Towards Science Unification Through Number Theory

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Abstract

The mass concept introduces a symmetry between gravitation and quantum physics in the Keplerian Diophantine equation. This connects the Bohr's orbits with the Permanent Holographic Oscillatory Bang model (PHOB), showing up the mean DNA bi-codon mass which connects to 0.1 ppm with the Topological Axis. The local c-observable base 2 Universe appears as a gauge boson in the base 3 Tachyonic Cosmos with massive photon and graviton. This induces a scalar boson mass confirmed both by the geometric extension of the Weinberg triangle, which correlates the three gauge couplings U(1)-SU(2)-SU(3) and by the 9D crystallographic symmetry, identified with the canonic 9D string reduction. The mass concept is associated to a number of cristallographic symmetries. This is tied to the Euler's maximal suitable number, connected with the generalized Riemann hypothesis, confirming Eddington's Fundamental Theory and justifying the U(1)-SU(2) gauge partition. The running number of the string dimension series of the Topological Axis is identified with the orbital quantum number, associating the spin 1/2 and the string dimension 2. This excludes the concepts of Multiverse, Continuum, Infinity, Locality and Zero mass Particle, and leads to stringent predictions.

***K*eywords** Computation Principle · Holographic Principle · Holic Principle · Cosmology · Number theory · String Theory · Bit-String Physics · DNA nucleotides · Crystallographic Symmetry · Periodic Table · Sporadic Groups.

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1 Introduction: the *invariant* Hubble radius

Despite many tremendous advances in Physics unification, the standard model of particles, after decades of intense research, is always unable to integrate gravitation. On the other hand, the string theory shows up a spin 2 graviton and introduces the elegant multi-dimensional Holographic Principle, but seems unable to connect with the reality. In particular, the Holographic Principle is not applicable to the usual 4D space-time, due to the official temporal variation of R, the Hubble horizon radius. According to Schwarz : *"this turns out to be surprisingly difficult"* [1].

In fact, the present-day value for R introduces a dramatic crisis. In spite of an optimisation of six parameters, the standard ΛCDM cosmologic model leads to the present-day Hubble constant $H_0 = c/R \approx 67.74 \pm 0.46$ km/s by Megaparsec [2], while, since several years, direct measurements, using different methods, confirm a significant discrepancy. In particular, the latest measurement is 69.8 ± 0.8 [3].

This discrepancy is widely discussed, but this last value was predicted long ago, by the mere application of c-free MLT dimensional analysis [4]. This value was later justified by the model of the Gravitational Hydrogen Molecule [5], which is also the limit of a star radius when its atomic number is reduced to unity, [6]. With Giga light-year (Gly) unit, where $\lambda\!\!\!^{-}_e = \hbar/m_e c$ is the electron reduced wavelength, this is:

$$R = 2a_G\ \lambda\!\!\!^{-}_e = 2\hbar^2/Gm_e m_p m_H \approx 13.811977\ \ Gly \tag{1}$$

corresponding to $H_0 \approx 70.790\ \ kms^{-1}Mpc^{-1}$, compatible with the above last measurement. Moreover, this value connects within 10^{-3} with the reduced topological function $g(k) = exp(2^{k+1/2})/k$, for $k = 6$ $(d = 2 + 4k = 26)$, see Graphics 3, where the dimension 26 is the privileged dimension of the string bosonic theory [7]:

$$R \approx g(6)\ \lambda\!\!\!^{-}_e = 13.82\ \ Gly \tag{2}$$

The Topological Axis (Graphics 3) rehabilitates the Large Number Correlations, whose the *single* official justification is the Anthropic Principle, which also is used to justify the so-called "biologic fine-tuning" through the Multiverse model [8]. But such a rough argument cannot explain the above precision, so *the Topological Axis recover the ancestral idea of an unique Universe.*

According to Russel *"The most surprising in the modern science is its come back to Pythagorism"* [9]. The aim of this article is to confirm this, in particular the pertinence of the Holic Principle, the diophantine form of the Holographic Principle, supporting an invariant Hubble radius. Also we show that the DNA characteristic mass is a central physical parameter, via 10^{-7} fine-tunnig with the mean dimension 16 of the Topologic Axis, which so was indeed *predictive*.

The standard cosmological model is not completely refuted, since the so-called "Universe age" is so close to R/c. Considering matter as a very rapid matter-antimatter vibration, a "Permanent Holographic Oscillatory Bang model" (PHOB) has been proposed, where the Planck wall is reduced by a factor 10^{61} [10], and the "dark matter" would be π-phased oscillation. The sum of baryons and anti-phased dark baryons has *the invariant relative density 3/10* [7], as confirmed by a formula in Table 4, which rehabilitates the Eddington's Baryon Large Number 136×2^{256} [11].

The Table 4 shows 55 simple formulas for the Hubble radius R. This recalls the 14 molecular formula of Jean Perrin that established definitely the existence of atoms, based on 6 different theories. In this table also, numeric terms from different approaches are used: [6], [8], [11], [12] and [27]. But the most significant is the Holic Theory, recalled below, characterised by the "primal factorial 7", the number $210 = 2 \times 3 \times 5 \times 7$. The appearance of 210^{210} and μ^{μ} resolves at last a complete mystery in the standard model: the *arithmetic* origin of the muon/electron mass ratio. Note that the approximation $\mu \approx 210$ is central in the Bit-String Physics [17], and will be confirmed below.

In this article, the search for precision correlations uses the precisely defined values obtained by the "maximal correlation principle" [7]. The optimized gravitation constant is $G \approx 6.675453718$ (SI). By respect to the electron mass, the Planck mass is $P \approx 2.389015908 \times 10^{22}$, the weak bosons masses are $W \approx 157340.1093$ and $Z \approx 178451.7402$, *in ppb accordance* with the 4^{th} formula of table 2. The leptons masses are $\mu \approx 206.7682869$ and $\tau \approx 3477.441701$, obtained trought the so elegant Koide empiric relation [13]. The *inverse* coupling constants are noted $a = 1/\alpha$ (electric coupling), $\sqrt{a_w} \approx 573007.3652$ (weak coupling), $a_s = a_w/2\pi(pH)^{3/2} \approx 8.4354502914$ (strong coupling), $a_G = hc/Gm_p m_H \approx 1.691936 \times 10^{38}$ (gravitational coupling). Note that $a_w \approx (2 \times 137\Gamma)^3$ was very precisely related to the Atiyah's constant $\Gamma = \gamma a/\pi$.

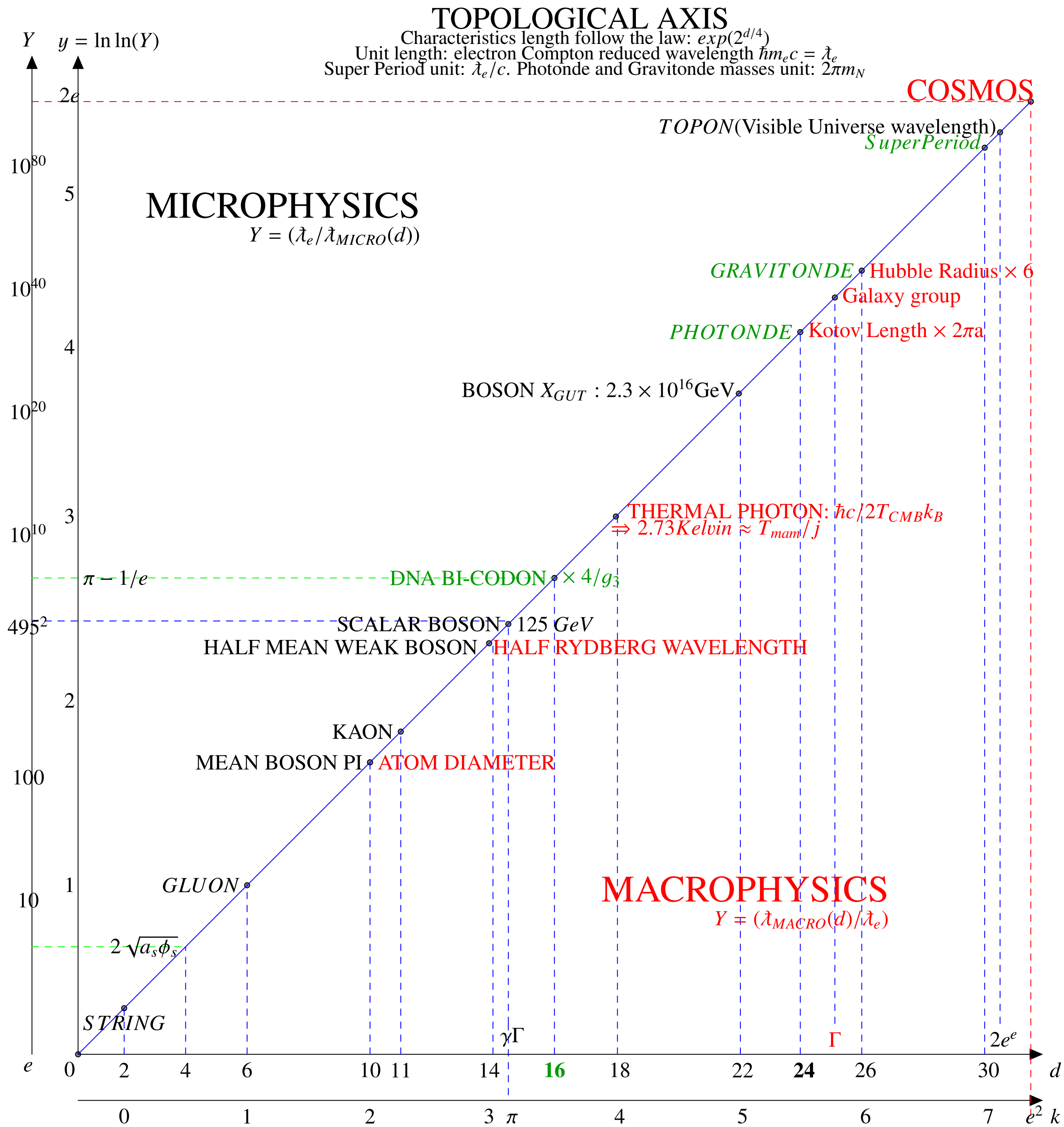
The above "maximal correlation principle" between "free parameters" is consistent with the identification of the latter with "computation basis". This supports the pertinence of the "economic numbers" a^a, 210^{210} and μ^{μ}. Moreover it was shown that the "basic economic numbers" of the form exp(exp(exp... plays a central role in these computation basis, entering a formula of Table 4.

Table 1: Adimensional primary constants

name	symbol	value	imp (ppb)
Euler-Napier constant	e	2.718281828459042	"exact"
Archimedes constant	π	3.14159265358979	"exact"
Euler-Mascheroni constant	γ	0.57721566490153	"exact"
Electric coupling constant	a	137.035999084(21)	0.15
Excess Electron Magnetic moment	d_e	1.00115965218128	0.15
Atiyah constant	$\Gamma = \gamma a/\pi$	25.17809724196	0.15
Optimized massive scalar boson/Electron mass ratio	$H^{(0)}$	$495^2 = 245025$	exact, $H^{(0)}_{mes}$: 245000(250)
Optimized charged weak boson/Electron mass ratio	W	157340.1093	ppb [5] W_{mes}: 157297(24)
Optimized neutral weak boson/Electron mass ratio	Z	178451.7524	ppb [5] Z_{mes}: 178450(4)
DNA Adenine-Thymine couple principal mass/m_H	o_1	612.31268	[23]
DNA Guanine-Cytosine couple principal mass/m_H	o_2	613.300194	[23]
Lucas Large Prime Number	N_L	$2^{127} - 1$	exact
Eddington Large Number	N_E	136×2^{256}	exact
Proton/Electron mass ratio m_p/m_e	p	1836.15267343	0.06
Hydrogen/Electron mass ratio	H	1837.15266014	0.06
Relativistic correction factor $1/(H-p)$	β	1.000026597	0.1
Neutron/Electron mass ratio	n_t	1838.6836617	0.5
Optimized Muon/Electron mass ratio	μ	206.7682869	0.1 μ_{mes}: 206.7682830
Optimized Koide Tau/Electron mass ratio	τ	3477.441701	0.1 τ_{mes}: 3477(2)
Fermi Sanchez-Atiyah mass ratio:	F	573007.3652	0.22 F_{mes}: 573007.362
Planck ratio m_P/m_e	P	$2.389015907 \times 10^{22}$	ppb [5]
Gravitational proton ratio $PN_L^{-1/2}$	p_G	1831.531181	ppb [5]
Gravitational coupling constant $R/2\lambda\!\!\!^{-}_e = P^2/pH$	a_G	$1.691936467 \times 10^{38}$	ppb [5]
Electroweak coupling constant $F^2 = (2\gamma \times 137)^3$	a_w	$3.283374406 \times 10^{11}$	ppb [5]
Optimised Strong coupling constant $a_w/2\pi(pH)^{3/2}$	a_s	8.434502906	ppb [5] $a_{s(mes)}$:8.47(7)
Scale-factor $8\pi^2/ln2$	j	113.9106346	[?]

Table 2: Table of Physical constants

name	Symbol	unit	Value	imprecision (ppb)
Reduced Planck constant $h/2\pi$	$\hbar$	J s	$1.05457181 \times 10^{-34}$	"exact"
Optimized Gravitation constant	G	$kg^{-1}m^3s^{-1}$	$6.67545375 \times 10^{-11}$	[5] G_{mes}: 6.67430
Relativity speed	c	ms^{-1}	299792458	exact
Fermi constant	G_F	Jm^3	$61.435851 \times 10^{-62}$	500
Electron mass $m_e = m_p/p = m_H/H = m_n/n_t$	m_e	kg	$9.1093837015 \times 10^{-31}$	0.3
Electron reduced wavelength $\hbar/m_ec$	$\lambda\!\!\!^{-}_e$	m	$3.861592675 \times 10^{-13}$	0.3
Electron classical radius $\hbar/am_ec$	r_e	m	$2.817940322 \times 10^{-15}$	0.45
Hydrogen Bohr radius $a(1+1/p)\lambda\!\!\!^{-}_e$	r_H	m	$5.294654092 \times 10^{-15}$	0.45
Cosmos (CMB) temperature	T_C	K	2.725820138	[5], $T_{CMB(mes)}$ 2.7255(6)
Cosmos (CMB) reduced wavelength	$\lambda\!\!\!^{-}_{CMB} = \hbar c/k_BT_C$	m	$8.400716621 \times 10^{-4}$	[5]
Cosmos (CMB) Wien wavelength	λ_{CMB}/ω_i	m	$1.063082472 \times 10^{-3}$	[5]
Non-Local length	l_{nl}	m	$2.878184911 \times 10^{12}$	[5]
Hubble length (Universe radius)	R	m	$1.306713894 \times 10^{26}$	[5]
Mono-electron Universe radius	R_1	m	$1.492365473 \times 10^{26}$	[5]
Cosmos holographic radius	R_N	m	$1.712894163 \times 10^{26}$	[5]
Cosmos radius	R_C	m	$9.075773376 \times 10^{86}$	[5]
Universe mass	M	kg	8.7965248×10^{52}	[5]
Cosmos holographic mass	$M_N = M'$	kg	$1.15308454 \times 10^{53}$	[5]
Cosmos mass	M_C	kg	2.247604×10^{113}	[5]



Bosonic string dimensions: $d = 4k + 2$ from k=0 to k=7 (Bott sequence). From k= 0 to k = 4: spectroscopic numbers

Table 3: The Topological Axis follows the law $exp(2^{d/4})$ for the main physical characteristics lengths, with unit length the electron Compton reduced wavelength: $\hbar/m_e c = \lambda_e$. This is the reunion of height 2D-1D holographic relations, hence the name 'Topological Axis', and the extrapolation towards smaller numbers of the Eddington's Large Number correlations. The double natural logarithms y = ln(ln(Y)) of the main dimensionless physical quantities (Y) corresponds to the special string dimension series, which identifies with the spectroscopic series with spin 2, where k is the orbital quantum number, d = 4k + 2, from k = 0 to k = 7, characteristics of a Bott octonion sequence, as anticipated by Atiyah, whose constant $\Gamma = \gamma a/\pi$ is central. The mean value $d = 16$ connects directly with the DNA bi-codon, decisive in the Holic Cosmology, where g_3 is the Weinberg-Sanchez SU(3) unification factor (Eq(4)), ϕ_s the golden number, $m_N = am_e$ the Nambu mass, and $j = 8\pi^2/ln2$ the scale factor [7].

Table 4: 55 formula for the Hubble radius R

Formula	Value (Gly)	Remarks
$2\hbar^2/Gm_em_pm_H$	13.81197676	Gravitational Hydrogen Molecule radius [5]
$(H/p)R_{1H}$	13.81197676	From mono-atomic star limit radius R_{1H} [6]
$\bar{\lambda}_w(t_{nl}/t_e)^2$	13.81197676	Identification predicting $t_{nl} \approx 9600.591457s$ (Eq.(5))
$(\bar{\lambda}_p\bar{\lambda}_H)^{1/2}(WZ)^4$	13.81197676	Symetrising the published relation $a_G \approx W^8$ [8]
$\bar{\lambda}_e 2^{128}/d_e^2(m_H/m_p)^6$	13.81197676	Empirical, from the Combinatorial Hierarchy Lucas Large Number [12]
$(n_t p_W/1836p)^{1/2}\bar{\lambda}_e(2^{18}/\pi\sqrt{a})^{10}$	13.81197711	From $(2g_3)^{21} \approx s_0/2\pi$ and $s_0 \approx (2\pi)^2\sqrt{a}$
$(1/g_3)\bar{\lambda}_H(R_C/2^{128}l_P)^{1/2}$	13.81197801	From $1/g_3 = 1+g_1^2+g_2^2 \approx 2a^3/pp_G$, with $p_G = P/2^{127/2}$
$(\bar{\lambda}_p\bar{\lambda}_H)^{1/2}a_w^{7/2}a/2\sqrt{5}$	13.81197801	From $(WZ)^8 \approx a^2a_w^7/20$
$\bar{\lambda}_e(p_{hol}/p)^2E^{\sqrt{a}/2}$	13.811984	Liaison between $E = exp(exp(e))$ and $p_{hol}^2 = 4a^3/3$
$(\pi a\beta/6\times137)\bar{\lambda}_e(2g_3)^{210}$	13.811959	From $(2g_3)^{21} \approx s_0/2\pi$ and $s_0 \approx (2\pi)^2\sqrt{a}$
$4a^4\bar{\lambda}_e(m_{bc}^{(0)}/m_H)^9(p/p_W)^2$	13.811964	DNA bi-codon mass as calculation basis
$\bar{\lambda}_e(WH/Zp)^{1/2}C_y^{(0)32}/6f(26)$	13.811948	Cytosine topologic pertinence $C_y^{(0)} \approx f(10) = f(26)^{1/16}$
$R_C(a_se^{2/a}210^{-210})^{1/8}$	13.81198	With the holic number 210^{210}, confirming the couple Universe- Cosmos
$l_P\sqrt{a}(\mu^\mu\ Wp/ZH)^{1/8}$	13.81198	Shows the pertinence of the computational term μ^μ
$12F_{137}^2 l_P(m_{bc}^{(0)}/m_e)^{3/2}/pH\beta^2$	13.81194	With the Fibonacci prime number F_{137} and the bi-codon mass
$12d_e(p/p_W)^4F_{137}^2l_P(m_{bc}^{(0)}/m_e)^{1/2}/\beta^2$	13.81195	With the Fibonacci prime number F_{137} and the bi-codon mass
$F_{137}(2l_Pf(26)\bar{\lambda}_e(m_{bc}^{(0)}/m_e+1)^{1/2})^{1/2}$	13.81208	With the Fibonacci prime number F_{137} and the bi-codon mass
$(H/p)(ad_e/137)^4R_{R_c,M_N}$	13.81203	From the geo-adimensional Cosmos-Universe (Fig.1)
$(137\beta/a)^2R_cR_el_P^2/a_wl_{nl}^2\bar{\lambda}_{bc}^{(0)}$	13.81194	Holic Principle, with the reduced wavelength of the DNA bi-codon.
$(136/137)2l_{nl}a^3f(16)$	13.8120	Empirical, with the central value $f(16) = e^{16}$
$(137^4/ap^2)\lambda_{WCMB}(e^a/4\pi)^{1/2}$	13.8120	Confirming the Wien CMB wavelength, from $4\pi(R_e/\lambda_{WCMB})^2 \approx e^a$
$(\bar{\lambda}_p\bar{\lambda}_H)^{1/2}a_w^4a^{14}/137^{16}$	13.8119	$137, a, a_w$ as computation basis
$(2a/137)q^2Z^{16}\bar{\lambda}_p\bar{\lambda}_H/2^{127}\bar{\lambda}_e$	13.8221	Cosmic role of electric charge $q = g_1\cos\theta = g_2\sin\theta$
$4o_2\sqrt{Z}\bar{\lambda}_e\lambda_{CMB}/l_P$	13.8129	Confirms the cosmic thermal bath and the couple GC with mass o_2m_H
$(H/p)(Gm_n/c^2)(10N_{Ed}/3)$	13.8125	From the Eddington Number 136×2^{256} and the gravitational parameter 10/3 [7]
$R_e(f(-2)/exp(exp(-g_1))^{128}/d_e^3$	13.8117	Symmetry R_e/R associated to symmetry f(2)-g_1 (string-SU(1) gauge coupling)
$2\bar{\lambda}_e(1836+s_0/2)^{\sqrt{a}}$	13.8123	Pertinence of the symmetry 1836-1848 (Eq.(25))
$(\bar{\lambda}_p\bar{\lambda}_H)^{1/2}(P/a^{13/2})^5/2\sqrt{5}$	13.8124	From the relation $a_w^7 \approx P^{3+7}/a^{(7+127)/2}$ [7]
$6(\bar{\lambda}_e^2/\bar{\lambda}_w)(a/\pi)^{16}$	13.8124	From the Topologial Axis: $f(18) \approx H^3 \approx (a/\pi)^4(6^{1/2}a_w)^{1/2}$
$4\bar{\lambda}_eaWZa_w^{1/2}(pH)^3$	13.817	Empirical
$4l_{nl}(pH/d_e)^2$	13.815	Empirical
$\bar{\lambda}_e(2/d_e^8)^{128}/(1-2g_3)$	13.815	Empirical, from the Combinatorial Hierarchy Lucas Large Number [12]
$2\bar{\lambda}_H2^{210}(a_w/P)^2$	13.811	Pertinence of the holic term 2^{210}
$\bar{\lambda}_ee^a/p^6\Gamma$	13.811	confirms the pertinence of the Atiyah constant Γ
$\bar{\lambda}_{CMB}^6(sin\ \theta)^{2\sqrt{a}}/R_Cl_P^4$	13.809	Liaison Cosmos-CMB thermal background
$l_P(\pi210^{210}/8)^{1/8}$	13.81	Pertinence of the holic term 210^{210}
R_ea^a/Π_{heur}	13.81	with the product of the 20 happy sporadic groups $\Pi_{heur} \approx e^{674.5210287}$
$2\hbar^2/Gm_em_pm_n$	13.80	c-free dimensional analysis [10]
$l_{nl}2p^3H/d_e$	13.82	p and H as computation basis
$(2\pi^2a^3)^5\bar{\lambda}_e$	13.80	5D holography in the gravitational Hydrogen molecule [7]
$(\lambda_{CMB}/(j+1))^2/l_P$	13.80	Central role of mammal temperature: $T_{mam} \approx jT_{CMB}$, with $j = 8\pi^2/ln2$
$2\bar{\lambda}_e(1/sin\theta)^{10d_e\sqrt{137}}$	13.80	Corresponds to $(1/sin\theta) \approx 3/\sqrt{2}$
$\bar{\lambda}_e\pi^{155/2}$	13.80	π calculation basis: $2^{1/155} \approx \pi^{1/16^2} \approx (2\pi)^{1/3\times137}$
$2\bar{\lambda}_eE^{2\sqrt{a_s}}$	13.79	Pertinence of the basic economic number $E = e^{e^e}$
$\bar{\lambda}_e(6/\pi)^{r_H/\bar{\lambda}_e}$	13.78	$6/\pi$ calculation basis
$\bar{\lambda}_e\Gamma^{55/2}$	13.77	Atiyah's constant Γ calculation basis
$g(6)\bar{\lambda}_e$	13.82	with the reduced topological function $g(k) = exp(2^{k+1/2})/k$, for k = 6, d = 26
$2l_{nl}(a\mu)^3$	13.84	Empirical
$\bar{\lambda}_eexp(2a_w/3\times7\times(137a)^2)$	13.85	Confirms that the Universe is a cosmic gauge boson, acting by its power 7
$(2l_{nl}^3/r_e)^{1/2}$	13.75	2D-3D Holography with the non-local length l_{nl}
$(r_e^2R_C)^{2/3}/l_{nl}$	13.75	Confirms the Cosmos non-locality
R_1^2/R_e	13.75	With the single electron radius R_1 [7], which specifies the approach $a \approx lna_G$ [8]
$(2\bar{\lambda}_e/3)(\bar{\lambda}_{CMB}/\bar{\lambda}_{H_2})^2$	13.90	2D-3D holography in the hydrogen molecule
$(4\pi\lambda_{CMB})^4/r_H^3$	13.78	Empirical, confirming the CMB invariance
$(l_P/2)(\bar{\lambda}_{CMB}^2\bar{\lambda}_p/\lambda_{CNB}r_e^2)^6$	13.7	Complementarity of photons and neutrinos backgrounds

2 The Computing Principle: the Tachyonic Holographic Cosmos

The classical theory associates conservation law with symmetry. More generally, a conservation law can be seen as the result of a computation. So, the main hypothesis is that the Cosmos, considered as a black-hole of radius R_C is a super-computer [5]. This is a very controversial domain. In particular, Y. Ng [14] introduces wrongly a "quantum foam" of dimension $(Rl_P^2)^{1/3} \approx 10^{-15}$ m, a length independantly found through the Computation Principle [5]. With the above hypothesis of an *invariant* Hubble radius R, this can be rather associated with the *invariant* electron classical radius. Hence, the "classical radius" of the Universe was defined as *the radius which is the simplest elimination of c between the classical electron radius and the Planck length formula. Indeed the speed c is dramaticaly too small a speed to inter-connect a so vast Cosmos*:

$$R_e = 2r_e^3/l_P^2 = 2\hbar^2/Gm_N^3 \qquad M_N = m_P^4/m_N^3 \tag{3}$$

The factor 2 comes from the Schwarzschild horizon formula $R_e = 2GM_N/c^2$, implying the Nambu mass $m_N = am_e$, central in particle physics [15]. *This considers the mass concept as primordial, specially the Panck mass m_P, excluding any Zero mass particle concept* [7].

This radius R_e connects with R by the following 10^{-7} precise relation, implying the "Weinberg-Sanchez" natural geometric extension of the Weinberg triangle [26], $1/g_3 = 1 + g_1^2 + g_2^2 = 1 + (Z/H^{(0)})^2$, identified as the inverse SU3 coupling where $g_1 = W/H^{(0)}$ is the U(1) coupling and $g_2 = Z sin\ \theta/H^{(0)}$ is the SU2 coupling, with $cos\theta = W/Z$ and the BE-Higgs scalar boson mass ratio, by respect to electron $H^{(0)} = 495^2$ (125.208 GeV) [7]. With $p_G = P/2^{127/2}$ (from the Combinatorial Hierarchy [12]), to 10^{-7}:

$$\begin{cases} q = g_1 cos\theta = g_2 sin\theta = (4\pi/a)^{1/2} \\ 1/g_3 = 1 + g_1^2 + g_2^2 = 1 + (Z/H^{(0)})^2 \\ R_e \approx 2R\ H/g_3 p_G \end{cases} \tag{4}$$

This radius R_e is about 30% larger than R, so *R_e was identifed to the holographic reduced Cosmos radius [7], defined by the Bekenstein-Hawking entropy of the sphere with radius R_e* [16]: $\pi(R_e/l_P)^2 = 2\pi R_C/l_P$, so:

$$R_c = R_e^2/2l_P \approx 2^{128} l_P (R/\eta\lambda\!\!\!^{-}_H)^2 \approx 9.075773 \times 10^{86} m \tag{5}$$

The Table 5 presents 22 formula confirming this Cosmos radius.

There is an unambigous numerical relation between the Cosmos, its holographic reduced sphere and the Universe (fig.1), separating the mass concept from the cinematik one. The Universe seems characterised by the constant c in the equivalence between the spatial and temporal terms, hence the following 2 factor [7]:

$$(ln(R_c/\lambda\!\!\!^{-}_e))^2 \approx (ln(M_e/m_e)^2 + 2(ln(R/\lambda\!\!\!^{-}_e)^2) \tag{6}$$

This will be related to the following Holic Principle (Section 3).

The Topological Axis rehabilitates the bosonic part of the string theory which has the apparent imperfection it includes tachyons. In fact, it is rather an advantage in order to explain the quasar non-Doppler oscillation, introducing a non-local period $t_{nl} \approx 9600, 06(2)$ s [18]. Indeed, the ratio of this period and the electron period $t_e = h/m_e c^2$ is given by the elimination of c between the electro-weak constant a_w and the inverse gravitational coupling $a_G = R/\lambda\!\!\!^{-}_e$:

$$t_{nl}/t_e \approx (a_G a_w)^{1/2} \tag{7}$$

This gives a G value precise to 10^{-6}, compatible with the BIPM 10^{-5} precise measurement [20]. This implies that the official value of G, *the incongruous mean between incompatible measurements*, is too weak by 8 σ.

By analogy with the practical holography, which is a two-step process, it was introduced a two-step interaction procedure, with a precursor speed $C = R_C/R$ much greater than c, leading to the following masses for the photon and graviton [7]:

$$\begin{cases} m_{ph} = \hbar/c^2 t_{nl} \\ m_{gr} = m_{ph}/a_w \end{cases} \tag{8}$$

Table 5: 22 formula for the Cosmos radius R_C

Formula	Value (10^{86}m)	Remarks
$R_e^2/2l_P$	9.07577	1D-2D Holographic Principle with R_e [5]
$2^{128}l_P(g_3R/\lambda\!\!\!^{-}_H)^2$	9.07577	with $\eta = 1 + g_1^2 + g_2^2 = 1 + (Z/H^{(0)})^2$
$\lambda\!\!\!^{-}_e\ exp(exp(exp(exp(exp(-g_2)))))$	9.07577	The final log of $R_c/\lambda\!\!\!^{-}_e$ is a SU(2) coupling: $g_2 \approx W/(495^2 + (\tau/\mu)^2)$
$2(a/137\beta)^2 l_{nl}^4 \lambda\!\!\!^{-(0)}_{bc}/R_e\lambda\!\!\!^{-}_e l_P^2$	9.07580	Holic Principle, with the reduced wavelength of the DNA bi-codon.
$l_P\ (\sqrt{a_w}(a)^2)^{16}$	9.07568	Computation with the nuclear coupling a_w and a_s
$l_P\ (210^{210}(8e)^{-1/2})^{1/4}$	9.07585	Holic central term 210^{210}
$2^{136}R(g_3\lambda_{CMB}/\lambda\!\!\!^{-}_e)^2$	9.080	Connection with the CMB thermal background, with $\eta = 1 + g_1^2 + g_2^2$
$Rl_{nl}^2 a_w/R_N r_{bc}^{(0)}$	9.081	Holic Principle, Eq.(15), with $r_{bc}^{(0)} = Gm_{bc}^{(0)}/c^2$
$2r_H 3^{210}/1830$	9.076	Empiric, base 3 Holic term, with $1830 = (60 \times 61)/2$
$l_P(p/H)(R\Pi_{26}/R_e)^{1/3}$	9.076	with the product of orders of the 26 sporadic groups $e^{674.5210287}$ [5]
$\lambda\!\!\!^{-}_P\ (210^{210}/\sqrt{(8e)})^{1/4}$	9.076	Empirical, using the Holic Number $210^{210} \approx \tau^{2\mu/3} \approx e^{e2\mu}$
$\lambda\!\!\!^{-}_e g(7)(H/p)^2P/6$	9.076	with the reduced topologic function for $d = 30 : g(7) = f(30)/7$ [5]
$24\lambda\!\!\!^{-}_e\pi^{210}/a^3$	9.077	Empiric, with base π holic term
$a^2\lambda_{Wien}^4/(p_K l_P)^3$	9.078	Empiric, confirms T_{CMB} with $p_K = (1 + \mu + \tau)/2$ [13]
$\sqrt{3}l_{nl}^3/r_e l_P$	9.07	with the non-local length l_{nl}
$\lambda\!\!\!^{-}_e g(7)(a^2pp_G)^2$	9.08	Empiric [5]
$\lambda\!\!\!^{-}_e e^{(p_0+1/2)/8}$	9.09	natural base e, with $p_0 = (60 \times 61)/2$
$\lambda\!\!\!^{-}_e e^{e^{2e}+1}/2\pi$	9.11	natural base e in the Topological Axis
$(ln(R_c/\lambda\!\!\!^{-}_e))2 \approx (ln(M_e/m_e)^2 + 2(ln(R/\lambda\!\!\!^{-}_e)^2)$	9.12	c-observable Universe Cosmos couple ($R/\lambda\!\!\!^{-}_e = t/t_e$) , fig
$\lambda\!\!\!^{-}_e g(7)(\lambda\!\!\!^{-}_{CMB}/r_H)^3$	9.1	Empirical, invariance of the thermal background [5]
$(Rl_{nl})^{3/2}/r_e^2$	9.2	From non-local holography [5]
$l_P\ \mu^{\mu R_e/3R}$	9.0	μ calculation basis, close to holic base 210

In the Topological axis, these masses correspond to the special string dimensions 24 (transverse dimensions) and 26 (main dimension), and will be determinant in the following section. Note that, with r_H the Bohr's radius and the relativistic factor $1/\beta = H - p$ (1 ppm):

$$f(24)^{1/26} \approx d_e(r_H/\beta\lambda\!\!\!^{-}_e)^{1/2} \tag{9}$$

showing that the electric parameter $a = (p/H)r_H/\lambda\!\!\!^{-}_e$ is central in the Topological Axis.

3 The Holic Principle

The string theory considers space-time as a secondary property [19], so the concepts of mass, length and time are, in final, related to pure numbers. Indeed an arithmetic-physical synthesis has been anticipated by the Holic Principle [34], a simplified form of the Holographic Principle. Recall that holistic equations are prefered to differential ones, in order to eliminate free parameters. In any Diophantine equation, this Holographic Principle allows to discriminate a temporal ratio T, acting by its square, from a spatial ratios L, acting by its cube (due to the 3D space). Indeed, *the simplest Diophantine Equation, which implies a 2-dimensional Time*, $T^2 = L^3 = n^6$ is the Diophantine form of the third law of Kepler, it implies: $L_n = r_n/r_1 = n^2$ (the Bohr's orbit law) and $T_n = t_n/t_1 = n^3$. Hence, with $v_n = r_n/t_n$:

$$\begin{cases} r_n v_n^2 = r_1 v_1^2 = Gm_G \\ r_n v_n = n r_1 v_1 = n\hbar/m_\hbar \end{cases} \tag{10}$$

These gravito-quantum equations introduce an "hyper-symmetry" between the universal constants G and ħ, by respect to the mass concept: the undefined masses m_G and $m_\hbar$. So, this defines the conceptual trajectories:

$$\begin{cases} r_n = n^2 r_1 \\ r_1 = \hbar^2/Gm_G m_\hbar^2 \end{cases} \tag{11}$$

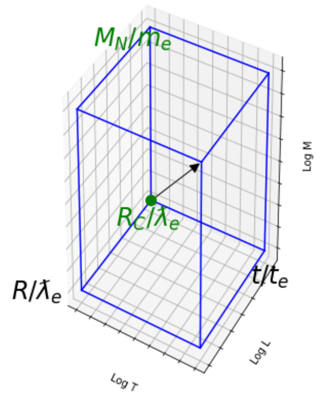


Figure 1: *Geo-adimensional Cosmos-Universe couple*, with unit length the Electron Compton reduced wavelength. In a 3D Super-space, logarithms of physical ratios are considered vectors. The Cosmos radius R_C appears as the norm of the vector using for length and time projections the same value $R/\lambda_e = t/t_e$. For the mass projection it is M_N/m_e where M_N is the critical mass in the Cosmos *reduced spherical hologram* of radius R_e. This is a dramatic geometrical confirmation (not dependant of the base for logarithms) of the Extended (2D-1D) Holographic Principle applied to the Bekenstein-Hawking Universe entropy. So the Universe is characterised by the c-equivalence $R/\lambda_e = t/t_e$, where t is the Hubble time (no relation with any "Universe age").

With $m_G = m_e^{(red)} = m_e m_p/(m_e + m_p)$, the classical electron reduced mass and $m_\hbar = m_P/\sqrt{a}$, this is the Bohr's orbits distribution. The above PHOB Cosmology includes the following 6 more special cases (Table 4), using the main masses, plus a new one m_{bc}, close to m_H^2/m_e, *which identifies with the DNA bi-codon mass*, studied in the next section.

So, the PHOB Cosmology is tied to the couple $G, \hbar$, while the classical quantum theory uses in fact the "photonde" couple $\hbar, c$, and the gravitation theory the "gravitonde" couple G, c. These three couples define the "Trihedra of Constants" (fig2).

Extrapolating the above simplest Diophantine equation with the prime numbers 5 and 7 which follow the prime couple 2,3, the Holic Principle proposes the exponent 5 for a mass ratio, and 7 for a field ratio (note that the lifetime of a particle depends effectively to the power 5 of its mass):

$$T^2 = L^3 = M^5 = F^7 = n^{210} \tag{12}$$

Indeed, the Hubble radius "holic key" is singular, to 15 ppm, while the base 2 is confirmed to 0.3 ppm, and the base 3 to 60 ppb :

$$\begin{cases} (R/\lambda_e)^{1/210} \approx 2R/R_e \\ (P^2/a_w)^2/p)^{1/210} \approx 2 \\ ((p_G/2an_t)R_C/\lambda_p)^{1/210} \approx 3 \end{cases} \tag{13}$$

with p the proton-electron mass ratio and n_t the proton-electron mass ratio. *Note that 3 is the optimal integer base, the closest integer to e* [22].

4 The DNA bi-codon

By respect to the hydrogen mass H, the masses of the 4 DNA nucleotides are, using the main isotopes: ${}^1_1H^{(0)} = H$, ${}^6_{12}C = C^{(0)}$, ${}^7_{14}N = N^{(0)}$, ${}^8_{16}O = O^{(0)}$, ${}^{15}_{31}P = P^{(0)}$ [23], close to the Fermi mass ratio: $\sqrt{a_w/pH} \approx 311.9846$

Table 6: PHOB cosmology, Eq. (9)

m_G	$m_\hbar$	$r_1 = \hbar^2/Gm_Gm_\hbar^2$	Precision	Arithmetic Property
m_e	m_P	$\lambda\!\!\!^{-}_e$: Electron reduced wavelength	exact	
$m_e^{(red)}$	$m_P/\sqrt{a}$	r_H: Bohr's radius	exact	$r_H/\lambda\!\!\!^{-}_e \approx 137 = 2^7+2^3+2^0$
m_N	m_N	$R_e/2$: half cosmos reduced holographic radius	exact	$R_N/\lambda\!\!\!^{-}_e \approx (3^3)^{3^3}$
$m_{bc}^{(0)}$	$m_{bc}^{(0)}$	$2l_{cc}$: double non-local length	-6.3×10^{-3}	$l_{cc}/\lambda\!\!\!^{-}_e \approx \pi^{50}$
m_Pa^3	$\sqrt{m_pm_H}$	λ_{Wn}: Wien CMB wavelength (background thermal radiation)	-3.2×10^{-4}	$\lambda_{Wn}/l_P \approx \pi^{64}$
m_e	$\sqrt{m_pm_H}$	$R/2$: half Universe radius	exact	$R/\lambda\!\!\!^{-}_e \approx g(6) \approx 2^{2^7} \approx (2R/R_N)^{210}$
$m_{bc}^{(0)}R_e/R$	$\sqrt{m_{ph}m_{gr}}$	R_C: Cosmos radius $= RC/c = (R/2)m_N^3/m_{bc}m_{ph}m_{gr}$	4.7×10^{-4}	$R_C/\lambda\!\!\!^{-}_e \approx e^{e^{2e}} \approx 6^{2^7} \approx (2R/R_N)^{64a_s}$

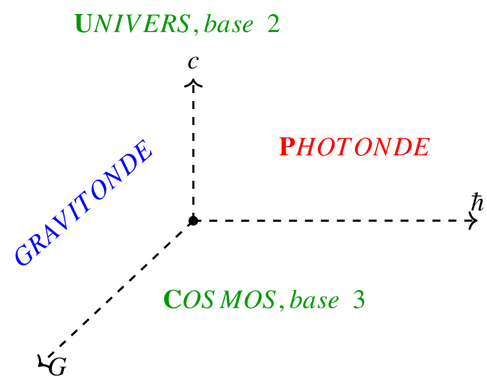


Figure 2: The Trihedra of Constants $\hbar Gc$. The c-local visible Universe is a Cosmos bosonic "immergence"

$$\begin{cases} Cytosine: & C_9^{(0)}H_{12}^{(0)}N_3^{(0)}O_6^{(0)}P^{(0)}\ (150pr.+139nt.):\ C_y^{(0)} \approx 286.8021362 \approx 495(a^3/n_t^2)^2 \approx WH/4an_t \\ Thymine: & C_{10}^{(0)}H_{13}^{(0)}N_2^{(0)}O_7^{(0)}P^{(0)}\ (158pr.+146nt.):\ T_h^{(0)} \approx 301.68553403 \approx \sqrt{a_w}\,\Pi_0/H\Pi_+ \\ Adenine: & C_{10}^{(0)}H_{12}^{(0)}N_5^{(0)}O_5^{(0)}P^{(0)}:\ (162pr.+151nt.):\ A_d^{(0)} \approx 310.6269397 \approx \sqrt{a_w}/pd_e^4 \\ Guanine: & C_{10}^{(0)}H_{12}^{(0)}N_5^{(0)}O_6^{(0)}P^{(0}\ (170p+159n_t):\ G_u^{(0)} \approx 326.4976654 \approx 495(137a/\beta)^{1/2}/\mu d_e^4 \approx Zp/2H\Pi_+ \end{cases} \tag{14}$$

The mean masses of the effective couples are close to $H/3 \approx 612.3842155$:

$$\begin{cases} Couple\ AT: & A_d^{(0)} + T_h^{(0)} = o_1 \approx 612.312280 \\ Couple\ GC: & G_u^{(0)} + C_y^{(0)} = o_2 \approx 613.299802 \end{cases} \tag{15}$$

The bi-codon minimal mass uses the three couples AT, so is very close to Hm_H. Since $o_2 \approx o_1 + 1$, the other masses are of type $(H+i)m_H$, with i = 1, 2 or 3: *the DNA seems a base 3 computer, like the Cosmos.*

The mean nucleotide mass is $(o_1+o_2)/4 \approx 306.4032199$, close to $\pi^5 \approx 306.02$. The mean DNA bi-codon mass is $m_{bc}^{(0)}/m_H = (6/4)(o_1+o_2) \approx 1838.418122$, close to $p_W = 6\pi^5$, the Lenz-Wyler proton-electron mass ratio [28]. Note that the Wyler formula $6\pi^5$ shows a geometric property: it is the product of the area by the volume of a cube of side π. More precisely, with n_t/p the neutron-proton mass ratio, d_e the electron magnetic excess, and $f(16) = e^{16}$, the central term of the Topological Axis (150 ppb and 55 ppb):

$$\begin{cases} m_{bc}^{(0)}/m_H \approx H(n_t/p)^{1/2} \\ f(16) \approx 2(d_em_{cd}^{(0)}/m_e)^2/a^3 \end{cases} \tag{16}$$

Table 5 : Number of Crystallographic Ponctual Symmetry operations PSO_{Cr}

E_d	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}	E_{13}
N_d	2	6	10	24	38	78	118	224	$\underline{330}$	584	838	1420	2002
K_d (positive PSO_{Cr})	1	5	5	19	19	59	59	165	$\underline{165}$	419	419	1001	1001
$q_d = \binom{3+d}{4}$ (pentapope nbr)	1	5	15	35	70	126	210	330	$\underline{495}$	715	1001	1365	1820
$Q_d = q_d + d$ (extended q_d)	5	10	21	42	78	$\underline{135}$	220	341	507	728	1015	1380	$\underline{1836}$
$S_d = \Sigma_1^d(N_d/2)$	1	4	9	21	40	79	$\underline{138}$	250	415	707	1126	$\underline{1836}$	2837
$T_d = \sum_{d-1}^{d+1} K_d$	7	11	29	$\underline{43}$	97	$\underline{137}$	283	389	749	1003	$\underline{1839}$	2421	4259

Thus, this central value $f(16)$ establishes the lacking connection (0.1 ppm) between the Topological Axis and the main masses: electron, proton, Hydrogene, neutron. It shows also the following Keplerian holic relation, implying the leptons ratios:

$$e^{16} = f(12)^2 \approx (f(4(1+\sqrt{2}))^3 \quad \to f(4(1+\sqrt{2}) \approx \mu \quad f(12) = e^8 \approx 6\tau/7 \tag{17}$$

where $1+\sqrt{2}$ is the Pell-Fermat generator. Since $a^{1/32} \approx 7/6$ this implies the terminal term $e^{16} = f(32)$ of the Topological Axis. The analysis shows, to 4 ppm:

$$(\tau - 1)^{32}/f(32) \approx a^2/137 \tag{18}$$

So, the terminal dimension 32 of the Topological Axis is associated to τ, the terminal lepton. The number three of particle families is thus confirmed.

One notes the direct correlation implying *the product* of the nucleotide mass ratios:

$$4d_eG_u^{(0)}C_y^{(0)} \approx 4d_eA_d^{(0)}T_h^{(0)} \approx (H/3)^2 \approx H^{(0)}/g_3 = H^{(0)} + Z^2/H^{(0)} \tag{19}$$

This induces the symmetrical relation implying $s_0 = 1848$, the last Euler number

$$\begin{cases} H^{(0)} + Z^2/H^{(0)} \approx (H/3)^2 \\ H^{(0)} + W^2/H^{(0)} \approx (s_0/\pi)^2 \end{cases} \tag{20}$$

The pertinence of s_0 will be confirmed in the following.

5 The Multi-Dimensional Crystallography

The main problem of string theory is the connection between the usual 4D time-space with the favored theoretical dimensions: 26 for the bosonic theory, 24 for the transverse dimensions, 10 for the superstring theory, 11 for the supergravity.

As recalled above, *conservation is tied to both symmetry and computation*. So, this section is devoted to connections between the Multi-Dimensional Crystallography and the main particle mass ratios.

Carl Hermann [24] calculated the number of crystallographic point symmetries N_d for dimensions from 1 to 8. *This number N_d is the number of monic polynomials (i.e. first term $1x^d$) with roots on the unit cercle.*

The Weigel team [25] (Table 5) extended this calculation for higher dimensions, up to $d = 70$, focusing on *the positive symmetry number*, noted K_d, which defines N_d via:

$$\begin{cases} N_{2n+1}/2 = K_{2n+1} = K_{2n} \\ \\ N_{2n} = K_{2n} + K_{2n-2} \end{cases} \tag{21}$$

These recurrence rules are non sufficient to defines the series. This implies to look for specific recurrences.

5.1 The *Positive Crystallographic Function* and the Scalar Boson

The method of least square leads to the following polynomial, where the coefficients clearly correlate with the physical parameters, with emphasis to the scalar boson - electron mass ratio $H^{(0)} = 495^2$ predicted by the Topological Axis and the Atiyah constant $\Gamma = \gamma\, a/\pi$ (Table 1).

$$\begin{cases} \quad d \approx (lnN_d)^2/A + BlnN_d + 1/C \\ \\ A \approx 11.4672 \approx 2 \times 137\,\sqrt{6}/5\,\sqrt{a} \approx 137a495/\sqrt{2a_w} \approx 2\pi^2 WZ/a^5 \\ B \approx 1.1812 \approx 495/K_{10} \approx N_{10}d_e/495 \\ C \approx 43.9290 \approx 495^2 \sin\theta/\sqrt{2} \times 9\mu \approx 495^2 \cos\theta/\sqrt{2} \times \tau \end{cases} \tag{22}$$

Here μ and τ are the leptons relative masses, $\cos\theta = W/Z$, and $d_e \approx 1.00116$ is the electron magnetic excess.

The two fist terms are close for $d = 32$, which specifies the Topological Axis symmetry, from $k = 0$ to $k = 7$, and the characteristics of the string group SO(32), whose dimension is the third perfect number 496:

$$d_k + d_{7-k} = 32 \qquad d(SO(32)) = \binom{32}{2} = 496 \tag{23}$$

From the above double relation for B, the following property of the scalar boson emerges, showing a special recurrence relation between the dimensions 10 and 9, showing also a connection with $S_{26} = 381540$, to 48 ppm:

$$\begin{cases} H^{(0)} = 495^2 = K_9\mathbf{N}_9 = K_{10}N_{10} + N_9 - 1 \approx \sqrt{WS_{26}} \\ \\ 495 = \binom{12}{4} = \binom{11}{3} + \binom{11}{4} = 3\binom{11}{3} = 3K_9 = \binom{32}{2} - 1 = 496 - 1 \end{cases} \tag{24}$$

where $\mathbf{N}_9 = 9K_9$ is the total number of positive zeros on the unit circle for the central string reduction dimension 9, and $N_9 - 1$ the number of non-trivial $9D$ symmetries. This is clearly related to the fact that $q_8 = N_9$, $q_{11} = K_{12}$, $q_9 \approx \sqrt{K_{10}N_{10}}$, showing a kind of symmetry between N and K. Note that 495 is the odd part of the first Mathieu group order 16×495, and the couple 495- 496 is the third perfect couple. In such a couple, the first number is the sum of the *non-trivial* divisors of the second. Since $496 = \binom{32}{2}$ is the dimension number of the group SO(32), and $495 = \binom{12}{4}$ *, this leads to the conjecture : *the third co-perfect number 495 could be the single one being a non-trivial binomial number*. Note that N_9 and K_9, as well as $210 = \binom{10}{4}$ are Euler suitable numbers, whose pertinence is confirmed below.

5.2 The Proton-Electron mass ratio and the Euler maximal suitable number

The equivalent relation for dimension 4, implying $\mathbf{N}_4 = 4K_4$ shows up a relation between the brute proton-electron mass ratio 1836 with the Euler maximal suitable number $s_0 = 1848 = 43^2 - 1$:

$$N_4(\mathbf{N}_4 + 1/2) = (K_4 + N_4)^2 - 1 - N_4/2 = s_0 - 12 = 1836 \tag{25}$$

This shows up a kind of symmetry between the additive and multiplicative operations. The maximal Euler's suitable number is very close to the Eddingtons's prediction [11] for the proton/electron mass ratio, 1847.599459, as the ratio of the roots of the equation $10x^2 - 136x + 1 = 0$.

In the Particle standard model, the scalar boson is necessary to explain the non-Zero mass of particles. Indeed, in the above procedure, the connexion between the scalar boson and the 9D crystallography is immediate, while it is not so for the above decisive 4D relation. But the first one has induced the latter one by analogic induction.

Thus the central role of the scalar boson is confirmed, and the mass concept is identified to a number of cristallographic symmetries.

There is another connection between 1836 and 1848 : they are both the area of a integer-sided triangle which is 6 times its perimeter, opening new further study.

The connection with the pentapope number q_{13} is immediate:

*Number of groups of 4 elements taken among 12 ones.

$$q_{13} + 16 = \binom{16}{4} + 16 = 1836 \tag{26}$$

meaning that 1836 is the sum of crossings from 16 points, *including those points*. Note the parallel with the above defition of 137, the maximal number of zones defined by 16 lines in a plane.

Moreover with $\Pi_0 = \bar{\lambda}_e/\lambda_{\Pi_0}$, and the associated term Π_+ for the charged Pion, $p_{hol} = (4(r_H/\bar{\lambda}_e)^3/3)^{1/2}$ and the 137^{th} Fibonacci (prime) number:

$$\begin{cases} s_0\Pi_+\Pi_0 \approx m_{cd}/m_e \\ s_0 + 1/2 \approx (4\pi)^2\sqrt{a} \approx F_{137}/96a_w^2 \approx (q^2a/4)^2H^{(0)}/p_{hol} \\ (s_0 + 1/2)/2 = \binom{12}{6} + 1/4 \approx (210^{210}/\mu^{\mu})^{1/3} \end{cases} \tag{27}$$

The last formula is deduced from the relation with the Monster Group (section 5.7) confirming the connection $\mu \approx 210$. The $+1/2$ term comes from taking account of the dimension 0 in the half sum of symmetry numbers, as confirmed below. The involved precise value for π is very particular, opening further study.

It is shown [29] that a single Euler suitable number could exist beyond s_0, and if not, i.e. if s_0 is really the maximal one, then the generalized Riemann conjecture would be confirmed. So the proton-electron ratio is at the heart of Number Theory.

Thus the string canonical $9D$ dimension reduction is identified with the $9D$ crystallographic symmetries. This implies the elimination of the continuum in theoretical physics, in conformity with the Computing Principle. This could unblock the present dilemma of string theories which lead to an enormous number 10^{500} of solutions for dimension reduction, an anomaly which is claimed to sustain the unscientific Multiverse model.

Moreover the number 1839, which is close to the neutron/electron mass ratio, appears also in the Table 5, and so implies the following:

$$1839 = \sum_{10}^{12} K_d = \sum_{1}^{12}(N_d/2) + 3 \quad \Rightarrow \quad \sum_{2}^{9} K_d = (K_{12} - 9)/2 = 496 \tag{28}$$

This identifies the partial sum up to the 9D space with the above dimension of the canonic group SO(32), central in string theory. Thus the couple proton-neutron is represented by the crystallogaphic symmetries.

With the electric charge $q = Wsin\theta/H^{(0)}$, the computer shows up the following relations, in the ppb domain:

$$\tau\sqrt{a_w}/Wq \approx K_3K_5K_9/3 \tag{29}$$

Note that $1 + K_3K_5/3K_9 = 4181 \approx a_w^{1/2}/a$, showing the 19^{th} term of the Fibonacci series, the first non prime number of order prime. Moreover, the U(1) coupling $g_1 = Z\sin\theta/H^{(0)}$ is confirmed in the ppb domain by:

$$f(26) = f(2)^{32} \approx (H/p)(2/g_1^2d_e)^{16} \tag{30}$$

This confirms the central role of the string dimension 26.

5.3 The Eddington-Atiyah's inverse brute electric coupling 137

The number 137 is the Eddington's inverse brute electric coupling, and has been unambigously connected with the Lucas-Lehmer series [7]. Atiyah recently associated this number with three algebra: the octonion, quaternion and real ones, associated to the number $273 \approx m_{\Pi_+}/m_e$ is again one of the Euler's suitable numbers:

$$137 = 2^7 + 2^3 + 2^0 \qquad 2\times 137 - 1 = 273 = 2^8 + 2^4 + 2^0 \tag{31}$$

Strangely enough, it seems that nobody have looked for the prime numbers that appear in the harmonic series, which is the single pole of the Rieman series, precisely known to inform about the distribution of prime numbers. The six first prime numbers appearing are the following, showing a symmetry of 11 around 137, showing the 11 supergravity dimensions and the usual 4 ones:

$$3; 11; 5; 137; 7; 11 \qquad \Rightarrow 137 = 11^2 + 4^2 \tag{32}$$

Note that, while $137 = l_{16}$, the 16^{th} Lazy Caterer number (maximal number of zones in a plane defined by n lines), $11 = l_4$ and $4 = l_2$. This "arithmetic monster" 137 appears in the Table 5:

$$137 = \sum_6^8 K_d = \sum_1^7 (N_d/2) - 1 \quad \Rightarrow \quad \sum_1^4 K_d = (K_7 + 1)/2 = d_7 \tag{33}$$

This identifies the 4D term $\sum_1^4 K_d = d_7 = 30$ in the brute U(1)-SU(2) gauge partition 137 = 107 + 30 [26]. Extrapolating to the superstring dimensions 10 and 11, this connects with the holic term 210, itself connecting with $26 = d_6$:

$$\begin{cases} (K_7 + 1)/2 = d_7 = 2 \times 3 \times 5 = 30 \\ (K_{11} + 1)/2 = d_{2d_6} = 2 \times 3 \times 5 \times 7 = 210 \end{cases} \tag{34}$$

This connects the main dimension 30 of the Topological Axis with the dimension 210 of the Holic principle.

5.4 The *precise* U(1)-SU(2) gauge partition

Taking account of the dimension zero, the above sum becomes $S_{12} = 1836.5$, close to the mean proton-Hydrogen mean, and the gauge separation could imply rather $n_7 + 1/2 = 30.5$, which is close to 196 ppm with the real U(1)-SU(2) gauge partition term $a(\sin\ \theta)^2 \approx 30.505983$, and more precisely:

$$d_7 + 1/2 \approx 137^2/ad_e - (a_w^2)/Z^4 \approx a_w^{1/2}/a^2 \tag{35}$$

Moreover, this number connects again with the holic term 210:

$$2(d_7 + 1/2)^2 = 9 \times 210 - (d_7 - 1/2) \tag{36}$$

The above proximity between μ and 210 materializes in the following 44 ppb determination of μ, with a 23 ppm correlation with τ :

$$(a/137)(2(137^2/(ad_e - (a_w^2)/Z^4)^2) \approx 9\ \mu \approx \tau\ tg\theta \tag{37}$$

So the U(1)-SU(2) gauge partition is at the heart of the optimal computation process.

5.5 The String dimension partition 26 = 22 + 4

In the string theory, the 26 dimensions reduce to the usual 4D by separating 22 hidden dimensions. Indeed, one observes:

$$N_{22} = K_{20} + K_{22} = (20 \times 22) \times 137 \tag{38}$$

where 137 is the above Eddington value for the electric coupling. The same relation applies also to the 4D usual space:

$$N_4 = K_2 + K_4 = (2 \times 4) \times 3 \tag{39}$$

The computer shows up another case, which involves in a symmetrical way, the four usual dimensions d = 1, 2, 3, 4:

$$N_{13} = 2K_{11} = (2 \times 11 \times 13) \times 7 = N_6 N_8 N_9 / N_1 N_2 N_3 N_4 \tag{40}$$

The sum of the implied dimensions is the same: 23 = 1+2+3+4+13 = 6+8+9.

The other string partition is 26 = 10 + 16. One observes the following precise relations with the 3 couplings, electric, electroweak and gravitational (1 ppb and 10 ppb):

$$K_{10}/(K_{10} - 2) \approx a_w^{5/2}/Pa^3 \approx e^{1/(210-1)} \tag{41}$$

This could be tied to the two trivial symmetries, identity and point inversion.

5.6 The Connections with the Periodic Table

The string dimensions special series $d = 2+4k$ identifies both with the Topological Axis one and with the spectroscopic one, *so, the string dimension 2 identifies with the spin 1/2 degeneracy, where k identifies with the orbital number, running in the octonion series, between 0 and 7.*

The theoretical total number of elements untill the n^{th} raw, where n is the principal quantum number is:

$$n_n = \sum_{j=1}^{n} \sum_{k=0}^{k=j-1} = 2 \sum_{j=1}^{n} n^2 \tag{42}$$

There is a particularity for the 7^{th} row, due to the association symmetry-computation where the central dimension is 16: indeed $2 \times 16 = 32 = 2 + 30 = 6 + 26 = 10 + 22 = 14 + 18$:

$$\sum_{k=0}^{k=7} d_k = 2^7 \qquad \Rightarrow \qquad \sum_{k=0}^{k=7} d_k + \sum_{0}^{1} d_k + 1 = 137 \tag{43}$$

where the term 137-127 = 3 + 7 shows the first terms of the Combinatorial Hierarchy [12]. The height numbers are all of the form "prime - 1", except $d_i = 14$ and 26, the later being the critical dimension which verifies: $d_{26} = d_{d_6} = 106$, so justifying the "reduced" Atiyah sum, with the octonion term (2^7) and the quaternion one (2^3). This identifies with the reduced U(1)-SU(2) gauge partition, where 136 is the initial Eddington's electric coupling, the number of elements in the symmetrical matrix 16×16:

$$\sum_{k=0}^{k=7} (d_k + 1) = 2^7 + 2^3 = 136 = 30 + 106 = d_7 + d_{d_6} \tag{44}$$

There is a particularity for the 4^{th} row which is effectively used in the Periodic Table, corresponding to the famous spectroscopic numbers, called by Friedrich Hund "sharp" ($s = 2$), "principal" ($p = 6$), "diffuse" ($d_i = 10$) and "fundamental" ($f = 14$). The 7^{th} row of the Periodic Table terminates in the Oganesson, recently synthetised [30], of atomic number 118, *which is precisely the Herman number for d = 7.* The involved coefficients, ruling the *symmetrical* distribution of the spectroscopic groups s, p, d_i, f are the following:

$$\sum_{k=0}^{k=3} c_k d_k = 118 \qquad \rightarrow c_k = (7, 6, 4, 2) \tag{45}$$

The above variation of one unity, connected to prime numbers, leads to

$$\sum_{k=0}^{k=3} c_k(d_k + 1) = 137 = 2^7 + 2^3 + 2^0 = 107 + 30 \tag{46}$$

which recovers the complete Atiyah sum, including the "real algebra" term 2^0, and, since the last "fundamental" term is $2 \times 15 = 30$, coming back to the above brute U(1)-SU(2) gauge partition.

This Atiyah series presents an imperfection: the absence of the term 2^1, corresponding to the complex algebra. One observes that the total sum taking account of the four algebra is $139 \approx i^{(}\pi/i) = e^{\pi^2/2}$. So the origin of 137 would be the mean between 139 and 135, the latter being the product of the two co-perfect numbers 5 and 3^3, very close to $16a_s$. Indeed, one oberves, in the ppb domain:

$$137 = (16a_s + i^{\pi/i})/2 - 1/d_e + 2^0 \tag{47}$$

So the optimized value of $a_s \approx a_w/2\pi(pH)^{3/2}$ is confirmed in the ppb domain. This tight connection with the electron excess magnetic moment $d_e \approx 1.001159652$, which is the best confirmation of the quantum theory, opens future research.

6 The Sporadic Groups Connections

The 26 sporadic groups include 20 "happy" groups tied to the Monster, and 6 "pariah" groups. Many relations with the physical parameters was published [7], two of them implying formula for R and R_c (Tables 2 and 3). One observes the relations tying the electric, strong and weak couplings $a, a_s, a_s a_w$, to 7, 150 and 300 ppm :

$$\sqrt{a_w}/aa_s \approx 495 \times 2^{1/(24\times20)} \approx K_{26}/f(10) \approx O_M^{1/20} \tag{48}$$

with $K_{26} = 141877$. Now $f(10)^{10} \approx l_{nl}/\lambda_e$ and K_{26}^{20} is of order R_C/λ_e. This implies again a pertinence for the canonic string dimensions 26 and 10, calling for further study.

The order of the Monster group connects with the Lepton masse ratios:

$$O_M^9 \approx \tau^{137} \approx \mu^\mu s_0^2/\sqrt{2} \approx 4\sqrt{2}\,210^{210}/s_0 \tag{49}$$

This implies the above Eq.(32) proving that $\mu_0 = 2 \times 3 \times 5 \times 7 = 210$ is the pertinent arithmetic approximation of μ. With the symmetric approximation $\tau_0 = (2 + 3 + 5 + 7) \times 2 \times 3 \times 5 \times 7$:

$$(p/n_t d_e)(\tau/\tau_0)^{137} \approx \sqrt{2}p^3/a_s^2 H^2(H-p) \approx \pi^\pi \tag{50}$$

confirming to the ppb range the Koide tau value [13], where n_t/p is the mass ratio neutron-proton.

So the sporadic groups are at the heart of the overall unification.

7 Conclusions and Predictions

The Topological Axis, with its *invariant* Hubble radius is the key for debunking theoretical physics. It permits to connect the main "free" physical parameters with different domains of the Number Theory, with emphasis on the multi-dimensional crystallography. It rehabilitates several discarded physical theories: those of Eddington [11], Noyes [17], Wyler [28] and Atiyah [27]. It is inferred that these parameters are calculation basis in the computing Cosmos. Indeed high powers of them appear in the tables 1 and 2, in formula for both the Hubble radius R and the Cosmos one R_c, with special importance of the holic power 210, confirming the pertinence of the Holic Principle.

The tachyonic character of the Cosmos is of paramount importance, interpreting at last the non-Doppler quasar power oscillation, rehabilitating the string bosonic theory and integrating at last the "quantum holism", the manifestation of quantum non-locality by introducing a super-celerity C. The Toponic Holographic Principle breaks down the Planck wall by the factor C/c, which explains at last the giant factor for the vacuum quantum energy. *Instead of ignoring such an "incomprehensible" non-Doppler phenomena, the astrophysicists ought to study this intensively, specially the phase differences fram a quasar to the other, with emphasis on the determination of the tachyon celerity C or its intermediate gravitational value $C/P \approx 10^{38}c$* [7].

The String Theory connects at last with Reality, but it must be wholly reconsidered, by replacing the continuum by a "quantinuum", based on the "Topon", the reduced wavelength of the Universe, and adopting a *massive* string, as predicted by the Topological Axis. Also *massive* gluons, photon and graviton must be included in the Particle standard model. The later must also include the Koide formula, whose associated leptons masses μ and τ connect so precisely with the other data.

The Cosmology must be completely re-interpreted, with the unifying concept of "Permanent Holographic Oscillation Bang Matter-Antimatter". The future giant telescopes must observe an invariant background (CMB) temperature, as well as an invariant value 3/10 for the baryon+dark matter density, the later being an anti-phase oscillation of normal baryons.

The DNA bi-codon mass is central in the Cosmos, confirming again, and with high precision, the pertinence of the dimension 16, showing how the Topological Axis has been predictive. So, the DNA molecule would be more that the simple memory anticipated by Schrödinger [31]. It must be a bio-computer, probably activated by real holography. Indeed, electric current is observed in DNA [32]. So physical laws are identical to biological ones, again ruling out the Multiverse model. The total number of particles (protons + neutrons + electrons) involved in the four nucleotides is 1863, which after separating the 4×4 trivial ones from the Helium, reduces to 1847 = 435 + 446 + 471 + 495, at one unity from s_0. The presence of 495 for the Guanine could not be due to hasard. Indeed while its atomic massis is 329, at one unity from $N_6 = 330$, its number of particles is about $(3/2) \times 330 = 495$, *d*ue to the electrical neutrality, so the factor 3/2 in the Table 5 is justified, opening further study.

So, the relation, for k = 4, between the cosmic temperature and the mammal one $T_{mam} \approx jT_{CMB}$, where $j = 8\pi^2/ln2$ is the scale constant [7] takes a renewed importance, as well as the relations with the triple points of Hydrogen, Oxygen and Water. It is foreseen that future theory will be able to calculate these triple points, a task nowadays impossible.

The overwhelming connections between the HVW crystallographic series, the string theory, and the sporadic groups confirms that *the pure mathematics must now pursue unification, by concentrating on the mathematical properties of physical parameters, looking for the connection with the generalized Riemann hypothesis through the maximal Euler suitable number*.

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