratic expression under the integral referring to the field.

The potentials of the field can't be used in the expression of the action $S_{free\ field}$ because they are not defined in one manner (univocal manner and this univocal manner have no importance in the definition of $S_{interaction}$). We conclude that $S_{free\ field}$ is an integral of the tensor F_{ik} of the electromagnetic field. But because the action should be a scalar , it should be the integral of a scalar. The unique scalar existent in this case is the product $F_{ik}F^{ik}$. The function under the sign of the integral in the expression of the action $S_{free\ field}$ shouldn't contain any derivative of F_{ik} because that the Lagrange function can't contain except the coordinates of the system ,only the firsts derivatives according to time. The role of *coordinates* (i.e. the variables according to them we execute the variations of principle of least action) is assumed here by the potentials A_k of the field. Reminder that in classical mechanics the Lagrange function of a mechanical system contain only the coordinates of corpuscles and their firsts derivatives according to time.

Concerning the quantity $\epsilon^{iklm}F_{ik}F_{lm}$ it represents the total four-dimensional divergence and its insertion in the expression of the $S_{free\ field}$ doesn't affect the equations of motion. This quantity is excluded from the expression of the action independently of the fact that is a pseudo-scalar. This pseudo scalar can be represented as a form of four-divergence $\epsilon^{iklm}F_{ik}F_{lm} = 4\frac{\partial}{\partial x^i}(\epsilon^{iklm}A_k\frac{\partial}{\partial x^l}A_m)$ which can be easily verified because ϵ^{iklm} is anti-symmetric.

So the action of fields is as:

$$S_{free\ field} = -\sqrt{\frac{\kappa}{2048.\pi^3.G}}\int F_{ik}F^{ik}d\Omega \qquad (323)$$

Where κ & G are positive constants to choose one and determine the other.

The total action is:

$$S = -\{\int mcds + \frac{1}{c^2}\int A_i j^i\, d\Omega + \sqrt{\frac{\kappa}{2048.\pi^3.G}}\int F_{ik}F^{ik}d\Omega\} \quad (324)$$

For many charges the action is the sum of equation (324).

2-2-2-6-3)The second pair of Maxwell equations:

When we search to establish the equation of the field from the principle of least action , we are obligated to pose that the motion of the charges are given and to vary only the potentials of the field (which play in this case the role of *coordinates* of the system).In the inverse sense to establish the equations of motion, we had pose that the field is given and we vary only the trajectory of the corpuscle.

In consequence the variation of the first term of equation (324) is maintained equal to zero but in the second term we should only vary the current j^i.So:

$$\delta S = -\frac{1}{c}\int[\frac{1}{c}j^i\delta A_i + \sqrt{\frac{\kappa.c^2}{512.\pi^3.G}}\int F_{ik}\delta F^{ik}]d\Omega \quad (325)$$

For equation (325) take in consideration that $F_{ik}\delta F^{ik} \equiv F^{ik}\delta F_{ik}$.

Substitute in (325) $F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}$ we get:

$$\delta S = -\frac{1}{c}\int\{\frac{1}{c}j^i\delta A_i + \sqrt{\frac{\kappa.c^2}{512.\pi^3.G}}F^{ik}\frac{\partial}{\partial x^i}\delta A_k - \sqrt{\frac{\kappa.c^2}{512.\pi^3.G}}F^{ik}\frac{\partial}{\partial x^k}\delta A_i\}d\Omega \quad (326)$$

Permute in the second term of (326) the indices i & k on which we do the summation and replace F_{ki} by $-F_{ik}$:

$$\delta S = -\frac{1}{c}\int\{\frac{1}{c}j^i\delta A_i - \sqrt{\frac{\kappa.c^2}{128.\pi^3.G}}F^{ik}\frac{\partial}{\partial x^k}\delta A_i\}d\Omega \quad (327)$$

Integrate by party the second integral which means apply the theorem of Gauss:

$$\delta S = -\frac{1}{c}\int\{\frac{1}{c}j^i + \sqrt{\frac{\kappa.c^2}{128.\pi^3.G}}\frac{\partial F^{ik}}{\partial x^k}\}\delta A_i d\Omega - \sqrt{\frac{\kappa.c^2}{128.\pi^3.G}}\int F^{ik}\delta A_i dS_k \quad (328)$$

In the second term we should take its value in the limits of integration. The limits of integration on the coordinates are extended to the infinite because the field disappear in the infinite. In the limits of integration on time i.e. in the initial and final instants given the variation of the potentials is equal to zero because according to the principle of least action those potentials are known in those instants. In consequence the second term of (328) is equal to zero and thus we get:

$$\int\{\frac{1}{c}j^i + \sqrt{\frac{\kappa.c^2}{128.\pi^3.G}}\frac{\partial F^{ik}}{\partial x^k}\}\delta A_i d\Omega = 0 \quad (329)$$

As the principle of least action implies that the variations δA_i are arbitrary, the coefficient of δA_i in (329) should be equal to zero:

$$\frac{\partial F^{ik}}{\partial x^k} = -\frac{1}{c^2}\sqrt{\frac{128.\pi^3.G}{\kappa}}\,j^i \qquad (330)$$

Rewrite those equations $(i = 0,1,2,3)$ in three-dimensional form .

For $i = 1$ we have:

$$\frac{1}{c}\frac{\partial F^{10}}{\partial t} + \frac{\partial F^{11}}{\partial x} + \frac{\partial F^{12}}{\partial y} + \frac{\partial F^{13}}{\partial z} = -\frac{1}{c^2}\sqrt{\frac{128.\pi^3.G}{\kappa}}\,j^1 \qquad (331)$$

By substituting the values of the components of the tensor F^{ik} we get:

$$\frac{1}{c}\frac{\partial E_x}{\partial t} - c.\frac{\partial B_z}{\partial y} + c.\frac{\partial B_y}{\partial z} = -\frac{\gamma}{c^2}\sqrt{\frac{128.\pi^3.G}{\kappa}}\,j_x \qquad (332)$$

The equation (332) and the succeeded equations for $(i = 2,3)$ can be written as a unique vector form:

$$rot\boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{\gamma}{c^3}\sqrt{\frac{128.\pi^3.G}{\kappa}}\,\boldsymbol{j} \qquad (333)$$

Finally for $i = 0$ we have:

$$\frac{\partial F^{00}}{\partial x^0} + \frac{\partial F^{01}}{\partial x^1} + \frac{\partial F^{02}}{\partial x^2} + \frac{\partial F^{03}}{\partial x^3} = -\frac{1}{c^2}\sqrt{\frac{128.\pi^3.G}{\kappa}}\,j^0 \qquad (334)$$

By substituting the values of the tensor F^{ik} and the current j^0 we get:

$$div\boldsymbol{E} = \gamma\frac{\rho}{c}\sqrt{\frac{128.\pi^3.G}{\kappa}} \qquad (335)$$

The equations (333) and (335) are the second pair of Maxwell equations as formulated by H.A.Lorentz for the electromagnetic field in vacuum contained punctual charges.

We can write equation (335) as:

$$div\boldsymbol{E} = 4\pi\sigma\rho \qquad (336)$$

With σ constant. So we have:

$$\gamma = \sigma c \sqrt{\frac{\kappa}{8.\pi.G}} \qquad (337)$$

In fact don't forget that conversion coefficient γ is a product of two conversion coefficients & ε . If we resolve the problem for one coefficient, it remains unsolved for the other so the second group of Maxwell equations remains inhomogeneous equations .

To resolve the problem for one coefficient we had build a theory so there is experiences to do & so there is new technologies to rise.

Resolving the problem for the second conversion coefficient needs a new theory to build, new experiences to do and new technologies will rise but we will notice that this coefficient is in fact a product of two coefficients....etc. So the science will be never had an end: welcome to the city of science.

2-2-2-6-4)Continuity equation for charges:

The variation of the charge contained in a given volume is represented by the derivative $\frac{\partial}{\partial t}\int \rho dV$.

In other hand the variation per unit of time depends on the quantity of charge going out or in this volume. The quantity of charges going in this volume per unit time among the element $\boldsymbol{df}$ of the surface bordering this volume is equal to $\rho\boldsymbol{v}d\boldsymbol{f}$ where $\boldsymbol{v}$ is the speed of displacement of the charge in the point of space where exist the element $d\boldsymbol{f}$. As it is useful in usage , the vector $d\boldsymbol{f}$ is directed in the same sense of the extern vector orthogonal to this surface i.e. in the same sense of the orthogonal vector directed out the considered volume. So the quantity $\rho\boldsymbol{v}d\boldsymbol{f}$ is positive if the charge go out the volume and negative if the charge go in the volume. The total charge going out per unit time of a given volume is equal to $\oint \rho\boldsymbol{v}d\boldsymbol{f}$ where the integral is extended for all the closed surface bordering this volume. So we have:

$$\frac{\partial}{\partial t}\int \rho dV = -\oint \rho\boldsymbol{v}d\boldsymbol{f} \qquad (338)$$

The minus sign before the second member of (338) is introduced to consider the first member positive when the total charge in the volume is augmenting. The equation (338) is the conservation law of continuous charge called also *continuity equation* written in an integral form.

We remark that $\rho\boldsymbol{v}$ is the density of current ,we can write the equation (338) as: $\frac{\partial}{\partial t}\int \rho dV = -\oint \boldsymbol{j}d\boldsymbol{f}$ (339)

In a differential form we apply Gauss theorem for the second member of (339) :

$$\oint \boldsymbol{j}d\boldsymbol{f} = \int div\boldsymbol{j}dV \qquad (340)$$

We get:

$$\int \left(div\boldsymbol{j} + \frac{\partial \rho}{\partial t}\right) dV = 0 \qquad (341)$$

Equation (341) should be verified by integrate in any volume , so we should have:

$$div\boldsymbol{j} + \frac{\partial \rho}{\partial t} = 0 \qquad (342)$$

(342) is the differential form of the continuity equation.

We can insure that the expression (314) which give ρ as a function of δ verify automatically the equation (342).

Let's suppose that it exist only on charge as $\rho = e\delta(\boldsymbol{r} - \boldsymbol{r_0})$,the current is $\boldsymbol{j} = e\boldsymbol{v}\delta(\boldsymbol{r} - \boldsymbol{r_0})$ where $\boldsymbol{v}$ is the speed of the charge. Calculate the derivative $\frac{\partial \rho}{\partial t}$. When the charge move its coordinates varies so $\boldsymbol{r_0}$ vary. We have:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial \boldsymbol{r_0}}\frac{\partial \boldsymbol{r_0}}{\partial t}$$

Or $\frac{\partial \boldsymbol{r_0}}{\partial t}$ is the speed $\boldsymbol{v}$ of the charge . In the other hand as ρ is a function of $\boldsymbol{r} - \boldsymbol{r_0}$ we have $\frac{\partial \rho}{\partial \boldsymbol{r_0}} = -\frac{\partial \rho}{\partial \boldsymbol{r}}$ and so in consequence :

$\frac{\partial \rho}{\partial t} = -\boldsymbol{v}grad\rho = -div(\rho\boldsymbol{v}) = -div\boldsymbol{j}$ considering that $\boldsymbol{v}$ as independent from $\boldsymbol{r}$. So we get the equation (342).

In a four-dimensional form the equation (342) is obtained by putting equal to zero the four-divergence of the four-current :

$$\frac{\partial j^i}{\partial x^i} = 0 \qquad (343)$$

In equation (319) we had established that the total charge can represented as $\frac{1}{c}\int j^i\, dS_i$ where the integration is extended to the hyper-plan $x^0 = const$. In another instant the total charge can be represented by a similar integral extended to another hyper-plan orthogonal to the axle x^0 . We can easily verify that the law of charge conservation is coming from the equation (343) i.e. the integral $\int j^i\, dS_i$ is the same for any hyper-plan of integration $x^0 = const.$

The difference between the integrals $\int j^i\, dS_i$ taken on two hyper-surfaces $x^0 = const$ can be written as $\oint j^i dS_i$ where the integral is extended to the closed hyper-surface bordering the four-volume existing between the considered hyper-plans (this integral is different from the difference obtained by an integral extended to the *lateral* hyper-surface localised at the infinite which is eliminated because there is no charge in the infinite). With Gauss theorem we can transform this one on an integral in the four-volume existing between the two hyper-plans and be insured that :

$$\oint j^i dS_i = \int \frac{\partial j^i}{\partial x^i} d\Omega = 0 \qquad (344).$$

CQFD.

This demonstration remains available for two integrals $\int j^i dS_i$ extended to two hyper-surfaces infinites chosen arbitrary (and not only for hyper-plans $x^0 = const$) which bordering all the three-dimensional space. Those considerations shows that the integral $\frac{1}{c}\int j^i\, dS_i$ only a unique value (equal to the total charge contained in the space) for any integration hyper-surface .

We had seen in equation (247) that the gauge invariance of equations implies the conservation of charge . In the equation of motion (324) let's replace A_i by $A_i \frac{\partial f}{\partial x^i}$, the integral $\frac{1}{c^2}\int j^i \frac{\partial f}{\partial x^i} d\Omega$ will be added to the second term of (324): this is the conservation of charge given by the continuity equation (343) which allow us to write the expression under the symbol of integration as a four-divergence $\frac{\partial}{\partial x^i}(f j^i)$ and with Gauss theorem this integral will be transformed as an integral on the hyper-surfaces bordering the four-volume .Those integrals will be eliminated when we vary the action and thus will not affect the equations of motion.

2-2-3)The meaning of the constant $''a''$:

From equation (43) we have in Cartesian coordinates:

$$\tau^2.\left(1-\frac{\dot{x}^2+\dot{y}^2+\dot{z}^2}{c^2}\right)=\tau_0^2 \quad (345)$$

Differentiate (345) we will get :

$$2.\tau.\left(1-\frac{\dot{x}^2+\dot{y}^2+\dot{z}^2}{c^2}\right)+\tau^2.\left(\frac{-2}{c^2}.[\dot{x}.\ddot{x}+\dot{y}.\ddot{y}+\dot{z}.\ddot{z}]\right)=0 \quad (346)$$

Let's consider one dimension the abscissa coordinate:

$$\left(1-\frac{\dot{x}^2}{c^2}\right)-\frac{\tau_0}{c^2.\sqrt{1-\frac{\dot{x}^2}{c^2}}}(\dot{x}.\ddot{x})=0 \quad (347)$$

So we get :

$$\left(1-\frac{\dot{x}^2}{c^2}\right)^{\frac{3}{2}}-\frac{\tau_0}{c^2}.\dot{x}.\ddot{x}=0 \qquad (348)$$

Let's suppose that the speed of the corpuscle tends to $''c''$ so from equation (348) we deduce that the acceleration tends to zero : we can't apply on the corpuscle any force , there is a maximum force to apply . Also the power to transmit to the corpuscle had a superior limit: we can't exchange any amount of energy with the corpuscle instantaneously, there is always a delay time to exchange energy and that's which translate this new constant.

2-2-4)Wave-corpuscle duality:

For $m=0$ it correspond to light which have a peed equal to $''c''$. It is possible that we can treat corpuscles and waves as the same thing.

If we can treat a corpuscle as a wave , we can represent it by a plane wave as the following:

$$\psi(\boldsymbol{x},t)=A.\exp(i\boldsymbol{k}.\boldsymbol{x}-i\omega.t) \quad (349)$$

Where :

A: the amplitude of the wave function;

$\boldsymbol{k}$: the wave-vector;

ω: the frequency of the wave.

The principle of relativity implies the invariance of phase of the wave i.e. we have:

$$x.k - \omega.t = x'.k' - \omega'.t' \qquad (350)$$

Let's suppose that the corpuscle is in rest in the referential R', so we consider that it's wave-vector in this referential is equal to zero (nothing is in propagation):

$$k' = 0 \qquad (351)$$

And so we deduce that:

$$\boldsymbol{k} = \frac{\omega'.\boldsymbol{V}/c^2}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (352)$$

$$\omega = \frac{\omega'}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (353)$$

So:

$$\boldsymbol{k} = \frac{\omega'}{m.c^2}.\boldsymbol{p} \qquad (354)$$

$$\omega = \frac{\omega'}{m.c^2}.E \qquad (355)$$

$$\boldsymbol{k} = \frac{\omega}{c^2}.\boldsymbol{V} \qquad (356)$$

We generalise those equations for every speed of the corpuscle.

But in this conception there is a major contradiction: how do we accept that a corpuscle which limited in space-time extension can be represented by a plane wave which is present everywhere. In 1927 deBroglie had found the solution by applying the principle of non-contradiction: the corpuscle is a packet of waves which reinforce each other in a limited region of space-time and annihilate each other above. The group speed of waves is identified to the corpuscle speed.

2-2-5) Unity-multiplicity duality:

As a consequence of de Broglie conception the corpuscle is considered as a unique object and also multiple of superposing waves. The wave function associated to the corpuscle has a quasi-monochromatic frequency ω and a wave vector k verifying the principle of uncertainty as follows:

$$\Delta k.\Delta x \geq 1 \qquad (357)$$

$$\Delta\omega.\Delta t \geq 1 \qquad (358)$$

The group speed v_g of the packet of waves is the speed with which the energy is transmitted .Its definition is as follows:

$$\frac{1}{v_g} = \frac{dk}{d\omega} \qquad (359)$$

From (354) and (355) the uncertainties are as the following:

$$\Delta k = \frac{\omega\prime}{m.c^2}.\Delta p \qquad (360)$$

$$\Delta\omega = \frac{\omega\prime}{m.c^2}.\Delta E \qquad (367)$$

We have:

$$\Delta k.\Delta x = \frac{\omega\prime}{m.c^2}.\Delta p.\Delta x \qquad (368)$$

$$\Delta\omega.\Delta t = \frac{\omega\prime}{m.c^2}.\Delta E.\Delta t \qquad (369)$$

From equations (357) & (358) we deduce that:

$$\Delta p.\Delta x \geq \frac{m.c^2}{\omega\prime} \qquad (370)$$

$$\Delta E.\Delta t \geq \frac{m.c^2}{\omega\prime} \qquad (371)$$

The constant $\frac{m.c^2}{\omega\prime}$ should be independent from any corpuscle. We declare it an universal constant.

We put:

$$\frac{m.c^2}{\omega\prime} = \hbar \qquad (372)$$

The constant $\hbar$ should have a very low value in the MKS system.

Equations (355) & (356) becomes:

$$\hbar.\boldsymbol{k} = \boldsymbol{p} \qquad (373)$$

$$\hbar.\omega = E \qquad (374)$$

It is very easy to verify that:

$$\frac{1}{v_g} = \frac{dk}{d\omega} = \frac{1}{v} \qquad (375)$$

And that what is CQFD.

2-2-6)Viscosity-dispersion duality:

The equation of propagation of the wave function (Klein-Gordon equation) is:

$$\frac{1}{c^2}.\frac{\partial^2}{\partial t^2}\psi(\boldsymbol{x},t) - \nabla^2\psi(\boldsymbol{x},t) = -\frac{m^2\,c^2}{\hbar^2}\psi(\boldsymbol{x},t) \qquad (376)$$

We define the following operator called also d’Alembertian:

$$\Box \equiv \frac{1}{c^2}.\frac{\partial^2}{\partial t^2} - \nabla^2 \qquad (377)$$

Equation (93) can be written also as the following:

$$\Box\psi(\boldsymbol{x},t) = -\frac{m^2\,c^2}{\hbar^2}\psi(\boldsymbol{x},t) \qquad (378)$$

The d’Alembertian of the wave function is not equal to zero so there is dispersion of the wave function. The medium in which the packet of waves is in propagation is a dispersive medium: there is attenuation of the packet of waves with absorption of energy.

A dispersive medium for waves correspond for corpuscles to a viscous medium. The equation of motion of the corpuscle is:

$$\frac{d\boldsymbol{p}}{dt} = \boldsymbol{f} - a.\boldsymbol{v} \qquad (379)$$

Where:

$\boldsymbol{f}$: all unknown forces which act on the corpuscle;

$-a.\boldsymbol{v}$: an universal friction force which act on the opposite side of motion of the corpuscle;

a : friction coefficient of the space-time (or mechanical impedance of vacuum).

So we can conclude that for wave-corpuscle duality there is another duality which is viscosity-dispersion duality of space-time .

Equation (379) is the same equation (4).

We define the moment of the corpuscle as:

$$\boldsymbol{p} = \xi.\boldsymbol{v} \qquad (380)$$

Where :

ξ : the inertia of the corpuscle;

$\boldsymbol{v}$: the speed of the corpuscle.

From equation (379) we deduce that:

$$\dot{\xi}.\boldsymbol{v} + \xi.\frac{d\boldsymbol{v}}{dt} = \boldsymbol{f} - a.\boldsymbol{v} \qquad (381)$$

We put that:

$\dot{\xi} = a$ (382): if the energy of the corpuscle is varying;

$\dot{\xi} = 0$ (383): if the energy of the corpuscle is constant.

So we deduce from (382) &(383) that :

$$\xi = a.\tau \qquad (384)$$

Where:

τ: is the inertial time of the corpuscle;

With:

$d\tau = dt$ (385): if the energy of the corpuscle is varying;

$d\tau = 0$ (386): if the energy of the corpuscle is constant.

If the corpuscle is in rest that we associate to it an inertial time in rest as:

$$m = a.\tau_0 \qquad (387)$$

So we get from (381):

$$lim(\frac{d\boldsymbol{v}}{dt})_{t\to+\infty} = lim(\frac{\boldsymbol{f}-2.a.\boldsymbol{v}}{a.\tau})_{\tau\to+\infty} = \boldsymbol{o} \qquad (388)$$

So the speed of the corpuscle tends to a constant $''c''$ and this constant is declared as an universal constant: we know that it is the speed of light in vacuum.

We get the transformations of space and time (8) & (9). Also we have the transformations of momentum and inertia as the following:

$$\xi' = \frac{\xi-\boldsymbol{p}.\boldsymbol{V}/c^2}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (389)$$

$$\boldsymbol{p}' = \frac{\boldsymbol{p}-\xi.\boldsymbol{V}}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (390)$$

$\boldsymbol{p'}$and ξ' are respectively the momentum and the inertia of the corpuscle according to the reference R'. There is always invariants in inertial references to conserve the same speed of light. We have here an invariant which is:

$$\xi^2 - (\frac{p}{c})^2 = m^2 \qquad (391)$$

Now we can write equation (379) as:

$$\boldsymbol{f} = \frac{d^2(\xi.\boldsymbol{X})}{dt^2} \qquad (392)$$

Where :

$\boldsymbol{X}$**:** the position of the corpuscle.

We can take this formulae as a beginning for the equation of motion of a corpuscle and we get the same results. Let's develop equation (392):

$$\boldsymbol{f} = \ddot{\xi}.\boldsymbol{X} + 2.\dot{\xi}.\boldsymbol{v} + \xi.\ddot{\boldsymbol{X}} \qquad (393)$$

In a referential inertia equation (393) should be independent from the choice of its origin so we get from (393):

$$\ddot{\xi} = 0 \qquad (394)$$

So:

$$\dot{\xi} = a \qquad (394)$$

And so on.

2-2) Black body radiation:

This experiment is the black body radiation [12] .Let's have a cavity in a temperature T with a little hole from which we measure the energy and power of the emergent radiation.

2-2-1)Black body thermal equilibrium:

The number of photons of the frequency ν at the thermal equilibrium is :

$$n = \frac{1}{\exp\left(\frac{h.\nu}{k.T}\right)-1} \qquad (395)\text{: Boltzmann formulae}$$

Where $h = 6.626\ 10^{-34}\ j.s$: the constant of Planck

$k = 1.380\ 10^{-23} j.°K^{-1}$: the constant of Boltzmann

T: the temperature of the black body

The energy of photons at the frequency ν is :

$$E_\nu = n.h.\nu = \frac{h.\nu}{\exp\left(\frac{h.\nu}{k.T}\right)-1} \qquad (396)$$

The power of photons at the frequency ν is :

$$P_\nu = n.a.c^2 = \frac{a.c^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1} \quad (397)$$

2-2-2)Number of modes contained in the interval of frequency $\delta\nu$:

For a length L there is a stationary polarized wave if we have:

$$L = j.\frac{\lambda}{2} = j.\frac{\pi}{k_j} \quad (398)$$

So:

$$k_j = j.\frac{\pi}{L} \quad (399)$$

j: integer , λ: wave length , k_j: wave vector.

The interval between two successive waves numbers is :

$$\delta k = \frac{\pi}{L} \quad (400)$$

The number of values of k included in an interval δk is very high than δk. This number is :

$$\frac{\delta k}{(\frac{\pi}{L})} = \delta k.\frac{L}{\pi} \quad (401)$$

A stationary wave contains two waves . The number of modes δM is the half of the number of values of k so:

$$\delta M = \frac{\delta k.L}{2.\pi} \quad (402)$$

In three dimensions we get:

$$\delta M = \delta M_x.\delta M_y.\delta M_z = \frac{L_x.L_y.L_z.\delta k_x.\delta k_y.\delta k_z}{(2\pi)^3} \quad (403)$$

i.e.:

$$\delta M = \frac{V.\delta k^3}{(2\pi)^3} \quad (404)$$

With $V = L_x.L_y.L_z$

The photon had two states of possible polarisation so:

$$\delta M = \frac{2.V.\delta k^3}{(2\pi)^3} \qquad (405)$$

δk^3 is the spherical volume interval in the space of k and it is equal to :

$$4\pi.k^2.\delta k = \delta k^3 \qquad (406)$$

So:

$$\delta M = \frac{8\pi.V.k^2.\delta k}{(2\pi)^3} = \frac{V}{\pi^2}.k^2.\delta k \qquad (407)$$

With $k = \frac{2\pi\nu}{c}$: the wave vector.

So we get:

$$\delta M = 8\pi.V.\frac{\nu^2}{c^3}.\delta\nu \qquad (408)$$

2-2-3)Black body volume power density:

The cavity of the black body englobe δM modes which everyone contains the power given in (397).

So the power which is contained in the interval of frequency $\delta\nu$ is :

$$\delta P = P_\nu.\delta M = 8\pi.V.\frac{\nu^2}{c^3}.\frac{a.c^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.\delta\nu \qquad (409)$$

Here k is the Boltzmann constant.

The volume power per frequency interval $\delta\nu$ is:

$$dP = \frac{\delta P}{V} = 8\pi.\frac{\nu^2}{c^3}.\frac{a.c^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.d\nu \qquad (410)$$

Integrate (409) for all frequencies:

$$P = \int_0^\infty 8\pi.\frac{\nu^2}{c^3}.\frac{a.c^2}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.d\nu \qquad (411)$$

Replace $x = \frac{h.\nu}{k.T}$ in (411) we get:

$$P = \int_0^\infty 8\pi.\frac{1}{c^3}.\frac{k^2.T^2}{h^2}.x^2\frac{a.c^2}{\exp(x)-1}.\frac{k.T}{h}dx \quad (412)$$

So:

$$P = \int_0^\infty \frac{8\pi.a.k^3.T^3}{c.h^3}.\frac{x^2}{\exp(x)-1}.dx = \frac{30.\zeta(3).\sigma.a.c^2}{\pi^4.k}.T^3 \quad (413)$$

With: $\zeta(3) = 1.202056 \ldots.$, $\zeta(x) = \sum_{n=1}^{\infty}\frac{1}{n^x}$ $(x > 1)$: Riemann function (or Zeta function).

$$\sigma = \frac{8.\pi^5.k^4}{15.h^3.c^3} = 7.56\ 10^{-16}\ j.m^{-3}.°K^{-4}$$

$$P\ have\ a\ dimension\ of [W.m^{-3}]$$

Than P is a linear function of T^3 where the linearity coefficient contains $''a''$ so we can determinate it.

2-2-4)Volume energy of the black body:

The energy contained in the interval of frequency $\delta\nu$ is:

$$\delta U = E_\nu.\delta M = \frac{8\pi.V.\nu^2}{c^3}.\frac{h.\nu}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.\delta\nu \quad (414)$$

The volume energy per interval of frequency $d\nu$ is:

$$dU = \frac{\delta E}{V} = \frac{8\pi.h.\nu^3}{c^3}.\frac{1}{\exp\left(\frac{h.\nu}{k.T}\right)-1}.d\nu \quad (415)$$

By integrate (415) we found:

$$U = \frac{8\pi.k^4}{c^3h^3}.T^4.\int_0^\infty \frac{\left(\frac{h\nu}{kT}\right)^3}{\exp\left(\frac{h\nu}{kT}\right)-1}.d\left(\frac{h\nu}{kT}\right) = \sigma.T^4 \quad (416)$$

Because we have:

$$\int_0^{\infty} \frac{x^3}{\exp(x)-1}.dx = \frac{\pi^4}{15} \qquad (417)$$

$$U\ have\ a\ dimension\ of\ [j.m^{-3}]$$

The power of radiation per surface unit is:

$$R = \frac{c}{4}.U = \frac{2.\pi^5.k^4}{15.c^2h^3}.T^4 \qquad (418)$$

$$R\ have\ a\ dimension\ of\ [W.m^{-2}]$$

2-3)Limits of the constant $''a''$:

If the Universe is dominated by an isotropic radiation than we have its state equation as [13]:

$$Pressure\ of\ radiation = \frac{R}{3} \qquad (419)$$

The total power is :

$$P_{tot} = P.V = \frac{R}{3}.(L_z.L_y + L_z.L_x + L_x.L_y) \qquad (420)$$

So we have:

$$\frac{30.\zeta(3).\sigma.a.c^2}{\pi^4.k}.T^3.L_x.L_y.L_z = \frac{2.\pi^5.k^4}{45.c^2h^3}.T^4.(L_z.L_y + L_z.L_x + L_x.L_y) \qquad (421)$$

Than:

$$\frac{30.\zeta(3).a.c^2}{\pi^4.k}.\frac{8.\pi^5.k^4}{15.h^3.c^3} = \frac{2.\pi^5.k^4}{45.c^2h^3}.T.(\frac{1}{L_x} + \frac{1}{L_y} + \frac{1}{L_z}) \qquad (422)$$

So:

$$360.\zeta(3).a.c^2 = k.c.T.\pi^4.(\frac{1}{L_x} + \frac{1}{L_y} + \frac{1}{L_z}) \qquad (423)$$

Let's design by D a characteristic dimension of the Universe, we have:

$$360.\zeta(3).a.c^2 = k.c.T.\pi^4.\frac{3}{D} \qquad (424)$$

Than we will have that:

$$a = \frac{k.T.\pi^4}{120.\zeta(3).c.D} \qquad (425)$$

If we take for $D = 1000\ Mpc = 3.1\ 10^{25}\ m$ a distance to study the properties of homogenous Universe expanding slowly [14] and a temperature $T = 2.725\ °K$ [15] than we get:

$$a = \frac{1.38\ 10^{-23}.2{,}725.\pi^4}{120.\zeta(3).3\ 10^8.3{,}1\ 10^{25}} \approx\ 2.8\ 10^{-57}\ kg.s^{-1} \qquad (426)$$

In the other hand with the meshing done for space-time by equations (5), (6) & (7) the density of vacuum is a mass M in a volume of L^3 than we have:

$$\rho_0 = \frac{a^2}{c.\hbar} \qquad (427)$$

The energy density of vacuum is :

$$E_0 = \rho_0.c^2 = \frac{a^2.c}{\hbar} \qquad (428)$$

Cosmologists find by observations that the vacuum density is approximately as [16]:

$$\varrho_0 \approx 10^{-29}\ g.cm^{-3} \qquad (429)$$

So we deduce that :

$$a \approx 2\ \ 10^{-26}\ kg.s^{-1} \qquad (430)$$

We can say that constant $''a''$ is:

$$2.8\ 10^{-57} kg.s^{-1} \leq a \leq 2\ 10^{-26}\ kg.s^{-1} \qquad (431)$$

We need another experience to slice about the value of constant $''a''$ such as the photoelectric experience or the black body radiation experience otherwise we will have the same history of the lamda constant in General Relativity Theory.

We can get a more accurate interval for the constant "a". A mole of perfect gas occupy a volume of $22.4\ liters$ at the normal conditions of pressure and temperature $(20°C@1\ bar)$.

So the distance $D = (22.4\ liters)^{\frac{1}{3}} = 0.282\ m$ and we have $kT = 404.34\ \ 10^{-23} joule$ than:

$$a \approx 323.3\ \ 10^{-31}\, kg.s^{-1} \qquad (432)$$

So:

$$323.3\ \ 10^{-31}\, kg.s^{-1} \leq a \ll 2\ 10^{-26}\, kg.s^{-1} \qquad (433)$$

We should expect that the number of corpuscles in this volume should be near the Avogadro number ($N_A = 6.02\ 10^{23}$) if of course Boltzman statistics still valid for photons (but it is wrong because photons are relativist corpuscle and there is another statistics which is Bose-Einstein statistics or Fermi-Dirac statistics).

From equation (413) we deduce the density of photons:

$$n = \frac{P}{a.c^2} = \frac{30.\zeta(3).\sigma}{\pi^4.k}.T^3 \qquad (434)$$

So for $T = 20\ °C$ & $V = 22.4\ liters$ the number of photons is:

$$N = n.V = 1.14\ \ 10^{13} \neq 6.02\ 10^{23} \qquad (435)$$

The most important in equation (425) is the ratio $\frac{kT}{D}$ which we should search the meaning. The reader is invited to do this.

3)Vacuum energy levels:

3-1)The energy of the corpuscle as an exchange energy with vacuum:

The work of the friction force between two points A & B of the trajectory of the corpuscle is as follows:

$$\varepsilon_{AB} = \int_A^B -a.\boldsymbol{v}.d\boldsymbol{x} = \int_A^B -a.v^2.d\tau =$$

$$= -a.c^2.(\tau_B - \tau_A) - a.c^2.\tau_0^2.\left(\frac{1}{\tau_B} - \frac{1}{\tau_A}\right)$$

$$= -a.c^2.(\tau_B - \tau_A).(1 - \frac{\tau_0^2}{\tau_A.\tau_B}) \qquad (436)$$

We take the origin of the energy as the rest state of the corpuscle so:

$$\varepsilon_{AB} = \varepsilon_B - \varepsilon_A \qquad (437)$$

With:

$$\varepsilon_B = -a.c^2.(\tau_B - \tau_0).\left(1 - \frac{\tau_0}{\tau_B}\right) = -a.c^2.\tau_B.\left(1 - \frac{\tau_0}{\tau_B}\right)^2 \quad (438)$$

Idem for ε_A.

We define the energy exchanged by the corpuscle with vacuum as:

$$\varepsilon = a.c^2.\tau.(1 - \frac{\tau_0}{\tau})^2 = \xi.c^2.(1 - \frac{m}{\xi})^2 \quad (439)$$

In general this energy can be positive of negative.

If the speed of the corpuscle tends to the celerity of light than we have from (440):

$$\varepsilon \approx \xi.c^2 \quad (441)$$

The energy exchanged with vacuum (436) corresponds exactly to the energy exchanged with vacuum by light i.e. by a corpuscle which has a mass equal to zero. We can deduce that the energy of a corpuscle of a mass m is approximately as follows:

$$E \approx \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}} \quad (442)$$

Its moment is as follows :

$$\boldsymbol{p} = \frac{E}{c^2}.\boldsymbol{v} \approx \frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}} \quad (443)$$

Its Lagrangian is as follows :

$$L = \boldsymbol{p}.\boldsymbol{v} - E \approx -m.c^2.\sqrt{1 - \frac{v^2}{c^2}} \quad (444)$$

3-2)Vacuum energy levels :

In the interval $(x \pm \Delta x, t \pm \Delta t)$the corpuscle is a superposition of many monochromatic waves at every point of this interval and also above. If we choose a certain discernible number of positions in this interval we can accept that the exchanging energy with vacuum is approximately:

$$\varepsilon_n = n.\varepsilon = n.\xi.c^2.(1-\frac{m}{\xi})^2 \qquad (445)$$

Where : is an integer which can be positive or negative (the corpuscle can take energy from vacuum or loose energy for vacuum).

With the condition that $\varepsilon_n \to 0$ when $n \to \pm\infty$: the total energy of the corpuscle should be finite.

The total energy of the corpuscle is:

$$E = \xi.c^2 + \varepsilon_n = \xi.c^2 + n.\xi.c^2.(1-\frac{m}{\xi})^2 \qquad (446)$$

With the condition that $\xi \to m$ when $n \to \pm\infty$ (the momentum is very well defined and it tends to zero but the position is bad defined).

This image is that the corpuscle is like an harmonic oscillator maintained in oscillation by a force $\boldsymbol{f} = a.\boldsymbol{v}$ where $''a''$ is a coefficient of a mechanical impedance.

The ‘’stability’’ for any mechanical system is in general defined as when its energy is an extremum i.e. in our case that :

$$\frac{dE}{d\xi} = 0 \qquad (447)$$

So we get:

$$\xi = \pm m.\sqrt{\frac{n}{n+1}} \qquad (448)$$

Replace (448) in (446) we get:

$$E_{(+)} = 2.m.c^2.\sqrt{n}.(\sqrt{n+1} - \sqrt{n}) \qquad (449)$$

$$E_{(-)} = -2.m.c^2.\sqrt{n}.(\sqrt{n+1} - \sqrt{n}) \qquad (450)$$

With :n positive integer

We can draw the curve $E = function(\xi)$ and we find that it had a minimum given by equation (444) for positive ξ and that there is no inflexion point because $\frac{d^2E}{d\xi^2}$ doesn't change in sign.

If we consider the rest state as the origin of energy, the exchanged energy is:

$$\Delta E = E - m.c^2 \qquad (451)$$

For vacuum we take the mass m as equal in equation (5) and by equations (449) , (450) & (451) we get many levels of vacuum energy and exchanged energy with vacuum.

For every level of vacuum energy we can define a certain mass as for example $\frac{E_{(+)}}{c^2}$ and from this origin we get infinite other levels and so on.

4)Conclusion:

In equation (133) we had concluded that the energy of corpuscle is as:

$$E = \beta.\omega \qquad (452)$$

If we take two electric charge separated by a distance R the Coulomb force is:

$$f = \sigma.\frac{e^2}{R^2} \qquad (453)$$

The question is what is the value of constant β. We can take the way of Planck which is that:

$$\beta = \hbar \qquad (454)$$

Where the Planck constant $\hbar$ is determined by thermodynamics experiment (black body radiation).

There is another way which to have a force $f = ac$ acting between charges separated with the distance $R = \sqrt{\frac{\beta}{a}}$ so we have :

$$\beta = \sigma.\frac{e^2}{c} \qquad (455)$$

The two constants should be equal to avoid any contradiction, so there is an universal constant e as :

$$e = \sqrt{\frac{\hbar.c}{\sigma}} \qquad (456)$$

Which is equivalent to:

$$\sigma.\frac{e^2}{\hbar c} = 1 \qquad (457)$$

From equation (228) & (445) we have:

$$\gamma = a.\sqrt{\frac{\sigma.c}{\hbar}} \qquad (458)$$

Don't forget that conversion factor γ is a product of two conversion factors.

It is clear that conversion factors open us for more theories so more experiments & so more technologies. The same problem will be found in thermodynamics because we have the conversion relationship $\hbar\omega \ll kT$ or $\hbar\omega \gg kT$ and we see the relationship (458).

The most important thing done here is the unification of fields in a Minkowski space-time i.e. in inertial referenctials where Lorentz transformations are available. But physics experiments should be independent from the choice of the referential. The reader is invited to rewrite this paper in a Riemann space-time (any transformations of space & time between referentials). The reader can take the document 49089264 of Pierre Paillere [17]available on the internet and rewrite the 115 pages available with the same spirit of this paper. Join to this document the paper of C.LANZANOS [18].

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