# New Nomenclature in Operators

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Abstract

In this new nomenclature of operators, I show how it is possible to invent four new types of operators. I define some of their properties and I show a practical application solving theoretically the problem of how many primes there are less than a given number.



(1) $$\sum_{n=a}^{b} f(x)=x_a+x_{a+1}+x_2+...+x_{b-1}+x_b$$

And we define also the product of sequences as a series of products.

(2) $$\prod_{n=a}^{b} f(x)=x_a\cdot x_{a+1}\cdot x_{a+2}\cdot ...\cdot x_{b-1}\cdot x_b$$

Following this logic we can define series of subtraction (Subtractory or Rho notation) and series of division (Divisory or Delta notation).

(3) $$\underset{n=a}{\overset{b}{\mathrm{P}}} x=-x_a-x_{a+1}-x_{a+2}-...-x_b$$

(4) $$\underset{n=a}{\overset{b}{\Delta}} x=x_a\div x_{a+1}\div x_{a+2}\div ...\div x_b$$

We can write some properties:

(5) $$\sum_{n=-a}^{a} f(x)=0$$

(6) $$\sum_{n=-a}^{-b} f(x)=\underset{n=a}{\overset{b}{\mathrm{P}}} f(x)$$

(7) $$\sum_{n=a}^{b} f(x)=-\underset{n=a}{\overset{b}{\mathrm{P}}} f(x)$$

(8) $$\underset{n=a}{\overset{b}{\mathrm{P}}} f(x)=-\sum_{n=a}^{b} f(x)$$

(9) $$\underset{n=a}{\overset{b}{\Delta}} x=\frac{1}{\prod_{n=a}^{b} x}$$

(10) $$\prod_{n=a}^{b} x=\frac{1}{\underset{n=a}{\overset{b}{\Delta}} x}$$

(11) $$\frac{\prod_{n=a}^{b} x}{\underset{n=a}{\overset{b}{\Delta}} x}=\left(\prod_{n=a}^{b} x\right)^2=\left(\underset{n=a}{\overset{b}{\Delta}} x\right)^2$$

(12) $$\sum_{n=a}^{b} f(x)+\underset{n=a}{\overset{b}{\mathrm{P}}} f(x)=0$$

(13) $$\sum_{n=a}^{b} f(x)-\underset{n=a}{\overset{b}{\mathrm{P}}} f(x)=2\sum_{n=a}^{b} f(x)=-2\underset{n=a}{\overset{b}{\mathrm{P}}} f(x)$$

(14) $$\sum_{n=a}^{b} f(x)\div\underset{n=a}{\overset{b}{\mathrm{P}}} f(x)=-1$$

(15) $$\left(\underset{n=a}{\overset{b}{\Delta}} x\right)\cdot\left(\prod_{n=a}^{b} x\right)=1$$

With this tools we can find the first term of a succession:

(16) $$\sum_{n=a}^{b} f(x) + \prod_{n=a+1}^{b} f(x) = a \cdot f(x)$$

And the last term of a succession,

(17) $$\sum_{n=a}^{b} f(x) + \prod_{n=a}^{b-1} f(x) = b \cdot f(x)$$

If we want the succession advance in a way that is not the real numbers we can define an interval.

(18) $$\sum_{n=a(c)}^{b} f(x)$$

Where:
a= first term.
b=last term.
c= interval of the succession.

To make a full interval succession it must be true that:

(19) $$\frac{b-a}{c} = n \rightarrow n \in \mathbb{Z}$$

Some examples:

(20) $$\sum_{n=1(0,5)}^{4} x = 1{,}0 + 1{,}5 + 2{,}0 + 2{,}5 + 3{,}0 + 3{,}5 + 4{,}0 = 17{,}5$$

(21) $$\prod_{n=2(0,5)}^{4} x = -2{,}0 - 2{,}5 - 3{,}0 - 3{,}5 - 4{,}0 = -15$$

(22) $$\Pi_{n=5(0,25)}^{7} x = 5{,}00 \cdot 5{,}25 \cdot 5{,}50 \cdot 5{,}75 \cdot 6{,}00 \cdot 6{,}25 \cdot 6{,}50 \cdot 6{,}75 \cdot 7{,}00 = 9561065{,}186$$

(23) $$\Delta_{n=5(1,5)}^{8} x = \left(\frac{5}{6{,}5}\right) \div 8 = 5/52$$

We define a series of exponentials as a Exponentory or Theta notation.

(24) $$\Theta_{n=a}^{b} k^{f(x)} = (((((k^{a})^{a+1})^{a+2})^{\cdots})^{(b-1)})^{b}$$

Some examples (normal and with interval):

(25) $$\underset{n=1}{\overset{3}{\Theta}}\, 5^{x}=(((5^{1}))^{2})^{3}=15625$$

(26) $$\underset{n=2(0,5)}{\overset{3}{\Theta}}\, 2^{(3x-3)}=((2^{(3\cdot2-3)})^{(3\cdot2,5-3)})^{(3\cdot3-3)}=((2^{3})^{(4,5)})^{(5)}=2,41\cdot10^{24}$$

We define a series of roots as a Rootory or a Zeta notation.

(27) $$\underset{n=a}{\overset{b}{Z}}\, \sqrt[f(x)]{k}=\sqrt[b]{ANS}\sqrt[b-1]{ANS}\sqrt[\cdots]{ANS}\sqrt[a+2]{ANS}\sqrt[a+1]{ANS}\sqrt[a]{k}$$

Some examples:

(28) $$\underset{n=3(1)}{\overset{5}{Z}}\, \sqrt[f(x)]{3}=\sqrt[5]{ANS}\sqrt[4]{ANS}\sqrt[3]{3}=1,01847$$

(29) $$\underset{n=1(2)}{\overset{7}{Z}}\, \sqrt[f(x)]{5}=\sqrt[7]{ANS}\sqrt[5]{ANS}\sqrt[3]{ANS}\sqrt[1]{5}=1,01544$$

(30) $$\underset{n=3(5)}{\overset{13}{Z}}\, \sqrt[4x+2]{59538}=\sqrt[4\cdot13+2]{ANS}\sqrt[4\cdot8+2]{ANS}\sqrt[4\cdot3+2]{59538}=\sqrt[54]{ANS}\sqrt[34]{ANS}\sqrt[14]{59538}=1,0042$$

Operators without variable, instead a function we introduce a constant to give us a set:

(31) $$\underset{n=a}{\overset{b}{\Sigma}}\, c=[c+a,c+(a+1),c+(a+2),\ldots,c+b]$$

(32) $$\underset{n=a}{\overset{b}{P}}\, c=[c-a,c-(a+1),c-(a+2),\ldots,c-b]$$

(33) $$\underset{n=a}{\overset{b}{\Pi}}\, c=[c\cdot a,c\cdot(a+1),c\cdot(a+2),\ldots,c\cdot b]$$

(34) $$\underset{n=a}{\overset{b}{\Delta}}\, c=[c\div a,c\div(a+1),c\div(a+2),\ldots,c\div b]$$

Divisory without variable and prim numbers, we use the definition of number prim to analyze a number:

(35) $\underset{n=2}{\overset{a-1}{\Delta}}\, a=[b_2,b_3,\ldots,b_{a-1}]$ “a” is a prim if (36) $\forall b_x \nexists b \in \mathbb{N}$

Some easy examples:

(37) $\underset{n=2}{\overset{4-1}{\Delta}} 4 = \underset{n=2}{\overset{3}{\Delta}} 4 = [4/2, 4/3] = [2, 4/3]$ $\rightarrow$ 4 is not prim has a natural number in b.

(38) $\underset{n=2}{\overset{7-1}{\Delta}} 7 = \underset{n=2}{\overset{6}{\Delta}} 7 = [7/2, 7/3, 7/4, 7/5, 7/6]$ $\rightarrow$ 7 is prim has not a natural number in b.

Divisory without variable and perfect numbers.

(39) $\underset{n=2}{\overset{a}{\Delta}} a = [b_2, b_3, \ldots, b_a]$ the number a is a perfect number if

(40) $\Sigma b_x = a \quad \forall b_x \in \mathbb{N}$

Divisory without variable and amicable numbers.

(41) $\underset{n=2}{\overset{a}{\Delta}} a = [b_2, b_3, \ldots, b_a]$ (42) $\underset{n=2}{\overset{c}{\Delta}} c = [d_2, d_3, \ldots, d_a]$

The numbers a and b are amicable if

(43) $\Sigma b_x = c \quad \forall b_x \in \mathbb{N}$
(44) $\Sigma d_x = a \quad \forall d_x \in \mathbb{N}$

Solution for the problem quantity of prim numbers less than a number:

We assign 1 to the (35) set if (36) is true, and we assign 0 to the set (35) if
(45) $\forall b_x \exists b \in \mathbb{N}$ is true.

The we define the function f(x),

(46) $f(x) = \underset{n=2}{\overset{(2-1)}{\Delta}} 2 + \underset{n=2}{\overset{(3-1)}{\Delta}} 3 + \ldots + \underset{n=2}{\overset{((a-1)-1)}{\Delta}} a + \underset{n=2}{\overset{(a-1)}{\Delta}} a = \sum_{n=2}^{a} \left( \underset{n=2}{\overset{(a-1)}{\Delta}} a \right) = n$

n= quantity of prim numbers < than a.