Does General Relativity Properly Describe the Perihelion Shift and Black Holes?

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Abstract

The paper proves that General Relativity (GR) is not needed to explain the effects mentioned in the title. It will be shown that Newton's laws are sufficient if the influence of the far distant masses of the universe is included into the calculation, i.e., if Mach's principle is truly obeyed. The quantitative result with the perihelion shift of Mercury is very close to that of the GR, whereas there is a small, but possibly measurable, difference in the size of the event horizon of static black holes.

Key words: perihelion shift, black holes, event horizon, Newton's laws and the universe, action-at-a-distance, Mach's Principle, General Relativity, cosmology, fundamental physics

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1. Introduction

It is commonly accepted within the community of physicists that the correct figures for the event horizon of black holes and for the perihelion shift can only be found within the framework of General Relativity (GR). The calculation based solely on Newton's laws yields the wrong values of the perihelion shift if compared to experimental figures. In the case of Mercury, the calculated shift deviates about 43'' (arc seconds) from the experimental figure. It was one of Einstein's first and amazing successes when he could calculate the correct value using his GR. This was and is one of the reasons that Einstein's GR is considered powerful and as one of the fundamental theories of physics.

But there is an important deficit if planetary or cosmic systems are calculated on the basis of Newton's laws alone: the influence of (in cosmological scales) far distant masses has never been considered up to now. If, for instance, the perihelion shift of the planet mercury is calculated, only the influences of the other planets of the solar system have been and are taken into account.

In an earlier paper[1)], we determined that including the far distant masses in the calculations with Newton's laws leads to decisive and measurable changes compared to the results without this inclusion. For the behavior of a specimen mass positioned somewhere in the universe, one finds exactly the formulas of the Special Theory of Relativity (SR), provided that the mass distribution is homogeneous, i.e. there is no single mass in the close vicinity of the specimen mass. The contrary case that other masses are in the vicinity was mentioned within the paper[1)] but only briefly.

In the present paper we will go into details for two of these cases, namely for the problem of the perihelion shift and for a static black hole. We shall see that the result for the motion of planets leads to particle trajectories that are no longer closed, even if the influence of the other planets within the solar system are not taken into account. For the planet Mercury, the resulting perihelion shift is identical to the value found by the GR. For a static black hole, one finds similar qualitative properties as with GR, but there are small deviations in the size of the event horizon.

2. Short description of the theory applied

We hark back in this paper to a theory developed earlier (see [1),2),3)]). Because this theory is not widely known, we will present the main elements here briefly. The consideration may appear trivial at first view, but its consequences are in no way trivial.

If a meteorite impacts the surface of the earth, its kinetic energy is converted into heat. The meteorite has gained this kinetic energy on its part by the conversion of its potential energy, which it had initially at far distance from earth. We can see that the energy content of the earth after the impact has grown by exactly the same amount as the potential energy of the meteorite has been reduced. If the meteorite does not impinge the surface of the earth but first falls into a hole directed to the center of the earth where it is then stopped, then the total potential energy that the meteorite has lost is given by [4)]:

$$\Delta E_{pot\ tot} = 2\pi\ G\ m_0\ \rho\ R_o^2. \tag{2.1}$$

With the abbreviation

$$b_0^2 = 2\pi\ G\ \rho\ R_o^2 \tag{2.2}$$

this can be written in the form:

$$E_0 = m_0\ b_0^2, \tag{2.3}$$

where G is the gravitational constant, ρ the average mass density of the earth, and R_0 its radius. The relation (2.1) is valid for a homogeneously distributed mass of the earth. In case of a non-homogeneous distribution, e.g. in case of a number of point masses m_i, distributed randomly at the locations r_i, one finds instead of (2.2) and (2.3) [1)]:

$$b_0^2 = G \sum_i \frac{m_i}{r_i} \tag{2.2a}$$

and

$$E_0 = m_0\ b_0^2 = m_0\ G \sum_i \frac{m_i}{r_i}\ . \tag{2.3a}$$

The physical essence of these relations can be described as follows: The (gravitational) energy content of the earth with respect to a mass m_0 at rest at its origin is given by the sum of the losses of the potential energies of the masses m_i having moved from infinity to r_i and being at rest there [1)].

Let us now leave the earth and consider the universe instead, and let us take the following model as a basis: The universe shall be finite in the three dimensional space and shall form a sphere with the radius R_0. On very large scales, the masses shall be distributed homogeneously, i.e. the universe shall show (on this scale) a mean density ρ. In this case, the energy content of the universe, including the mass m_0 at its center, is described by exactly the same formula as described above for a meteorite that was fallen to the center of the earth. Of course, the values ρ and R_0 of the universe have to be set now.

It is important to stress that the energy (2.3) is not merely "assigned" to the mass m_0, but that it is **existentially united** to it: It does not exist, if the mass m_0 is lacking, and it must be imperatively present, if the mass m_0 exists at the origin of the universe and is at rest there. Hence, we can describe this physical state also by saying: A mass at rest being located at the center of the universe "has" (or "possesses") a "rest energy" E_0.

Let us now consider the circumstances with a moving mass. And let us assume the general validity of Newton's law of inertia:

$$F = \dot{p} = m_{in}\dot{v} + \dot{m}_{in}v\,. \tag{2.4}$$

Here m_{in} represents the inert mass. F shall be an arbitrary external force, but not a gravitational one (for instance, the mass m_{in} can carry a charge and move under the influence of an electrical field). Other possibly existing masses m_j shall be so small or at far distances that their gravitational forces are very small compared to F and can therefore be neglected in (2.4). If we multiply (2.4) on both sides with the infinitesimal shift ds produced by the force F

$$Fds = \dot{p}\,ds = (m_{in}\dot{v} + \dot{m}_{in}v)ds, \tag{2.5}$$

then we find the energy increase dE when the system "mass within the universe" changes (caused by F) from the status "mass at rest" to the status "mass in motion". According to the consideration above (equations (2.1) to (2.3)), the rest energy of the mass m within the universe is given by $E = m(v{=}0)b_0^2 = m_0\, b_0^2$, where m is the gravitational mass $m{=}m_g$. We know that the inert and the gravitational mass are proportional to each other (or equal if suitable units are chosen). Therefore, the essential relation is valid:

$$m = m_g = m_{in}\,. \tag{2.6}$$

This equivalence principle is experimentally very well confirmed for arbitrary speeds.

Therefore, we can rewrite (2.5) into the form

$$dE = (m\dot{v} + \dot{m}v)\,ds. \tag{2.7}$$

For a mass $m = m_0$ at rest the total energy E_0 is given by (see (2.3))

$$E_0 = m_0\, b_0^2. \tag{2.3a}$$

The following ansatz comes to mind for the system "universe plus moving mass":

$$E = m\, b_0^2. \tag{2.8}$$

Then, accordingly

$$dE = dm\, b_0^2. \tag{2.9}$$

By inserting (2.9) into (2.7) and after some minor conversions we find eventually:

$$dm\, b_0^2 = dm\, v^2 + m\, v\, dv$$

or

$$\frac{dm}{m} = \frac{v\, dv}{b_0^2 - v^2} \,. \tag{2.10}$$

Of course, instead of (2.8) we can also try any other ansatz at will. But such another ansatz would then only represent a solution if it would fulfill the differential equation (2.7), and if the rest energy (2.3a) would result for the limit case v = 0. Obviously, the ansatz (2.8) fulfills these requirements.

By integrating (2.10) one ends up with

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{b_0^2}}} \,, \tag{2.11}$$

and with (2.8)

$$E = \frac{m_0}{\sqrt{1 - \frac{v^2}{b_0^2}}}\, b_0^2 \,. \tag{2.12}$$

Surprisingly, the same formulas for a moving mass and its energy result as in the Special Theory of Relativity (SR). In contrast to the SR, the constancy of the light velocity in inertial systems, uniformly moved against each other, does **not** have to be postulated here. Light and the light velocity play no role in the derivation. But we find also a maximum velocity b_0 for moving masses. This is the consequence of the fact that we have taken the distant masses of the universe into consideration. The question as to whether this maximum velocity b_0 and the light velocity c are identical is investigated in the papers [1), 2)] and [3)]. Due to several reasons, this seems to be mandatory.

The results above are remarkable and, of course, have a number of consequences that have already been investigated to some extent in the papers [1), 2)] and [3)]. They are valid under the prerequisite of a homogeneous mass distribution of the universe. In the paper [1)] we mentioned briefly that this mass distribution is generally, of course, not homogeneous and the theory has to be generalized for inhomogeneous distributions. In the following, we perform one step towards such a generalization. We will consider the special case of adding a single mass M, which causes an inhomogeneity in the vicinity of the mass m. As is known, this case of a spherical symmetric mass distribution can be solved exactly in the GR (Schwarzschild metric).

Though our studies [1), 2)] and [3)] suggest almost imperatively that b_0 is identical with the light velocity c, in the following we will continue to use b_0 in order to recall that our theory does not at all need Einstein's postulate of the constancy of the light velocity. Rather, the maximum velocity $v_{max} = b_0$ is already a consequence of Newton's laws if the distant masses of the universe are taken into account appropriately.

3. Extension of the theory to an inhomogeneous mass distribution

As before, we start with the consideration of the forces involved, but allow the existence of a mass M located at distance r on the positive x-axis to the right of the origin (M conceived to be at rest at the beginning). Now, the influence of this additional mass M shall not be negligeble compared to the other forces.

a) Special case: central force

The force F shall act into the positive x – axis, i.e. into the direction of M. Then, there will be no angular momentum caused by the beginning motion, and we find instead of (2.4):

$$-\mathrm{m}\dot{\mathrm{v}} - \dot{\mathrm{m}}\mathrm{v} + \mathrm{F} + \mathrm{m}\frac{\mathrm{MG}}{\mathrm{r}^2} = 0\,. \tag{3.1}$$

Also, in this case the validity of the equivalence principle (2.6) is implied, but again in its "weak form" $m = m_g = m_{in}$ (see (2.6)).

Due to our choice of the direction of F we have ds = dr and we obtain the following by multiplying (3.1) with dr:

$$-\mathrm{dm}\,\mathrm{v}^2 - \mathrm{m}\,\mathrm{v}\,\mathrm{dv} + \mathrm{Fdr} + \mathrm{m}\frac{\mathrm{MG}}{\mathrm{r}^2}\mathrm{dr} = 0\,. \tag{3.2}$$

Corresponding to the relation (2.3a) we find for the rest energy of the mass m (being located at the origin)

$$E_0 = m_0\, G\Big(\sum_i \frac{m_i}{r_i} + \frac{M}{r}\Big) = m_0\,(b_0^2 + \frac{MG}{r}). \tag{3.3}$$

Again, it is self-evident to try the following ansatz for the total energy of the system, consisting of the masses m and M and the distant masses of the universe (completely analogous to (2.8)):

$$E = m\,(b_0^2 + \frac{MG}{r}) = m\,b_0^2\,(1 + \frac{MG}{rb_0^2}). \tag{3.4}$$

Every change of this total energy is then given by

$$dE = dm\,(b_0^2 + \frac{MG}{r}) - m\frac{MG}{r^2}dr. \tag{3.5}$$

Such a change of the total energy is caused by the external force F, hence it is

$$Fdr = dE = dm\,(b_0^2 + \frac{MG}{r}) - m\frac{MG}{r^2}dr. \tag{3.6}$$

Inserting (3.6) into (3.2) leads to

$$dm\,(b_0^2 - v^2 + \frac{MG}{r}) = \mathrm{m\,v\,dv} \tag{3.7}$$

or

$$\frac{\mathrm{dm}}{\mathrm{m}} = \frac{\mathrm{v\,dv}}{(b_0^2 - v^2 + \frac{MG}{r})}\,. \tag{3.8}$$

If we integrate again from m = m_0 to m = m and from v = 0 to v = v we arrive at

$$m = m_0 \frac{\sqrt{(1+x)}}{\sqrt{(1-\frac{v^2}{{b_0}^2}+x)}} \tag{3.9}$$

and eventually we find for the total energy

$$E_r = m_0 {b_0}^2 \frac{\sqrt{(1+x)}}{\sqrt{(1-\frac{v^2}{{b_0}^2}+x)}} (1+x). \tag{3.10}$$

There we have used the abbreviation $x = \frac{MG}{rb_0^2}$. In the GR it is common to use the Schwarzschild radius $r_s = \frac{2MG}{b_0^2}$. Therefore we can also write:

$$x = \frac{MG}{rb_0^2} = \frac{1}{2}\frac{r_s}{r} . \tag{3.11}$$

The result (3.10) describes the energy of the system "universe plus mass m", whereas m is moving along the connecting line between m and the mass M at rest. This formula for the energy is therefore also applicable for a static black hole, where only the gravitational attraction between m and M is acting and no angular momentum has to be taken into account. In contrast to the classical Newton consideration, now, besides the influence of the mass M, the distant masses of the universe are also effective.

Before we start to apply the formula (3.10) to a black hole, we should briefly touch on some of the properties of this formula:

a) $v = 0$: $\quad E(v=0) = m_0 {b_0}^2 (1 + x) = E_0$: correct

b) $r \to \infty$: $\quad E(x=0) = \frac{m_0}{\sqrt{1-\frac{v^2}{{b_0}^2}}} {b_0}^2$: correct

c) $r \to r_s$ $\quad E(r=r_s) = m_0 {b_0}^2 \frac{\sqrt{(\frac{3}{2})}}{\sqrt{(\frac{3}{2}-\frac{v^2}{{b_0}^2})}} (\frac{3}{2})$. : no singularity !

d) $r \to 0$ $\quad E(r\to 0) \to \infty$: correct

The total energy shows a singularity for $r\to 0$. Of course, this is to be expected because of Newton's law of gravity. But it does **not** show a singularity for $r \to r_s$. We recognize here a divergence from the solution of the vacuum field equations in GR. In fact, the spatial spherical symmetric solution of the field equations in the GR is also time-independent (Birkhoff-Theorem). However, there is the difficulty that the exact solution in the Schwarzschild-metric becomes singular at $r = r_s$. As a so-called "co-ordinate"- singularity it is removable, e.g. by the Kruskal-transformation. But in the Kruskal form, the Schwarzschild metric then becomes time dependent. We will not go into a deeper discussion of this circumstance here, but will only point to it at the moment. At first glance, it might be an advantage of the theory presented and applied here.

Let us now determine the size of the event horizon of a static black hole on the basis of formula (3.10), and yet follow the thought already formulated by John Mitchell in 1783 [5)]. A mass m, which tries to escape the gravitational attraction of a mass M, can obtain at most the velocity b_0. The kinetic energy is then given by

$$E_{kin} = E\ (v= b_0) – E(v = 0). \tag{3.12}$$

The mass m can only escape when the amount of E_{kin} exceeds the difference of the potential energy

$$\Delta E_{pot} = m(r,\ v= b_0)\ \frac{MG}{r} – m(r\rightarrow \infty, v=0)\ \frac{MG}{r\rightarrow\infty} = m(r,\ v= b_0)\ \frac{MG}{r}\ . \tag{3.13}$$

Therefore, the following has to be fulfilled:

$$E(v= b_0) – E(v = 0) \geq\ \Delta E_{pot}\ . \tag{3.14}$$

The mass $m(r, v=b_0)$ is given by (3.9)

$$m(r,\ v= b_0) = m_0\ \frac{\sqrt{(1+x)}}{\sqrt{(x)}}\ , \tag{3.15}$$

and one finds (for the equal sign in (3.14), i.e. for the maximum value of x)

$$m_0\ b_0^2\ \frac{\sqrt{(1+x)}}{\sqrt{(x)}}\ (1 + x) -\ m_0\ b_0^2\ (1 + x) = m_0\ \frac{\sqrt{(1+x)}}{\sqrt{(x)}}\ \frac{MG}{r} \tag{3.16}$$

or

$$\frac{\sqrt{(1+x)}}{\sqrt{(x)}}\ (1+x) -\ (1+x) = \frac{\sqrt{(1+x)}}{\sqrt{(x)}}\ x \tag{3.17}$$

and eventually

$$x^3 + 2x – 1 = 0\ . \tag{3.18}$$

The solution of (3.18) is x = 0,4534 and with (3.11):

$$r_{EH} = \frac{1}{2}\ \frac{r_s}{x} = 1{,}10\ r_s. \tag{3.19}$$

One can see that we find a solution without using the GR solely based on Newton's laws and Mach's principle. It qualitatively describes the essential property of a black hole, namely the existence of an event horizon. The quantitative value for the size of this event horizon is close to that resulting from GR ($r = r_s$). The difference amounts to about 10%. But we have not yet taken account of one of the basic results of the theory applied here, namely that the maximum velocity of a mass m depends on its position in the universe (see [1)]). If we take this into account, we find smaller values for r_{EH} if the black hole is not near the center of the universe. If, for instance, it is located near the edge of the universe, one finds a value of r_{EH} = 0,775 r_s. The GR does not show this deviation since the GR is founded, inter alia, on the postulate that the light velocity is a natural constant, the value of which is the same throughout the whole universe (with the exception of local deviations in local coordinate systems due to local masses).

We are not going to investigate this deviation from the result of the GR in more detail within this paper or to place a value on it. But the size of the event horizon should be measurable in principle, thus opening the possibility of falsifying the GR or the theory presented here.

If the dependency from the position of a black hole in the universe is correct, then the size of r_{EH} could possibly be used for the determination of cosmological distances, too, provided there are other properties of black holes like periodic alterations, which make them suitable as “standard candles”. In any event, it is questionable whether static black holes (without angular momentum) can be found in reality. In general, they will show angular momentum, and the above consideration has to be generalized, respectively, in order to allow the comparison between theory and experiment. But in any case, recent experiments have shown that black holes are much more complex than a simple static black hole and possess other components like magnetic fields, dust clouds or accretion disks, which will possibly impede the determination of the size of an event horizon to a higher accuracy than 20%. Therefore, let’s leave the theoretical consideration at that for the moment.

b) Special case: Force perpendicular to the connecting line

Now, we will investigate the case that the force F = $F\varphi$, which acts onto the mass m, is oriented perpendicular to the connecting line between m and M. In this case, the force $F\varphi$ effects a change of the location of m into the azimuthal direction ds = r $d\varphi$. We allow an additional force F_r to act on m, which shall always be oriented into the direction of M. Newton’s laws of motion have then to be written in the form:

for the motion in φ – direction

$$- \dot{m}v_{\varphi} - m\,\dot{v}_{\varphi} + F_{\varphi} = 0 \qquad (3.20)$$

and in the r – direction

$$- \dot{m}v_r - m\,\dot{v}_r + F_r - Z(v_{\varphi}) + m\frac{MG}{r^2} = 0. \qquad (3.21)$$

Here $Z(v_{\varphi})$ is the centrifugal force at the velocity v_{φ}. Let us now choose the amount of F_r in such a way, that it compensates for the difference between the forces $Z(v_{\varphi})$ and m $\frac{MG}{r^2}$ for each φ. This could be effectuated e.g. by charging the mass m electrically and letting it move within a magnetic field of suitable strength, which is also oriented perpendicular to the r – φ – plane. In this case, a motion into the radial direction is not possible (dr, v_r and $\dot{v}_r$ = 0). Therefore, an energy alteration through forces into that direction is not possible. The radial forces are also always directed perpendicular to the direction of ds and cannot cause an energy change into this direction, too. F_r becomes zero when the system has reached the stationary state. Then, the centrifugal force completely compensates the attraction of M onto m.

If we again multiply (3.20) with ds, we find:

$$-dm\, v_{\varphi}{}^2 - m v_{\varphi}\, dv_{\varphi} + F_{\varphi}\, dr = 0\,. \qquad (3.22)$$

The only one of the forces appearing in (3.21) and (3.22) which is able to add energy to the system is F_{φ}. It causes the energy alteration F_{φ} dr = dE. And we can, analogous to (3.4), try again the ansatz

$$F_\varphi \, dr = dE = m\,(b_0^2 + \frac{MG}{r}), \quad (3.23)$$

since we have to set again the following, of course, for the rest energy:

$$E_0 = m_0 \, G(\sum_i \frac{m_i}{r_i} + \frac{M}{r}) = m_0 \,(b_0^2 + \frac{MG}{r}). \quad (3.3)$$

(3.23) in (3.22) yields:

$$dm\,(b_0^2 - v_\varphi{}^2 + \frac{MG}{r}) = m\, v_\varphi \, dv_\varphi \quad (3.24)$$

or, if we replace for simplicity v_φ by v:

$$\frac{dm}{m} = \frac{v\,dv}{(b_0^2 - v^2 + \frac{MG}{r})} \,. \quad (3.25)$$

This is identical to (3.8), and for m and E there are again the relations (3.9) and (3.10). The only difference consists of the direction of v, which is now perpendicular to the connecting line between m and M. We denote the respective energy by $E\varphi$.

c) General Case: Force and motion in arbitrary direction

Until now we have treated the two cases of a pure radial and a pure azimuthal motion completely separately. The total energy of a mixed radial and azimuthal motion is not the plain addition of E_r and $E\varphi$, because the rest energy is contained in both energy forms and may not be counted twice. Hence, we have to subtract the rest energy once, when adding E_r and $E\varphi$, to compose the total energy:

$$E_g = m_0 \, b_0^2 \,(1+x) \, \left(\frac{\sqrt{(1+x)}}{\sqrt{(1-\frac{v_r{}^2}{b_0{}^2}+x)}} + \frac{\sqrt{(1+x)}}{\sqrt{(1-\frac{v_\varphi{}^2}{b_0{}^2}+x)}} \right) - m_0 \, b_0^2 (1+x). \quad (3.10a)$$

For the total velocity v the following relation is valid:

$$v^2 = v_r{}^2 + v_\varphi{}^2. \quad (3.10b)$$

The energy E_g described by (3.10a) fulfills the requirements

$$E_g\,(v_\varphi{=}0) = E_r \quad \text{und} \quad E_g\,(v_r{=}0) = E\varphi. \quad (3.26)$$

For a differential alteration of the total energy, the following requirement has to also be satisfied:

$$dE_g = \frac{\partial E_g}{\partial v_r} dv_r + \frac{\partial E_g}{\partial v_\varphi} dv_\varphi, \quad (3.27)$$

or respectively

$$\int dEg = \int \frac{\partial E}{\partial v_r} \, dv_r + \int \frac{\partial E}{\partial v_\varphi} \, dv_\varphi \,, \quad (3.28)$$

which is also met by E_g defined by (3.10a).

Since F_r is a central force, it has no influence on the amount of the angular momentum J, defined by

$$m\, r^2\, \dot{\varphi} = J. \tag{3.29}$$

J can only be changed by $F\varphi$ and is determined by v_φ or $\dot{\varphi}$ respectively (see (3.22).

As usual we define J = mh, then (3.29) assumes the shape

$$r^2\, \dot{\varphi} = h. \tag{3.30}$$

This can also be written in the form $$dt = \frac{r^2}{h}\, d\varphi. \tag{3.31}$$

We would like now to gain the path curvature of the mass m around the central mass M. For this purpose, we transform (3.10a):

$$\left(\frac{Eg}{m_0 b_0{}^2} + 1 + x\right)(1+x)^{-3/2} - \frac{1}{\sqrt{\left(1-\frac{v_\varphi{}^2}{b_0{}^2}+x\right)}} = \frac{1}{\sqrt{\left(1-\frac{v_r{}^2}{b_0{}^2}+x\right)}}\,. \tag{3.32}$$

To carry out the calculation clearly arranged, we introduce the abbreviations Γ_1 and Γ_2:

$$\Gamma_1 = \left(\frac{Eg}{m_0 b_0{}^2} + 1 + x\right)(1+x)^{-3/2} - \frac{1}{\sqrt{\left(1-\frac{v_\varphi{}^2}{b_0{}^2}+x\right)}}\,, \tag{3.33}$$

$$\Gamma_2 = \frac{1}{\sqrt{\left(1-\frac{v_r{}^2}{b_0{}^2}+x\right)}}\,. \tag{3.34}$$

Then (3.32) turns into the simple form

$$\Gamma_1 = \Gamma_2. \tag{3.32a}$$

With the definition $$\frac{v_r{}^2}{b_0{}^2} = y^2 \tag{3.35}$$

we find $$\frac{1}{\Gamma_2{}^2} = 1 - y^2 + x, \tag{3.36}$$

or with (3.32a): $$y^2 = \frac{(1+x)\,\Gamma_1{}^2 - 1}{\Gamma_1{}^2}\,, \tag{3.37}$$

or in other form $$\frac{1}{y} = \frac{\Gamma_1}{\sqrt{(1+x)\,\Gamma_1{}^2 - 1}}\,. \tag{3.38}$$

Using (3.31) and (3.35) we arrive at

$$\frac{d\varphi}{dr} = \frac{h}{r^2 b_0}\,\frac{\Gamma_1}{\sqrt{(1+x)\,\Gamma_1{}^2 - 1}}\,. \tag{3.39}$$

This relation is still fully exact. In order to identify the quantitative amount of the perihelion shift of the planet Mercury, we start now to carry out a series expansion. During this procedure we have to evaluate which terms may be neglected. Therefore, we first compile the order of magnitude for the relevant terms related to the conditions of the planet Mercury (to be found in [6)]).

$$\overline{v_r} \approx 6{,}2\ \frac{km}{s}\ ,\qquad \dot{\varphi} \approx 8{,}3\ 10^{-7}\ \frac{1}{s}\ ,\qquad \frac{{v_r}^2}{{b_0}^2} = y^2 \approx 4{,}3\ 10^{-10}\ ,\qquad \frac{{v_\varphi}^2}{{b_0}^2} = z^2 \approx 2{,}5\ 10^{-8}\ ,$$

$$\bar{h} = \bar{r}\ \overline{v_\varphi} \approx 2{,}8\ 10^9\ \frac{km^2}{s}\ ,\qquad \frac{\bar{h}^{\,2}}{\bar{r}^{\,2}{b_0}^2} \approx 2{,}5\ 10^{-8}\ ,\qquad r_s \approx 2{,}9\ 10^3\ m\ ,\qquad \frac{r_s}{\bar{r}} \approx 5{,}1\ 10^{-8}\ ,$$

$$x \approx 2{,}5\ 10^{-8}\ ,\qquad v^2 \approx 2{,}3\ 10^3\ \frac{km^2}{s^2}\ . \tag{3.40}$$

Using, additionally, the abbreviation $E' = \frac{Eg}{m_0 {b_0}^2}$, we find the series expansion for Γ_1:

$$\Gamma_1 \approx E' - \frac{3}{2} E'x - \frac{1}{2} z^2 + \frac{3}{4} z^2 x\ . \tag{3.41}$$

In this relation, we have neglected terms of higher order than z^2 and x, because, due to (3.40), we find as a very good approximation: $z^4 \ll z^2$ und $x^2 \ll x$.

If we neglect higher order terms in the same way, we get:

$$\Gamma_1^{\,2} \approx E' \,(E' - 3\,E'x - z^2 + 3\,z^2x) \tag{3.42}$$

and

$$\Gamma_1^{\,2}(1+x) \approx E' \,(E' - 2\,E'x - z^2 + 2\,z^2x). \tag{3.43}$$

Inserting this into (3.39) yields eventually:

$$\frac{d\varphi}{dr} = \pm\,\frac{h}{r^2 b_0}\,\frac{E' - \frac{3}{2}E'x - \frac{1}{2}z^2 + \frac{3}{4}z^2x}{\sqrt{E'}\sqrt{(E' - 2\,E'x - z^2 + 2\,z^2x) - \frac{1}{E'}}}\ . \tag{3.44}$$

We would like to further simplify this relation. For this purpose we divide E_g into the constituent parts “rest energy” and “kinetic energy”:

$$E_g = m_0\,{b_0}^2\,(1+x) + E_{kin}\ . \tag{3.45}$$

Then we can write

$$E' = 1+x + \frac{E_{kin}}{m_0 {b_0}^2} = 1+x+E''. \tag{3.46}$$

With this definition, we continue the series expansion in (3.44):

$$\frac{d\varphi}{dr} = \pm\,\frac{h}{r^2}\,\frac{\left(1-\frac{1}{2}x-\frac{1}{2}E''\right)(1+x+E'')\,\left(1-\frac{3}{2}x-\frac{1}{2}\frac{z^2}{E'}+\frac{3}{4}\frac{z^2}{E'}x\right)}{\sqrt{2E''b^2 - 2\,E''b^2x - {b_0}^2z^2 + 2{b_0}^2z^2\,x}}\ . \tag{3.47}$$

With (3.10a), (3.45), (3.46) and the quantities (3.40) for the circumstances of Mercury, we can see that E‘ is of the order of 1, whereas x and E“ are smaller by a factor of 10^{-8}. If we substitute now r by 1/u and x by $\frac{1}{2}\,\frac{r_s}{r}$ (see (3.11)), we can write in a very good approximation:

$$d\varphi = \pm \frac{h\,du}{\sqrt{2E''{b_0}^2 - E''{b_0}^2 r_s u - h^2u^2 + h^2 r_s u^3}}. \tag{3.48}$$

We would like to compare this result with the path curvature of a planet, which is calculated solely on the basis of Newton's laws **without consideration of the other planets and the distant masses**. It is given by

$$d\varphi = \pm \frac{h\,du}{\sqrt{\frac{2E_N}{m_0} + {b_0}^2 r_s u - h^2u^2}}. \tag{3.49}$$

Here E_N is Newton's total energy

$$E_N = \frac{1}{2}\, m_0\,({v_r}^2 + {v_\varphi}^2) - \frac{m_0 MG}{r} = \frac{1}{2}\, m_0\, v^2 - \frac{m_0 MG}{r}\,,$$

or written in another form

$$\frac{1}{2}\, m_0\, v^2 = E_N + \frac{m_0 MG}{r}\,. \tag{3.50}$$

To compare (3.49) with (3.48) we remember, how we defined E" in (3.46):

$$E'' = \frac{E_{kin}}{m_0 {b_0}^2}\,. \tag{3.46a}$$

Here E_{kin} has to be determined according to (3.45) referring to the total energy E_g (3.10a). If we again neglect terms of higher order then $\frac{{v_r}^2}{{b_0}^2}$, $\frac{{v_\varphi}^2}{{b_0}^2}$ and x (which due to (3.40) is again a good approximation), we find:

$$E_g - m_0\, {b_0}^2\,(1+x) = \quad E_{kin} = \frac{1}{2}\, m_0 v^2 + \frac{3}{4}\, m_0 v^2 x \approx \frac{1}{2}\, m_0 v^2. \tag{3.51}$$

Hence, the kinetic energy calculated on the basis of the theory used here is, in case of Mercury, in very good accord with the classical theory of Newton. Because of (3.50) and (3.51) we, therefore, can write:

$$E'' = \frac{E_N}{m_0 {b_0}^2} + \frac{MG}{r {b_0}^2} = \frac{E_N}{m_0 {b_0}^2} + x. \tag{3.52}$$

If we insert this in (3.48) we find:

$$d\varphi = \pm \frac{h\,du}{\sqrt{2(\frac{E_N}{m_0 {b_0}^2} + x){b_0}^2 - (\frac{E_N}{m_0 {b_0}^2} + x\,){b_0}^2 r_s u - h^2u^2 + h^2 r_s u^3}},$$

or

$$d\varphi = \pm \frac{h\,du}{\sqrt{2\frac{E_N}{m_0} + {b_0}^2 r_s u - (x{b_0}^2 + \frac{E_N}{m_0})\, r_s u - h^2u^2 + h^2 r_s u^3}}\,. \tag{3.53}$$

We assess again the order of magnitude for the circumstances of Mercury, here concerning the term $(xb_0{}^2+\frac{E_N}{m_0})$:

$$(xb_0{}^2+\frac{E_N}{m_0}) = \frac{1}{2}\frac{r_s}{r}b_0{}^2 + \frac{1}{2}v^2 - \frac{1}{2}\frac{2MG}{rb_0{}^2}b_0{}^2 = \frac{1}{2}b_0{}^2\left(\frac{v^2}{b_0{}^2}\right) . \qquad (3.54)$$

It is to be seen that the third term under the root (3.53) is to a factor of 10^{-8} smaller than the second one and we can neglect it. It would additionally take account of a small influence, which can be attributed to the influence of the SR alone [8], which is as well incorporated in the theory presented here (see section 2 above and [1]).

With this final approximation, we find eventually:

$$d\varphi = \pm \frac{h\,du}{\sqrt{2\frac{E_N}{m_0} + b_0{}^2 r_s u - h^2u^2 + h^2 r_s u^3}} . \qquad (3.55)$$

It can be seen that this relation migrates for small u (i.e. large r) into the classical Newtonian case, and it is also in very good accordance with the respective relation of the GR [7), 8)]. It contains the important fourth term, which brings in a dependency of u^3 and, therefore, a closed path is no longer possible. Such a closed path is allowed if Newton's theory is applied without considering the influence of the other planets and also neglecting the influence of the distant masses. For Mercury, the term with u^3 is responsible for the famous additional perihelion shift of 43.03'' per century (3.8" for the earth and 8.6" for Venus) [7), 8)]. These are exactly the same figures that result from the theory presented here.

4. Summary and conclusion

It is shown that General Relativity is not needed for the explanation and quantitative description of the perihelion shift of the planet Mercury as well as of the event horizon of black holes. This explanation succeeds on the basis of Newton's laws alone if the distant masses of the universe are included into the consideration, i.e. if Mach's principle is duly respected. Quantitative calculations partly show deviations to the respective results of the GR. In the case of black holes, there is a small, but probably measurable, difference with the size of the event horizon. The result for the perihelion shift of Mercury obtained here is nearly identical with that of the GR.

The connections revealed here, together with the results found in [1), 2)] and [3)], lead to the conclusion that the theory applied here digs deeper than the Theory of Relativity, and that means both the Special as well as the General Theory. Compared to the SR and the GR, the theory applied here exhibits the advantage that the constancy of the light velocity does **not** have to be postulated. Rather, this constancy is one of the results of the theory, and it turns out to be not generally valid but only valid within spatial volumes that are small compared to the extent of the universe.

A further and fundamental difference to the SR and GR appears in the fact that the long-range-order (or action-at-a-distance principle) of Newton's force of gravity constitutes an essential part of the theory here, though it leads to a maximum velocity for moving masses.

Hence, the theory is, with respect to the fundamental law of gravitation and the distant masses, non-local, but it describes some essential properties of the nature as local, becoming manifest with the maximum velocity of moving masses and light. Thus, it seems not to be in contrast to the non-local character of nature, which has been demonstrated in a huge number of experiments with entangled quantum states (recently even on cosmic dimensions [9)]). Yet, the principle of causality is preserved in the theory, due to the maximum velocity for masses and light (see [8)], p. 29).

The theory applied in the present paper introduces, together with the preceding papers [1), 2)] and [3)], the basic principles of a fundamental theory which, apparently, forges ahead to a deeper cognizance than the Theory of Relativity. Still, the geometrical description of the nature by the GR is, of course, mathematically elegant as well as aesthetic. However, it seems that the GR is not exactly valid, but it will prove as an approximation of an even better-grounded theory. The instantaneous influence of the distant masses of the universe, i.e. the observance of Mach's principle, seems to be fundamental in this respect.

It has to be investigated whether and how the theory demonstrated and applied here for some (but important) examples could be better formalized and generalized (e.g. as a field theory as in [2)]). And of course, it has to be further discussed whether it represents a deeper insight compared to the Theory of Relativity.

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