The Haug Quantum Wave Equation Combined with Pauli Operator

Espen Gaarder Haug
orcid.org/0000-0001-5712-6091
Norwegian University of Life Sciences, Norway
e-mail espenhaug@mac.com

August 24, 2020

Abstract

In this short note we will show the Haug-1 quantum wave equation in relation to where we incorporate spin following the Pauli route. This lead to a similar equation to the Schrödinger-Pauli equation, but our equation is relativistic while the Schrödinger-Pauli equation is non-relativistic, our equation is also simpler in terms of for example the time and space are on the same order (first derivatives), while in the Schrödinger-Pauli equation time is on first order and spatial-spin dimensions on second order. Comments are welcome.

Key Words: quantum mechanics, spin, Pauli operator.

1 Haug equation following the Pauli Route

The Haug-1 wave-equation [1] is given by

$$i\hbar\frac{\partial\psi}{\partial t} = (-ci\hbar\nabla + V)\,\psi \tag{1}$$

where V is the potential energy, and $-i\hbar\nabla$ is the kinetic Compton momentum operator.

The Hamiltonian operator in our new wave equation when following the route of Pauli will be

$$\hat{H} = c\boldsymbol{\sigma}\cdot(\boldsymbol{p}_k - q\boldsymbol{A}) + q\phi \tag{2}$$

Because incorporation of the Pauli operator the Hamilton operator is now a 2×2 matrix. The Pauli operator $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is formed by Pauli matrixes:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3}$$

so the Haug equation can then be re-written as

$$i\hbar\frac{\partial}{\partial t}\left|\psi\right\rangle = -c\boldsymbol{\sigma}\cdot(\boldsymbol{p}_k - q\boldsymbol{A})\left|\psi\right\rangle - q\phi\left|\psi\right\rangle \tag{4}$$

Where $|\psi\rangle$ is a two-component spinor wave function or a coulum vector $|\psi\rangle = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$ and $\boldsymbol{p}_k = i\hbar\nabla$ is the kinetic Compton momentum operator, and $\boldsymbol{A}$ is the the magnetic vector potential and ϕ the electric scalar potential. Equation 4 can be seen as a parallell to the Schrödinger-Pauli equation [2, 3], but this equation is relativistic while the Schrödinger-Pauli equation is non-relativistic. This equation is also in general simpler, as time and space dimensions are of the same derivatives order, while in the Schrödinger-Pauli equation the time derivative is of first order and the space dimension of second order (derivatives).

References

[1] E. G. Haug. Deeper insight on existing and new wave equations in quantum mechanics. *ViXrA or Research-gate, submitted for review in a journal*, 2020. URL https://vixra.org/abs/2008.0010.

[2] E. Schrödinger. An undulatory theory of the mechanics of atoms and molecules. *Physical Review*, 28(6): 104–1070, 1926. URL https://doi.org/10.1103/PhysRev.28.1049.

[3] W. Pauli. Zur quantenmechanik des magnetischen elektrons". zeitschrift fr physik. *Zeitschrift für Physik*, 43: 601, 1927. URL https://doi.org/10.1007/BF01397326.