Towards Science Unification Through Number Theory

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Abstract

The Number Theory comes back as the heart of unified Science, in a Computing Cosmos using the bases 2;3;5;7 whose two symmetric combinaisons explain the main lepton mass ratios. The corresponding Holic Principle induces a symmetry between the Newton and Planck constants which confirms the Permanent Sweeping Holography Bang Cosmology, with invariant baryon density 3/10, the dark baryons being dephased matter-antimatter oscillation. This implies the DNA bi-codon mean isotopic mass, confirming to 0.1 ppm the electron-based Topological Axis, whose terminal boson is the base 2 *c*-observable Universe in the base 3 Cosmos. The physical parameters involve the Euler ideonal numbers and the special Fermat primes of Wieferich (bases 2) and Mirimanoff (base 3). The prime numbers and crystallographic symmetries are related to the 4-fold structure of the DNA bi-codon. The forgotten Eddington's proton-tau symmetry is rehabilitated, renewing the supersymmetry quest. This excludes the concepts of Multiverse, Continuum, Infinity, Locality and Zero-mass Particle, leading to stringent predictions in Cosmology, Particle Physics and Biology.

***K*eywords** Number theory · Optimal Computation Principle · Holic Principle · Cosmology · Supersymmetry · String Theory · Bit-String Physics · Cellular Automaton · DNA nucleotides · Crystallography · Sporadic Groups.

1 Introduction

Since the Poincaré-Hilbert opposition, mathematicians separated in intuitionnists and formalists. Poincaré convinced at last the reticent communauty that there is no means to escape the quantization of the field-matter interaction, claiming a return to Pythagorism [1]. In particular, he explained that the cosmology cannot be founded on differential equations, because of the involved "free parameters" in an *unique* Univers [2]. However, the mainstream has followed Hilbert and his differential equations, leading nowadays to a blockage of the 6 "free parameters" standard cosmology, with its "mysterious" so-called "dark energy" with the relative present density about 0.7 [3], and the negation of the Universe unicity [4], the Multiverse concept. We showed [5] that the latter is misleading and that the missing energy density is *invariant and exactly* 7/10, as recalled below.

The situation is even worse in the particle physics, with about 20 adimensional "free parameters". The Table 1 presents the main ones, and the Table 2 recalls the main physical constants [3]. For specifying some values, an Optimal Correlation Principle has been applied, interpreting the "free parameters" as computation bases in a Computing Cosmos [6]. This article shows, more fundamentally, the symmetry between the four basic primes: $2;3;5;7$, considered as the cosmic computation bases. Indeed, the symmetric combinations $2\times3\times5\times7 = 210$ and $(2+3+5+7)(2\times3\times5\times7) = 3570$ give the brute mass ratios, relative to electron, of the Muon and Tau.

The standard Supersymmetry receives no confirmation from the CERN super-collisionner. This article shows that a number of symmetries have been overlooked, in particular a meson eta-lepton tau symmetry, which could enter a future

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Corrected Supersymmetry, and also the Eddington's proton-tau symmetry, which could enter a future "Intrasymmetry", connecting particles of the same spin. It has been forgotten that Eddington predicted the tau fermion, 35 years before its surprising discovery, with a correct estimation of its mass. Indeed, starting from *a non-standard electron-proton symmetry*, Eddington deduced a "mesotron-heavy mesotron" one, the old name for the couple muon-tau. This article rehabilitates these symmetries, in liaison with an Optimal Computation Principle (OCP), which implies the last ideonal Euler number, connected with the Riemann hypothesis. This OCP favors the form x^x and the numbers exp(exp(... characterizing at last the "free" parameters, in connection with the Holic Principle, recalled in Section 3.

The table 1 shows the pertinence of the most famous large prime number in Mathematics history, the Lucas Number, which enters the last term of the Combinatorial Hierarchy [7] [8]. Indeed, it defines a gravitational proton-mass ratio p_G, connected with u_{30}, where u_n is the Bisection of Rule 23 cellular automaton Wolfram series [9]. The dimension $d = 30$ is the last dimension of the Topological Axis, corresponding to the terminal term $k = 7$ of the Bott octonion sequence of the special string dimensions (Table 3). This article shows that p_G enters also a 0.1 ppm correlation with the Higgs-electron mass ratio $H^{(0)} = 495^2$ [5], leading to a connection between gravitation and the gauge couplings of the standards groups U(1),SU(2) and SU(3). *This is on the way to resolve the "hierarchy" problem, the huge gap between gravitation and electroweak couplings.*

The Eddington's Fundamental Theory [10] has been disregarded. Indeed, at his epoch, it was strange to interconnect cosmology and microphysics, and even stranger to introduce a so simple Large Number $N_{Ed} = 136 \times 2^{256}$ as the number of hydrogen atoms in the Universe. Nowadays, nobody realizes that it is precisely the number of *neutron masses* in 30 % of the Universe critical mass, using the following simple formula involving the Planck mass and the four basic masses of electron, proton, Hydrogen and neutron, *revealing a 4-fold symmetry*:

$$M/m_n = m_P^4/m_e m_p m_H m_n \approx (10/3)N_{Ed} \tag{1}$$

This gives a critical radius $R = 2GM/c^2 \approx 13.80$ Giga light-years, at 5×10^{-4} from the Hubble radius given by the following Eddington-Sanchez's symmetrical double relation, which involves the electric-gravitation force ratio in the hydrogen atom [5]:

$$R/2\lambda\!\!\!^{-}_H = (M/m_e^{(red)})^{1/2} = \hbar c/Gm_e m_p \tag{2}$$

where $m_e^{(red)} = m_e m_p/m_H$ is the formal reduced electron mass in the hydrogen atom. Such a pertinence of the Lucas Number and the Eddington Number means that the Pythagorean Natural Philosophy comes back, called now "Number Theory": this is the main goal of the present article to confirm this. According to Russel "The most surprising in the modern science is its come back to Pythagorism" [11]. It is a big surprise indeed, since the formalist mainstream rather followed Dirac when he interpreted the Cosmic Large Number correlation as age-varying, with a gravitation constant G varying with time. It was forgotten that Poincare clearly specified that Physics is not possible with varying constants [2]. Also, *an excess of reductionism has blocked the formalists, unable to imagine that cosmology may be simpler than microphysics, and do not really need particles with mass inferior to the electron one.*

The String Theory uses already the Number Theory. However, nobody understand why 496, the string group SO(32) dimension, is the third perfect number. This article shows the liaison with 495, the square root of the scalar boson/electron mass ratio. With its continuum space, the string theory compactification process results in an enormous multiplicity of solutions, of order $(10)^{500}$ [12]. By replacing the continuum by a "quantinuum", this article eliminates this multiplicity, using the above celular automaton series, to comes back to the predicted 1D Sweeping Holography process: *in an absolute space a moving object sees a variation of its internal period, explaining the local Relativity and the Parity violation* [13].

Any cosmology founded on *integral* equations must take into account the well-known Newtonian gravitational potential energy $E_{pot} = -(3/5)GM^2/R$ of an homogeneous ball of mass M and radius R. For the critical radius $R = 2GM/c^2$, this is the energy $E_{pot} = -(3/10)Mc^2$, which is exactly the opposite of the *non-relativist* kinetic energy E_{kin} of the receding galaxies in the steady-state model, [5] which is so characterised by the following classical integral equation, explaining directly the relative density 0.3 of the combinaison matter + dark matter, so justifting the complement 0.7:

$$E_{kin} = -E_{pot} = (3/10)Mc^2 \tag{3}$$

The deepest mystery of the standard cosmology is the clearest evidence for its opponent simplest model, the non-relativist permanent Newtonian model, proving that the standard cosmology is unduly complicated. Recall that Bondi [14] showed that the Newtonian Cosmology is sufficient to obtain the principal results of General Relativity. In particular the above critical formula, corresponding to an overall euclidean geometry, $R = 2GM/c^2$ states simply that the Newtonian liberation speed at the Hubble surface, $(2GM/R)^{1/2}$ is identified to c. *So one may consider that receding far galaxies reach the speed c and exceed it when they disappear at the Hubble horizon.*

This means that the Restricted Relativity was unduly applied to cosmology. Indeed, it is a *local* theory, based on the concept of an inertial referential, which cannot be defined by the theory. By contrast, the above Newton Absolute

Space is observationally evident through the Cosmic Microwave Background (CMB). In liaison with the on-going production of Helium in the stars, such a thermal bath was predicted by the steady-state model, with the correct temperature (contrary to the erroneous prediction of the Big Bang tenants : 10K for George Gamow, 30 K for James Peebles) [15]. Even worse, the precise thermal Planck spectrum was contradictory with the initial standard Big Bang model, which was obliged to introduce the monstruous inflation step, bringing as many problems as it resolves [16]. Moreover, one of the three conditions of Sakharov [17] for the elimination of antimatter during the Big Bang is precisely the rupture of thermal equilibrium. Thus, *the Cosmic Microwave Background is the refutation of the Big Bang, but it is unduly presented as its confirmation.* Note that the tenants of the steady-state model failed to find a thermostatic agent able to transform star light into microwave radiation. It is much simpler to admit that the CMB is simply the emanation of the *external* Cosmos [5].

Also, the steady-state model has predicted an *exponential* galaxy recession. In the standard model, it was a shocking surprise to observe an acceleration of the galactic recession (unduly interpretated as a Space dilatation). *The prediction of the accelerated galaxy recession from the concurrent model has been simply forgotten.* Such a conandrum comes from the fact that every phenomena which cannot be explained by the standard model is not seriously considered. In particular, this is the case for the non-Doppler power oscillation of several quasars and the sun [18]. This directly proves the existence of tachyonic physics, which is a necessity since the speed c is far too small to connect a so vast Universe. Indeed, the simplest application of the primary method in Physics, the *c-free* dimensional analysis, gives half the Hubble radius [19] [20]. *The foundators of the modern cosmology have missed the most elementary calculation*. Even Eddington did not emphatize that his above formula induces a R value independent of c, see Eq. (5). This is due to the catastrophic use of the so-called "natural units" where $c = 1$, assimilating Time to Space. In his basic text founding Restricted Relativity, Poincaré [21] insisted that the 4D space-time concept must *not* be systematised.

It is claimed that the present epoch is marked by the domination of matter on radiation. This is true for the energy, but the reverse holds for the particle population ratio. Despite the fact that the latter is considered as a standard key parameter, called "Critical Entropy", nobody, apart us [22], signaled the following tight correlation, where the factor 2 is the polarisation factor:

$$Mc^2/(E_{CMB} + E_{CNB}) \approx (2n_{ph}/n_H)^{1/2} \quad ; \quad n_{ph} = (4\pi/3)(R/l_{phCMB})^3 \; ; \; n_H = M/m_H \tag{4}$$

Note the formal similitude with Eq(2): *this shows a fundamental but non-standard Cosmic Matter-Field Symmetry*. This is specified in the Table 2, which presents these two adimensional nearby (0.3 %) quantities K_E and K_n which, in the standard cosmology, are believed to follow different temporal evolutions. *The ratio K_E shows a double expression specifying the electric charge to ppm precision*. This is a strong argument in favor of a synthesis of the steady-state model and the standard Big Bang one, since both the Cosmic Microwave Background (CMB) and the Cosmic Neutrino background (CNB) are invoked. Such a model was proposed: the Permanent Sweeping Holographic Oscillatory Bang (PSHOB), where the matter is considered as a very rapid matter-antimatter oscillation, and dark matter a π-dephased oscillation [23] [24].

The general tendency of the *formalist mainstream* has been to neglect such correlations, invoking either the hazard or a multiplicity of Universes. This much controversed Multiverse has been associated to an incongruous application of the so-called "anthropic principle" [4]. Using the argument of an Universe with a finite age of several giga years, it was argued [25] that this age is comparable with that of a star, which by the novae process, forms the heavy elements necessary to Life, so the main Eddington's correlation would be a neccesity. This neglects completely the second Eddington's correlation, which leads, by a simple statistical argument to define the Hubble radius, from the simple, but non-standad, hypothesis of an electron-proton symmetry.

It is confounding that this fallacious "anthropic" argument was universaly accepted by the physics communauty. The reason for this success is that, more or less conciously, people wants to reintegrate physics and biology. In this respect, it is even more stupefious that nobody seems to realize that the masses of the two DNA nucleotides couples are the same, within an hydrogen atom [23]. So the mass of the bi-codon (3 nucleotides couples) is defined to 3 hydrogen, and very close to 1837, which is the hydrogen-electron mass ratio. This article shows, by considering the pure isotopic masses, that the precision reaches 0.1 ppm. Moreover, this bi-codon mass connects also with the central dimension 16 of the Topological Axis, to again 0.1 ppm precision.

This Topological Axis (Graph 3) is the extension of the double Eddington Large Number Correlations, leading to a Bott octonion sequence of dimensions, which rehabilitates the bosonic String Theory [5]. The latter was *unduly* discarded because it involves a tachyon. Quite the contrary, the tachyon interprets the above *non-Doppler* quasar power oscillation.

A great hope in theoretical physics is the Holographic Principle, which is precisely of integral type (not differential). However, the standard model cannot use it on the Hubble sphere because its radius is considered as variable. It suffices to admit its invariance, and apply this Holographic Principle with the Topon, the wavelength of the whole Universe,

to demonstrate the critical cosmic character. So, there is no need to the standard inflation, which appears as an ad-hoc "epicycle". This could not have been seen before, because this Topon rejects the Planck wall by a factor 10^{60}. *But it is very surprising that cosmologists, who consider the Universes as a whole, have not introduced the Universe wavelength (Topon), which justifies at last the* 10^{120} *quantum vacuum/Universe energy ratio* [5].

The Section 2 recalls how the application of the c exclusion and the Holographic Principle lead to the PSHOB cosmology and introduces the connection between gravitation, CMB, and the gauge couplings g_1, g_2 and g_3 of the groups U(1), SU(2) and SU(3). The Hubble radius invariance is definitely proved by the Table 4 showing 55 simple formula, with 8 in the 0.1 ppm domain, while the Table 5 confirms the Cosmos radius by 54 formula, with 10 in the 3 ppm domain.

The Section 3 applies the Holic Principle, the degenerate diophantian form of the Holographic Principle. It shows how the mass concept introduces a symmetry between gravitation and quantum physics in the Keplerian Diophantine Equation. This connects the Bohr's orbits with the PSHOB model, showing up the mean DNA bi-codon mass which connects to 0.1 ppm with the Topological Axis. *The local c-observable base 2 Universe appears as a gauge boson in the base 3 Tachyonic Cosmos with massive photon and graviton*. These masses are obtained from the *two-step* interaction, involving a high speed C precursor, by analogy with the two-step real holography technique, using the above non-local quasar period. Note that the concept of a "fuzzy-ball" photon is non necessary [26]. *What is general is the symmetry Field-Matter: the propagation is wavy (Shrodinger equation), but the reception is a quantum process (wave packet reduction). All that is incomprehensible without the above cosmic precursor concept.* The Section 4 is devoted to the properties of the DNA, showing outstanding 0.1 ppm connections with the Topological Axis and the main "free parameters".

The Section 5 studies the multi-dimensional cristallography, with emphasis to its relation with the "rule 23" celular automaton Wolfram bisection series. By characterising the main integer parameters 137 and 1836, this leads to a liaison with the string compactification problem. The nomber 1836 is connected with the maximal Euler suitable number, tied to the generalized Riemann hypothesis. The base 2 Wieferich last prime permits to define the fine-structure constant within its 0.15 ppb indetermination. Considering the connection between multi-dimensional crystallography and the Periodic Table, the running number of the string dimension series of the Topological Axis is identified with the orbital quantum number, associating the spin 1/2 and the string dimension 2. The conclusion, Section 6, resumes the advances and presents a series of stringent predictions in cosmology, particle physics, chemistry and biology.

2 The Permanent Sweeping Holographic Oscillatory Bang Cosmology (PSHOB)

2.1 The Topological Axis, the *invariant* Hubble radius and the Economic Computation Principle

The present-day value for R introduces a dramatic crisis. Despite an optimisation of six parameters, the standard ΛCDM cosmologic model(Cold Dark Matter with cosmological constant) leads to the so-called present-day Hubble constant $H_0 = c/R \approx 67.74 \pm 0.46$ km/s by Megaparsec [3], while, since several years, direct measurements, using different methods, confirm a significant discrepancy. In particular, the latest measurement is 69.8 ± 0.8 [27].

This discrepancy is widely discussed, but nobody signals this last value was predicted long ago, by the mere application of c-free MLT dimensional analysis [19] [20]. This value was later justified by the model of the Gravitational Hydrogen Molecule [23], which is also the limit of a star radius when its atomic number is reduced to unity [28] and verifies the above double relation Eq.(2). With the Giga light-year unit (Gly), and the electron reduced wavelength $\lambda\!\!\!^{-}_e = \hbar/m_e c$, this Hubble radius is:

$$R = 2a_G \, \lambda\!\!\!^{-}_e = 2\hbar^2/Gm_e m_p m_H \approx 13.811977 \;\; Gly \tag{5}$$

corresponding to $H_0 \approx 70.790 \;\; kms^{-1}Mpc^{-1}$, compatible with the above last measurement. Moreover, this value connects within 10^{-3} with the reduced topological function $g(k) = exp(2^{k+1/2})/k$, for $k = 6$, see Graphics 3. This value $k = 6$ corresponds to the privileged dimension 26 of the string bosonic theory:

$$R \approx g(6) \, \lambda\!\!\!^{-}_e = 13.82 \;\; Gly \tag{6}$$

This Topological Axis generalizes the Large Number Correlations, whose the *single* official justification is the Anthropic Principle, also used to justify the so-called "biologic fine-tuning" through the Multiverse model [4]. But such a rough argument cannot explain the above precision, so *the Topological Axis recovers the ancestral idea of an unique Universe. The above Optimal Correlation Principle (OCP) favors the "economic" of* x^x *form*. In particular, with $e_2 = e^e$ and $e_3 = e^{e^e}$, one observes:

$$e_3 \approx aHe_2 \approx p_{W1}p_{W2} \approx \eta\tau \tag{7}$$

showing the Wieferich prime couple: $p_{W1} = 1093$, $p_{W2} = 3511$, [29]. Ths is the *two-numbers-only* series A001220 in the On-line Encyclopedia of Integers sequences (OEIS), showing an outstanding non-standard symmetry meson eta-lepton tau. This leads below to an expression for the inverse fine-structure constant a in its 0.15 ppb indetermination (section 5.3).

The Hubble-Table 4 recalls the 14 molecular formula of Jean Perrin that definitely established the existence of atoms, based on 6 different theories. In this table also, numeric terms from *different approaches* are used: [4], [10], [7] [28], and [30]. But the most significant is the Holic Theory, recalled below, characterized by "primeval 7", the number $210 = 2 \times 3 \times 5 \times 7$. The appearance of 210^{210} and μ^{μ} resolves at last a complete mystery in the standard model: the *arithmetic* origin of the muon/electron mass and tau/electron mass ratios. Note that the approximation $\mu \approx 210$ is central in the Bit-String Physics [8], and will be confirmed below.

2.2 The Classical Universe Radius and the Cosmic Gravity-CMB-Microphysics connection

The standard theory associates conservation law with symmetry. However, a conservation law can be seen as the result of a computation. Considering that the Universe is a computing black-hole of radius R, this introduces the length $(Rl_P^2)^{1/3} \approx 10^{-15}$ m [23]. The identification with the *invariant* electron classical radius defines the Classical Universe Radius R_e. *It is the radius eliminating c between the classical electron radius and the Planck length formula. This radius presents a very precise dramatic holographic property involving the CMB Wien wavelength*:

$$\begin{cases} R_e = 2r_e^3/l_P^2 = 2\hbar^2/Gm_N^3 & M_N = m_P^4/m_N^3 \\ 4\pi(R_e/\lambda_{WCMB})^2 \approx e^a \end{cases} \tag{8}$$

where $M_N = R_e c^2/2G$ is the associated critical mass, implying the Nambu mass $m_N = am_e$, central in particle physics [31]. The OCP introduces the "Weinberg-Sanchez" natural geometric extension of the Weinberg triangle [32], $1/g_0 = 1 + g_1^2 + g_2^2 = 1 + (Z/H^{(0)})^2$. With the BE-Higgs scalar boson mass ratio, by respect to electron: $H^{(0)} = 495^2$ (125.208 GeV) [5], the radius ratio R_e/R obeys the following 10^{-7} precise relations, involving G through $p_G = P/2^{127/2}$ (from the Combinatorial Hierarchy [7]) and the Wien factor w_i. From the OCP, the following coupling g_3 and the electric charge q are proposed:

$$\begin{cases} 1/g_0 = 1 + g_1^2 + g_2^2 \approx 2R/R_e \approx 2(H/p_t)(\beta\sqrt{a_s}/w_i)^{1/2} \\ tg\theta = g_1/g_2 = g_0/g_3 \\ g_1 cos\theta = g_2 sin\theta = (4\pi_q/a)^{1/2} \approx y\Pi_+/2d_e\sqrt{F} \quad ; \quad 3^{11-1} \approx 6(2\pi_q)^5 \approx \pi a^2 \end{cases} \tag{9}$$

Note that the electric charge is tied to the Mirimanoff prime 11 [33], showing the connection supersymetry-supergravity 10 = 11 - 1. The small difference between g_0 and g_2 induces formula in the Hubble and Cosmos Tables. In this manner, the gauge couplings are connected with the Wien factor w_i, whose pertinence is not reckognized in the present standard model. Moreover, there is a direct confirmation, precise to 0.1 ppm, of the CMB temperature invariance:

$$K_n = (32\xi(3)/3\pi)^{1/2} Rl_P/(\lambda\!\!\!^{-3}_{CMB}\lambda\!\!\!^{-}_H)^{1/2} \approx K_E(1 - 1/12^2)^{-1/2} \tag{10}$$

This means there is a symmetry between radiation and matter, totally unexpected in the present standard theory. This relation has the same form than that of Eddington : $\sqrt{M/m_e} = R/2\lambda\!\!\!^{-}_H$*, equivalent to Eq. (5).*

2.3 The Tachyonic Holographic Cosmos

The radius R_e, about 30% larger than R, was identifed to the holographic reduced Cosmos radius [19], defined by the Bekenstein-Hawking entropy of the R_e-radius sphere [34]: $\pi(R_e/l_P)^2 = 2\pi R_C/l_P$, so:

$$R_c = R_e^2/2l_P \approx 2^{128} l_P(Rg_0/\lambda\!\!\!^{-}_H)^2 \approx 9.075773 \times 10^{86} m \tag{11}$$

The Table 5 presents 54 simple formula confirming this Cosmos radius R_c. In particular, there is the dramatic geometrical property, the "Geo-adimensional Cosmos-Universe couple" (Fig.1):

$$(ln(R_c/\lambda\!\!\!^{-}_e))^2 \approx (ln(M_N/m_e)^2 + 2(ln(R/\lambda\!\!\!^{-}_e))^2 \tag{12}$$

where the 2 factor comes from *the local character of the speed c* [5].

Table 1: Adimensional primary constants

name	symbol	value	remarks
Euler-Napier constant	e	2.718281828459042	optimal base
Archimedes constant	π	3.14159265358979	
Euler-Mascheroni constant	γ	0.57721566490153	
Golden ratio	Φ	1.61803398874990	
Wien factor $w_i = hc/k_BT\lambda_{Wien} = 5(1-e^{-w_i})$	w_i	4.96514245	
Relative Radiation Ratio $u_{CMB+CNB}/u_{CMB} = 1 + 3(7/8)(4/11)^{4/3} \approx p_t n_t/Hp_{W1}$	y	1.681321953	$p_{W1} = 1093$ (A001220)
Scale-factor $8\pi^2/ln2$	j	113.9106346	[5]
Lucas Large Prime Number	n_L	$2^{127}-1$	[7]
Eddington Large Number	n_{Ed}	136×2^{256}	[10]
Maximal Euler Ideonal ("Suitable") Number	s_{65}	1848	OEIS (A000926)
Extended Maximal Number of Intersection of Diagonals from a 16-gone	p_{00}	1836	OEIS (A000332)
Proton/Electron mass ratio m_p/m_e	p_t	1836.15267343	0.06 ppb
Wyler Proton/Electron mass ratio $6\pi^5$	p_W	1836.118109	[40]
Hydrogen/Electron mass ratio	H	1837.15266014	0.06 ppb
Relativistic correction factor $1/(H-p_t)$	β	1.000026597	0.1 ppb
Neutron/Electron mass ratio $n_t \approx (\pi^2 w_i/4)^3 \approx p_t + 1 + 1/g_0$	n_t	1838.6836617	0.5 ppb
Planck ratio $m_P/m_e \approx 2^{2^6}(a/4\pi)^3 p_t\beta^2/n_t$	P	$2.389015907 \times 10^{22}$	ppb [5]
Lucas Gravitational Proton/Electron mass ratio $P/\sqrt{n_L} \approx \sqrt{2}(a/137)^2 p_t/p_W q^6$	p_G	1831.531181	$\approx \beta u_{30} a/137$
Bisection of Rule 23 Wolfram series (R23W) $u_d = 1 + d(2d+1)$ for $d = 30$	u_{30}	1831	EOIS(A266438)
Combined R23W: $v_d = u_{2D=2(d+3)} = 1 + 2(d+3) + 8(d+3)^2$ for d = 12	v_{12}	1831	30D to 12D
R23W for $d = 8$: Eddington's formula [10]: $u_8 = 136 + 1$	u_8	137	$v_8 = 495 + 496$
Combined R23W $v_1 = u_{2\times(1+3)}$ for d = 1: Hol Sweeping 1D Absolute Space	v_1	137	12D to 1D
$u_{2d} = 1 + 2d(4d+1)$ Bissection of odd central polygonal numbers for d = 4	a_{00}	137	OEIS (A188135)
Eddington-Atiyah constant $2^7 + 2^3 + 2^0$	a_{00}	137	from three algebra [30]
Maximal number of zones defined by 16 straight lines in a plane	a_{00}	137	OEIS (AA000124)
Number of partitions of 20 that do not contain 1 as a part	a_{00}	137	OEIS (A002865)
Square root of $105^2 + 88^2$, where 105 = Part(19) , 88 = Part(18)	a_{00}	137	$88 \times 105 = 5 \times s_{65}$
Sum $v_1 = u_1^2 + u_2^2$ implying usual 4D and supergravity 11D	a_{00}	137	$u_1 = 4$; $u_2 = 11$
Third term of Combinatorial Hierarchy $(2^2-1) + (2^3-1) + (2^7-1)$	a_{00}	137	[7]
Electric coupling constant (α^{-1})	a	137.035999084(21)	0.15 ppb
Optimised series with the last Wieferich prime $137 + 1/28 + 1/p_{W2}$	a_0	137.035999104	$p_{W2} = 3511$
Electron Excess Magnetic moment	d_e	1.00115965218128	0.15 ppb
Atiyah constant $\gamma a/\pi$	Γ	25.17809724196	$\pi/\gamma \approx (g_2/g_1^2) n_t p_W/pH$
Optimised massive scalar boson/Electron mass ratio	$H^{(0)}$	$495^2 = 245025$	$H^{(0)}_{mes}$: 245000(250)
Optimized charged weak boson/Electron mass ratio	W	157340.1093	[5] W_{mes}: 157297(24)
Optimized neutral weak boson/Electron mass ratio	Z	178451.7524	[5] Z_{mes}: 178450(4)
Optimised Weak-mixing ratio W/Z	$cos\theta$	0.88169557	0.1 ppm
Optimised U(1) Gauge coupling $Zsin\theta/H^{(0)} \approx (\gamma/y)(n_t/1837) \approx p_W\sqrt{2}/Hf(2)$	g_1	0.3436256	$f(2) = e^{\sqrt{2}}$
Adimensional Electric charge $q \approx (6^2/\beta d_e)^{-1/3}$	q	0.30297311	$6 \approx 2^{137/53}$
Optimised SU(2) Gauge coupling $W/H^{(0)}$	g_2	0.6421390	$g_2 \approx y/\Phi^2 \approx \pi g_1/y$
Gravitational Coupling $p_t p_G/2a^3$	g_0	0.6534149985	1 ppb, this work
Optimised SU(3) Gauge coupling $g_0 g_2/g_1 \approx (p_t^2/n_t H)\, a_s\, (95a/Z)^2$	g_3	1.2210476	0.1 ppm, this work
Optimized charged Pion/Electron mass ratio	Π_+	273.1328373	Π_{+mes}: 273.13288(47)
Optimized neutral Pion/Electron mass ratio	Π_0	264.143971	Π_{0mes}: 264.14341(97)
Optimized eta Meson/Electron mass ratio $4(\Pi_0\Pi_+)^{1/2}(137/a)^8$	η	1072.147344	η_{mes}: 1072.139(33)
Optimized "Squared Effective (m_Z) Weak-mixing angle" $e/32sin\theta(cos\theta)^2$	$(\sin\theta_1)^2$	0.2315965	$(\sin\theta_1)^2_{mes}$: 0.23153(4)
DNA Adenine-Thymine couple main isotopic mass/m_H	o_1	612.312280	[36]
DNA Guanine-Cytosine couple main isotopic mass/m_H	o_2	613.299802	[36]
Optimized Muon/Electron mass ratio	μ	206.7682869	μ_{mes}: 206.7682830
Optimized Koide Tau ratio, with $p_K = (1+\mu+\tau)/2 = (1+\sqrt{\mu}+\sqrt{\tau})^2/3$ [35]	τ	3477.441701	τ_{mes}: 3477(2)
Koide constant $p_K \approx \beta\tau ln(2+3+5+7)/ln(2\times3\times5\times7)$	p_K	1842.604994	bases 2;3;5;7
Fermi mass ratio: $\sqrt{a_w} \approx (4\pi/3)(lnP)^3 \approx (y\Pi_+/2d_e q)^2 \approx (p_t/n_t)(137aa_s)^2/4\pi\tau$ [5]	F	573007.3652	F_{mes}: 573007.362
Eta-Tau mass ratios product	$\eta\tau$	3.728×10^6	
Product of the Wieferich primes 1093×3511	$p_{W1}p_{W2}$	3.837×10^6	$p_{W1}/\eta \approx (p_{W2}\beta/\tau)^2$
Third Primary Economic Number e_3	e^{e^e}	3.814×10^6	$e_3/e_2 \approx aH$
Optimised Gravitational (inverse) coupling constant $R/2\lambda\!\!\!^{-}_e = P^2/pH$	a_G	$1.691936467 \times 10^{38}$	ppb [5]
Optimised Electroweak (inverse) coupling constant $(2\Gamma\times137)^3 \approx (6(\lambda\!\!\!^{-}_{CMB}/\lambda\!\!\!^{-}_e)^3)^{2/5}$	a_w	$3.283374406 \times 10^{11}$	ppb [5]
Optimised Strong (inverse) coupling constant $a_w/2\pi(pH)^{3/2}$	a_s	8.434502906	[5] $a_s \approx g_1^{-2}(p_t/H)^4$

Table 2: Physical constants [3]

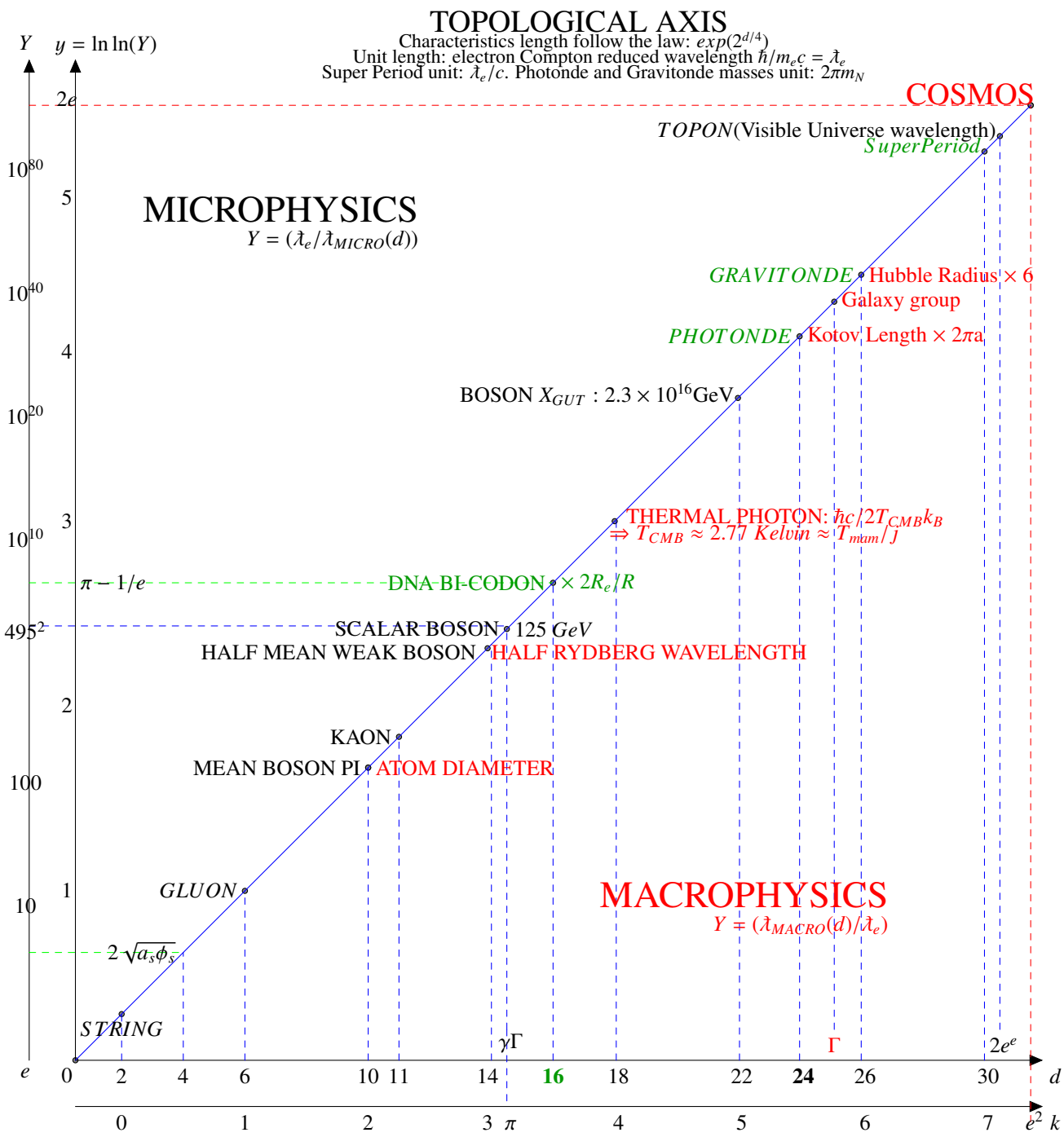
name	Symbol	unit	Value	remarks
Reduced Planck constant $h/2\pi$	$\hbar$	$J\ s$	$1.05457181 \times 10^{-34}$	"exact"
Official Gravitation constant	G_{off}	$kg^{-1}m^3s^{-1}$	6.67430×10^{-11}	contested
Optimized Gravitation constant	G	$kg^{-1}m^3s^{-1}$	$6.67545375 \times 10^{-11}$	[5]
Relativity local speed	c	$m\ s^{-1}$	299792458	exact
Boltzmann *conversion* constant	k_B	JK^{-1}	1.380649×10^{-23}	exact
Rydbergh reduced wavelength $\lambda\!\!\!^{-}_e(2aH/p_t)^2$	$\lambda\!\!\!^{-}_{Ryd}$	m	$1.45190673 \times 10^{-8}$	$\lambda\!\!\!^{-}_{Ryd}/l_P \approx \sqrt{O_M}$ (Monster Group order)
Fermi constant	G_F	$J\ m^3$	$61.435851 \times 10^{-62}$	500 ppb
Electron mass $m_e = m_p/p_t = m_H/H = m_n/n_t$	m_e	kg	$9.1093837015 \times 10^{-31}$	0.3 ppb
Electron reduced wavelength $\hbar/m_ec$	$\lambda\!\!\!^{-}_e$	m	$3.861592675 \times 10^{-13}$	0.3 ppb
Electron classical radius $\hbar/am_ec$	r_e	m	$2.817940322 \times 10^{-15}$	0.45 ppb
Hydrogen Bohr radius $a(1 + 1/p_t)\lambda\!\!\!^{-}_e$	r_H	m	$5.294654092 \times 10^{-15}$	0.45 ppb
Planck length $(\hbar G/c^3)^{1/2}$	l_P	m	1.616395×10^{-35}	ppb [5]
Critical density $3/8\pi t^2$	ρ_{cr}	$kg\ m^{-3}$	$9.41197996 \times 10^{-27}$	with $t = R/c$, ρ_{cr} independent of c
Optimised CMB temperature	T_{CMB}	K	2.7258204	[5], $T_{CMB(mes)}$ 2.7255(6)
CMB wavelength hc/k_BT_{CMB}	λ_{CMB}	m	$5.278325924 \times 10^{-3}$	[5] $\lambda_{CMB} \approx 2f(18)\lambda\!\!\!^{-}_e((a-136)a/137)^{1/2}$
CMB red. wav. $\lambda_{CMB}/2\pi$; $\pi_{CMB} \approx 3 + 2(a - 137^2/a)$	$\lambda\!\!\!^{-}_{CMB}$	m	8.4007166×10^{-4}	$(4\pi_{CMB}/3)(\lambda\!\!\!^{-}_{CMB}/\lambda\!\!\!^{-}_e)^3 \approx e^{\sqrt{137a}}$
CMB Wien wavelength λ_{CMB}/w_i	λ_{WCMB}	m	1.0630825×10^{-3}	[5]
CMB photon length $\lambda_{CMB}(16\pi\xi(3))^{-1/3}$	l_{phCMB}	m	1.345131×10^{-3}	[5]
CMB energy density $(\pi^2/15)\hbar c/\lambda\!\!\!^{-4}_{CMB}$	$u_{CMB} =$	$J\ m^{-3}$	$4.1767647 \times 10^{-14}$	[5]
Radiation density energy $u_{CMB} + u_{CNB} = yu_{CMB}$	u_{rad}	J^{-3}	$7.0224862 \times 10^{-14}$	[5]
Critical Energy versus radiation ratio $\rho_{cr}c^2/u_{CMB+CNB}$	K_E	-	$\underline{12045.685}$	$\approx p_tH/8\beta\pi^2 q\sqrt{a} \approx p_t^3/(4\pi)^2\sqrt{\pi}p_W$
Square root of critical entropy $(2Mn_{ph}/m_H)^{1/2}$	K_n	-	$\underline{12087.731}$	$(K_EK_np_t/n_t)^{1/2} \approx g_2a^2$
Optimised CNB temperature T_{CMB}/t_0	T_{CNB}	K	1.9455976	$t_0 = (11/4)^{1/3}$ [3]
Neutrino (CNB) reduced wavelength	$\lambda\!\!\!^{-}_{CNB}$	m	1.176957×10^{-3}	
Non-Local (Kotov) length ct_{nl}	l_{nl}	m	$2.878184911 \times 10^{12}$	[5]
Hubble length (Universe radius)	R	m	$1.306713894 \times 10^{26}$	[5]
Cosmos holographic radius	R_e	m	$1.712894163 \times 10^{26}$	[5]
Cosmos radius	R_C	m	$9.075773376 \times 10^{86}$	[5]
Universe mass $m_P^4/m_em_pm_H$	M	kg	8.7965248×10^{52}	[5]
Baryonic Neutron Number $(3/10)m_P^4/m_em_pm_Hm_n$	$n_n^{(bar)}$	-	$\underline{136.068464 \times 2^{256}}$	[5]
Eddington Large Number	n_{Ed}	-	$\underline{136 \times 2^{256}}$	[10]
Cosmos holographic reduced mass m_P^4/m_N^3	M_N	kg	$1.15308454 \times 10^{53}$	[5] Nambu mass $m_N = am_e$

2.4 The Non-Local Period: the Photon and Graviton masses

The Topological Axis rehabilitates the bosonic part of the string theory which has the apparent imperfection it includes tachyons. In fact, it is rather an advantage in order to explain the quasar non-Doppler oscillation, introducing a non-local period $t_{nl} \approx 9600,06(2)$ s [18]. Indeed, the ratio of this period and the electron period $t_e = h/m_ec^2$ is precisely given by the elimination of c between the electro-weak constant a_w and the inverse gravitational coupling $a_G = R/\lambda\!\!\!^{-}_e$: $t_{nl}/t_e \approx (a_Ga_w)^{1/2}$. This gives a G value precise to 10^{-6}, compatible with the BIPM 10^{-5} precise measurement [37]. This implies that the official value of G, *the incongruous mean between incompatible measurements*, is dramatically too small by 8 σ. By analogy with the practical holography, which is a two-step process, it was introduced a two-step interaction procedure, with *a precursor speed* $C = cR_C/R$ much greater than c, leading to the following masses for the photon and graviton [5]:

$$\begin{cases} m_{ph} = \hbar/c^2t_{nl} \\ m_{gr} = m_{ph}/a_w \end{cases} \tag{13}$$

In the Topological axis, these masses correspond to the special string dimensions 24 (transverse dimensions) and 26 (main dimension), and will be determinant in the following section. Note the relation, precise to 1 ppm, between the Bohr's radius r_H and the relativistic factor $1/\beta = H - p$: $f(24)^{1/26} \approx d_e(r_H/\beta\lambda\!\!\!^{-}_e)^{1/2}$, showing that the electric parameter $a = (p/H)r_H/\lambda\!\!\!^{-}_e$ is central in the Topological Axis.



Bosonic string dimensions: $d = 4k + 2$ from k=0 to k=7 (Bott sequence). From k= 0 to k = 3: spectroscopic numbers

Table 3: The Topological Axis follows the law $exp(2^{d/4})$ for the main physical characteristics lengths, with unit length the electron Compton reduced wavelength: $\hbar/m_e c = \lambda\!\!\!^{-}_e$. This is the extrapolation towards smaller numbers of the Eddington's Large Number correlations, a reunion of height 2D-1D holographic relations, whose micro-macrophysics imbrication explains the double log, hence the name 'Topological Axis'. The double natural logarithms $y = ln(ln(Y))$ of the main dimensionless physical quantities (Y) corresponds to the special string dimension series, which identifies with the spectroscopic series with spin 1/2, where k is the orbital quantum number, $d = 4k + 2$, from $k = 0$ to $k = 7$, characteristics of a Bott octonion sequence, as anticipated by Atiyah, whose constant $\Gamma = \gamma a/\pi$ is central. The mean value $d = 16$ connects directly with the DNA bi-codon, decisive in the Holic Cosmology, where R_e is the holographic reduced Cosmos radius and R the Hubble radius, whose ratio is connected with the Weinberg-Sanchez gauge couplings (Eq(4), ϕ_s a special value of the golden number, $m_N = am_e$ the Nambu mass, the Kotov length ct_{nl} and $j = 8\pi^2/ln2$ the scale factor [19]. In this rehabilitation of the Bosonic String Theory, the Universe appears as the final boson in the Cosmos.

Table 4: 56 formula for the Hubble radius R

Formula	Value (Gly)	Remarks
$2\hbar^2/Gm_em_pm_H$	13.81197676	Gravitational Hydrogen Molecule radius [5]
$(H/p_t)\ R_{1H}$	13.81197676	From mono-atomic star limit radius R_{1H} [28]
$\lambda\!\!\!^{-}_w\ (t_{nl}/t_e)^2$	13.81197676	Identification predicting $t_{nl} \approx 9600.591457s$ (Eq.(5))
$(\lambda\!\!\!^{-}_p\lambda\!\!\!^{-}_H)^{1/2}\ (WZ)^4$	13.81197676	Symetrising the published relation $a_G \approx W^8$ [4]
$\lambda\!\!\!^{-}_e\ 2^{128}/d_e^2(m_H/m_p)^6$	13.81197676	Empirical, from the Combinatorial Hierarchy Lucas Large Number [7]
$(n_tp_W/1836p)^{1/2}\lambda\!\!\!^{-}_e(2^{18}/\pi\sqrt{a})^{10}$	13.811977	From $(2g_3)^{21} \approx s_{65}/2\pi$; $p_W = 6\pi^5$ and $s_{65} \approx (2\pi)^2\sqrt{a}$
$(1/g_0)\ \lambda\!\!\!^{-}_H\ (R_C/2^{128}l_P)^{1/2}$	13.811978	From $1/g_0 = 1 + g_1^2 + g_2^2 \approx 2a^3/pp_G$, with $p_G = P/2^{127/2}$
$(\lambda\!\!\!^{-}_p\lambda\!\!\!^{-}_H)^{1/2}\ a_w^{7/2}\ a/2\sqrt{5}$	13.811978	From $(WZ)^8 \approx a^2a_w^7/20$
$\lambda\!\!\!^{-}_e\ (p_{hol}/p)^2\ e_3^{\sqrt{a}/2}$	13.81198	Liaison between $e_3 = exp(exp(e))$ and $p_{hol}^2 = 4a^3/3$
$(32\beta^2/\pi^3)\ \lambda\!\!\!^{-3}_{CMB}/\lambda\!\!\!^{-2}_Z$	13.81200	from the holographic relation $2\pi R/\lambda\!\!\!^{-}_e \approx (4\pi/3)(\lambda\!\!\!^{-}_{CMB}/\lambda\!\!\!^{-}_{H_2})^3$ [5]
$(H/p_W)\ (2\pi^2a^3)^5\ \lambda\!\!\!^{-}_e$	13.81196	with $p_W = 6\pi^5$, 5D holography in the gravitational Hydrogen molecule [19]
$\lambda_p\ (pH)^{3a_s/4}$	13.81193	confirms the strong coupling a_s
$4\ a^4\ \lambda\!\!\!^{-}_e\ (m_{bc}^{(0)}/m_H)^9(p_t/p_W)^2$	13.81196	DNA bi-codon mass as calculation basis
$\lambda\!\!\!^{-}_e\ (WH/Zp)^{1/2}\ C_y^{(0)32}/6f(26)$	13.81195	Cytosine topologic pertinence $C_y^{(0)} \approx f(10) = f(26)^{1/16}$
$\lambda\!\!\!^{-}_e\ (N_{ph}\ (n_t/p_t)^2/\pi\sqrt{g_0})^{1/7}$	13.81197	with N_{ph} the Cosmos photon number, confirms that the Universe is a cosmic boson
$R_C\ (a_se^{2/a}210^{-210})^{1/8}$	13.81198	With the holic number 210^{210}, confirming the couple Universe- Cosmos
$l_P\ \sqrt{a}\ (\mu^\mu\ Wp/ZH)^{1/8}$	13.81198	Shows the pertinence of the computational term μ^μ
$(H/p_t)\ (ad_e/137)^4\ R_{R_c,M_N}$	13.81203	From the geo-adimensional Cosmos-Universe (Fig.1)
$(137\beta/a)^2\ R_c\ R_e\ l_P^2/a_wl_{nl}^2\lambda\!\!\!^{-(0)}_{bc}$	13.81194	Holic Principle, with the reduced wavelength of the DNA bi-codon.
$(\pi\ a/6\times137)\ \lambda\!\!\!^{-}_e\ (1/g_0)^{210}$	13.81185	From $(1/g_0) \approx 2R/R_e \approx (R/\lambda\!\!\!^{-}_e)^{1/210}$ (Eq. (14))
$(136/137)\ 2\ l_{nl}\ a^3\ f(16)$	13.8120	Empirical, with the central value $f(16) = e^{16}$
$(137^4/ap_t^2)\ \lambda_{WCMB}\ (e^a/4\pi)^{1/2}$	13.8120	Confirming the Wien CMB wavelength, from $4\pi(R_e/\lambda_{WCMB})^2 \approx e^a$
$\lambda\!\!\!^{-}_e\ (1/q)^{3\pi a_s}/496$	13.8124	Confirms the strong coupling a_s and the electric charge q
$(\lambda\!\!\!^{-}_p\ \lambda\!\!\!^{-}_H)^{1/2}\ a_w^4\ a^{14}/137^{16}$	13.8119	$137, a, a_w$ as computation basis
$(2a/137)\ q^2\ Z^{16}\lambda\!\!\!^{-}_p\lambda\!\!\!^{-}_H/2^{127}\lambda\!\!\!^{-}_e$	13.8221	Cosmic role of electric charge $q = g_1\cos\theta = g_2\sin\theta$
$4\ o_2\ \sqrt{Z}\ \lambda\!\!\!^{-}_e\lambda_{CMB}/l_P$	13.8129	Confirms the cosmic thermal bath and the couple GC with mass o_2m_H
$(H/p_t)\ (Gm_n/c^2)\ (10N_{Ed}/3)$	13.8125	From the Eddington Number 136×2^{256} and the gravitational parameter 10/3 [19]
$R_e\ (f(-2)/exp(exp(-g_1))^{128}/d_e^3$	13.8117	Symmetry R_e/R associated to symmetry f(2)-g_1 (string-SU(1) gauge coupling)
$2\ \lambda\!\!\!^{-}_e\ ((1836+s_{65})/2)^{\sqrt{a}}$	13.8123	Pertinence of the symmetry 1836-1848 (Eq.(25))
$(\lambda\!\!\!^{-}_p\ \lambda\!\!\!^{-}_H)^{1/2}\ (P/a^{13/2})^5/2\sqrt{5}$	13.8124	From the relation $a_w^7 \approx P^{3+7}/a^{(7+127)/2}$ [19]
$6\ (\lambda\!\!\!^{-2}_e/\lambda\!\!\!^{-}_w)\ (a/\pi)^{16}$	13.8124	From the Topologial Axis: $f(18) \approx H^3 \approx (a/\pi)^4(6^{1/2}a_w)^{1/2}$
$(1/q)^{3\pi a_s}\ \lambda\!\!\!^{-}_e/496$	13.8124	Confirms the strong coupling a_s and the electric charge q
$4\ \lambda\!\!\!^{-}_e\ a\ W\ Z\ F\ (p_tH)^3$	13.817	Shows a symmetry $a\ W\ Z\ F$
$4\ l_{nl}\ (p_t\ H/d_e)^2$	13.815	Confirms the non-local Kotov length
$\lambda\!\!\!^{-}_e\ (2/d_e^8)^{128}/(2g_0-1)$	13.815	From the Combinatorial Hierarchy Lucas Large Number [12]
$2\ \lambda\!\!\!^{-}_H2^{210}\ (a_w/P)^2$	13.811	Pertinence of the holic term 2^{210}
$\lambda\!\!\!^{-}_e\ e^a/p_t^6\Gamma$	13.811	confirms the pertinence of the Atiyah constant Γ
$l_P\ (\pi210^{210}/8)^{1/8}$	13.81	Pertinence of the holic term 210^{210}
$R_e\ a^a/\Pi_{heur}$	13.81	with the product of the 20 happy sporadic groups $\Pi_{heur} \approx e^{674.5210287}$
$2\ \hbar^2/Gm_em_pm_n$	13.80	c-free dimensional analysis [24]
$(\Pi_+/\Pi_0)\lambda\!\!\!^{-}_e\ e^{1/(g_0-g_2)}$	13.82	Confirms the pertinence of g_0 and g_2
$l_{nl}\ 2\ p_t^3\ H/d_e$	13.82	p and H as computation basis
$(\lambda_{CMB}/(j+1))^2/l_P$	13.80	Central role of mammal temperature: $T_{mam} \approx jT_{CMB}$, with $j = 8\pi^2/ln2$
$2\ \lambda\!\!\!^{-}_e\ (1/sin\theta)^{10d_e\sqrt{137}}$	13.80	Corresponds to $(1/sin\theta) \approx 3/\sqrt{2} \approx p^{1/10}$
$\lambda\!\!\!^{-}_e\ \pi^{155/2}$	13.80	π calculation basis: $2^{1/155} \approx \pi^{1/16^2} \approx (2\pi)^{1/3\times137}$
$2\ \lambda\!\!\!^{-}_e\ e_3^{2\sqrt{a_s}}$	13.79	Pertinence of the basic economic number $e_3 = e^{e^e}$
$\lambda\!\!\!^{-}_e\ (6/\pi)^{r_H/\lambda\!\!\!^{-}_e}$	13.78	$6/\pi$ calculation basis
$\lambda\!\!\!^{-}_e\ \Gamma^{55/2}$	13.77	Atiyah's constant Γ calculation basis
$g(6)\ \lambda\!\!\!^{-}_e$	13.82	with the reduced topological function $g(k) = exp(2^{k+1/2})/k$, for k = 6, d = 26
$2\ l_{nl}\ (a\mu)^3$	13.84	Confirms the non-local Kotov length
$(2l_{nl}^3/r_e)^{1/2}$	13.75	2D-3D Holography with the non-local length l_{nl}
$(r_e^2R_C)^{2/3}/l_{nl}$	13.75	Confirms the Cosmos non-locality
$(2\ \lambda\!\!\!^{-}_e/3)\ (\lambda\!\!\!^{-}_{CMB}/\lambda\!\!\!^{-}_{H_2})^2$	13.90	2D-3D holography in the hydrogen molecule
$(4\ \pi\ \lambda_{CMB})^4/r_H^3$	13.78	Confirms the CMB invariance
$(l_P/2)\ (\lambda\!\!\!^{-2}_{CMB}\lambda\!\!\!^{-}_p/\lambda_{CNB}r_e^2)^6$	13.7	Complementarity of photons and neutrinos backgrounds

Table 5: 55 formula for the Cosmos radius R_C

Formula	Value (10^{86}m)	Remarks
$R_e^2/2l_P$	9.07577	1D-2D Holographic Principle with R_e [5]
$\lambda\!\!\!^{-}_e\ exp(exp(exp(exp(exp(-g_2)))))$	9.07577	The final log of $R_c/\lambda\!\!\!^{-}_e$ is a SU(2) coupling: $g_2 \approx W/(495^2 + (\tau/\mu)^2)$
$(\lambda\!\!\!^{-}_W \lambda\!\!\!^{-}_Z)^{1/2} e^{1/(a-9)}\ exp(exp(exp(exp(exp(-g_0)))))$	9.07579	from the connections $g_0 \approx g_2$ and $a - 9 \approx 2^7$
$e^{210}\ R\ (f(16)e^{e^6}9\mu/n_t)^{-1/6}$	9.07577	confirms the role of the central topologic term $f(16) = e^{16}$
$e^{210}\ l_{Wien}\beta(\sin\theta_1)^4$	9.07577	confirms the Holic Principle and the CMB temperature invariance
$2^{128}\ l_P(g_0R/\lambda\!\!\!^{-}_H)^2$	9.07577	confirms $1/g_0 = 1 + g_1^2 + g_2^2 = 1 + (Z/H^{(0)})^2$
$R\ p_t(6d_eP^{\sqrt{a}-4}/\pi)^{1/3}$	9.07574	from the cosmic atomic mass $M_c/m_H \approx P^{\sqrt{a}}$
$l_P(R_C/\lambda\!\!\!^{-}_e^{g_3}\ (p_G/2a)^{1/2}$	9.07574	confirms the SU(3) coupling g_3
$\lambda\!\!\!^{-}_e\ (a^{3/2}/1837)y^{64a_s}$	9.07579	from $y^{a_s} \approx 3^4 - 1$
$\lambda\!\!\!^{-}_e\ 14p_t^2/p_W n_t(a/4)^{64}$	9.07579	empirical, with $p_W = 6\pi^5$
$2\lambda\!\!\!^{-}_e\ (sin_Z\theta_{eff})^{-4}(4a_s)^{64}$	9.07777	empirical, implying a relation between a and a_s
$l_P\ (\Pi_+/18)3^{256}$	9.07586	confirms the cosmic base 3
$R\ a^6\ (12^{32}/P)^4 1839/4H$	9.07587	base 12
$g_2\lambda\!\!\!^{-}_e\ (60H/61p_t)\ (1/q)^{2^6}$	9.07548	confirms the electric charge q; $61/60 \approx (n_t/p_t)^{12}$
$l_P\ (2/\sqrt{5})(80)^{64}$	9.07510	implies the musical property (16 ppb): $((3/2)^4/5)^{256} \approx 24(a/137)^{32}(p_t/p_W)^{1/2}$
$e^{\mu}\ \lambda\!\!\!^{-}_{CMB}(R_e/R)^2(p_t/p_{t0})^4$	9.07584	confirms the CMB temperature invariance, with $p_{t0} = 1836$
$e^{\mu}\ \lambda_{Wien}e/2\beta^2\ \sqrt{d_e}$	9.07575	confirms the CMB temperature invariance, through the Wien wavelength
$e^{\mu}\ l_{phCMB}\ ((a-136)p_t/p_{t0})^2$	9.07573	confirms the CMB temperature invariance, through its photon length l_{phCMB}
$e^{\mu}\ \lambda\!\!\!^{-}_e\ \beta^{1/2}(p_t/n_t)^{1/4}$	9.07576	confirms the CMB temperature invariance
$2(a/137\beta)^2\ l_{nl}^4\ \lambda\!\!\!^{-(0)}_{bc}/R_e\lambda\!\!\!^{-}_e l_P^2$	9.07580	Holic Principle, with the reduced wavelength of the DNA bi-codon
$l_P\ (\sqrt{a_w}(a)^2)^{16}$	9.07568	bases a_w and a_s, the nuclear couplings
$l_P\ \sqrt{1 + (sin\theta_1)^2}\ a^{j/2}$	9.07566	base a the electric coupling, with j the scale factor [5]
$l_P\ (210^{210}(8e)^{-1/2})^{1/4}$	9.07585	Holic central term 210^{210}
$l_P\ \sqrt{2}\ (p_t/n_t)^6 e^{280}$	9.0767	base e
$l_P\ (Zd_e/W)\ 7^{12^2}$	9.075	base 7
$\lambda\!\!\!^{-}_e\ 6^{128}/(1 + 1/\sqrt{2})$	9.075	base 6 ; $6/(1 + 1/\sqrt{2}) \approx \mu d_e^2/\sqrt{\tau}$
$\lambda\!\!\!^{-}_e\ e^{1/2a}6^{p_G/\sqrt{a}}$	9.076	base 6
$2r_H\ 3^{210}/1830$	9.076	base 3 Holic term, with $1830 = (60 \times 61)/2$
$\lambda\!\!\!^{-}_e\ (1 + 1/\sqrt{2})^{6a_s^2+1}$	9.085	base $1 + 1/\sqrt{2}$
$\lambda\!\!\!^{-}_e\ 5^{2a_s^2+1}/6$	9.082	base 5
$l_P\ (7/6)^{(1836p)^{1/2}}/2e^2$	9.085	base $7/6 \approx a^{1/32}$
$\lambda\!\!\!^{-}_e\ ((1/q)^a\ W/aF)^2$	9.082	base $1/q$
$l_P\ e^{s_{65}n_tq/2p_W}$	9.077	confirms the terminal Euler number $s_{65} = 1848$
$l_P\ e^{27aq/4}((2W + Z)/3W)^2(n_t/p_t)^2$	9.076	confirms the charge q
$l_P\ (p/H)\ (R\Pi_{26}/R_e)^{1/3}$	9.076	with the product of orders of the 26 sporadic groups $e^{674.5210287}$ [5]
$\lambda\!\!\!^{-}_P\ (210^{210}/\sqrt{(8e)})^{1/4}$	9.076	Confirm the Holic Number $210^{210} \approx \tau^{2\mu/3} \approx e^{e2\mu}$
$\lambda\!\!\!^{-}_e g(7)\ (H/p)^2 P/6$	9.076	with the reduced topologic function for $d = 30 : g(7) = f(30)/7$ [5]
$24\lambda\!\!\!^{-}_e\ \pi^{210}/a^3$	9.077	Confirms the base π holic term
$a^2\ \lambda_{Wien}^4/(p_K l_P)^3$	9.078	Confirms T_{CMB} with $p_K = (1 + \mu + \tau)/2$ [35]
$\lambda\!\!\!^{-}_e\ ((3/4\pi)(R_1/\lambda\!\!\!^{-}_e)^7)^{1/3}$	9.078	comes from the de photons nomber, using the mono-electron radius R_1 [5]
$\lambda\!\!\!^{-}_e\ 137^{1836/(2\pi)^2}$	9.078	base 137
$\sqrt{3}\ l_{nl}^3/r_e l_P$	9.07	with the non-local length l_{nl}
$\lambda\!\!\!^{-}_e\ g(7)\ (a^2 p_t p_G)^2$	9.08	[5]
$l_P\ j^{60}\ e_3^{1/4}$	9.06	base j, the scale factor
$l_P\ e^{(n_t/a)^2/g_2}\ (3/\pi)^{1/2}$	9.06	base e, confirms g_2
$\lambda\!\!\!^{-}_e\ e^{(p_{00}+1/2)/8}$	9.09	natural base e, with $p_{00} = (60 \times 61)/2$
$l_P\ (n_t^2/l_p)^{2/(sin\theta_1)^4}$	9.06	bases p_t and n_t
$\lambda\!\!\!^{-}_e$	9.11	natural base e in the Topological Axis
$(ln(R_c/\lambda\!\!\!^{-}_e))^2 \approx (ln(M_e/m_e)^2 + 2(ln(R/\lambda\!\!\!^{-}_e)^2)$	9.12	c-observable Universe Cosmos couple $(R/\lambda\!\!\!^{-}_e = t/t_e)$, fig 1
$\lambda\!\!\!^{-}_P\ (6/\pi)^{\pi a}$	9.14	base $6/\pi$
$\lambda\!\!\!^{-}_e\ g(7)\ (\lambda\!\!\!^{-}_{CMB}/r_H)^3$	9.1	Confirms the invariance of the thermal background [5]
$(Rl_{nl})^{3/2}/r_e^2$	9.2	From non-local holography [5]
$l_P\ \mu^{\mu R_e/3R}$	9.0	μ calculation basis, close to holic base 210

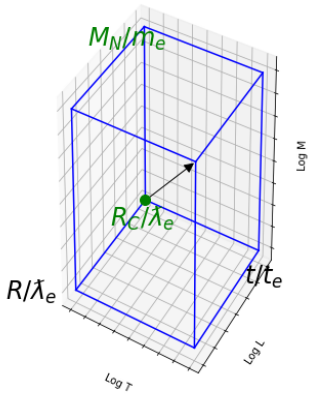


Figure 1: *Geo-adimensional Cosmos-Universe couple*, with unit length the Electron Compton reduced wavelength. In a 3D Super-space, logarithms of physical ratios are considered vectors. The Cosmos radius R_C appears as the norm of the vector using for length and time projections the same value $R/\lambda\!\!\!^{-}_e = t/t_e$. For the mass projection it is M_N/m_e where M_N is the critical mass in the Cosmos *reduced spherical hologram* of radius R_e. This is a dramatic geometrical confirmation (independent of the base for logarithms) of the Extended (2D-1D) Holographic Principle applied to the Bekenstein-Hawking Universe entropy. So the Universe is characterised by the c-equivalence $R/\lambda\!\!\!^{-}_e = t/t_e$, where t is the Hubble time (no relation with any "Universe age").

3 The Holic Principle

The string theory considers space-time as a secondary property [38], so the concepts of mass, length and time are, in final, related to pure numbers. Indeed an arithmetic-physical synthesis has been anticipated by the Holic Principle [13], a simplified form of the Holographic Principle.

Recall that holistic equations are prefered to differential ones, in order to eliminate free parameters. The systematic use of differential equations in the standard physics is the origin of the proliferation of free parameters.

In any Diophantine equation, this Holic Principle allows to discriminate a temporal ratio T, acting by its square, from a spatial ratios L, acting by its cube (due to the 3D space). Indeed, *the simplest Diophantine Equation, which implies a 2-dimensional Time*, $T^2 = L^3 = n^6$, with n a natural integer, is the Diophantine form of the third law of Kepler, and implies: $L_n = r_n/r_1 = n^2$ (the Bohr's orbit law) and $T_n = t_n/t_1 = n^3$. Hence, with $v_n = r_n/t_n$:

$$\begin{cases} r_n v_n^2 = r_1 v_1^2 = G m_G \\ r_n v_n = n r_1 v_1 = n\hbar/m_\hbar \end{cases} \tag{14}$$

These gravito-quantum equations introduce an "hyper-symmetry" between the universal constants G and ħ, by respect to the mass concept: the undefined masses m_G and $m_\hbar$. So, this defines the conceptual trajectories:

$$\begin{cases} r_n = n^2 r_1 \\ r_1 = \hbar^2/G m_G m_\hbar^2 \end{cases} \tag{15}$$

With $m_G = m_e^{(red)} = m_e m_p/(m_e + m_p)$, the classical electron reduced mass and $m_\hbar = m_P/\sqrt{a}$, this is the Bohr's orbits distribution. The above PSHOB Cosmology includes the following 6 more special cases (Table 6), using the main masses, plus a new one: m_{bc}, close to m_H^2/m_e, *which identifies with the DNA bi-codon mass*, studied in the next section.

So, the PSHOB Cosmology is tied to the couple $G, \hbar$, while the classical quantum theory uses in fact the "photonde" couple $\hbar, c$, and the gravitation theory the "gravitonde" couple G, c. These three couples define the "Trihedra of Constants"(Fig. 2).

These neologisms 'photonde" and "gravitonde" are introduced to recall that only waves propagate, not the particles: this is a main cause of misinterpretations in quantum physics.

Table 6: PSHOB cosmology (Eq.(12))

m_G	$m_\hbar$	$r_1 = \hbar^2/Gm_Gm_\hbar^2$	Precision	Arithmetic Property
m_e	m_P	$\lambda\!\!\!^{-}_e$: Electron reduced wavelength	exact	
$m_e^{(red)}$	$m_P/\sqrt{a}$	r_H: Bohr's radius	exact	$r_H/\lambda\!\!\!^{-}_e \approx 137 = 2^7 + 2^3 + 2^0$
m_N	m_N	$R_e/2$: half cosmos reduced holographic radius	exact	$R_e/\lambda\!\!\!^{-}_e \approx (3^3)^{3^3}$
$m_{bc}^{(0)}$	$m_{bc}^{(0)}$	$2l_{cc}$: double non-local length	-6.3×10^{-3}	$l_{cc}/\lambda\!\!\!^{-}_e \approx \pi^{50}$
$m_P a^3$	$\sqrt{m_p m_H}$	λ_{Wn}: Wien CMB wavelength (thermal background)	-3.2×10^{-4}	$\lambda_{Wn}/l_P \approx \pi^{64}$
m_e	$\sqrt{m_p m_H}$	$R/2$: half Universe radius	exact	$R/\lambda\!\!\!^{-}_e \approx g(6) \approx 2^{2^7} \approx (2R/R_e)^{210}$
$m_{bc}^{(0)}R_e/R$	$\sqrt{m_{ph}m_{gr}}$	R_C: Cosmos radius = $RC/c = (R/2)m_N^3/m_{bc}m_{ph}m_{gr}$	4.7×10^{-4}	$R_C/\lambda\!\!\!^{-}_e \approx e^{e^{2e}} \approx 6^{2^7} \approx (2R/R_e)^{64a_s}$

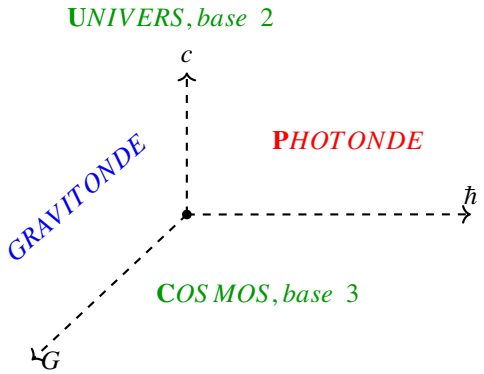


Figure 2: The Trihedra of Constants $\hbar - G - c$. The c-local visible Universe is a Cosmos bosonic "immergence"

Extrapolating the above simplest Diophantine equation with the prime numbers 5 and 7 which follow the basic prime couple 2;3, the Holic Principle proposes the exponent 5 for a mass ratio, and 7 for a field ratio (note that the lifetime of a particle depends effectively to the power 5 of its mass). So, the general resolution is:

$$T^2 = L^3 = M^5 = F^7 = n^{210} \tag{16}$$

Note that the primes 2;3;5;7 are the terms of the two simplest solutions of the Pell-Fermat equation, which has been connected with the metric equation [13].

Indeed, the Hubble radius "holic key" is singular, to 15 ppm, while the base 2 is confirmed to 0.3 ppm, and the base 3 to 60 ppb in the following relations:

$$\begin{cases} (R/\lambda\!\!\!^{-}_e)^{1/210} \approx 2R/R_e = p_G/Hg_0 \\ (P^2/a_w)^2/p_t)^{1/210} \approx 2 \\ ((p_G/2an_t)R_C/\lambda\!\!\!^{-}_p)^{1/210} \approx 3 \end{cases} \tag{17}$$

with p_t the proton-electron mass ratio and n_t the neutron-electron mass ratio. Note that 3 is the optimal integer base, the closest integer to e [39]. This is tied to the economic functional definition of e, whose square interconnects the main musical ratios, using the primes 2;3;5, while *the ratio 7/6, unknown in classical music, connects with a*. Recall that, according to Euler "music is an inconscious calculation":

$$\begin{cases} x^{1/x} \ \ maximal \ \ for \ \ x = e \\ e^2 \approx (3/2)^5 \approx (4/3)^7 \approx (5/4)^9 \approx (6/5)^{11} \approx (7/6)^{13} \approx a^{13/32} \approx \mu^{3/16} \end{cases} \tag{18}$$

So the base 7 is also pertinent. It is foreseen that special music could use also the base 7. A symmetric use of these four bases 2;3;5;7 explains the above role of the global base 210, so justifies the brute muon mass.

4 The DNA bi-codon

Using the main isotopes: ${}^{1}_{1}H^{(0)} = H$, ${}^{6}_{12}C = C^{(0)}$, ${}^{7}_{14}N = N^{(0)}$, ${}^{8}_{16}O = O^{(0)}$, ${}^{15}_{31}P = P^{(0)}$ [36], the masses of the 4 DNA nucleotides, by respect to the hydrogen mass H are close to the Fermi mass ratio: $\sqrt{a_w/pH} \approx 311.9846$, very close to 312, the 53^{th} ideonal Euler number, whose importance will appear in the next Section.

$$\left\{\begin{array}{ll} Cytosine: & C_9^{(0)}\,H_{12}^{(0)}\,N_3^{(0)}\,O_6^{(0)}\,P^{(0)}\,(150\ pr.+139\ nt.):\ C_y^{(0)} \approx 286.8021362 \approx 495(a^3/n_t^2)^2 \approx WH/4an_t \\ Thymine: & C_{10}^{(0)}\,H_{13}^{(0)}\,N_2^{(0)}\,O_7^{(0)}\,P^{(0)}\,(158\ pr.+146\ nt.):\ T_h^{(0)} \approx 301.68553403 \approx \sqrt{a_w}\,\Pi_0/H\Pi_+ \\ Adenine: & C_{10}^{(0)}\,H_{12}^{(0)}\,N_5^{(0)}\,O_5^{(0)}\,P^{(0)}:\ (162\ pr.+151\ nt.):\ A_d^{(0)} \approx 310.6269397 \approx \sqrt{a_w}/p_t d_e^4 \\ Guanine: & C_{10}^{(0)}\,H_{12}^{(0)}\,N_5^{(0)}\,O_6^{(0)}\,P^{(0}\,(170\ pr.+159\ nt.):\ G_u^{(0)} \approx 326.4976654 \approx 495(137a/\beta)^{1/2}/\mu d_e^4 \approx Zp_t/2H\Pi_+ \end{array}\right. \tag{19}$$

The mean masses of the effective couples are close to $H/3 \approx 612.3842155$:

$$\left\{\begin{array}{ll} Couple\ AT: & A_d^{(0)} + T_h^{(0)} = o_1 \approx 612.312280 \approx (Z/sin\theta)^{1/2}(p_t/H)^{1/8} \\ Couple\ GC: & G_u^{(0)} + C_y^{(0)} = o_2 \approx 613.299802 \approx (Z/sin\theta)^{1/2}(p_t/H)^{1/5} \end{array}\right. \tag{20}$$

The bi-codon minimal mass uses the three couples AT, so is very close to Hm_H. Since $o_2 \approx o_1 + 1$, the other masses are of type $(H + n)m_H$, with n = 1, 2 or 3: *the DNA seems a base 3 computer, like the Cosmos*.

The mean nucleotide mass is $(o_1 + o_2)/6 \approx 306.4032199$, close to $\pi^5 \approx 306.02$, the sixth part of the Lenz-Wyler proton-electron mass ratio [40] $6\pi^5$, which shows a geometric property: it is the product of the area by the volume of a cube of side π. The mean DNA bi-codon mass $m_{bc}^{(0)}/m_H = (6/4)(o_1 + o_2) \approx 1838.418122$ connects very precisely (150 ppb and 50 ppb) with the following main parameters and the central term $f(16) = e^{16}$ of the Topological Axis, and also with the Relative Radiation Ratio y (5 ppm), the second Wieferich prime $p_{W2} = 3511$ being, while e^8 connects directly to canonic numbers (0.6 ppm):

$$m_{bc}^{(0)} \approx (m_H^2/m_e)(n_t/p_t)^{1/2} \approx m_e e^8 a^{3/2}/d_e\,\sqrt{2} \approx m_H p_{W2} Z/yF \quad ; \quad e^8/8e \approx (a\beta)^2/137 \tag{21}$$

So, the DNA mass establishes the lacking connection (0.1 ppm) between the main masses: electron, proton, Hydrogen, neutron with $f(16)$, the central value of the Topological Axis $e^{16} = f(16)$, which shows also the following Keplerian holic relation, implying the leptons ratios:

$$e^{16} = f(12)^2 \approx (f(4(1+\sqrt{2}))^3 \quad \rightarrow f(4(1+\sqrt{2}) \approx \mu \quad ; \quad f(12) = e^8 \approx 6\tau/7 \tag{22}$$

where $(1 + \sqrt{2})$ is the Pell-Fermat generator. Since $a^{1/32} \approx 7/6$, this implies the terminal term $f(32)$ of the Topological Axis. The analysis shows, to 4 ppm:

$$(\tau - 1)^{32}/f(32) \approx a^2/137 \tag{23}$$

So, the terminal dimension 32 of the Topological Axis is associated to τ, the terminal lepton. The number three of particle families is therefore confirmed.

One notes the direct correlation implying *the product* of the nucleotide mass ratios: $4d_e G_u^{(0)} C_y^{(0)} \approx 4d_e A_d^{(0)} T_h^{(0)} \approx (H/3)^2 \approx H^{(0)}/g_0 = H^{(0)} + Z^2/H^{(0)}$. This induces the symmetrical relation implying $s_{65} = 1848$, the last Euler number:

$$\left\{\begin{array}{l} H^{(0)} + Z^2/H^{(0)} \approx (H/3)^2 \\ H^{(0)} + W^2/H^{(0)} \approx (s_{65}/\pi)^2 \end{array}\right. \tag{24}$$

This shows a symmetry between 1836 and 1848, which are the 34^{th} and 35^{th} areas of integer-sided triangles whose area equals 6 times their perimeter (A332879 in OEIS).

Table 7 : Multi-dimensional Crystallography and Number Theory

E_d	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}
N_d	2	6	10	24	38	78	118	224	330	584	838	1420	2002
K_d (positive)	1	5	5	19	19	59	59	165	165	419	419	1001	1001
$N_d + K_d$	3	11	15	43	57	137	177	389	495	1003	1257	2421	3003
$\sigma_{(d/2)^2} + \sigma_{(d/2)^2-1}$	-	1	-	17	-	137	-	611	-	1839	-	4405	-
$T_d = \sum_{d-1}^{d+1} K_d$	7	11	29	43	97	137	283	389	749	1003	1839	2421	4259
$S_d = \Sigma_1^d (N_d/2)$	1	4	9	21	40	79	138	250	415	707	1126	1836	2837
$q_d = \binom{3+d}{4}$	1	5	15	35	70	126	210	330	495	715	1001	1365	1820
$Q_d = q_d + d$	5	10	21	42	78	135	220	341	507	728	1015	1380	1836
$u_d = 1 + d(2d + 1)$	4	11	22	37	56	79	106	137	172	211	254	301	352
$u_{2d} = 1 + 2d(4d + 1)$	11	37	79	137	211	301	407	529	667	821	991	1177	1379
$v_d = u_{2D=2(d+3)}$	137	211	301	407	529	667	821	991	1177	1379	1597	1831	2081
13 final ideonal nbrs	312	330	345	357	385	408	462	520	760	840	1320	1365	1848

Table 7: Crystallographic Ponctual Symmetry Operation numbers N_d and the positive ones K_d. With the sum of primes including unity σ_n (EOIS A014284): $N_6 = \sigma_9$ and $K_6 = \sigma_8$. The sum $N_6 + K_6$ is the Eddington-Atiyah constant 137. Identifying $9 = (6/2)^2$, the corresponding sum for the supersymmetric dimension $d = 10$, is 1839, the closest integer to the neutron-electron mass ratio. The same numbers 137 and 1839 are given for a triplet combinaison of nearby K, for $d = 6$, and $d = 11$, the supergravity dimension. The sum of N_d for $d = 12$ (half the 24 transverse dimensions) is 1836, the integer closest to the proton-electron mass ratio. For $d = 9$, which is the number of compactified dimensions in string theory, $N_9 + K_9$ identifies both with 495, the square root of the Higgs boson-electron mass ratio and the 9^{th} pentapope number. Moreover, the relation $S_9 + 4 = K_{10}$ seems fundamental, since $419/417 \approx F^5/Pa^3$ to ppb precision. The natural extension of the 13^{th} pentapope number, where 13 is half the 26 main bosonic dimensions, is again 1836. The bissection of the Rule 23 Wolfram series shows the dimensions $u_1 = 4$ and $u_2 = 11$, which are those of the usual Space-time and the Supergravity, with $u_1^2 + u_2^2 = u_8 = 137$. Reducing the dimensions by a 2 factor shows that $u_{22} = 991 = 495 + 496$, the third "perfect couple", where 496 is the dimension of the string SO(32) group, while $11 = 5 + 6$ involves the first "perfect couple". The combined Rule 23 Wofram Series shows 137 for the dimension unity, which was predicted in a Sweeping Universe, and, for $d = 12$, the number 1831, the closest integer to the Lucas gravity proton-electron mass ratio, which is also u_{30}, showing the reduction from the Topological Axis dimension 30 to 12. If the Riemann conjecture is right, the final Euler ideonal number is $s_{65} = 1848 = 1836 + 12$, which shows dramatic connections with the electric constant a, the tau-electron mass ratio, the Topological Function and the DNA mass. The Fermi atomic number and that of a nucleotide are close to $s_{53} = 312$, while $53 = u_7/2 \approx 2\pi a_s$.

5 The Multi-Dimensional Crystallography

The main problem of string theory is the connection between the usual 4D time-space with the favored theoretical dimensions: 26 for the bosonic theory, 24 for the transverse dimensions, 10 for the superstring theory, 11 for the supergravity.

As recalled above, *conservation is tied to both symmetry and computation*. So, this section is devoted to connections between the Multi-Dimensional Crystallography, the Number Theory and the main particle mass ratios.

Carl Hermann [41] calculated the number of crystallographic point symmetries N_d for dimensions from 1 to 8. *This number N_d is the number of monic polynomials (i.e. with first term $1x^d$) with roots on the unit cercle: there must be a connection with the electromagnetic goup U(1) of complex numbers with modulus 1.*

The Weigel team [42] (Table 7) extended this calculation for higher dimensions, up to $d = 70$, focusing on *the positive symmetry number*, noted K_d, which defines N_d via:

$$\begin{cases} N_{2n+1}/2 = K_{2n+1} = K_{2n} \\ N_{2n} = K_{2n} + K_{2n-2} \end{cases} \tag{25}$$

These recurrence rules are non sufficient to defines the series. This implies to look for specific recurrences, characteristic of the Number Theory and the standard "free parameters", in particular $(a; p_t; n_t)$ with whole values (137;1836;1839).

5.1 The Prime Number series Connections

Considering the Prime Number Series including the unity $\sigma(n)$ (EOIS A014284), the connections are unambiguous (Table 7), showing a partition of 137, the Eddington-Atiyah constant. *This partition is characteristic of the Periodic Table (Section 5.7)*:

$$\begin{cases} \sigma_9 = 78 = N_6 \\ \sigma_8 = 59 = K_6 \quad \Rightarrow \quad N_6 + K_6 = 137 = \sigma_{3^2} + \sigma_{3^2-1} \\ \sigma_{2^2} + \sigma_{2^2-1} = \sigma_{2^2+1} - 1 = 2^{2^2} + 1 \\ \sigma_{4^2} + \sigma_{4^2-1} = 329 + 282 = 611 = 1833/3 \\ \sigma_{5^2} + \sigma_{5^2-1} = 964 + 875 = 1839 = 3 \times 613 = N_9 + K_9 + N_7 \approx 137 \times 4\pi\, ln(1/g_1) \\ \sigma_{5^2} + \sigma_{5^2-1} - 3 = 3(\sigma_{4^2} + \sigma_{4^2-1} + 1) = 1836 \end{cases} \tag{26}$$

The crystallographic partition of 137 induces the double partition of 1836, which is the arithmetic origin of the DNA bi-codon partition in 4 nucleotides. The number of nucleons in the Guanine is 329, see below, while $N_9 + K_9 = 495 = \sqrt{H^{(0)}}$, and $N_7 = 137 - K_4$ a partition tied to the Periodic Table (following section).

5.2 The *Positive Crystallographic Function* and the Scalar Boson

The method of least square leads to the following polynomial, where the coefficients clearly correlate with the physical parameters, with emphasis to the scalar boson - electron mass ratio $H^{(0)} = 495^2$ predicted by the Topological Axis and the Atiyah constant $\Gamma = \gamma\, a/\pi$ (Graph 3).

$$\begin{cases} \quad d \approx (lnN_d)^2/A + BlnN_d + 1/C \\ \\ A \approx 11.4672 \approx 2 \times 137\,\sqrt{6}/5\,\sqrt{a} \approx 137a495/\,\sqrt{2a_w} \approx 2\pi^2 WZ/a^5 \\ B \approx 1.1812 \approx 495/K_{10} \approx N_{10}d_e/495 \\ C \approx 43.9290 \approx 495^2 \sin\theta/\,\sqrt{2} \times 9\mu \approx 495^2 \cos\theta/\,\sqrt{2} \times \tau \end{cases} \tag{27}$$

Here μ and τ are the leptons relative masses, $\cos\theta = W/Z$, and $d_e \approx 1.00116$ is the electron magnetic excess.

The two fist terms are close for $d = 32$, which specifies the Topological Axis symmetry, from $k = 0$ to $k = 7$, and the characteristics of the string group SO(32), whose dimension is the third perfect number 496:

$$d_k + d_{7-k} = 32 \qquad d(SO(32)) = \binom{32}{2} = 496 \tag{28}$$

From the above double relation for B, the following property of the scalar boson emerges, with a special recurrence relation between the dimensions 10 and 9, showing also a connection with $S_{26} = 381540$, to 48 ppm:

$$\begin{cases} H^{(0)} = 495^2 = K_9\mathbf{N}_9 = K_{10}N_{10} + N_9 - 1 \approx \sqrt{WS_{26}} \\ \\ 495 = \binom{12}{4} = \binom{11}{3} + \binom{11}{4} = 3\binom{11}{3} = 3K_9 = \binom{32}{2} - 1 = 496 - 1 \end{cases} \tag{29}$$

where $\mathbf{N}_9 = 9K_9$ is the total number of positive zeros on the unit circle for the central string reduction dimension 9, and $N_9 - 1$ the number of non-trivial $9D$ symmetries. This is clearly related to the relation with pentapope numbers: $q_8 = N_9$, $q_{11} = K_{12}$, $q_9 \approx \sqrt{K_{10}N_{10}}$, showing a kind of symmetry between N and K. Note that 495 is the odd part of the first Mathieu group order 16×495, and the couple 495- 496 is the third perfect couple. In such a couple, the first number is the sum of the *non-trivial* divisors of the second. Since $496 = \binom{32}{2}$ is the dimension number of the group SO(32), and $495 = \binom{12}{4}$, this leads to the conjecture : *the third co-perfect number 495 could be the single one being a non-trivial binomial number*.

The most striking fact is the following connection between the Guanine and the couple $N_9 = 2K_9$, the factor 2 being identified to the duality proton-neutron, and the following factor 3 to a symmetry proton-neutron-electron, meaning that what counts is the number of particle, independently of their nature: this number is 499 in the Guanine molecule, which means 495 + 4, the latter 4 attributed to the Helium atom:

$$\begin{cases} N_9 = 330 = n_{Gu}^{(nucl)} + 1 \\ (3/2)N_9 = 495 \end{cases} \tag{30}$$

Note that N_9 and $K_9 = N_9/2$, as well as $210 = \binom{10}{4}$ are Euler suitable numbers, whose pertinence is confirmed below.

The associated nucleotide to the Guanine is the Cytosine, which is clearly tied with the topological function f(10), giving rise to a formula in the Hubble Table. The other couple (AT), as shown before (Eq. 19), is associated to the Fermi constant.

5.3 The "free parameters" and the Euler ideonal numbers

The equivalent relation for dimension 4, implying $\mathbf{N}_4 = 4K_4$ shows up a relation between the brute proton-electron mass ratio 1836 with the Euler maximal suitable number $s_{65} = 1848 = 43^2 - 1 \approx (4\pi)^2\sqrt{137}$:

$$N_4(\mathbf{N}_4 + 1/2) = (K_4 + N_4)^2 - 1 - N_4/2 = s_{65} - 12 = 1836 \tag{31}$$

This shows up a kind of symmetry between the additive and multiplicative operations in the 4D space. The maximal Euler's suitable number is very close to the Eddingtons's prediction [10] for the proton/electron mass ratio, $p_E \approx 1847.599459$, as the ratio of the roots of the equation $10x^2 - 136x + 1 = 0$. Note that to 10^{-4} and 23 ppm:

$$\begin{cases} \tau \approx p_E(2 - g_1^2) \approx (2 - 1/a_s)(4\pi)^2\sqrt{137} \\ s_{63} \approx (a/10)^2\mu\tau^7/p^8 \end{cases} \tag{32}$$

While a_s is tied to the SU(3) group, this shows a tight liaison with the U(1) group, which is rather logical for the lepton tau. *Moreover, this confirms the Eddington's prediction of the tau fermion, 35 years before its surprising discovery as an "heavy mesotron", based on a non-standad proton-tau symmetry* [10]. *This could unlock the, presently sterile, supersymmetry partner research*.

Note the dramatic properties of the ideonal numbers preceeding s_{65} (the unambigous factor 5 being unexplained):

$$\begin{cases} s_{64}/5 = 273 = s_{51} = q_{12}/5 \approx \Pi_+ \\ s_{63}/5 = 264 \approx 4qn_t/a_s \approx \Pi_0 \end{cases} \tag{33}$$

The total number of particles (protons + neutrons + electrons) involved in the four nucleotides is $1863 = 9 \times 207$, where 207 is the second approximation for μ in the bit-string model [8]. After separating the 4×4 trivial ones from Helium, this reduces to $1847 = 435 + 446 + 471 + 495$, at one unity from s_{65}. The presence of 495 for the Guanine could not be due to hasard. Indeed while its atomic massis is 329, at one unity from $N_6 = 330$, its number of particles is about $(3/2) \times 330 = 495$, *due to the electrical neutrality*, so *the factor 3/2 in the Table 5 is justified*. In the four nucleotides, counting the elementary particles (electrons + quarks) leads to $1863 \times 7/3 = 3 \times 7 \times 207 \approx 5\tau/4$, to 45 ppm, leading to further research.

In the Particle standard model, the scalar boson is necessary to explain the non-Zero mass of particles. Indeed, in the above procedure, the connection between the scalar boson and the 9D crystallography is clear, while it is not so for the above decisive 4D relation. But the first one has induced the latter one by analogic induction. *Thus the central role of the scalar boson is confirmed, and the mass concept is tied to a number of cristallographic symmetries.*

There is another connection between 1836 and 1848: they are both the area of an integer-sided triangle which is 6 times its perimeter, opening new further study. The connection with the pentapope number q_{13} is immediate:

$$q_{13} + 16 = \binom{16}{4} + 16 = 1836 \tag{34}$$

meaning that *1*836 is the sum of crossings from 16 points including those points, in parallel with the definition of 137, the maximal number of zones defined by 16 straight lines in a plane, as recalled below.

Moreover with Π_0, the neutral Pion-electron mass ratio, and the associated term Π_+ for the charged Pion, $p_{hol} = (4(r_H/\lambda\!\!\!^{-}_e)^3/3)^{1/2}$ and the 137^{th} Fibonacci (prime) number:

$$\begin{cases} \Pi_+\Pi_0 \approx 1838^2/4\sqrt{a} \approx (2\pi)^2 m_{cd}/m_e \\ s_{65} + 1/2 \approx (4\pi)^2\sqrt{a} \approx F_{137}/96a_w^2 \approx (q^2a/4)^2 H^{(0)}/p_{hol} \\ (s_{65} + 1/2)/2 = \binom{12}{6} + 1/4 \approx (210^{210}/\mu^\mu)^{1/3} \end{cases} \tag{35}$$

The last formula is deduced from the relation with the Monster Group (section 5.7) confirming the connection $\mu \approx 210$. The $+1/2$ term comes from taking account of the dimension 0 in the half sum of symmetry numbers, as confirmed below. The involved precise value for $\pi \approx 3 + 1/(7 + 9/137)$ is very particular, opening further study.

This number s_{65} enters the correlations:

$$s_{65}/2\pi \approx 8\pi\sqrt{a} \approx (R_e/R)^{1/21} \tag{36}$$

Comparing this with the above holic relation $R/\lambda\!\!\!^{-}_e \approx (2R/R_e)^{210}$, this leads to $R/\lambda\!\!\!^{-}_e \approx (2^{18}/\pi\sqrt{a})^{10}$ which is also, according to the tabulated holographic relation: $R/\lambda\!\!\!^{-}_e \approx (2\pi^2 a^3)^5$. Their ratio involves $a/137$, leading to:

$$\begin{cases} 137/\pi^4 \approx (a/2^7)^5 \approx \sqrt{2} \\ (ad_e/2^7)^{10} \approx 1 + d_e \end{cases} \tag{37}$$

where 2^7 is the Combinatorial Hierarchy brute value of 137 [7], and also the effective value for a at Fermi energy.

It is shown [43] that a single Euler suitable number could exist beyond s_{65}, and if not, i.e. if s_{65} is really the maximal one, then the generalized Riemann conjecture would be confirmed. So the proton-electron ratio is at the heart of Number Theory.

Thus the string canonical $9D$ dimension reduction is correlated with the $9D$ crystallographic symmetries. This confirms the elimination of the continuum in theoretical physics, in conformity with the Computing Principle. This could unlock the present dilemma of string theories which lead to an enormous number (10^{500}) of solutions for dimension reduction, an anomaly which is claimed to sustain the unscientific Multiverse model.

With the electric charge $q = W sin\theta/H^{(0)}$, the computer shows up the following relations, in the ppb domain:

$$\tau F/Wq \approx K_3 K_5 K_9/3 \tag{38}$$

Note that $1 + K_3 K_5/3K_9 = 4181 \approx F/a$, showing the 19^{th} term of the Fibonacci series, the first composite number of order prime. Moreover, the U(1) coupling $g_1 = Z \sin\theta/H^{(0)}$ is confirmed in the ppb domain by:

$$f(26) = f(2)^{32} \approx (H/p_t)(2/g_1^2 d_e)^{16} \tag{39}$$

This confirms the central role of the string dimension 26.

The above adopted value of the coupling $g_3 = g_2 g_1/g_0$ shows a dramatic connection (0.5 ppm) with the terminal Euler ideonal number:

$$g_3 \approx 3 + 2a/s_{65} \tag{40}$$

This means that the Cosmos uses approximations for π, which is quite natural when a "quantinum" replaces the standard continuum. Another formal approximation of π appears in the Adimensional Electrical Charge (Table 1), which confirms g_3:

$$\begin{cases} g_1 cos\theta = g_2 sin\theta = 4\pi_q^2/a & \rightarrow \quad 3^{10} \approx 6(2\pi_q)^5 \approx \pi a^2 \\ 2\pi_q q g_2 \approx g_3 (a/137)^8 \approx e\sqrt{2}/\pi \end{cases} \tag{41}$$

This confirms the central role of the base 3 Mirimanoff property of the supergravity and superstring dimensions 11 and 10. The definition of the first Mirimanoff number 11 is that $3^{11-1} - 1$ is a multiple of 11^2 [33]. This recalls that [13]:

$$3^{10} \approx \pi a^2 \approx \Phi^{137/6} \approx (l_{phCMB}/\lambda\!\!\!^{-}_e)^{1/2} \tag{42}$$

involving the Golden ratio in the old chinese musical scale of 60 notes per octavus. The lenth l_{phCMB} is the side of a cube containing a single CMB photon. The total number of photons in the Hubble sphere and in the Cosmos shows dramatic particularities (190 and 4 ppm):

$$\begin{cases} n_{phCMB} = (4\pi/3)((R/l_{phCMB})^3 \approx 2(R/(\pi^2 a^4 \lambda\!\!\!^{-}_e)^{3/2} exp(e^6/4) \\ (4\pi/3)((R_c/(\pi^2 a^4 \lambda\!\!\!^{-}_e)^3 \approx (a/137)\pi\sqrt{g_0}(R/\lambda\!\!\!^{-}_e)^7 \end{cases} \tag{43}$$

This direct liaison with the Number Theory confirms that the c-Universe acts as a Cosmic boson, acting by the seventh power, in conformity with the Topological Axis and the Holic Principle [13].

5.4 The Eddington-Atiyah's inverse brute electric coupling 137, an *Arithmetical Monster*

The number 137 is the Eddington's inverse brute electric coupling, and has been unambigously connected with the Lucas-Lehmer series [5]. Atiyah recently associated this number with three algebra: the octonion, quaternion and real ones, associated to the number $273 \approx m_{\Pi_+}/m_e$, which is again one of the Euler's suitable numbers:

$$137 = 2^7 + 2^3 + 2^0 \qquad 2 \times 137 - 1 = 273 = 2^8 + 2^4 + 2^0 \tag{44}$$

Strangely enough, it seems that nobody have looked for the prime numbers that appear in the harmonic series, which is the single pole of the Rieman series, precisely known to inform about the distribution of prime numbers. The six

first prime numbers appearing are the following, showing a symmetry of 11 around 137, showing the 11 supergravity dimensions and the usual 4 ones:

$$3; 11; 5; 137; 7; 11 \qquad \Rightarrow 137 = 11^2 + 4^2 \tag{45}$$

Note that, while $137 = l_{16}$, the 16^{th} Lazy Caterer number (maximal number of zones in a plane defined by n straight lines), $11 = l_4$ and $4 = l_2$. Moreover, with the Rule 23 cellular automaton Wolfram series $u_n = 1 + n(2n + 1)$, and its combined form $v_n = u_{2(d+3)}$ (Table 7):

$$\begin{cases} u_1 = 4 \\ u_2 = 11 \\ u_8 = 137 = v_1 = u_1^2 + u_2^2 \\ v_8 = 991 = 495 + 496 \\ u_{11} = 2(2^7 - 1) = N_{11} - N_{10} \end{cases} \tag{46}$$

This series has been deduced from the fact that $u_{30} = v_{12} = Q_{13} - 5 = 1831 \approx p_G$ (Table 1). This shows clearly that the compactification operates from the Topologic dimension 30, by groups of 3 and 4 dimensions, where 12 and 13 are the half of the 24 transverse and the 26 = 30 - 4 main dimensions. The 4D appears as 3D + 1D, separating the Space from the Time. *The latter 1D is interpreted as the predicted Cosmic Hol Sweeping Absolute Time* [13].

The corresponding Pythagorean triangle has the sides 88,105,137, i.e. the number of partitions of 18, 19 and 20, with elements greater than 1 (OEIS A002865). Its perimeter is $330 = K_9$ and its area 10 s_{65}:

$$\begin{cases} P_{137} = K_9 = 330 = 137 + 105 + 88 \\ A_{137} = 14P_{137} = 10s_{65} \\ g_2 a \approx 88 \\ g_2 a^2 \approx (105\pi/3)^2 \end{cases} \tag{47}$$

This connects the 9D crystallography with the maximal Euler number $s_{65} = 1848$. The above Pythagorian triangle has a radius 28 for the internal circle, while $a \approx 137 + 1/28$. The next term in the development is 3511, the second Fermat-Wieferich number: [29]

$$a \approx 137 + 1/28 + 1/3511 \tag{48}$$

This defines a in its 0.15 ppb indetermination (Table 2).

The only known couple of Wieferich numbers are $p_{W1} = 1093 = 1 + 4(16^2 + 16 + 1) = 1 + 4(136 + 137) \approx 4\Pi_+$ and $p_{W2} = 3511 = 1 + 6(8^3 + 8^2 + 8 + 1)$. This induces the *supersymmetric* couple (meson η, fermion τ). They connect with the only known couple of Mirimanoff numbers [33], which uses the base 3 instead of the Wieferich base 2: $p_{M1} = 11$ and $p_{M2} = 1006003 = 1003^2 - 6$, where $1003 = K_9 + K_{10} + K_{11}$ shows up in the Crystallographic Table 7. One notes *an 0.1 ppm relation between p_{M2} and the supersymmetric electron-proton-neutron triplet*:

$$\begin{cases} p_{W1}p_{W2} \approx e^7 \times e^{3e} \approx \eta\tau \approx e^{e^e} \approx e^e aH \\ p_{M2} = (K_{p_{M1}} + K_{p_{M1}-1} + K_{p_{M1}-2})^2 - 6 \approx 4a\sqrt{p_r n_t / d_e} \end{cases} \tag{49}$$

confirming the pertinence of the basic economic number e_3.

This "arithmetic monster" 137 appears twice in the Crystallographic Table:

$$137 = \sum_6^8 K_d = \sum_1^7 (N_d/2) - 1 \quad \Rightarrow \quad \sum_1^4 K_d = (K_7 + 1)/2 = d_7 \tag{50}$$

This identifies the 4D term $\sum_1^4 K_d = d_7 = 30$ in the brute U(1)-SU(2) gauge partition 137 = 107 + 30 [32]. Extrapolating to the superstring dimensions 10 and 11, this connects with the holic term 210, itself connecting with $26 = d_6$:

$$\begin{cases} (K_7 + 1)/2 = d_7 = 2 \times 3 \times 5 = 30 \\ (K_{11} + 1)/2 = d_{2d_6} = 2 \times 3 \times 5 \times 7 = 210 \end{cases} \tag{51}$$

This connects the main dimension 30 of the Topological Axis with the dimension 210 of the Holic principle.

As recalled above, 137 is the number of partitions of 20 with integers superior to 1. This seems connected to the Golden ratio Φ through:

$$\begin{cases} \sqrt{a}/2 \approx (1 + 2cos\theta)/sin\theta \approx (a/20) - 1 \approx \Phi^4 - 1 \\ (1 + 2cos\theta) \approx (4p_t/n_t)^{1/2} g_2 g_3 \approx (cos\theta/2e)(137/sin\theta)^{1/2} \end{cases} \tag{52}$$

where $1 + 2cos\theta \approx \pi$, involving the sum $Z + W_+ + W_- = Z + 2W$, *showing another non-standard particle symmetry*.

5.5 The *precise* U(1)-SU(2) gauge partition

Taking account of the dimension zero, the above sum (Table 7) becomes $S_{12} = 1836.5$, close to the mean proton-Hydrogen mean, and the gauge separation could imply rather $n_7 + 1/2 = 30.5$, which is close to 196 ppm with the real U(1)-SU(2) gauge partition term $a(\sin\ \theta)^2 \approx 30.505983$, and more precisely:

$$d_7 + 1/2 \approx 137^2/ad_e - (a_w^2)/Z^4 \approx a_w^{1/2}/a^2 \tag{53}$$

Moreover, this number connects again with the holic term 210:

$$2(d_7 + 1/2)^2 = 9 \times 210 - (d_7 - 1/2) \tag{54}$$

The above proximity between μ and 210 materializes in the following 44 ppb determination of μ, with a 23 ppm correlation with τ :

$$(a/137)(2(137^2/(ad_e - (a_w^2)/Z^4)^2) \approx 9\,\mu \approx \tau\ tg\theta \tag{55}$$

So the U(1)-SU(2) gauge partition is at the heart of the optimal computation process.

5.6 The String dimension partition 26 = 22 + 4

In the string theory, the 26 dimensions reduce to the usual 4D by separating 22 hidden dimensions. Indeed, one observes:

$$N_{22} = K_{20} + K_{22} = (20 \times 22) \times 137 \tag{56}$$

maybe the most incredible property of the Arithmetical Monster 137. The same relation applies also to the 4D usual space:

$$N_4 = K_2 + K_4 = (2 \times 4) \times 3 \tag{57}$$

The computer shows up another case, which involves the four usual dimensions d = 1, 2, 3, 4 in a symmetrical way, :

$$N_{13} = 2K_{11} = (2 \times 11 \times 13) \times 7 = N_6N_8N_9/N_1N_2N_3N_4 \tag{58}$$

The sum of the implied dimensions is the same: 23 = 1+2+3+4+13 = 6+8+9.

The other string partition is 26 = 10 + 16. One observes the following precise relations with the 3 couplings, electric, electroweak and gravitational (1 ppb and 10 ppb):

$$K_{10}/(K_{10} - 2) = K_{10}/(S_9 + 2) \approx F^5/Pa^3 \approx e^{1/(210-1)} \tag{59}$$

This could be tied to the two trivial symmetries, identity and point inversion.

5.7 The Connections with the Periodic Table

The string dimensions special series $d = 2{+}4k$ identifies both with the Topological Axis one and with the spectroscopic one, so *the string dimension 2 identifies with the spin 1/2 degeneracy, where k identifies with the orbital number, running in the octonion series, between 0 and 7.*

In the Periodic Table of elements, the total number of elements untill the n^{th} raw, where n is the principal quantum number is:

$$n_n = \sum_{j=1}^{n} \sum_{k=0}^{k=j-1} = 2\sum_{j=1}^{n} n^2 \tag{60}$$

There is a particularity for the 7^{th} row, due to the association symmetry-computation where the central dimension is 16: indeed $2 \times 16 = 32 = 2 + 30 = 6 + 26 = 10 + 22 = 14 + 18$:

$$\sum_{k=0}^{k=7} d_k = 2^7 \qquad \Rightarrow \quad \sum_{k=0}^{k=7} d_k + \sum_{0}^{1} d_k + 1 = 137 \tag{61}$$

which shows the Atiyah formula [30]. The height numbers are all of the form "prime - 1", except $d_i = 14$ and 26, the later being the critical dimension which verifies: $d_{26} = d_{d_6} = 106$, so justifying the "reduced" Atiyah sum, with the octonion term (2^7) and the quaternion one (2^3). This identifies with the reduced U(1)-SU(2) gauge partition, where 136 is the initial Eddington's electric coupling, the number of elements in the symmetrical matrix 16×16:

$$\sum_{k=0}^{k=7} (d_k + 1) = 2^7 + 2^3 = 136 = 30 + 106 = d_7 + d_{d_6} \tag{62}$$

There is a particularity for the 4^{th} row which is effectively used in the Periodic Table, corresponding to the famous spectroscopic numbers, called by Friedrich Hund "sharp" ($s = 2$), "principal" ($p = 6$), "diffuse" ($d_i = 10$) and "fundamental" ($f = 14$). The 7^{th} row of the Periodic Table terminates in the Oganesson, recently synthetised [45], of atomic number 118, *which is precisely the Herman number for d = 7*. By adding the following group of 18 (orbital quantum number 4), the Periodic Table would attain 136 elements. Apart one unity, since $118 = 2 \times 59$, this corresponds to the above crystallographic partition $137 = 118 + 19 = 78 + 59$. The Oganesson involved coefficients, with the *symmetrical* distribution of the spectroscopic groups s, p, d_i, f are the following:

$$\sum_{k=0}^{k=3} c_k d_k = 118 \quad \rightarrow c_k = (7, 6, 4, 2) \tag{63}$$

The above variation of one unity, connected to prime numbers, leads to

$$\sum_{k=0}^{k=3} c_k (d_k + 1) = 137 = 2^7 + 2^3 + 2^0 = 107 + 30 \tag{64}$$

which recovers the complete Atiyah sum, including the "real algebra" term 2^0, and, since the last "fundamental" (an anticipated judicious name) term is $2 \times 15 = 30$, coming back to the above brute U(1)-SU(2) gauge partition. Note that the four basic primes $p_1 = 2; p_2 = 3; p_3 = 5; p_4 = 7$ enters the following development, particularizing the dimension 4D:

$$7(N_1 + p_1 + N_2 + p_2) + N_3 + p_3 + N_4 + p_4 = 91 + 15 + 31 = 106 + 31 = 137 \tag{65}$$

However, this Atiyah series presents an imperfection: the absence of the term 2^1, corresponding to the complex algebra. One observes that the total sum taking account of the four algebra is $139 \approx i^{\pi/i} = e^{\pi^2/2}$. So the origin of 137 would be the mean between 139 and 135, the latter being the product of the two co-perfect numbers 5 and 3^3, very close to $16a_s$. Indeed, one oberves, in the ppb domain:

$$137 = (16a_s + i^{\pi/i})/2 - 1/d_e + 2^0 \tag{66}$$

So the optimized value of $a_s \approx a_w/2\pi(pH)^{3/2}$ is confirmed in the ppb domain. This tight connection with the electron excess magnetic moment $d_e \approx 1.001159652$, which is the best confirmation of the quantum theory, opens future research.

6 The Sporadic Groups Connections

The 26 sporadic groups include 20 "happy" groups tied to the Monster, and 6 "pariah" groups. Many relations with the physical parameters were published [19], two of them implying formula for R and R_c (Tables 2 and 3). The main connections implying the Monster Group order O_M are (3.8, 1.4 and 6 ppm):

$$\begin{cases} e^a \approx 2O_M Z^2 p/1839W \\ e^{1/2a} \approx O_M/(2a^2P^2)^2 \approx \mu^2 W/2a^2 Z \end{cases} \quad \Rightarrow \quad g_0 = R_e/2R \approx P^2 FW/O_M Z \tag{67}$$

Thus *The Monster Group is related to the CMB through the holographic term e^a and the Lucas Number through the gravitational coupling g_0*. Moreover, one observes the relations tying the electric, strong and weak couplings $a, a_s, and a_w = F^2$, to 10, 7, 150 and 300 ppm:

$$F/aa_s \approx (137/a)\tau^{3/2}/2\mu \approx 495 \times 2^{1/(24\times 20)} \approx K_{26}/f(10) \approx O_M^{1/20} \tag{68}$$

with $K_{26} = 141877$. Now $f(10)^{10} \approx l_{nl}/\lambda\!\!\!^{-}_e$ and K_{26}^{20} is of order $R_C/\lambda\!\!\!^{-}_e$. This implies again a pertinence for the canonic string dimensions 26 and 10, calling for further study. The order O_M of the Monster group connects with the Lepton mass ratios and the final Euler number s_{65}:

$$O_M^9 \approx \tau^{137} \approx \mu^\mu s_{65}^2/\sqrt{2} \approx 4\sqrt{2}\, 210^{210}/s_{65} \tag{69}$$

This confirms that $\mu_0 = 2 \times 3 \times 5 \times 7 = 210$ is the pertinent arithmetic approximation of μ. With the symmetric approximation $\tau_0 = (2 + 3 + 5 + 7) \times 2 \times 3 \times 5 \times 7$:

$$(p_t/n_t d_e)(\tau/\tau_0)^{137} \approx \sqrt{2}p^3/a_s^2 H^2(H - p) \approx \pi^\pi \tag{70}$$

confirming to the ppb range the Koide tau value [35], where n_t/p_t is the mass ratio neutron-proton. *So the sporadic groups are at the heart of the overall unification, opening further study*

7 Conclusions and Predictions

This article confirms the pertinence of the Topological Axis [5], with its *invariant* Hubble radius, as a key for debunking theoretical physics, by revealing two new decisive points. Fristly, the DNA bi-codon, imposed by the Holic Pinciple, corresponds to the central dimension $d = 16$. *This milits for a general cosmic DNA Life*. Secondly, the supergravity dimension $d = 11$, corresponding to the "strange" particle Kaon, is tied to the 10 superstring dimensions through the base 3 Fermat-Mirimanoff relation $11 - 1 = 10$. This implies unambigously the Arithmetic Monster 137, the Golden ratio, and the old chinese musical scale *This milits for a return to an harmonious Cosmos, meaning the generality of intelligent Life* [13].

This article permits to connect the main "free" physical parameters with different domains of the Number Theory. In particular, the two base 2 Fermat-Wieferich primes: 1093 and 3511 connect directly with the Relative Radiation Ratio, which connects also with the gauge couplings. This confirms the unification role of the Radiation Background (photons + neutrinos), common to the *c*-observable Universe and the Cosmos. *Besides the two Pillars of Physics, the third pillar, the Statistical Physics, shows a prominent role*.

This article rehabilitates several discarded physical theories: those of Eddington [10], Noyes [8], Wyler [40] and Atiyah [30]. It has been proved that the standad so-called standard "free" parameters are calculation bases in the computing Cosmos. Indeed high powers of them appear in the Hubble and Cosmos tables, with special importance of the symmetrical combinaison of the four basic primes 210, specially the term 210^{210}, confirming the pertinence of the Holic Principle and the Optimal Computation Principle. *This article shows clearly the arithmetical origin of the leptonic mass ratios from the main bases 2;3;5;7*.

The tachyonic character of the Cosmos is of paramount importance, interpreting at last the non-Doppler quasar power oscillation, rehabilitating the string bosonic theory and integrating the "quantum holism", the manifestation of quantum non-locality by introducing a super-celerity *C*. *Instead of ignoring such an "incomprehensible" non-Doppler phenomena, the astrophysicists ought to study this intensively, specially the phase differences fram a quasar to the other, with emphasis on the determination of the tachyon celerity C or its intermediate gravitational value* $C/P \approx 10^{38}c$ [5].

The String Theory connects at last with Reality, but it must be entirely reconsidered, by replacing the continuum by a "quantinuum", based on the "Topon", and adopting a *massive* string, as predicted by the Topological Axis. Also *massive* gluons, photon and graviton must be included in the Particle standard model. This means that another identification is needed for the scalar boson: not only the standard mass-generation role. The Particle Physics must also include *the Eddington's proton-tau "intersymmetry", the eta-tau supersymmetry and the elegant Koide formula, whose associated leptons masses* μ *and* τ *connect so precisely with the other data.*

The Cosmology must be completely re-interpreted, with the unifying concept of "Permanent Sweeping Holographic Oscillation Bang Matter-Antimatter Cosmology". Considering the visible Universe as a quantum entity, the simple consideration of its wavelength (Topon) leads to the Toponic Holography, which breaks down the Planck wall by the factor $C/c \approx 10^{60}$, explaining at last the giant factor (10^{120}) for the vacuum quantum energy. The future giant telescopes must observe an invariant background (CMB) temperature, as well as an invariant trivial value 3/10 [5] for the baryon+dark matter density, the latter being an anti-phase oscillation of normal baryons.

The DNA bi-codon mass is central in the Cosmos, confirming again, and with a high degree of precision, the pertinence of the dimension 16, showing how the Topological Axis has been predictive. Thus, the DNA molecule would be more than just a simple memory as anticipated by Schrödinger [46]. It must be a bio-computer, probably activated by real holography. Indeed, electric current is observed in DNA [47]. So physical laws are identical to biological ones, again ruling out the Multiverse model. *The DNA molecule would be, like the Cosmos, a 1D sweeping hologram, opening the way for "biocomputers"*.

The standard point of view, which considers Life as an "emergent" phenomena is incomprehensible and sterile. This study shows that, quite the contrary, Life, as well as the *c*-Universe, is an "immerging" cosmic phenomena. The fact that the term "immergence" is a perfect neologism proves the excess of reductionism that has been adopted by the standard formalists. So, the relation, for k = 4 (d = 18), between the cosmic temperature and the mammal one $T_{mam} \approx jT_{CMB}$, where $j = 8\pi^2/ln2$ is the scale constant [5] takes a renewed importance, as well as the relation swith the triple points of Hydrogen, Oxygen and Water. It is foreseen that future theory will be able to calculate these triple points, a task nowadays impossible.

The overwhelming connections confirm that the pure mathematics must now pursue unification, by concentrating on the mathematical properties of physical parameters. Such connections between apparently separated mathematical domains has been already introduced by Physics [48]. *In particular, research must concentrate on the Algebra of Eddington's E Numbers [10] [49], the Euler ideonal numbers, the Wieferich and Mirimanoff primes, the additive*

series of prime numbers, the Rule 23 Wolfram cellular automaton, the multi-dimensional crystallography and sporadic groups.

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References

[1] Poincaré H. (1912) Sur la théorie des quanta. Journal de physique, vol 2, p. 5.

[2] Poincaré H., (1913) Dernières Pensées. Conférence à l'Université de Londres, pp. 102-103 (Flammarion).

[3] Zyla. P.A. (2020) et al. Particle Data Group, Prog. Theor. Exp. Phys.

[4] Carr B.J. and Rees M. J. (1979) The anthropic principle and the structure of the physical world, Nature 278, 605. 123-142.

[5] F.M. Sanchez, V. Kotov, M. Grosmann, D. Weigel, R. Veysseyre, C. Bizouard, N. Flawisky, D. Gayral, L. Gueroult (2019) Back to Cosmos. Progress in Physics, vol. 15, issue 2.

[6] Sanchez F.M., Kotov V. and Bizouard C. (2013) Towards Coherent Cosmology, Galilean Electrodynamics, special issue, pp 63-80.

[7] Bastin T. and Kilmister C.W., (1995) Combinatorial Physics. World Scientific.

[8] Noyes P. (2001) Bit-String Physics: A Finite and Discrete Approach to Natural Philosophy. J. C. van den Berg (ed.) World Scientific. ISBN 978-981-02-4611-2.

[9] t'Hooft G. (2015) The Cellular Automaton Interpretation of Quantum Mechanics. ArXiv:1405.1548v3.

[10] Eddington A, (1949) Fundamental Theory, Cambridge University Press.

[11] Alcina C. (2013) La secte des nombres. Le thèorème de Pythagore, Images des Maths, p. 147.

[12] Schwarz (2004) J. H. String Theory: Past, Present, and Future. Séminaire Poincaré vol 1, p. 42.

[13] Sanchez F.M. (1995) Holic Principle, Entelechies, ANPA 16. Bowden K.G., 324-343.

[14] Bondi H. (1968) Cosmology, Cambridge University Press, p. 74.

[15] Hoyle F. et al. (2000) A Different Approach to Cosmology, C.U.P, Cambridge p. 83.

[16] Steinhardt P.J. (2011) The Inflation Debat: is the theory at the heart of cosmology deeply flawed? Sc. Am., p.88.

[17] A. D. Sakharov (1967) ZhETF Pis. Red. 5, 32 (1967) [JETP Lett. 5, 24

[18] Kotov V. A. and Lyuty V. M. (1990) The 160-min. Periodicity in the optical and X-ray observations of extragalactic objects. Compt. Rend. Acad. Sci. Paris 310, Ser. II, 743-748.

[19] Sanchez F.M. (2006) "Towards the grand unified Holic Theory". Current Issues in Cosmology. Ed. J.-C. Pecker and J. Narlikar. Cambridge Univ. Press, 257-260.

[20] Sanchez F.M., and Bizouard C. (2008) Radius invariance of the observable Universe, Galilean Electrodynamics 19, N 2, 39-40.

[21] Poincaré H. (1924) La mécanique nouvelle, Eds. Gauthiers-Villars, Jacques Gabay (1989).

[22] Sanchez F.M., Kotov V. and Bizouard C. (2009) Evidence for a steady-state, holographic, tachyonic and supersymmetric cosmology. Galilean Electrodynamics 20, Special Issues, No. 3, 43-53.

[23] Sanchez F. M. (2017) A Coherent Resonant Cosmology Approach and its Implications in Microphysics and Biophysics, Prog. Theor. Chem. and Phys, Springler, v. 30, p. 384, 375-407.

[24] Sanchez F.M., Kotov V. and Bizouard C. (2011) Towards a synthesis of two cosmologies: the steady- state flickering Universe. Journal of Cosmology, vol 17.

[25] Dicke, R. H. (1961) Dirac's Cosmology and Mach's Principle. Nature. 192 (4801): 440–441.

[26] Scully M.O. and Sargent M. (1972) The concept of the photon, Physics Today 25,3,38-47.

[27] Freedman W et al (2019) The Carnegie-Chicago Hubble program. ArXiv:1907.05922v1.

[28] Davies P.C.W. (1993) The Accidental Universe, Cambridge University Press p. 50.

[29] Wieferich A. (1909) Zum letzten Fermat'schen Theorem, J. Reine angew. Math., vol. 136, 293-302

[30] Atiyah M. (2018) Heidelberg Laureate Forum 24th https://hitsmediaweb.h-its.org/Mediasite/Play/35600dda1dec419cb4e99f706197a3951d.

[31] Nambu H. (1952) An Empirical Mass Spectrum of Elementary Particles, Prog. Theor. Phys. Vol 7, n°5, 595-6.

[32] Taylor John G. (1973) The New Physics. American Journal of Physics, Vol.41, p 1381-1382.

[33] Mirimanof D. (1910) Sur le dernier théorème de Fermat, C. R. Acad. Sci. Paris, 150 , 204-206.

[34] Bekenstein J. (1973) Black holes and entropy. Phys. Rev. D 7:2333-2346 Issue:8. Phys. Rev. D. 7.2333.

[35] Koide Y.(1982) Fermion-Boson Two-Body Model of Quarks and Leptons. Lett. Nuovo Cimento 34, 201 .

[36] Huang M., Audi G. Kondev F.G. Huang W.J., Naimi S. and Xu X. (2017) The Ame2016 mass evaluation. Chinese Physics, C41 03003.

[37] Quinn T, Speake C, Parks H, Davis R. (2014) The BIPM measurements of the Newtonian constant of gravitation. G. Phil.Trans. R. Soc. A372: 20140032. https://royalsocietypublishing.org/doi/10.1098/rsta.2014.0032.

[38] Seiberg N. Emergent Spacetime (2005) The Quantum Structure of Space and Time, Proccedings of the 23rd Sovay Conference on Physics, Brussels, Belgium, ed. Gross, Henneaux, Sevrin, World Scientific, 163-178.

[39] Hayes, Brian (2001). Third Base. A reprint from American Scientist, Volume 89, Number 6, pp. 490-494.

[40] Wyler A., (1969) "L'espace symetrique du groupe des equations de Maxwell" C. R. Acad. Sc. Paris, t. 269, 743-745. Wyler A. (1971) C.R. Acad. Sci, and t. 272, 186-188.

[41] Hermann (1949) C.Raumen betlieger Dimensionszahl. Acta. Cryst. 2, 139-145.

[42] Veysseyre R., Veysseyre H., and Weigel D. (1992) Counting, types and symbols of crystallographic Point Symmetry Operations of space E n AAECC 5, 53–70 DOI: 10.1007/BF01196625 ISBN: 0938-1279.

[43] Weinberger, P. (1973) Exponents of the class groups of complex quadratic fields. Acta Arith. 22, 117-124.

[44] Paulo Ribenboim 1979 13 Lectures on Fermat's Last Theorem, Springer, 23, 152-153.

[45] Oganessian Y. et al. (2002). Results from the first 249Cf + 48Ca experiment (PDF). JINR Com, Dubna.

[46] Schrödinger E. (1944) What is Life?, Macmillan.

[47] Montagnier L. et al, (2009) Electromagnetic Signals Are Produced by Aqueous Nanostructures Derived from Bacterial DNA Sequences Interdisciplinary Sciences Computational Life Sciences 1(2):81-90.

[48] Conway (1979) John Horton. Norton, Simon P. Monstrous Moonshine. Bull. London Math. Soc. 11 (3), 308–339.

[49] Salingaros N. (1985) Some Remarks on the Algebra of Eddington's E Numbers. Foundations of Physics, Volume 15, 6, 683–691.