# Dense Summary of New Developments in Quantum Mechanics 

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#### Abstract

In this note, we present a table with a summary of recent developments in quantum mechanics. If you not have studied our papers carefully already [1-4], then this paper will not make much sense and the equations could easily be misunderstood, so we strongly encourage the reader to become familiar with this material first. We will possibly write a long paper on this topic later, but it may be helpful to present the essence of both old and new QM, as we understand it today. This is useful for both tracing the history of ideas related to QM in mathematical form and for creating an opportunity to compare equations and analyze their similarities and differences.


Key Words: quantum mechanics, de Broglie wavelength, Compton wavelength.
To understand deeper quantum mechanics, one has to be both rigorous and open-minded; it is also essential to understand (at least) the following three points:

1. The Compton wave is the true matter wave and the de Broglie wave just is a mathematical derivative of this wave. The de Broglie wave, $\lambda_{b}$, is always given by $\lambda_{b}=\lambda \frac{c}{v}$. The de Broglie wave is not mathematically defined for $v=0$. The standard momentum is linked to the de Broglie wave, as it is $p=m \gamma=\frac{h}{\lambda_{b}}$. What we call total Compton momentum is given by $p_{t}=m c \gamma=\frac{h}{\lambda}$. The well-known wave equations such as KleinGordon, Sch\{" orodinger, and Dirac are all built from the standard momentum and are therefore linked to the de Broglie wave, even though they also contain the Compton wave in their last term. These equations are, therefore, unnecessary complex models of reality. It is a little like someone has taken the mass of a cow and multiplied the mass of the cow with the speed of light and divided by the mass of an apple and think what they have obtained represents a cow. Indeed, such a model contains information about the cow, but it is somewhat hidden if one does not know how it was constructed and is not clear on how one can can get down to the mass of the cow.
2. The kg rest-mass can always be described as $m=\frac{\hbar}{\lambda} \frac{1}{c}$. The standard mass is an incomplete picture of a mass; it is, at the depth of reality, a collision ratio, see [1]. This collision ratio does not contain information about the time it takes to collide, which is important for gravity and is indirectly embedded in all gravity physics through a multiplication of $G$ with $m$. However, as we have described in previous papers, $G$ is simply a composite constant of the form $G=\frac{l_{l}^{2} c^{3}}{\hbar}$ and the Planck length can be found independent on $G$ and $h$, see [2], and is actually also independent of $c$. The much more complete mass definition is given by $m=\frac{l_{p}}{c} \frac{l_{p}}{\lambda}$, which gives the collision time for any mass.
3. The rest-mass energy definition is also incomplete and should be reformulated as $\bar{E}=\bar{m} c$; this does not conflict in any way with $E=m c^{2}$, as discussed in [1]. If one use Joules as energy and kg as mass, one does need to have $E=m c^{2}$ as the relation between rest-mass energy and the mass. As noted above, it would be helpful, even essential to read our collision space-time paper to follow the analysis and see the original equations related to it.

In summary, the key takeaway here is that if you only work with point one, then you will already get greatly simplified quantum wave equations. If you also works with points two and three, you will arrive at the simplest possible quantum mechanics equations, where you can truly see that the critical components are nothing more than time, collision-time, empty space, and collision length in space; these elements constitute atomism at the deepest level and offer an array of opportunities for future research.

See Table 1:

## Overview QM wave equations, old insight and new insight:

## Defintions:

| Mass: | $m=\frac{\hbar}{\lambda} \frac{1}{c}$ | standard kg mass definition |  |
| :---: | :---: | :---: | :---: |
| Rest-mass energy: | $E=h f=m c^{2}$ | incomplete <br> incomplete |  |
| Scratching the surface QM | Wave equations: | Momentum operator on: | Comments : |
| Klein-Gordon (Spin 0) | $\frac{1}{c^{2}} \frac{\partial^{2} \psi}{\partial t^{2}}-\nabla^{2} \psi+\frac{m^{2} c^{2}}{\hbar^{2}} \psi=0$ | de Broglie momentum | Relativistic |
| deeper level | $\frac{\partial^{2} \psi}{\partial t^{2}}-c^{2} \nabla^{2} \psi+\frac{c^{2}}{\lambda^{2}} \psi=0$ |  |  |
| Dirac (Spin 1/2) | $i \hbar \frac{\partial \psi}{\partial t}-\left(c \sum_{i=n}^{3} \alpha_{n} \mathbf{p}_{n}-\beta m c^{2}\right) \psi=0$ | de Broglie momentum | Relativistic |
| deeper level | $i \frac{\partial \psi}{\partial t}-\left(\frac{c}{\hbar} \sum_{i=n}^{3} \alpha_{n} \mathbf{p}_{n}-\beta \frac{c}{\lambda}\right) \psi=0$ |  |  |
| Schrödinger | $i \frac{\partial \psi}{\partial t} \approx\left(\frac{-\hbar}{2 m} \nabla^{2}+\frac{m c^{2}}{\hbar}\right) \psi$ | de Broglie momentum | Non-relativistic |
| deeper level | $i \frac{\partial \psi}{\partial t} \approx\left(\frac{-c \bar{\lambda}}{2} \nabla^{2}+\frac{c}{\lambda}\right) \psi$ |  |  |

Deeper level, but still not deep enough
$\left.\begin{array}{|c|c|c|c}\hline \text { Haug-1: (Spin ?) } & \begin{array}{c}i \frac{\partial \psi}{\partial t}=\left(-i c \nabla+\frac{m c^{2}}{\hbar}\right) \psi \\ \text { deeper level }\end{array} & \begin{array}{c}\text { Kinetic Compton momentum }\end{array} & \text { Relativistic } \\ \hline \text { Haug-2: (Spin ?) } & \frac{\partial \psi}{\partial t}=\left(-i c \nabla+\frac{c}{\lambda}\right) \psi\end{array}\right)$

Much deeper level, but not yet at the bottom of the rabbit hole:

| CST Haug-1: (Spin ?) | $i \frac{\partial \psi}{\partial t}=\left(-i c \nabla+\frac{\bar{m} c^{2}}{l_{p}^{2}}\right) \psi$ | Kinetic Compton momentum | Relativistic |
| :---: | :---: | :---: | :---: |
| deeper level | $i \frac{\partial \psi}{\partial t}=\left(-i c \nabla+\frac{c}{\lambda}\right) \psi$ |  |  |
| CST Haug-2: (Spin ?) | $\frac{1}{c} \frac{\partial \psi}{\partial t}+\nabla \psi=0$ | Total Compton momentum | Relativistic |

## Definitions :

| Mass: | $\bar{m}=\frac{l_{p}}{c} \frac{l_{p}}{\lambda}$ | collision-time definition <br> collision-length | complete <br> complete |
| :---: | :---: | :---: | :---: |
| Rest-mass energy: | $\bar{E}=\bar{m} c=l_{p} \frac{l_{p}}{\lambda}$ | Momentum operator on: | Comments : |
| Scratching the surface CST | Wave equations: | de Broglie momentum | Relativistic |
| CST Klein-Gordon (Spin 0) | $\frac{\partial^{2} \psi}{\partial t^{2}}-\nabla^{2} \psi+\frac{\bar{m}^{2} c^{2}}{l_{p}^{4}} \psi=0$ |  |  |
| deeper level | $\frac{\partial^{2} \psi}{\partial t^{2}}-\nabla^{2} \psi+\frac{c^{2}}{\lambda^{2}} \psi=0$ | de Broglie momentum | Relativistic |
| CST Dirac (Spin 1/2) | $i l_{p}^{2} \frac{\partial \psi}{\partial t}-\left(\sum_{i=n}^{3} \alpha_{n} \mathbf{p}_{n}-\beta \bar{m} c\right) \psi=0$ |  |  |
| deeper level | $i \frac{\partial \psi}{\partial t}-\left(\frac{1}{l_{p}^{2}} \sum_{i=n}^{3} \alpha_{n} \mathbf{p}_{n}-\beta \frac{1}{\lambda}\right) \psi=0$ | Non-relativistic |  |
| CST Schrödinger | $i \frac{\partial \psi}{\partial t} \approx\left(\frac{-l_{p}^{2}}{2 \bar{m}^{2}} \nabla^{2}+\frac{\bar{m} c}{l_{p}^{2}}\right) \psi$ | de Broglie momentum |  |
| deeper level | $i \frac{\partial \psi}{\partial t} \approx\left(\frac{-\bar{\lambda}}{2} \nabla^{2}+\frac{1}{\lambda}\right) \psi$ |  | N |

Deepest level, welcome to the "bottom" of the rabbit hole!

| CST Haug-1: (Spin ?) | $i \frac{\partial \psi}{\partial t}=\left(-i \nabla+\frac{\bar{m} c}{l_{p}^{2}}\right) \psi$ | Kinetic Compton momentum | Relativistic |
| :---: | :---: | :---: | :---: |
| deeper level | $i \frac{\partial \psi}{\partial t}=\left(-i \nabla+\frac{1}{\lambda}\right) \psi$ |  |  |
| CST Haug-2: (Spin ?) | $\frac{\partial \psi}{\partial t}+\nabla \psi=0$ | Total Compton momentum | Relativistic |

Table 1: Well-known and new development in QM. CST stands for collision space-time, this is when we replace the kg mass with what we are convinced is the more complete mass $\bar{m}=\frac{l_{p}}{c} \frac{l_{p}}{\lambda}$.

We can alternatively use a mass operator instead of the Compton moment operator this gives. For some wave equations, we then need a new concept, namely kinetic mass. The kinetic mass we will define as

$$
\begin{equation*}
\mathbf{m}_{k}=m \gamma-m \tag{1}
\end{equation*}
$$

where $\gamma=1 / \sqrt{1-\frac{\mathrm{v}^{2}}{c^{2}}}$. and in terms of mass defined as collision-time, the kinetic mass will be

$$
\begin{equation*}
\overline{\mathbf{m}}_{k}=\bar{m} \gamma-\bar{m} \tag{2}
\end{equation*}
$$

Even if a new concept, a kinetic mass is simply a relativistic mass minus its rest-mass. A kinetic mass is no less real than standard momentum; it is actually more real, but possibly more of a mathematical construct than kinetic energy. Does it help us show something new? Yes and no; it can be useful to see existing equations in a different light. The kinetic mass is a vector, the total mass, which is the relativistic mass, is not necessarily always a vector, at least not under the standard mass definition because if $v=0$, we are then supposedly left with a point particle (that has an infinite or non- defined de Broglie wave). On the other hand, our new mass definition has likely also a directional extension in space, even when $v=0$, due to the fact that it is not a point particle.

| Definitions: |  |  |  |
| :---: | :---: | :---: | :---: |
| Mass: <br> Rest- mass energy: | $\begin{gathered} m=\frac{\hbar}{\lambda} \frac{1}{c} \\ E=h f=m c^{2} \end{gathered}$ | standard kg mass definition standard Joule energy def. | incomplete incomplete |
| Deep but, not deep enough | Wave equations: | operator on energy and: | Comments: |
| Haug-1: (Spin ?) deeper level | $\begin{gathered} i \frac{\partial \psi}{\partial t}=\left(-i c^{2} \nabla+\frac{m c^{2}}{\hbar)}\right) \psi \\ i \frac{\partial \psi}{\partial t}=\left(-i c^{2} \nabla+\frac{c}{\lambda}\right) \psi \end{gathered}$ | Kinetic mass | Relativistic |
| Haug-2: (Spin ?) | $\frac{\partial \psi}{\partial t}+c^{2} \nabla \psi=0$ | mass | Relativistic |

## Definitions:

| Mass: <br> Rest-mass energy: | $\bar{m}=\frac{l_{p}}{c} \frac{l_{p}}{\lambda}$ <br> $\breve{E}=\bar{m} c^{2}=l_{p} \frac{l_{p}}{\lambda} c$ | collision-time definition <br> collision-length times c | complete <br> derivative |
| :---: | :---: | :---: | :---: |
| Deeper level | Wave equations: | operator on energy and: | Comments: |
| CST Haug-1: (Spin ?) | $i \frac{\partial \psi}{\partial t}=\left(-i c^{2} \nabla+\frac{\bar{m} c^{2}}{l_{p}^{2}}\right) \psi$ | kinetic mass | Relativistic |
| deeper level | $i \frac{\partial \psi}{\partial t}=\left(-i c^{2} \nabla+\frac{c}{\lambda}\right) \psi$ |  |  |
| CST Haug-2: (Spin ?) | $\frac{\partial \psi}{\partial t}+c^{2} \nabla \psi=0$ | mass | Relativistic |

Definitions:

| Mass: <br> Rest-mass energy: | $\begin{gathered} \bar{m}=\frac{l_{p}}{c} \frac{l_{p}}{\lambda} \\ E=\bar{m} c=l_{p} \frac{l_{p}}{\lambda} \end{gathered}$ | collision-time definition collision-length | complete complete |
| :---: | :---: | :---: | :---: |
| Deepest level | Wave equations: | operator on energy and: | Comments: |
| CST Haug-1: (Spin ?) deeper level | $\begin{gathered} i \frac{\partial \psi}{\partial t}=\left(-i c \nabla+\frac{\bar{m} c}{l_{p}^{2}}\right) \psi \\ i \frac{\partial \psi}{\partial t}=\left(-i c \nabla+\frac{1}{\lambda}\right) \psi \end{gathered}$ | kinetic mass | Relativistic |
| CST Haug-2: (Spin ?) | $\frac{\partial \psi}{\partial t}+c \nabla \psi=0$ | mass | Relativistic |

Table 2: Working with mass operator instead of momentum operator.

Be aware when we work with energy at the deepest level, then the energy operator is identical to an operator on the Schwarzschild radius or, more precisely, half of it.

|  | standard kg mass | collision-time mass |
| :---: | :---: | :---: |
| Mass definition: | $m=\frac{\hbar}{\lambda} \frac{1}{c}$ | $m=\frac{l_{p}}{c} \frac{p}{\lambda}$ |
| Momentum operators: | $-\hbar i \nabla$ | $-l_{p}^{2} i \nabla$ |
| Energy operators: | $\hbar i \frac{\partial}{\partial t}$ | $l_{p}^{2} i \frac{\partial}{\partial t}$ |
| Mass operators: | $-\hbar i \nabla$ | $-l_{p}^{2} i \nabla$ |

Table 3: Overview of operators used. Using these operators will ensure that one gets the relativistic wave equations given, and all of them should be consistent with the relativistic energy momentum relation.

The Einstein relativistic energy momentum relation, which is linked to what we can call the de Broglie momentum, is given by

$$
\begin{equation*}
E^{2}=\mathbf{p}^{2} c^{2}+m^{2} c^{4} \tag{3}
\end{equation*}
$$

where $\mathbf{p}=m \mathbf{v} \gamma$, and the relativistic energy Compton momentum relation is given by

$$
\begin{equation*}
E=\mathbf{p}_{t} c=\mathbf{p}_{k} c+m c^{2} \tag{4}
\end{equation*}
$$

where $\mathbf{p}_{t}=m c \gamma$ and $\mathbf{p}_{k}=m c \gamma-m c$
The two relativistic energy momentum relations are two sides of the same coin, as is evident if we look closely at it

$$
\begin{align*}
E^{2} & =p^{2} c^{2}+m^{2} c^{4} \\
E^{2} & =p_{0}^{2} c^{2} \gamma^{2}+m^{2} c^{4} \\
E^{2} & =m^{2} v^{2} c^{2} \gamma^{2}+m^{2} c^{4} \\
E^{2} & =m^{2} c^{4} v^{2} / c^{2} \gamma^{2}+m^{2} c^{4} \\
E^{2}= & m^{2} c^{4}\left(\frac{v^{2}}{c^{2}}-1\right) \gamma^{2}+m^{2} c^{4} \gamma^{2}+m^{2} c^{4} \\
E^{2}= & \frac{m^{2} c^{4}\left(\frac{v^{2}}{c^{2}}-1\right)}{1-\frac{v^{2}}{c^{2}}}+m^{2} c^{4} \gamma^{2}+m^{2} c^{4} \\
& -1 \times m^{2} c^{4}\left(\frac{v^{2}}{c^{2}}-1\right) \\
E^{2}= & -1 \times\left(1-\frac{v^{2}}{c^{2}}\right) \\
= & m^{2} c^{4} \gamma^{2}+m^{2} c^{4} \\
E^{2}= & -m^{2} c^{4}+m^{2} c^{4} \gamma^{2}+m^{2} c^{4} \\
E^{2}= & m^{2} c^{4} \gamma^{2}  \tag{5}\\
E & =m c^{2} \gamma \\
E & =p_{t} c=p_{k} c+m c^{2}
\end{align*}
$$

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