
Cotes's Spiral Vortex in Extratropical Cyclone bomb South Atlantic Oceans

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The characteristic shape of hurricanes, cyclones, typhoons is a spiral. There are several types of turns, and determining the characteristic equation of which spiral the "cyclone bomb" (CB) fits into is the goal of the work. In mathematics, a spiral is a curve which emanates from a point, moving farther away as it revolves around the point. An "explosive extratropical cyclone" is an atmospheric phenomenon that occurs when there is a very rapid drop in central atmospheric pressure. This phenomenon, with its characteristic of rapidly lowering the pressure in its interior, generates very intense winds and for this reason it is called explosive cyclone, bomb cyclone. It was determined the mathematical equation of the shape of the extratropical cyclone, being in the shape of a spiral called "Cotes's Spiral." In the case of CB, which formed in the south of the Atlantic Ocean, and passed through the south coast of Brazil in July 2020, causing great damages in several cities in the State of Santa Catarina. With gusts recorded of 116 km/h, atmospheric phenomenon – "cyclone bomb" (CB) hit southern Brazil on June 30, the beginning of winter 2020, causing destruction in its influence over. In five hours the CB traveled a distance of 257.48 km (159.99 miles), at an average speed of 51.496 km/h (31.998 miles/h) 27.81 knots, moved towards ENE, with a low pressure center of 986 mbar, 07:20 UTC, approximate location 35°S45°W, and 5 hours after 12:20 UTC had already grown and had a low

pressure center of 972 mbar , approximate location 34°S 42°30'W.

1 Introduction

In mathematics, a spiral is a curve which emanates from a point, moving farther away as it revolves around the point. [1-4] The characteristic shape of hurricanes, cyclones, typhoons is a spiral [5-8]. There are several types of turns, and determining the characteristic equation of which spiral the cyclone bomb CB [9].

The work aims to determine the mathematical equation of the shape of the extratropical cyclone, in the case of CB [9], which formed in the south of the Atlantic Ocean, and passed through the south coast of Brazil in July 2020, causing great damages in several cities in the State of Santa Catarina. [9]

An "explosive extratropical cyclone" is an atmospheric phenomenon that occurs when there is a very rapid drop in central atmospheric pressure. This phenomenon, with its characteristic of rapidly lowering the pressure in its interior, generates very intense winds and for this reason it is called explosive cyclone, bomb cyclone. [5-9]

With winds of 100 km/h "explosive extratropical cyclone" left a trail of destruction in Santa Catarina, Paraná and Rio Grande do Sul on Tuesday, June 30, 2020. The phenomenon known as the "cyclone bomb" caused heavy rains, where gusts of wind destroyed houses, caused tree falls, debris and the destruction

of the energy network, [9] main Chapecó located 27°06'17"S 52°36'51"W, Santa Catarina [10], was the most affected by cyclone.

The occurrence of cyclones is relatively common for the region at this time of year, but the recent phenomenon has been exacerbated by other meteorological and atmospheric factors. This phenomenon, with this feature to lower the pressure inside quickly generates very strong winds and so that name of explosive cyclones. For large-scale occurs, the tropical cyclones influence and are influenced by the weather and other atmospheric phenomena point of view, the call synoptic condition. A very intense circulation of heat and humidity from the North region, with emphasis on the Amazon and Bolivia, increased the occurrence of the cyclone more sharply, reaching Paraguay, Uruguay and northern Argentina, as well as the south-Brazilian coast. [5-9]

2 The Anatomy of a Cyclone

Tropical cyclones are compact, circular storms, generally some 320 km (200 miles) in diameter, whose winds swirl around a central region of low atmospheric pressure. The winds are driven by this low-pressure core and by the rotation of the Earth, which deflects the path of the wind through a phenomenon known as the Coriolis force. As a result, tropical cyclones rotate in a counterclockwise (or cyclonic) direction in the Northern Hemisphere and in a clockwise (or anticyclonic) direction in the Southern Hemisphere. [9, 11-15]

The wind field of a tropical cyclone may be divided into three regions. First is a ring-shaped outer region, typically having an outer radius of about 160 km (100 miles) and an inner radius of about 30 to 50 km (20 to 30 miles). In this region the winds increase uniformly in speed toward the center. Wind speeds attain their maximum value at the second region, the eyewall, which is typically 15 to 30 km (10 to 20 miles) from the center of the storm. The eyewall in turn surrounds the interior region, called the eye, where wind speeds decrease rapidly and the air is often calm. These main structural regions are described in greater detail below. [9, 11-15]

3 Date of Synoptic charts of CB and satellite images

The cyclone pump (CB) [9] with a low pressure center of 986 mbar, 07:20 UTC, approximate location 35°S 45°W, and 5 hours after 12:20 UTC had already grown and had a low pressure center of 972 mbar, approximate location 34°S 42°30'W. The CB traveled a distance of 257.48 km (159.99 miles), at an average speed of 51.496 km/h (31.998 miles/h) 27.81 knots, Figure (3), moved towards ENE. To plot the graph of Figure

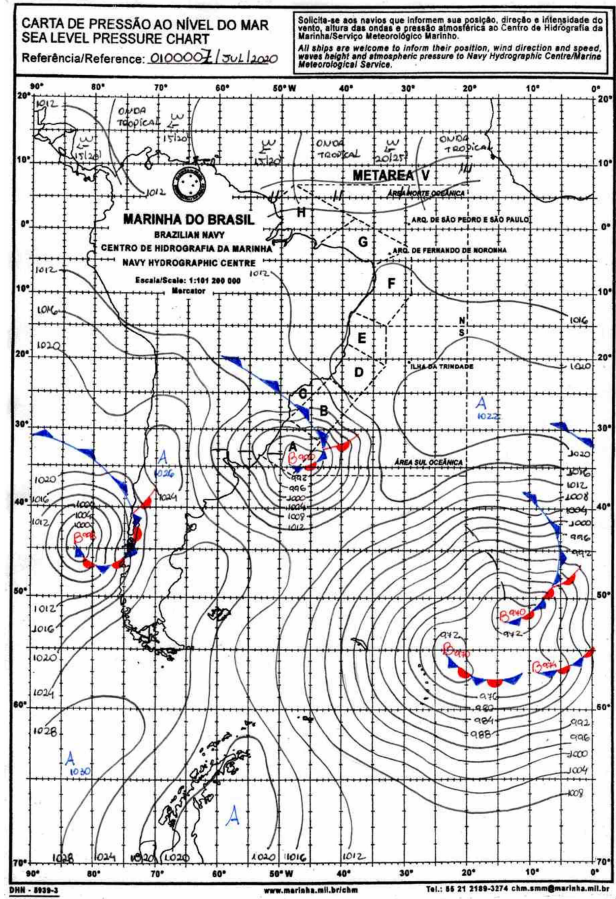


Figure 1: Synoptic Letters, from July 1, 2020, at 00h00 UTC. Navy Hydrography Center. Brazil's navy. Synoptic Letters. [22]

(5), Figures (1-3) were used to accompany the isobaric ones, where points of Figure (3) were used for 7:20 UTC and 12:20 UTC and in short straight lines.

The analysis of the images of the Figures (1-3) uses the methods [5-7, 15-21].

The Figures (1) and (2) show the Synoptic Letters, from July 1, 2020, at 00:00 UTC and 12:00 UTC, respectively, from Navy Hydrography Center, Brazil's navy. [22] The CB generated a low pressure 976 mbar inside it, 12:00 UTC, generating two atmospheric currents that moved at high speed. In a northwest-southeast direction, Bolivia and Paraguay, crossing the states of Parana and Santa Catarina, and this draft that hit the south of Brazil, which caused the destruction of the affected states. Another moving to Argentina, southwest-northeast direction, due to high area of high pressure (1022 mbar). Both enhanced the phenomenon.

In Figure (3) the image of the "bomb ciclone" moving to the Atlantic Ocean. Image in the infrared spectrum, July 1, 2020, for the 5 hour time slot, from 07:20 to 12:20 UTC. REDEMETS, Adapted [23].

The Figure (3) shows an image in the infrared spectrum of the CB moving towards the high seas, that is, the Atlantic Ocean. The eye of the cyclone is crisp,

Cotes's Spiral Vortex in
Extratropical Cyclone bomb
South Atlantic Oceans

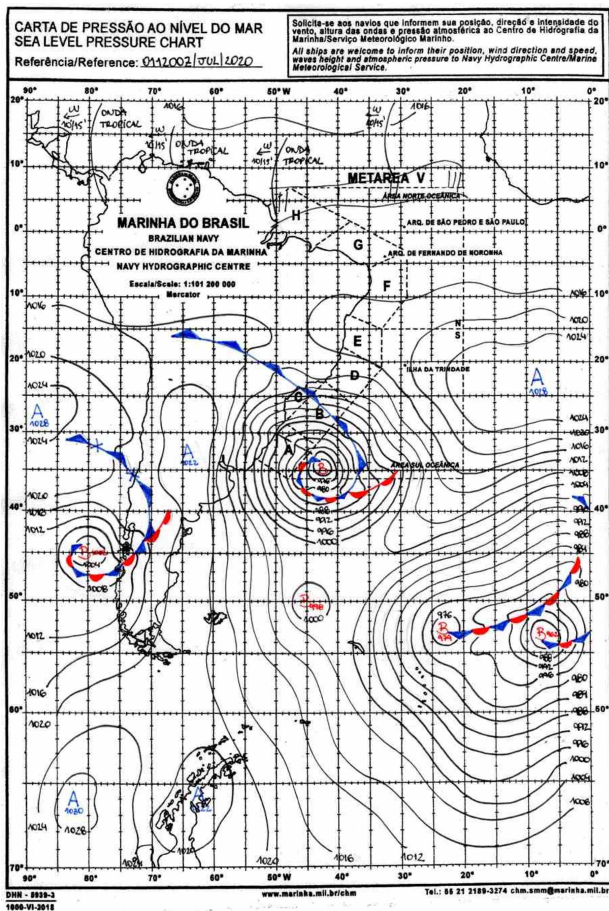


Figure 2: Synoptic Letters, from July 1, 2020, at 12h00 UTC. Navy Hydrography Center. Brazil's navy. Synoptic Letters. [22]

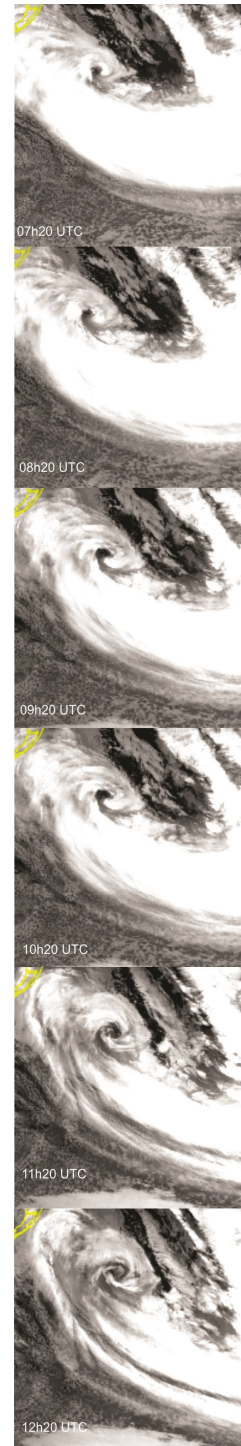


Figure 3: Image of the “bomb cyclone” moving to the Atlantic Ocean. Image in the infrared spectrum, July 1, 2020, for the 5 hour time slot, from 07:20 to 12:20 UTC, REDEMET. (Adapted) [23].

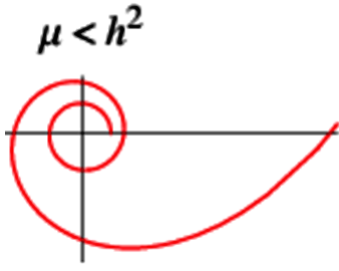


Figure 4: Spiral shape called "Cotes's Spiral." for $\mu < h^2$. [24, 25]

and gains intensity when advancing towards the ocean. Image of the CB moving to the Atlantic Ocean., July 1, 2020, 07:20, 08:20, 09:20, 10:20, 11:20 to 12:20 UTC.

A high pressure area 1026 mbar, Figure (2), over Argentina coordinates 33°S 65°W, moving in the direction to Paraguay. The synoptic chart in Figure (2) shows a 976 mbarmbar low pressure center, coordinates 35°S 34°W, next to Uruguay and Rio Grande do Sul coast, but away from the coast, 12:00 UTC on July 1, 2020.

The formation of the CB is clear, in Figures (3). An area of high pressure of 1022 mbar, over Argentina, with coordinates 35°S 65°W, continuing its movement towards Paraguay, acquiring greater amplitude and intensity.

4 "Cotes's Spiral" and Results

As stated in the introduction the characteristic shape of hurricanes, cyclones, typhoons is a spiral. [1-4] There are several types of turns, and determining the characteristic equation of which spiral the CB fits into is the goal of the work. In mathematics, a spiral is a curve which emanates from a point, moving farther away as it revolves around the point. After an analysis of the different types of spirals [26-29], it was found that the shape that came closest to the CB spiral, Figure (3), is a "Cotes's Spiral." [26] It was determined the mathematical equation of the shape of the extratropical cyclone, being in the shape of a spiral called "Cotes's Spiral" [24-26]

A spiral that gives the solution to the central orbit problem under a radial force law

$$\ddot{r} = -\mu[r]^{-3}\hat{r} \quad (1)$$

where μ is a positive constant. There are three solution regimes,

$$r = A \sec(k\theta + \epsilon),$$

where $k^2 = 1 - \frac{\mu}{h^2}$, when $\mu < h^2$

$$r = A \csc(k'\theta + \epsilon),$$

where $k^2 = \frac{\mu}{h^2} - 1$, when $\mu > h^2$

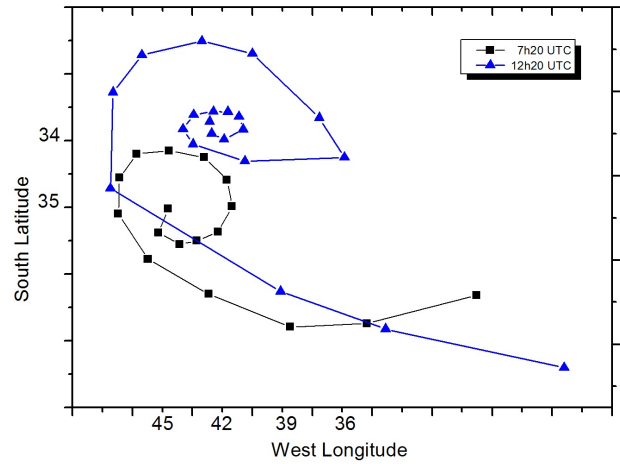


Figure 5: It represents the coordinates of the points collected from Figure (3), in Longitude and Latitude, using the Isobaric found in Figures (1) and (2). [Authors]

$$r = \frac{A}{\theta + \epsilon} \quad (2)$$

when $\mu = h^2$

where A and ϵ are constants, and h is the specific angular momentum [26]. The case $\mu > h^2$ gives an epispiral, while $\mu = h^2$ leads to a hyperbolic spiral.

Figure (5) represents the coordinates of the points collected from Figure (3), in Longitude and Latitude. With the location of the low pressure center (986 mbar) of the CB at 35°S 45°W, at 07:20 UTC, and the low pressure center (972 mbar) of the CB at 35°S 42°30'W, at 12:20 UTC, Figures (1) and (2). In this Figure (5) the shape of the CB is represented, using as a parameter the isobaric ones observed in Figures (1) and (2).

Analyzing the shape of the Spiral shape called "Cotes's Spiral." for $\mu < h^2$. [24, 25], it appears that adding two constants to Equation (3) makes the necessary adjustments for the Isobaric ones. In the case of CB, the spiral that gives the solution to a radial force law is given by Equation (1): $\ddot{r} = -\mu[r]^{-3}\hat{r}$; where:

$$r = A \sec(k\theta + \epsilon),$$

where

$$k^2 = 1 - \frac{\mu}{h^2}, \quad (3)$$

when $\mu < h^2$

An adjustment in Equation (3) is necessary to obtain the graph of Figure (5). Then, adding the constants $B \neq 0$ and C where for $\mu < h^2$.

$$r = BA \sec(k\theta + \epsilon) + C, \quad (4)$$

where

$$k^2 = 1 - \frac{\mu}{h^2}, \quad (5)$$

when $\mu < h^2$

5 Conclusion

The occurrence of cyclones is relatively common for the southern region of Brazil at this time of year, that is, winter, but the recent phenomenon is augmented by other meteorological and atmospheric factors.

CB generated a strong low pressure in its interior, creating two air streams which have worsened the phenomenon. An atmospheric current moved at high speed, in northwest-southeast direction, Bolivia, Paraguay, through the Paraná and Santa Catarina, and this airflow that hit the south of Brazil causing destruction. Another one traveled through Argentina in a southwest-northeast direction, colliding with the draft from Bolivia and Paraguay. Both of them clashed over the three southern states of Brazil. Only the outer edge of the CB reached the coast of the three states in the southern region of Brazil, Paraná, Santa Catarina and Rio Grande do Sul. After an analysis of the different types of spirals, it was found that the shape that came closest to the CB spiral, is a "Cotes's Spiral." It was determined the mathematical equation of the shape of the extratropical cyclone, being in the shape of a spiral called "Cotes's Spiral".

In five hours the CB traveled a distance of 257.48 km (159.99 miles), at an average speed of 51.496 km/h (31.998 miles/h) 27.81 knots, moved towards ENE, with a low pressure center of 986 mbar, 07:20 UTC, approximate location 35°S 45°W, and 5 hours after 12:20 UTC had already grown and had a low pressure center of 972 mbar, approximate location 34°S 42°30'W, using as a parameter the isobaric ones observed.

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*Cotes's Spiral Vortex in
Extratropical Cyclone bomb
South Atlantic Oceans*

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