

Cosmological Special Theory of Relativity

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ABSTRACT

In the Cosmological Special Relativity Theory, we study Maxwell equations, electromagnetic wave equation and function.

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1. Introduction

Our article's aim is that we make cosmological special theory of relativity.

At first, Robertson-Walker metric is

$$d\tau^2 = dt^2 - \frac{1}{c^2} \Omega^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right] \quad (1)$$

According to Λ CDM model, our universe's k is zero. In this time, if t_0 is cosmological present time,

$$k = 0, t = t_0 \gg \Delta t, \Delta t \text{ is period of matter's motion} \quad (2)$$

Hence, the proper time is in cosmological present time,

$$\begin{aligned} d\tau^2 &= dt^2 - \frac{1}{c^2} \Omega^2(t_0) [dr^2 + r^2 d\Omega^2] \\ &= dt^2 - \frac{1}{c^2} \Omega^2(t_0) [dx^2 + dy^2 + dz^2] \\ &= dt^2 \left(1 - \frac{1}{c^2} \Omega^2(t_0) V^2 \right), \quad V^2 = \frac{dx^2 + dy^2 + dz^2}{dt^2} \end{aligned} \quad (3)$$

In this time,

$$d\bar{t} = dt, d\bar{x} = \Omega(t_0) dx, d\bar{y} = \Omega(t_0) dy, d\bar{z} = \Omega(t_0) dz \quad (4)$$

Cosmological special theory of relativity's coordinate transformations are

$$\begin{aligned} c\bar{t} &= ct = \gamma \left(ct' + \frac{v_0}{c} \Omega(t_0) \bar{x}' \right) = \gamma \left(ct + \frac{v_0}{c} \Omega(t_0) x' \Omega(t_0) \right) \\ \bar{x} &= x \Omega(t_0) = \gamma \left(\bar{x}' + v_0 \Omega(t_0) \bar{t}' \right) = \gamma \left(\Omega(t_0) x' + v_0 \Omega(t_0) t' \right) \\ \bar{y} &= \Omega(t_0) y = \bar{y}' = \Omega(t_0) y', \\ \bar{z} &= \Omega(t_0) z = \bar{z}' = \Omega(t_0) z', \quad \gamma = 1 / \sqrt{1 - \frac{v_0^2}{c^2} \Omega^2(t_0)} \end{aligned} \quad (5)$$

Therefore, proper time is

$$\begin{aligned} d\tau^2 &= d\bar{t}^2 - \frac{1}{c^2} [d\bar{x}^2 + d\bar{y}^2 + d\bar{z}^2] \\ &= dt^2 - \frac{1}{c^2} \Omega^2(t_0) [dx^2 + dy^2 + dz^2] \\ &= dt^2 - \frac{1}{c^2} \Omega^2(t_0) [dx'^2 + dy'^2 + dz'^2] \\ &= d\bar{t}'^2 - \frac{1}{c^2} [d\bar{x}'^2 + d\bar{y}'^2 + d\bar{z}'^2] \end{aligned} \quad (6)$$

Hence, velocities are

$$\begin{aligned}\frac{dx}{dt} = V_x &= \frac{V_x' + v_0}{1 + \frac{\Omega^2(t_0)}{c^2} V_x' \cdot v_0}, V_x' = \frac{dx'}{dt'} \\ \frac{dy}{dt} = V_y &= \frac{V_y'}{\gamma(1 + \frac{\Omega^2(t_0)}{c^2} V_x' \cdot v_0)}, V_y' = \frac{dy'}{dt'} \\ \frac{dz}{dt} = V_z &= \frac{V_z'}{\gamma(1 + \frac{\Omega^2(t_0)}{c^2} V_x' \cdot v_0)}, V_z' = \frac{dz'}{dt'}\end{aligned}\quad (7)$$

In cosmological special theory of relativity(CSTR)'s differential operators are

$$\begin{aligned}\frac{1}{c} \frac{\partial}{\partial \bar{t}} &= \frac{1}{c} \frac{\partial}{\partial t} = \gamma \left(\frac{1}{c} \frac{\partial}{\partial \bar{t}'} - \frac{v_0}{c} \Omega(t_0) \frac{\partial}{\partial \bar{x}'} \right) \\ &= \gamma \left(\frac{1}{c} \frac{\partial}{\partial t'} - \frac{v_0}{c} \frac{\partial}{\partial x'} \right) \\ \frac{\partial}{\partial \bar{x}} &= \frac{\partial}{\partial x} \frac{1}{\Omega(t_0)} = \gamma \left(\frac{\partial}{\partial \bar{x}'} - \frac{v_0}{c} \Omega(t_0) \frac{1}{c} \frac{\partial}{\partial \bar{t}'} \right) \\ &= \gamma \left(\frac{\partial}{\partial x'} \frac{1}{\Omega(t_0)} - \frac{v_0}{c} \Omega(t_0) \frac{1}{c} \frac{\partial}{\partial t'} \right) \\ \frac{\partial}{\partial \bar{y}} &= \frac{\partial}{\partial y} \frac{1}{\Omega(t_0)} = \frac{\partial}{\partial \bar{y}'} = \frac{\partial}{\partial y'} \frac{1}{\Omega(t_0)} \\ \frac{\partial}{\partial \bar{z}} &= \frac{\partial}{\partial z} \frac{1}{\Omega(t_0)} = \frac{\partial}{\partial \bar{z}'} = \frac{\partial}{\partial z'} \frac{1}{\Omega(t_0)}, \gamma = 1 / \sqrt{1 - \frac{v_0^2}{c^2} \Omega^2(t_0)}\end{aligned}\quad (8)$$

Hence,

$$\begin{aligned}\frac{1}{c^2} \frac{\partial^2}{\partial \bar{t}^2} - \bar{\nabla}^2 &= \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{1}{\Omega^2(t_0)} \left\{ \left(\frac{\partial}{\partial x} \right)^2 + \left(\frac{\partial}{\partial y} \right)^2 + \left(\frac{\partial}{\partial z} \right)^2 \right\} \\ &= \frac{1}{c^2} \frac{\partial^2}{\partial t'^2} - \frac{1}{\Omega^2(t_0)} \left\{ \left(\frac{\partial}{\partial x'} \right)^2 + \left(\frac{\partial}{\partial y'} \right)^2 + \left(\frac{\partial}{\partial z'} \right)^2 \right\}\end{aligned}\quad (9)$$

The electric charge density ρ and the electric current density \vec{j} are

$$j^\mu = \rho_0 \frac{dx^\mu}{d\tau}, j^0 = c\rho = c\gamma\rho_0, j^i = \vec{j} = \rho\vec{u}, i = 1,2,3 \quad (10)$$

In CSTR, transformations of the electric charge density and the electric current density are likely as coordinate transformations are

$$\begin{aligned} c\bar{\rho} &= c\rho = \gamma(c\bar{\rho}' + \frac{V_0}{c}\Omega(t_0)\bar{j}_x') = \gamma(c\rho' + \frac{V_0}{c}\Omega(t_0)j_x'\Omega(t_0)) \\ \bar{j}_x &= j_x\Omega(t_0) = \gamma(\bar{j}_x' + v_0\Omega(t_0)\bar{\rho}') = \gamma(\Omega(t_0)j_x' + v_0\Omega(t_0)\rho') \\ \bar{j}_y &= \Omega(t_0)j_y = \bar{j}_y' = \Omega(t_0)j_y', \\ \bar{j}_z &= \Omega(t_0)j_z = \bar{j}_z' = \Omega(t_0)j_z' \end{aligned}, \quad \gamma = 1/\sqrt{1 - \frac{V_0^2}{c^2}\Omega^2(t_0)} \quad (11)$$

2. Electrodynamics in CSTR

The electromagnetic potential A^μ is 4-vector potential. Hence, transformations of A^μ are

$$\begin{aligned} \bar{\phi} &= \phi = \gamma(\bar{\phi}' + \frac{V_0}{c}\Omega(t_0)\bar{A}_x') = \gamma(\phi' + \frac{V_0}{c}\Omega(t_0)A_x'\Omega(t_0)) \\ \bar{A}_x &= A_x\Omega(t_0) = \gamma(\bar{A}_x' + \frac{V_0}{c}\Omega(t_0)\bar{\phi}') = \gamma(\Omega(t_0)A_x' + \frac{V_0}{c}\Omega(t_0)\phi') \\ \bar{A}_y &= \Omega(t_0)A_y = \bar{A}_y' = \Omega(t_0)A_y', \\ \bar{A}_z &= \Omega(t_0)A_z = \bar{A}_z' = \Omega(t_0)A_z' \end{aligned}, \quad \gamma = 1/\sqrt{1 - \frac{V_0^2}{c^2}\Omega^2(t_0)} \quad (12)$$

In CSTR, electric field $\vec{\bar{E}}$ and magnetic field $\vec{\bar{B}}$ have to satisfy Maxwell equations of special relativity theory. Hence, in CSRT, Maxwell equations are likely as special theory of relativity,

$$\vec{\nabla} \cdot \vec{\bar{E}} = 4\pi\bar{\rho} \quad (13-i)$$

$$\vec{\nabla} \cdot \vec{\bar{B}} = 0 \quad (13-ii)$$

$$\vec{\nabla} \times \vec{\bar{E}} = -\frac{1}{c} \frac{\partial \vec{\bar{B}}}{\partial t} \quad (13-iii)$$

$$\vec{\nabla} \times \vec{\bar{B}} = \frac{1}{c} \frac{\partial \vec{\bar{E}}}{\partial t} + \frac{4\pi}{c} \vec{j} \quad (13-iv)$$

In this time, Eq(13-i) is

$$\vec{\nabla} \cdot \vec{\bar{E}} = \frac{1}{\Omega(t_0)} \vec{\nabla} \cdot \vec{E} = 4\pi\bar{\rho} = 4\pi\rho \quad (14)$$

Hence, $\vec{\bar{E}} = \vec{E}\Omega(t_0)$. According to special relativity, $\vec{\bar{B}} = \vec{B}\Omega(t_0)$

Eq(13-ii) is

$$\vec{\nabla} \cdot \vec{\bar{B}} = \frac{1}{\Omega(t_0)} \vec{\nabla} \cdot \vec{B} \Omega(t_0) = \vec{\nabla} \cdot \vec{B} = 0 \quad (15)$$

Eq(13-iii) is

$$\vec{\nabla} \times \vec{\bar{E}} = \frac{1}{\Omega(t_0)} \vec{\nabla} \times \vec{E} \Omega(t_0) = \vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t} = -\frac{1}{c} \frac{\partial \vec{\bar{B}}}{\partial t} \Omega(t_0) \quad (16)$$

Eq(13-iv) is

$$\begin{aligned} \vec{\nabla} \times \vec{\bar{B}} &= \frac{1}{\Omega(t_0)} \vec{\nabla} \times \vec{B} \Omega(t_0) = \vec{\nabla} \times \vec{B} \\ &= \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j} = \Omega(t_0) \left(\frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j} \right) \end{aligned} \quad (17)$$

Hence, in CSTR, Maxwell equations are

$$\vec{\nabla} \cdot \vec{\bar{E}} = 4\pi\rho \quad (18-i)$$

$$\vec{\nabla} \cdot \vec{\bar{B}} = 0 \quad (18-ii)$$

$$\vec{\nabla} \times \vec{\bar{E}} = -\frac{1}{c} \frac{\partial \vec{\bar{B}}}{\partial t} \Omega(t_0) \quad (18-iii)$$

$$\vec{\nabla} \times \vec{\bar{B}} = \Omega(t_0) \left(\frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{j} \right) \quad (18-iv)$$

Therefore, in CSTR, the electric field $\vec{\bar{E}}$ and the magnetic field $\vec{\bar{B}}$ are

$$\begin{aligned} \vec{\bar{E}} &= \vec{E} \Omega(t_0) = \Omega(t_0) \left(-\vec{\nabla} \phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t} \right) \\ &= -\Omega(t_0) \vec{\nabla} \phi - \Omega(t_0) \frac{1}{c} \frac{\partial \vec{A}}{\partial t} = -\vec{\nabla} (\phi \Omega^2(t_0)) - \frac{1}{c} \frac{\partial \vec{A}}{\partial t} \end{aligned} \quad (19)$$

$$\vec{\bar{B}} = \vec{B} \Omega(t_0) = \Omega(t_0) \vec{\nabla} \times \vec{A} = \Omega(t_0) \vec{\nabla} \times \vec{A} \quad (20)$$

3. Electromagnetic Wave in CSTR

Electromagnetic wave equation is in CSTR,

$$\begin{aligned} \frac{1}{c} \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{\bar{E}}) &= -\Omega(t_0) \frac{1}{c^2} \frac{\partial^2 \vec{\bar{B}}}{\partial t^2} \\ &= \vec{\nabla} \times \left(\frac{1}{c} \frac{\partial \vec{\bar{E}}}{\partial t} \right) = \vec{\nabla} \times \left(\frac{1}{\Omega(t_0)} \vec{\nabla} \times \vec{\bar{B}} \right), \vec{\nabla} \times \vec{j} = \vec{0} \\ &= \frac{1}{\Omega(t_0)} \{ -\nabla^2 \vec{\bar{B}} + \vec{\nabla} (\vec{\nabla} \cdot \vec{\bar{B}}) \} = -\frac{1}{\Omega(t_0)} \nabla^2 \vec{\bar{B}} \end{aligned} \quad (21)$$

Hence, electromagnetic wave equation is

$$\Omega(t_0) \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} - \frac{1}{\Omega(t_0)} \nabla^2 \vec{B} = \vec{0} \quad (22)$$

And,

$$\begin{aligned} \frac{1}{c} \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) &= \Omega(t_0) \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}, \frac{1}{c} \frac{\partial \vec{j}}{\partial t} = \vec{0} \\ &= \vec{\nabla} \times \left(\frac{1}{c} \frac{\partial \vec{B}}{\partial t} \right) = \vec{\nabla} \times \left(-\frac{1}{\Omega(t_0)} \vec{\nabla} \times \vec{E} \right) \\ &= -\frac{1}{\Omega(t_0)} \{ -\nabla^2 \vec{E} + \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) \} = \frac{1}{\Omega(t_0)} \nabla^2 \vec{E}, \vec{\nabla}(4\pi\rho) = \vec{0} \end{aligned} \quad (23)$$

Hence, electromagnetic wave equation is

$$\Omega(t_0) \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} - \frac{1}{\Omega(t_0)} \nabla^2 \vec{E} = \vec{0} \quad (24)$$

In CSTR, electromagnetic wave functions are

$$\vec{E} = \vec{E}_0 \sin \Phi, \vec{B} = \vec{B}_0 \sin \Phi$$

$$\Phi = \omega \left\{ \frac{t}{\sqrt{\Omega(t_0)}} - \frac{\sqrt{\Omega(t_0)}}{c} (lx + my + nz) \right\} \quad (25)$$

Where,

$$l^2 + m^2 + n^2 = 1 \quad (26)$$

According to Maxwell equations are in CSTR,[1]

$$\begin{aligned} \Omega(t_0) \left\{ \frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{4\pi}{c} j_x \right\} &= \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right), & \Omega(t_0) \frac{1}{c} \frac{\partial B_x}{\partial t} &= \left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right) \\ \Omega(t_0) \left\{ \frac{1}{c} \frac{\partial E_y}{\partial t} + \frac{4\pi}{c} j_y \right\} &= \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right), & \Omega(t_0) \frac{1}{c} \frac{\partial B_y}{\partial t} &= \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right) \\ \Omega(t_0) \left\{ \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{4\pi}{c} j_z \right\} &= \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right), & \Omega(t_0) \frac{1}{c} \frac{\partial B_z}{\partial t} &= \left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) \end{aligned} \quad (27)$$

Where,

$$\begin{aligned} \Omega(t_0) \left\{ \frac{1}{c} \frac{\partial E_x'}{\partial t} + \frac{4\pi}{c} j_x \right\} &= \left\{ \frac{\partial}{\partial y} \gamma(B_z' + \frac{V_0}{c} \Omega(t_0) E_y') - \frac{\partial}{\partial z} \gamma(B_y' - \frac{V_0}{c} \Omega(t_0) E_z') \right\} \\ \Omega(t_0) \left\{ \frac{1}{c} \frac{\partial}{\partial t} \gamma(E_y' + \frac{V_0}{c} \Omega(t_0) B_z') + \frac{4\pi}{c} j_y \right\} &= \left\{ \frac{\partial B_x'}{\partial z} - \frac{\partial}{\partial x} \gamma(B_z' + \frac{V_0}{c} \Omega(t_0) E_y') \right\} \end{aligned}$$

$$\Omega(t_0) \left\{ \frac{1}{c} \frac{\partial}{\partial t} \gamma(E_z' - \frac{v_0}{c} \Omega(t_0) B_y') + \frac{4\pi}{c} j_z \right\} = \left\{ \frac{\partial}{\partial y} \gamma(B_y' - \frac{v_0}{c} \Omega(t_0) E_z') - \frac{\partial B_x'}{\partial z} \right\} \quad (28)$$

Where,[1]

$$\begin{aligned} \Omega(t_0) \frac{1}{c} \frac{\partial B_x'}{\partial t} &= \left\{ \frac{\partial}{\partial z} \gamma(E_y' + \frac{v_0}{c} \Omega(t_0) B_z') - \frac{\partial}{\partial y} \gamma(E_z' - \frac{v_0}{c} \Omega(t_0) B_y') \right\} \\ \Omega(t_0) \frac{1}{c} \frac{\partial}{\partial t} \gamma(B_y' - \frac{v_0}{c} \Omega(t_0) E_z') &= \left\{ \frac{\partial}{\partial x} \gamma(E_z' - \frac{v_0}{c} \Omega(t_0) B_y') - \frac{\partial E_x'}{\partial z} \right\} \\ \Omega(t_0) \frac{1}{c} \frac{\partial}{\partial t} \gamma(B_z' + \frac{v_0}{c} \Omega(t_0) E_y') &= \left\{ \frac{\partial E_x'}{\partial y} - \frac{\partial}{\partial x} \gamma(E_y' + \frac{v_0}{c} \Omega(t_0) B_z') \right\} \end{aligned} \quad (29)$$

Hence, in CSTR, transformations of electromagnetic field are

$$E_x = E_x', E_y = \gamma(E_y' + \frac{v_0}{c} \Omega(t_0) B_z'), E_z = \gamma(E_z' - \frac{v_0}{c} \Omega(t_0) B_y') \quad (30)$$

$$B_x = B_x', B_y = \gamma(B_y' - \frac{v_0}{c} \Omega(t_0) E_z'), B_z = \gamma(B_z' + \frac{v_0}{c} \Omega(t_0) E_y') \quad (31)$$

In CSTR, electromagnetic wave functions are

$$E_x' = E_{x0} \sin \Phi', E_y' = \gamma(E_{y0} - \frac{v_0}{c} \Omega(t_0) B_{z0}) \sin \Phi', E_z' = \gamma(E_{z0} + \frac{v_0}{c} \Omega(t_0) B_{y0}) \sin \Phi' \quad (32)$$

$$B_x' = B_{x0} \sin \Phi', B_y' = \gamma(B_{y0} + \frac{v_0}{c} \Omega(t_0) E_{z0}) \sin \Phi', B_z' = \gamma(B_{z0} - \frac{v_0}{c} \Omega(t_0) E_{y0}) \sin \Phi' \quad (33)$$

In this time,

$$\Phi' = \omega' \left\{ \frac{t'}{\sqrt{\Omega(t_0)}} - \frac{\sqrt{\Omega(t_0)}}{c} (l'x' + m'y' + n'z') \right\} \quad (34)$$

$$\Phi = \omega \left\{ \frac{t}{\sqrt{\Omega(t_0)}} - \frac{\sqrt{\Omega(t_0)}}{c} (lx + my + nz) \right\} \quad (35)$$

If we compare Eq(34) and Eq(35),

$$\omega' = \omega \gamma \left(1 - l \frac{v_0}{c} \right), \quad l = \frac{l - \frac{v_0}{c}}{1 - l \frac{v_0}{c}}, \quad m = \frac{m}{\gamma \left(1 - l \frac{v_0}{c} \right)}, \quad n = \frac{n}{\gamma \left(1 - l \frac{v_0}{c} \right)} \quad (36)$$

Where,

$$l^2 + m^2 + n^2 = 1 \quad (37)$$

4. Conclusion

We know Maxwell equations, electromagnetic wave equations and functions in Cosmological Special Theory of Relativity.

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