Supplement Formulas of the Fine-structure Constant α, New Formulas of Euler Number e and Their Relationships with Nuclides

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to the paper "Chen's Formulas of the Fine-structure Constant" (viXra:2002.0203) for giving some supplements. In the previous paper, many formulas of the fine-structure constant α based on the most important key number 112 had been given. In this paper, some new formulas of α based on a subsequent key number 173 were deduced, some formulas of α were expressed with large integers, some new formulas of Euler number e and their relationships with nuclides were given, the relationships of some formulas of 2π with nuclides were revealed, the relationships between some constants (e, γ, γ_c, γ_g and γ_{cg}) and nuclides were disclosed, a picture showing the unification of mathematics and physics through α was depicted, the most important formula of the speed of light in atomic unites c_{au} was revised to be more reasonable, a Fibonacci sequence containing 173 and its relationships with nuclides were proposed, the meanings of the numerical values of α and c_{au} were discovered, and some formulas of α based on 137, 83, 83^2 and 112×173 were presented.

Keywords: formulas of the fine-structure constant α, formulas of Euler number e; the speed of light; the unification of mathematics and physics; relationships; nuclides.

1. Construct formulas of α with 173 instead of 112

Referring to $_{112}Cn_{173}$, $_{137}Fy_{209}$ and $_{173}Ch_{261,262}$ presented in the previous paper "Chen's Formulas of the Fine-structure Constant" (viXra:2002.0203)[1], the natural end of elements is $_{112}Cn$, the Fynmann end of elements is $_{137}Fy$, and the end of ideal extended elements is $_{173}$Ch. We had already constructed formulas of α with 112 which is double of the most stable number 56 in the world of nuclides according to our Chirality and Poetry Model of Atomic Neclei[2], and 173 seems to be a subsequent stable number connected to 112, so it should be possible to construct some reasonable (but may less important) formulas of α with 173 instead of 112 as follows. And the factors in these formulas are supposed to relate to nuclides as in the previous paper[1].

$$\alpha_{1-1(173)}=\frac{8}{e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{20}{19})^{39}}}\frac{1}{173+\dfrac{1}{31}-\dfrac{1}{10\cdot 227-\dfrac{103}{7\cdot 193}\text{ or }\dfrac{60}{6\cdot 131+1}\text{ or }\dfrac{112}{13\cdot 113}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{39}_{19}K_{20}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{89}_{39}K_{50}$ ${}^{103}_{45}Rh_{58}$ ${}^{131}_{54}Xe_{77}$

${}^{191,193}_{77}Ir_{114,116}$ ${}^{171,173}_{70}Yb_{101,103}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{*}_{173}$ ${}^{2\cdot173}_{137}Fy^{ie}_{11\cdot19}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-1(173)-Wallis}=\frac{2}{(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{58}{59}\frac{4\cdot3\cdot5}{2\cdot29+1})}\frac{1}{173+\dfrac{1}{31}-\dfrac{1}{8\cdot11\cdot23+\dfrac{171}{2\cdot112}}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{100}_{44}Ru_{56}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{170-174}_{70}Yb_{100-104}$ ${}^{226}_{88}Ra^{*}_{138}$

${}^{283,15\cdot19}_{112}Cn^{*}_{171,173}$ ${}^{284,22\cdot13}_{113}Nh^{*}_{171,173}$ ${}^{2\cdot173,12\cdot29}_{137,138}Fy^{ie}_{11\cdot19,210}$ ${}^{33\cdot13,430}_{171}Ch^{ie}_{258,260}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

$$\alpha_{1-1(173)-GL}=\frac{1}{1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot19+1}}\frac{1}{173+\dfrac{1}{14}-\dfrac{1}{4\cdot61}+\dfrac{1}{2\cdot173\cdot191+\dfrac{1}{50}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{61}_{28}Ni_{33}$ ${}^{88}_{38}Sr_{50}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{131}_{54}Xe_{77}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{170,171,172,173,174,176}_{70}Yb_{100,101,102,103,104,106}$

${}^{191,193}_{77}Ir_{114,116}$ ${}^{4\cdot61}_{94}Pu^{*}_{150}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{*}_{173}$ ${}^{2\cdot173}_{137}Fy^{ie}_{11\cdot19}$ ${}^{402}_{158}Ch^{ie}_{4\cdot61}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-1(173)-NC}=\frac{8}{(2\pi)_{NC-1}}\frac{1}{173+\dfrac{1}{10}-\dfrac{1}{17\cdot31}+\dfrac{1}{2\cdot47\cdot53\cdot127}}=1/137.035999037435$$

$$=\frac{8}{6+\frac{1}{3}}\frac{1}{173+\dfrac{1}{10}-\dfrac{9\cdot19\cdot(3\cdot11\cdot112+1)}{2\cdot17\cdot31\cdot47\cdot53\cdot127}}=\frac{24}{19}\frac{1}{173+\dfrac{1}{10}-\dfrac{171\cdot(2\cdot43^2-1)}{2\cdot17\cdot31\cdot47\cdot53\cdot127}}$$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{47}_{22}Ti_{25}$ ${}^{53}_{24}Cr_{29}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{77}_{34}Se_{43}$

${}^{83}_{36}Kr_{47}$ ${}^{97}_{42}Mo_{53}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{127}_{53}I_{74}$ ${}^{131}_{54}Xe_{77}$ ${}^{148}_{62}Sm_{86}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$

${}^{170,171,172,173,174,176}_{70}Yb_{100,101,102,103,104,106}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{11\cdot19,210}_{83}Bi^{*}_{126,127}$ ${}^{6\cdot37}_{86}Rn^{*}_{136}$ ${}^{283,15\cdot19}_{112}Cn^{*}_{9\cdot19,173}$

${}^{284,15\cdot19,22\cdot13}_{113}Nh^{*}_{9\cdot19,4\cdot43,173}$ ${}^{6\cdot53,11\cdot29,320}_{127}Ch^{ie}_{191,192,193}$ ${}^{33\cdot13,430}_{171}Ch^{ie}_{6\cdot43,7\cdot37}$ ${}^{430,432}_{172}Ch^{ie}_{6\cdot43,260}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

Note: $(2\pi)_{NC-k}=6+\sum_{n=1}^{k}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}}$

$$\alpha_{1-15(173)}=\frac{7\cdot 17}{3\cdot 5\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot(6\cdot 47-1)}{6\cdot 11\cdot 17+1})^{3\cdot 7\cdot 107}}}\frac{1}{173+\frac{1}{20\cdot(138+1)\cdot 227}}$$

$=1/137.035999037435$

${}^{14,15}_{7}N_{7,8}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{45}_{21}Sc_{24}$ ${}^{47}_{22}Ti_{25}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{118,7\cdot 17}_{50}Sn_{68,69}$
${}^{121,123}_{51}Sb_{70,72}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{170,172,173,176}_{70}Yb_{100,102,103,106}$ ${}^{179}_{72}Hf_{107}$ ${}^{199}_{80}Hg_{7\cdot 17}$
${}^{227}_{89}Ac^*_{138}$ ${}^{15\cdot 19}_{112}Cn^*_{173}$ ${}^{22\cdot 13}_{113}Nh^*_{173}$ ${}^{2\cdot 149}_{119}Ch^{ie}_{179}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{348}_{139}Ch^{ie}_{209}$ ${}^{8\cdot 47}_{149}Ch^{ie}_{227}$

$$\alpha_{1-15(173)-Wallis}=\frac{7\cdot 17}{4\cdot 3\cdot 5\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{3370}{3371}\frac{4\cdot 3\cdot 281}{2\cdot 5\cdot 337+1})}\frac{1}{173+\frac{1}{11\cdot 103\cdot(18\cdot 31-1)+\frac{7}{10}}}$$

$=1/137.035999037435$

${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{170,171,172,173}_{70}Yb_{100,101,102,103}$ ${}^{281}_{110}Ds^*_{171}$ ${}^{281}_{111}Rg^*_{170}$ ${}^{283,15\cdot 19}_{112}Cn^*_{9\cdot 19,173}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,2\cdot 131}$

$$\alpha_{1-15(173)-GL}=\frac{7\cdot 17}{8\cdot 3\cdot 5\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 29\cdot 37+1})}\frac{1}{173+\frac{1}{2\cdot 19\cdot 23\cdot 157-\frac{10}{13}}}$$

$=1/137.035999037435$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{80}_{34}Se_{46}$ ${}^{7\cdot 17}_{50}Sn_{69}$ ${}^{199}_{80}Hg_{7\cdot 17}$ ${}^{257}_{100}Fm^*_{157}$

$$\alpha_{1-15(173)-NC}=\frac{7\cdot 17}{3\cdot 5\cdot(2\pi)_{NC-9}}\frac{1}{173+\frac{1}{2\cdot 41}-\frac{1}{2\cdot 9\cdot(2\cdot 199-1)-\frac{29}{139}}}$$

$=1/137.035999037435$

${}^{63,65}_{29}Cu_{34,36}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{138,140}_{58}Ce_{80,82}$ ${}^{199}_{80}Hg_{7\cdot 17}$ ${}^{12\cdot 29}_{139}Ch^{ie}_{209}$

$$\alpha_{1-59(173)}=\frac{4\cdot 9\cdot 13}{59\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot(8\cdot 13\cdot 137+1)}{3\cdot 7\cdot 23\cdot 59})^{5\cdot(2\cdot 41\cdot 139+1)}}}\frac{1}{173+\frac{2\cdot 9\cdot 11\cdot 59}{5\cdot 10^{11}}}$$

$=1/137.035999037435$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{93}_{41}Nb_{52}$ ${}^{105}_{46}Pd_{59}$ ${}^{118}_{50}Sn_{68}$ ${}^{136137,138}_{56}Ba_{80,81,82}$
${}^{138,139}_{57}La_{81,82}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{196}_{78}Pt_{118}$ ${}^{15\cdot 19}_{112}Cn^*_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210}$

$$\alpha_{1-59(173)-Wallis}=\frac{9\cdot 13}{59\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{85482}{85483}\frac{4\cdot 7\cdot 43\cdot 71}{2\cdot 27\cdot(2\cdot 7\cdot 113+1)+1})}\frac{1}{173+\frac{9\cdot 7\cdot 71}{4\cdot 10^{11}}}$$

$=1/137.035999037435$

${}^{27}_{13}Al_{14}$ ${}^{59}_{27}Co_{32}$ ${}^{171,172,173,174}_{70}Yb_{101,102,103,104}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{4\cdot 71,15\cdot 19,22\cdot 13}_{113}Nh^*_{9\cdot 19,4\cdot 43,173}$ ${}^{430,16\cdot 27}_{172}Ch^{ie}_{6\cdot 43,260}$

$$\alpha_{1-59(173)-GL}=\frac{9\cdot13}{2\cdot59\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot4\cdot3\cdot5\cdot(4\cdot227-1)+1})}\frac{1}{173+\frac{9\cdot79}{4\cdot10^{10}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{59}_{27}Co_{32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{89}_{39}Y_{50}$ ${}^{117,118,119}_{50}Sn_{67,68,69}$ ${}^{3\cdot47}_{59}\mathrm{Pr}_{82}$ ${}^{195,196,198}_{78}Pt_{117,118,120}$

${}^{171,172,173,174}_{70}Yb_{101,102,103,104}$ ${}^{197}_{79}Au_{2\cdot59}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{284,15\cdot19,22\cdot13}_{113}Nh^{*}_{9\cdot19,172,173}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$

$$\alpha_{1-59(173)-NC}=\frac{4\cdot9\cdot13}{59\cdot(2\pi)_{NC-23}}\frac{1}{173+\frac{1}{32\cdot9\cdot(10\cdot19+1)-\frac{29}{2\cdot17}}}=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{283,15\cdot19}_{112}Cn^{*}_{9\cdot19,173}$ ${}^{284,15\cdot19,22\cdot13}_{113}Nh^{*}_{9\cdot19,4\cdot43,173}$

${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{33\cdot13,430}_{171}Ch^{ie}_{6\cdot43,7\cdot37}$ ${}^{430,16\cdot27}_{172}Ch^{ie}_{258,260}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-1(173)}=\frac{e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\frac{e^{2}}{(\frac{4}{3})^{7}}\cdots\frac{e^{2}}{(\frac{37}{36})^{73}}}{5}\frac{1}{173-\frac{1}{5\cdot11}+\frac{1}{2\cdot7\cdot(2\cdot17^{2}-1)+\frac{2\cdot47}{3\cdot103}\text{ or }\frac{5\cdot23}{14\cdot27}}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47}_{22}Ti_{25}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{83,84}_{36}Kr_{47,48}$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{115,118,119,120,122,124}_{50}Sn_{65,68,69,70,72,74}$ ${}^{131}_{54}Xe_{77}$ ${}^{7\cdot19}_{55}Cs_{78}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$

${}^{170,171,172,173,176}_{70}Yb_{100,101,102,103,106}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{11\cdot19}_{83}Bi^{*}_{126}$ ${}^{262}_{103}Lr_{159}$

${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{288,17^{2}}_{115}Mc^{ie}_{173,174}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,11\cdot19,210}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-1(173)-Wallis}=\frac{4\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{108}{109}\frac{2\cdot5\cdot11}{2\cdot2\cdot27+1})}{5}\frac{1}{173-\frac{1}{7\cdot13}+\frac{1}{4\cdot25\cdot19\cdot23-\frac{10}{71}}}$$

$$=1/137.035999111818$$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{91}_{40}Zr_{51}$

${}^{99,100}_{44}Ru_{55,56}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{7\cdot19}_{55}Cs^{ie}_{78}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$ ${}^{169}_{69}Tm_{100}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{21\cdot11}_{91}Pa^{*}_{140}$

${}^{257}_{100}Fm^{*}_{157}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{4\cdot71,15\cdot19,22\cdot13}_{113}Nh^{ie}_{9\cdot19,4\cdot43,173}$ ${}^{6\cdot71}_{169}Ch^{ie}_{257}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-1(173)-GL}=\frac{8\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot4\cdot17+1})}{5}\frac{1}{173-\frac{1}{128\cdot7}+\frac{1}{5\cdot11^{2}\cdot19\cdot73}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{60,62}_{28}Ni_{32,34}$ ${}^{73}_{32}Ge_{41}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{125}_{52}Te_{73}$ ${}^{166,167,168}_{68}Er_{98,99,100}$

${}^{180,181}_{73}Ta_{107,108}$ ${}^{11\cdot19}_{83}Bi^{*}_{126}$ ${}^{11\cdot19}_{84}Po^{*}_{125}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$

$$\alpha_{2-1(173)-NC} = \frac{(2\pi)_{NC-3}}{5}\frac{1}{173-1+\frac{1}{2}-\frac{1}{10}+\frac{1}{2\cdot 5\cdot 23}-\frac{1}{4\cdot 9\cdot 7\cdot(400+1)+\frac{1}{10}}}$$

$=1/137.035999111818$

${}^{19}_{9}F_{10}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{82,93,84,86}_{36}Kr_{46,47,48,50}$ ${}^{125,126}_{52}Te_{73,74}$ ${}^{169}_{69}Tm_{100}$ ${}^{208}_{82}Pt_{126}$ ${}^{11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$
${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{288,289}_{115}Cn^{*}_{173,174}$

$$\alpha_{2-44(173)} = \frac{4\cdot 11\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\frac{e^{2}}{(\frac{4}{3})^{7}}\cdots\frac{e^{2}}{(\frac{3\cdot(8\cdot 131+1)}{2\cdot 11^{2}\cdot 13})^{7\cdot 29\cdot 31}}}{3\cdot 73}\frac{1}{173-\frac{1}{11\cdot 29\cdot(62\cdot 59+1)+\frac{5}{6}}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{100}_{44}Ru_{56}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{125}_{52}Te_{73}$ ${}^{131}_{54}Xe_{77}$ ${}^{171,173,174}_{70}Yb_{101,103,104}$ ${}^{180,181}_{73}Ta_{107,108}$
${}^{201}_{80}Hg_{121}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{302}_{121}Ch^{ie}_{181}$ ${}^{332}_{131}Ch^{ie}_{201}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210}$ ${}^{33\cdot 13}_{171}Ch^{ie}_{258}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,2\cdot 131}$

$$\alpha_{2-44(173)-Wallis} = \frac{16\cdot 11\cdot(2\frac{2}{3}\frac{4}{3}\cdots\frac{9440}{9441}\frac{9442}{2\cdot 16\cdot 5\cdot 59+1})}{3\cdot 73}\frac{1}{173-\frac{1}{59\cdot(2\cdot 3\cdot 7\cdot 13\cdot 17+1)-\frac{4}{11}}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{73}_{32}Ge_{41}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{107,109}_{47}Ag_{60,62}$
${}^{118,119}_{50}Sn_{68,69}$ ${}^{125}_{52}Te_{73}$ ${}^{3\cdot 47}_{59}Pr_{82}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{9\cdot 22,199}_{80}Hg_{2\cdot 59,7\cdot 17}$

$$\alpha_{2-44(173)-GL} = \frac{32\cdot 11\cdot(1-\frac{1}{3}+\frac{1}{5}-\cdots+\frac{1}{2\cdot 10\cdot(14\cdot 43-1)+1})}{3\cdot 73}\frac{1}{173-\frac{1}{(8\cdot 61-1)\cdot(42\cdot 23-1)}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{46,47,50}_{22}Ti_{24,25,28}$ ${}^{61}_{28}Ni_{33}$ ${}^{73}_{32}Ge_{41}$ ${}^{92,96,97,98,100}_{42}Mo_{50,54,55,56,58}$ ${}^{98,99}_{43}Tc^{*}_{55,56}$
${}^{99,100}_{44}Ru_{55,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{125}_{52}Te_{73}$ ${}^{138}_{56}Ba_{82}$ ${}^{2\cdot 73}_{61}Pm^{*}_{85}$ ${}^{169}_{69}Tm_{100}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{238}_{92}U^{*}_{2\cdot 73}$

$$\alpha_{2-44(173)-NC} = \frac{4\cdot 11\cdot(2\pi)_{11}}{3\cdot 73}\frac{1}{173-\frac{1}{7\cdot 113}+\frac{2\cdot 7\cdot 29\cdot 31\cdot 47}{25\cdot 10^{11}}} = 1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{47,49,50}_{22}Ti_{25,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{73}_{32}Ge_{41}$
${}^{99,100}_{44}Ru_{55,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{65,66}$ ${}^{125}_{52}Te_{73}$ ${}^{173}_{70}Yb_{103}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$
${}^{2\cdot 173,12\cdot 29}_{137,138}Fy^{ie}_{11\cdot 19,210}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,2\cdot 131}$

$$\alpha_{2-45(173)} = \frac{9\cdot5\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{8\cdot5\cdot27}{13\cdot83})^{17\cdot127}}}{2\cdot112}\frac{1}{173-\frac{1}{2\cdot151\cdot173-\frac{9}{8\cdot5}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{77}_{34}Se_{43}$ ${}^{83}_{36}Kr_{47}$ ${}^{90,91,92,94}_{40}Zr_{50,51,52,54}$
${}^{103}_{45}Rh_{58}$ ${}^{126,128}_{52}Te_{74,76}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{131}_{54}Xe_{77}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{173}_{70}Yb_{103}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{13\cdot19}_{96}Cm^{*}_{151}$
${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{2\cdot151}_{121}Ch^{ie}_{181}$ ${}^{318,319,320}_{127}Ch^{ie}_{191,192,193}$ ${}^{344,2\cdot173}_{136,137}Fy^{ie}_{16\cdot13,11\cdot19}$ ${}^{380}_{151}Ch^{ie}_{229}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-45(173)-Wallis} = \frac{9\cdot5\cdot(2\frac{2}{3}\frac{4}{3}\cdots\frac{3236}{3237}\frac{2\cdot(20\cdot81-1)}{2\cdot2\cdot(8\cdot101+1)+1})}{8\cdot7}\frac{1}{173-\frac{1}{8\cdot101\cdot(4\cdot9\cdot13-1)-\frac{2}{15}}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{45}_{21}Sc_{24}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{99}_{43}Tc^{*}_{56}$
${}^{100,101}_{44}Ru_{56,57}$ ${}^{103}_{45}Rh_{58}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{171,172,173}_{70}Yb_{101,102,103}$ ${}^{6\cdot43}_{101}Md^{*}_{157}$ ${}^{283,15\cdot19}_{112}Cn^{*}_{171,173}$
${}^{284,15\cdot19,22\cdot13}_{113}Nh^{ie}_{171,172,173}$ ${}^{8\cdot43,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{33\cdot13,430}_{171}Ch^{ie}_{6\cdot43,7\cdot37}$ ${}^{430,16\cdot27}_{172}Ch^{ie}_{6\cdot43,260}$
${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-45(173)-GL} = \frac{9\cdot5\cdot(1-\frac{1}{3}+\frac{1}{5}-\cdots+\frac{1}{2\cdot2\cdot(10\cdot103+1)+1})}{4\cdot7}\frac{1}{173-\frac{1}{4\cdot17\cdot(44\cdot13-1)-\frac{32}{3\cdot19}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{100,101}_{44}Ru_{56,57}$
${}^{103}_{45}Rh_{58}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{168,170}_{68}Er_{100,102}$ ${}^{170,171,172,173,174,176}_{70}Yb_{100,101,102,103,104,106}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$
${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{8\cdot43,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

$$\alpha_{2-45(173)-NC} = \frac{9\cdot5\cdot(2\pi)_7}{2\cdot112}\frac{1}{173-\frac{1}{3\cdot17\cdot59+\frac{121}{320}}} = \frac{9\cdot5\cdot(2\pi)_7}{2\cdot112}\frac{1}{173-\frac{1}{2\cdot5\cdot7\cdot43-\frac{199}{320}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{85,87}_{37}Rb_{48,59}$ ${}^{91}_{40}Zr_{51}$ ${}^{99}_{43}Tc^{*}_{56}$
${}^{105}_{46}Pd_{59}$ ${}^{121,123}_{51}Sb_{70,71}$ ${}^{112}_{48}Cd_{64}$ ${}^{185,11\cdot17}_{75}\mathrm{Re}_{110,112}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{173}_{70}Yb_{103}$ ${}^{199,200,201}_{80}Hg_{7\cdot19,120,121}$
${}^{222}_{86}Rn^{*}_{136}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{8\cdot43,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

$$\alpha_{2-87(173)}=\frac{3\cdot 29\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\,\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{341846}{341845})^{683691}}}{14\cdot 31-1}\,\frac{1}{173-\dfrac{14560}{10^{13}}}$$

$$=\frac{3\cdot 29\cdot e^2 e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\,\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{2\cdot 59\cdot(2\cdot 9\cdot 7\cdot 23-1)}{5\cdot 7\cdot(24\cdot 11\cdot 37-1)})^{3\cdot(4\cdot 163+1)\cdot(12\cdot 29+1)}}}{2\cdot 7\cdot 31-1}\,\frac{1}{173-\dfrac{112\cdot 13}{10^{12}}}$$

$$=1/137.035999111818$$

${}^{11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Cr_{32}$
${}^{50,52,,53,54}_{24}Cr_{26,28,29,30}$ ${}^{54,56,57,58}_{26}Pd_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{67}_{30}Zn_{37}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{85,87}_{37}Rb_{48,50}$
${}^{99,100}_{44}Ru_{55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{131}_{54}Xe_{77}$ ${}^{149}_{62}Sm_{87}$ ${}^{163}_{66}Dy_{97}$ ${}^{4\cdot 67}_{105}Db^{*}_{163}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

$$\alpha_{2-87(173)-Wallis}=\frac{4\cdot 87\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{1044088}{1044089}\frac{1044090}{1044089+1})}{433}\,\frac{1}{173-\dfrac{434\text{ or }435}{10^{13}}}$$

$$=\frac{4\cdot 3\cdot 29\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{1044088}{1044089}\frac{2\cdot 81\cdot 5\cdot(23\cdot 56+1)}{2\cdot 4\cdot 19\cdot(4\cdot 17\cdot 101+1)})}{2\cdot 7\cdot 31-1}\,\frac{1}{173-\dfrac{7\cdot 31}{5\cdot 10^{12}}\text{ or }\dfrac{3\cdot 29}{2\cdot 10^{12}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,57,58}_{26}Pd_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{97,99}_{43}Tc^{*}_{54,56}$ ${}^{79,81}_{45}Br_{44,46}$
${}^{100,101,102}_{44}Ru_{56,57,58}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{149}_{62}Sm_{87}$ ${}^{157}_{64}Gd_{93}$ ${}^{168,170}_{68}Er_{100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{9\cdot 19,172,173}_{70}Yb_{101,102,103}$
${}^{7\cdot 29,205}_{81}Tl_{122,124}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{258}_{101}Md^{*}_{157}$ ${}^{7\cdot 37}_{102}No^{*}_{157}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{284,286}_{113}Nh^{ie}_{9\cdot 19,173}$
${}^{288,17^2}_{115}Mc^{ie}_{173,2\cdot 87}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,11\cdot 19,210}$ ${}^{429,430}_{171}Ch^{ie}_{6\cdot 43,7\cdot 37}$ ${}^{430,16\cdot 27}_{172}Ch^{ie}_{6\cdot 43,260}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

$$\alpha_{2-87(173)-GL}=\frac{8\cdot 3\cdot 29\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 16\cdot(72\cdot(2\cdot 17^2-1)+1})}{2\cdot 7\cdot 31-1}\,\frac{1}{173-\dfrac{137}{125\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{149}_{62}Sm_{87}$ ${}^{223}_{87}Fr^{*}_{136}$
${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{288,17^2}_{115}Mc^{ie}_{173,2\cdot 87}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

$$\alpha_{2-87(173)-NC}=\frac{3\cdot 29\cdot(2\pi)_{NC-55}}{2\cdot 7\cdot 31-1}\,\frac{1}{173-\dfrac{1}{2\cdot 3\cdot 7\cdot 29\cdot 61+\dfrac{7}{4\cdot 3}}}=1/137.035999111818$$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{61}_{28}Ni_{33}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{87}_{38}Sr_{49}$ ${}^{107.109}_{47}Ag_{60,62}$ ${}^{97}_{42}Mo_{55}$ ${}^{133}_{55}Cs_{78}$
${}^{5\cdot 29}_{61}Pm^{*}_{84}$ ${}^{149}_{62}Sm_{87}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-89(173)}=\frac{89\cdot e^{2}\dfrac{e^{2}}{(\frac{2}{1})^{3}}\dfrac{e^{2}}{(\frac{3}{2})^{5}}\dfrac{e^{2}}{(\frac{4}{3})^{7}}\cdots\dfrac{e^{2}}{(\frac{3\cdot13\cdot41}{2\cdot17\cdot47})^{23\cdot(138+1)}}}{2\cdot13\cdot17+1}\;\frac{1}{173-\dfrac{1}{7\cdot17\cdot(4\cdot11\cdot29+1)+\dfrac{2}{25}}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,50}_{22}Ti_{24,25,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{80,82}_{34}Se_{46,48}$ ${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$

${}^{89}_{39}Y_{50}$ ${}^{93}_{41}Nb_{52}$ ${}^{100}_{44}Ru_{56}$ ${}^{116-120}_{50}Sn_{66-70}$ ${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{151,153}_{63}Eu_{88,90}$

${}^{170-174,176}_{70}Yb_{100-104,106}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{8\cdot43,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{12\cdot29}_{139}Ch^{ie}_{209}$

${}^{33\cdot13}_{171}Ch^{ie}_{258}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-89(173)-Wallis}=\frac{4\cdot89\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{4794}{4795}\frac{4\cdot11\cdot109}{2\cdot3\cdot17\cdot47+1})}{4\cdot3\cdot37-1}\;\frac{1}{173-\dfrac{1}{2\cdot49\cdot(32\cdot113+1)+\dfrac{7}{8}}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{74}_{32}Ge_{42}$ ${}^{86,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{100}_{44}Ru_{56}$ ${}^{107,109}_{47}Ag_{60,62}$

${}^{113,115}_{49}In_{65,66}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{183}_{74}W_{109}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{2\cdot173}_{137}Fy^{ie}_{11\cdot19}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$

$$\alpha_{2-89(173)-GL}=\frac{8\cdot89\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot4\cdot7\cdot109+1})}{4\cdot3\cdot37-1}\;\frac{1}{173-\dfrac{1}{9\cdot(4\cdot5\cdot49\cdot47+1)+\dfrac{3}{8}}}$$

$=1/137.035999111818$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{58,60,64}_{28}Ni_{30,32,36}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{103}_{45}Rh_{58}$

${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{65,66}$ ${}^{137}_{56}Ba_{81}$ ${}^{173}_{70}Yb_{103}$ ${}^{183}_{74}W_{109}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{6\cdot37}_{86}Rn^{*}_{136}$ ${}^{227}_{89}Ac^{*}_{138}$

${}^{262}_{103}Lr^{*}_{159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{288,289}_{115}Mc^{ie}_{173,174}$ ${}^{2\cdot173}_{137}Fy^{ie}_{209}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

$$\alpha_{2-89(173)-NC}=\frac{89\cdot(2\pi)_{NC-9}}{4\cdot3\cdot37-1}\;\frac{1}{173-\dfrac{1}{9\cdot25}+\dfrac{1}{13\cdot(2\cdot3\cdot7\cdot19^{2}-1)+\dfrac{6}{17}}}$$

$=1/137.035999111818$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{40,41,42}_{19}K_{20,21,22}$ ${}^{45}_{21}Sc_{24}$ ${}^{55}_{25}Mn_{30}$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{103}_{45}Rh_{58}$ ${}^{173}_{70}Yb_{103}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{288,289}_{115}Mc^{ie}_{173,174}$

${}^{2\cdot173}_{137}Fy^{ie}_{209}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

$$\alpha_{2-131(173)} = \frac{131\cdot e^2 \dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{4\cdot 3\cdot 13\cdot 59}{2\cdot 43\cdot 107+1})^{79\cdot(8\cdot 29+1)}}}{4\cdot 163}\frac{1}{173-\dfrac{4\cdot 59\cdot(2\cdot 11\cdot 19+1)}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{47}_{22}Ti_{25}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{116,117,118}_{50}Sn_{66,67,68}$ ${}^{131}_{54}Xe_{77}$

${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{163}_{66}Dy_{97}$ ${}^{2\cdot 97,195,196}_{78}Pt_{116,117,118}$ ${}^{197}_{79}Au_{118}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{270}_{107}Ac^{*}_{163}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{293}_{116}Lv^{ie}_{3\cdot 59}$

${}^{294}_{118}Og^{ie}_{16\cdot 11}$ ${}^{4\cdot 83}_{131}Ch^{ie}_{201}$ ${}^{8\cdot 43,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$ ${}^{33\cdot 13,430}_{171}Ch^{ie}_{6\cdot 43,260}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,2\cdot 131}$

$$\alpha_{2-131(173)-Wallis} = \frac{131\cdot(2\dfrac{2}{3}\dfrac{4}{3}\cdots\dfrac{27610}{27611}\dfrac{4\cdot 9\cdot 13\cdot 59}{2\cdot 2\cdot 5\cdot 11\cdot(4\cdot 9\cdot 7-1)+1})}{163}\frac{1}{173-\dfrac{8\cdot 67\cdot 157}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{59}_{27}Cr_{32}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{105}_{46}Pd_{59}$ ${}^{9\cdot 13,118}_{50}Sn_{67,68}$

${}^{131}_{54}Xe_{77}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{157}_{64}Gd_{93}$ ${}^{163}_{66}Dy_{97}$ ${}^{165}_{67}Ho_{98}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{4\cdot 67}_{105}Db^{*}_{163}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$

${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{294}_{118}Og^{ie}_{16\cdot 11}$ ${}^{4\cdot 83}_{131}Ch^{ie}_{3\cdot 67}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$ ${}^{435}_{173}Ch^{ie}_{2\cdot 131}$

$$\alpha_{2-131(173)-GL} = \frac{2\cdot 131\cdot(1-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{7}+\cdots+\dfrac{1}{2\cdot 2\cdot 11\cdot 17\cdot 47+1})}{163}\frac{1}{173-\dfrac{11\cdot 67\cdot 97}{4\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47}_{22}Ti_{25}$ ${}^{67}_{30}Zn_{37}$ ${}^{97}_{42}Mo_{55}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{131}_{54}Xe_{77}$ ${}^{163}_{66}Dy_{97}$ ${}^{15\cdot 11}_{67}Ho_{98}$ ${}^{247}_{97}Bk^{*}_{150}$

${}^{4\cdot 67}_{105}Db^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{4\cdot 83}_{131}Ch^{ie}_{3\cdot 67}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210}$ ${}^{410}_{163}Ch^{ie}_{247}$ ${}^{435}_{173}Ch^{ie}_{2\cdot 131}$

$$\alpha_{2-131(173)-NC} = \frac{131\cdot(2\pi)_{NC-17}}{4\cdot 163}\frac{1}{173-\dfrac{1}{20\cdot 64}+\dfrac{49\cdot(12\cdot 269+1)}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{113,115}_{49}In_{65,66}$ ${}^{131}_{54}Xe_{77}$ ${}^{163}_{66}Dy_{97}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{435}_{173}Ch^{ie}_{2\cdot 131}$

$$\alpha_{2-175(173)} = \frac{7\cdot 25\cdot e^2 \dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{4\cdot 3\cdot 11\cdot 47-1}{14\cdot(2\cdot 13\cdot 17+1)})^{15\cdot(14\cdot 59+1)}}}{13\cdot 67}\frac{1}{173-\dfrac{1}{2\cdot 167\cdot(32\cdot 3\cdot 49-1)}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$

${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{77,78}_{34}Se_{43,44}$ ${}^{79,81}_{35}Br_{44,46}$

${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{100}_{44}Ru_{56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{131,132,134}_{54}Xe_{77,78,80}$

${}^{114,115,116,117,118,119,120}_{50}Sn_{64,65,66,67,68,69,70}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{165}_{67}Ho_{98}$ ${}^{167,168}_{68}Er_{99,100}$ ${}^{169}_{69}Tm_{100}$ ${}^{175,176}_{71}Lu_{104,105}$

${}^{172,173,174,176}_{70}Yb_{102,103,104,106}$ ${}^{4\cdot 49}_{78}Pt_{118}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{6\cdot 49}_{118}Og^{ie}_{176}$

${}^{4\cdot 83}_{131}Ch^{ie}_{3\cdot 67}$ ${}^{2\cdot 167}_{132}Ch^{ie}_{202}$ ${}^{340}_{134}Ch^{ie}_{206}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$ ${}^{2\cdot 209}_{167}Ch^{ie}_{251}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-175(173)-Wallix} = \frac{4\cdot 7\cdot 25\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{18606}{19607}\frac{16\cdot(2\cdot 7\cdot 83+1)}{2\cdot 3\cdot 7\cdot(2\cdot 13\cdot 17+1)+1})}{13\cdot 67}\frac{1}{173-\dfrac{1}{5\cdot(16\cdot 9\cdot 5\cdot 17\cdot 41+1)}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{40,42,44,46,48}_{20}Ca_{20,22,24,26,28}$

${}^{45}_{21}Sc_{24}$ ${}^{48}_{22}Ti_{26}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{68,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$

${}^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{78,80,82,83,84,86}_{36}Kr_{42,44,46,47,48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{90,91,92}_{40}Zn_{50,51,52}$ ${}^{93}_{41}Nb_{52}$

${}^{103}_{45}Rh_{58}$ ${}^{115,117,118,119,120,122}_{50}Sn_{65,67,68,69,70,72}$ ${}^{129,130,131,132,134,136}_{54}Xe_{75,76,77,78,80,82}$ ${}^{134,136,137,138}_{56}Ba_{78,80,81,82}$

${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{170,172,173,174}_{70}Yb_{100,102,103,104}$ ${}^{204,206,207,208}_{82}Pb_{122,124,125,126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{262}_{103}Lr^{*}_{159}$

${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{4\cdot 83}_{131}Ch^{ie}_{3\cdot 67}$ ${}^{340}_{134}Ch^{ie}_{206}$ ${}^{344,2\cdot 173}_{136,137}Fy^{ie}_{208,209}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-175(173)-GL} = \frac{8\cdot 7\cdot 25\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot(2\cdot 9\cdot 7\cdot 47+1)+1})}{13\cdot 67}\frac{1}{173-\dfrac{1}{5\cdot 59\cdot(4\cdot 5\cdot 13\cdot 17+1)}}$$

$$=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47}_{22}Ti_{25}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{40,42,44,46,48}_{20}Ca_{20,22,24,26,28}$ ${}^{59}_{27}Co_{32}$ ${}^{115,117,118,120}_{50}Sn_{65,67,68,70}$

${}^{134,136,137,138}_{56}Ba_{78,80,81,82}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$

$$\alpha_{2-175(173)-NC} = \frac{7\cdot 25\cdot(2\pi)_{NC-15}}{13\cdot 67}\frac{1}{173-\dfrac{1}{2\cdot(3\cdot 128-1)}+\dfrac{1}{125\cdot 7\cdot 29^2}} = 1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{125,128}_{52}Te_{73,76}$ ${}^{145}_{60}Nd_{85}$

${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{7\cdot 29,205}_{81}Tl_{122,124}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{24\cdot 13}_{125}Ch^{ie}_{187}$ ${}^{326}_{128}Ch^{ie}_{198}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$

2. Formulas of $\alpha_{1\text{-}7\text{-}NC}$ and $\alpha_{2\text{-}13\text{-}NC}$ expressed with large integers

$$\alpha_{1-7-NC}=\frac{36}{7\cdot(2\pi)_{NC-3}}\frac{1}{112+\frac{1}{4\cdot 7}-\frac{1}{2\cdot 5\cdot(3\cdot 7)^2-\frac{113}{7\cdot 17}}}$$

$$=\frac{36}{7\cdot(6+\sum_{n=1}^{3}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}})}\frac{1}{112+\frac{1}{4\cdot 7}-\frac{1}{2\cdot 5\cdot(3\cdot 7)^2-\frac{113}{7\cdot 17}}}$$

$$=\frac{36}{7\cdot(6+\frac{1}{1\cdot\frac{3}{2}\cdot 2}-\frac{1}{2\cdot\frac{5}{2}\cdot 3}+\frac{1}{3\cdot\frac{7}{2}\cdot 4})}\frac{1}{112+\frac{1}{4\cdot 7}-\frac{1}{2\cdot 5\cdot(3\cdot 7)^2-\frac{113}{7\cdot 17}}}$$

$$=\frac{36}{7\cdot(6+\frac{1}{3}-\frac{1}{15}+\frac{1}{42})}\frac{1}{112+\frac{1}{4\cdot 7}-\frac{1}{2\cdot 5\cdot(3\cdot 7)^2-\frac{113}{7\cdot 17}}}$$

$$=\frac{36}{7\cdot(6+\frac{61}{210})}\frac{1}{112+\frac{1}{4\cdot 7}-\frac{1}{2\cdot 5\cdot(3\cdot 7)^2-\frac{113}{7\cdot 17}}}$$

$$=\frac{36}{7\cdot(\frac{44}{7}+\frac{1}{210})}\frac{1}{112+\frac{1}{4\cdot 7}-\frac{1}{2\cdot 5\cdot(3\cdot 7)^2-\frac{113}{7\cdot 17}}}$$

$$=\frac{36}{44+\frac{1}{30}}\frac{1}{112+\frac{1}{4\cdot 7}-\frac{7\cdot 17}{41\cdot 67\cdot 191}}$$

$$=\frac{1080}{1321}\frac{1}{112+\frac{5\cdot 11\cdot 9479}{4\cdot 7\cdot 41\cdot 67\cdot 191}}=\frac{1080}{1321}\frac{28\cdot 41\cdot 67\cdot 191}{81\cdot 89\cdot 229\cdot 997}$$

$$=\frac{2^5\cdot 5\cdot 7\cdot(2^3\cdot 5+1)(2\cdot 3\cdot 11+1)(2\cdot 5\cdot 19+1)}{3\cdot(2^3\cdot 11+1)(2^3\cdot 3\cdot 5\cdot 11+1)(4\cdot 3\cdot 19+1)(4\cdot 3\cdot 83+1)}$$

$$=1/137.035999037435$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$

${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$ ${}^{73}_{32}Ge_{41}$

${}^{79,81}_{35}Br_{44,46}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{89}_{39}Y_{50}$ ${}^{96,98,99,100,101,102,104}_{44}Ru_{52,54,55,56,57,58,60}$ ${}^{112,113}_{48}Cd_{64,65}$

${}^{113,115}_{49}In_{64,66}$ ${}^{112-120,122,124}_{50}Sn_{62-70,72,74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{165}_{67}Ho_{98}$

${}^{164,166,167,168,170}_{68}Er_{96,98,99,100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{188,9\cdot 21}_{76}Os_{112,113}$ ${}^{191,193}_{77}Ir_{114,116}$

${}^{199,200,201}_{80}Hg_{119,120,121}$ ${}^{208}_{82}Pt_{126}$ ${}^{11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$

${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{318,11\cdot 29,320}_{127}Ch^{*}_{191,192,193}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,11\cdot 19,210}$ ${}^{22\cdot 17}_{148}Ch^{*}_{226}$ ${}^{380}_{151}Ch^{*}_{229}$

$$\alpha_{2-13-NC}=\frac{13\cdot(2\pi)_{NC-5}}{100}\frac{1}{112-\frac{1}{4\cdot 9}+\frac{1}{1600}-\frac{1}{2(2\cdot 7\cdot 199\cdot 257+1)}}$$

$$=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{40,42,43,44,46,48}_{20}Ca_{20,22,23,24,26,28}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$
${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ ${}^{90}_{40}Zr_{50}$ ${}^{100}_{44}Ru_{56}$ ${}^{119}_{50}Sn_{69}$ ${}^{169}_{69}Tm_{100}$
${}^{199}_{80}Hg_{119}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{328}_{129}Ch^{ie}_{199}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$

$$(2\pi)_{NC-5}=6+\sum_{n=1}^{5}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}}=6+\frac{1}{1\cdot 3}-\frac{1}{3\cdot 5}+\frac{1}{2\cdot 3\cdot 7}-\frac{1}{2\cdot 3^2\cdot 5}+\frac{1}{3\cdot 5\cdot 11}$$

$$=6+\frac{2\cdot 3\cdot 5\cdot 7\cdot 11}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}-\frac{2\cdot 3\cdot 7\cdot 11}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}+\frac{3\cdot 5\cdot 11}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}-\frac{7\cdot 11}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}+\frac{2\cdot 3\cdot 7}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}$$

$$=6+\frac{2310-462+165-77+42}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}=6+\frac{1978}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}=6+\frac{989}{5\cdot 7\cdot 9\cdot 11}=\frac{29\cdot 751}{5\cdot 7\cdot 9\cdot 11}$$

$$\alpha_{2-13-NC}=\frac{13\cdot 29\cdot(2\cdot 3\cdot 5^3+1)}{2^2\cdot 5^3\cdot 7\cdot 9\cdot 11}\frac{1}{112-\frac{1}{4\cdot 9}+\frac{1}{1600}-\frac{1}{2(2\cdot 7\cdot 199\cdot 257+1)}}$$

$$=\frac{13\cdot 29\cdot(2\cdot 3\cdot 5^3+1)}{2^2\cdot 5^3\cdot 7\cdot 9\cdot 11}\frac{1}{112-\frac{1}{2}(\frac{1}{2\cdot 9}-\frac{1}{800}+\frac{1}{(2\cdot 7\cdot 199\cdot 257+1)})}$$

$$=\frac{13\cdot 29\cdot(2\cdot 3\cdot 5^3+1)}{2^2\cdot 3^2\cdot 5^3\cdot 7\cdot 11}\frac{1}{112-\frac{13\cdot 19\cdot(2\cdot 17^2\cdot 37\cdot 53+1)}{2^6\cdot 3^2\cdot 5^2\cdot(2\cdot 7\cdot 199\cdot 257+1)}}$$

$$=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{53}_{24}Cr_{29}$ ${}^{55}_{25}Mn_{30}$
${}^{63,65}_{29}Cu_{34,36}$ ${}^{67}_{30}Zn_{37}$ ${}^{75}_{33}As_{42}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{5\cdot 19}_{42}Mo_{53}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{127}_{53}I_{2\cdot 37}$ ${}^{7\cdot 19}_{55}Cs_{78}$ ${}^{163}_{66}Dy_{97}$
${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{199}_{80}Hg_{7\cdot 17}$ ${}^{11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{13\cdot 19}_{96}Cm^{*}_{151}$ ${}^{13\cdot 19}_{97}Bk^{*}_{50}$ ${}^{257}_{100}Fm^{*}_{157}$
${}^{269,271}_{106}Sg^{*}_{163,165}$ ${}^{17^2}_{115}Mc^{*}_{6\cdot 29}$ ${}^{24\cdot 13}_{125}Ch^{ie}_{11\cdot 17}$ ${}^{6\cdot 53,11\cdot 29,320}_{127}Ch^{ie}_{191,3\cdot 64,193}$ ${}^{328}_{129}Ch^{ie}_{199}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$ ${}^{426}_{169}Ch^{ie}_{257}$

3. Deduction of new formulas of e

Define $\infty=\lim\limits_{n\to\infty} n$

$$e=(1+\frac{1}{\infty})^{\infty}=1+\frac{\infty}{1!}\frac{1}{\infty}+\frac{\infty(\infty-1)}{2!}\frac{1}{\infty^2}+\frac{\infty(\infty-1)(\infty-2)}{3!}\frac{1}{\infty^3}+\cdots$$

$$=(1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots)+(\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+\cdots)\frac{1}{\infty}+$$

$$[\frac{1\cdot 2}{3!}+\frac{1\cdot 2+(1+2)3}{4!}+\frac{1\cdot 2+(1+2)3+(1+2+3)4}{5!}+\cdots]\frac{1}{\infty^2}+$$

$$\{\frac{1\cdot 2\cdot 3}{4!}+\frac{1\cdot 2\cdot 3+[1\cdot 2+(1+2)3]4}{5!}+\cdots\}\frac{1}{\infty^3}+\cdots$$

3-1. There are or should be some formulas as follows.

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} = 2.718281828459045\cdots$$

$$e = 2\left(\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + \cdots\right) = 2\sum_{k=2}^{\infty} \frac{\sum_{l=1}^{k-1} l}{k!} = 1 + \sum_{k=1}^{\infty} \frac{1}{k!}$$

$$e = \frac{24}{11}\left[\frac{1\cdot 2}{3!} + \frac{1\cdot 2 + (1+2)3}{4!} + \frac{1\cdot 2 + (1+2)3 + (1+2+3)4}{5!} + \cdots\right] = \frac{24}{11}\sum_{k=3}^{\infty} \frac{\sum_{l=2}^{k-1} l \sum_{m=1}^{l-1} m}{k!}$$

$$e = \frac{16}{7}\left\{\frac{1\cdot 2\cdot 3}{4!} + \frac{1\cdot 2\cdot 3 + [1\cdot 2 + (1+2)3)]4}{5!} + \cdots\right\} = \frac{16}{7}\sum_{k=4}^{\infty} \frac{\sum_{l_1=3}^{k-1} l_1 \sum_{l_2=2}^{l_1-1} l_2 \sum_{m=1}^{l_2-1} m}{k!}$$

$$\frac{1}{e} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots = 1 + \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} = 1/2.718281828459045\cdots$$

$$\frac{1}{e} = 2\left(\frac{1}{2!} - \frac{1+2}{3!} + \frac{1+2+3}{4!} - \cdots\right) = 2\sum_{k=2}^{\infty} \frac{(-1)^{k-2}\sum_{l=1}^{k-1} l}{k!} = 1 + \sum_{k=1}^{\infty} \frac{(-1)^k}{k!}$$

$$\frac{1}{e} = \frac{24}{5}\left[\frac{1\cdot 2}{3!} - \frac{1\cdot 2 + (1+2)3}{4!} + \frac{1\cdot 2 + (1+2)3 + (1+2+3)4}{5!} - \cdots\right] = \frac{24}{5}\sum_{k=3}^{\infty} \frac{(-1)^{k-3}\sum_{l=2}^{k-1} l \sum_{m=1}^{l-1} m}{k!}$$

$$\frac{1}{e} = \frac{48}{5}\left\{\frac{1\cdot 2\cdot 3}{4!} - \frac{1\cdot 2\cdot 3 + [1\cdot 2 + (1+2)3)]4}{5!} + \cdots\right\} = \frac{48}{5}\sum_{k=4}^{\infty} \frac{(-1)^{k-4}\sum_{l_1=3}^{k-1} l_1 \sum_{l_2=2}^{l_1-1} l_2 \sum_{m=1}^{l_2-1} m}{k!}$$

3-2. There are or should be some general formulas as follows.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = 1 + \sum_{k=1}^{\infty} \frac{x^k}{k!} = 1 + x\left(\frac{1}{1!} + \frac{x}{2!} + \frac{x^2}{3!} + \cdots\right) = 1 + x\sum_{k=1}^{\infty} \frac{x^{k-1}}{k!}$$

$$e^x = 2\left[\frac{1}{2!} + \frac{1+2}{3!}x + \frac{1+2+3}{4!}x^2 + \cdots\right] = 2\left(\frac{1}{2!} + \sum_{k=3}^{\infty} \frac{x^{k-2}\sum_{m=1}^{k-1} m}{k!}\right) = 2\sum_{k=2}^{\infty} \frac{x^{k-2}\sum_{m=1}^{k-1} m}{k!}$$

$$e^x = \frac{8}{x+8/3}\left[\frac{1\cdot 2}{3!} + \frac{1\cdot 2 + (1+2)3}{4!}x + \cdots\right] = \frac{8}{x+8/3}\sum_{k=3}^{\infty} \frac{x^{k-3}\sum_{l=2}^{k-1} l \sum_{m=1}^{l-1} m}{k!}$$

$$e^x = \frac{6\cdot 8}{(x+2)(x+6)}\left\{\frac{1\cdot 2\cdot 3}{4!} + \frac{1\cdot 2\cdot 3 + [1\cdot 2 + (1+2)3)]4}{5!}x + \cdots\right\}$$

$$= \frac{6\cdot 8}{(x+2)(x+6)}\sum_{k=4}^{\infty} \frac{x^{k-4}\sum_{l_1=3}^{k-1} l_1 \sum_{l_2=2}^{l_1-1} l_2 \sum_{m=1}^{l_2-1} m}{k!} \qquad (x \in \mathrm{R},\ 0^0 = 1)$$

3-3. There should be some special zero points for the above formulas as follows.

$$\sum_{k=3}^{\infty}\frac{(-\frac{8}{3})^{k-3}\sum_{l=2}^{k-1}l\sum_{m=1}^{l-1}m}{k!}=\frac{1\cdot 2}{3!}+\frac{1\cdot 2+(1+2)3}{4!}(-\frac{8}{3})+\frac{1\cdot 2+(1+2)3+(1+2+3)4}{5!}(-\frac{8}{3})^{2}+\cdots=0$$

$$\sum_{k=4}^{\infty}\frac{(-2)^{k-4}\sum_{l_1=3}^{k-1}l_1\sum_{l_2=2}^{l_1-1}l_2\sum_{m=1}^{l_2-1}m}{k!}=\frac{1\cdot 2\cdot 3}{4!}+\frac{1\cdot 2\cdot 3+[1\cdot 2+(1+2)3]4}{5!}(-2)+\frac{1\cdot 2\cdot 3+[1\cdot 2+(1+2)3)]4+[1\cdot 2+(1+2)3+(1+2+3)4]5}{6!}(-2)^{2}+\cdots=0$$

$$\sum_{k=4}^{\infty}\frac{(-6)^{k-3}\sum_{l_1=3}^{k-1}l_1\sum_{l_2=2}^{l_1-1}l_2\sum_{m=1}^{l_2-1}m}{k!}=\frac{1\cdot 2\cdot 3}{4!}+\frac{1\cdot 2\cdot 3+[1\cdot 2+(1+2)3)]4}{5!}(-6)+\frac{1\cdot 2\cdot 3+[1\cdot 2+(1+2)3)]4+[1\cdot 2+(1+2)3+(1+2+3)4]5}{6!}(-6)^{2}+\cdots=0$$

3-4. There should be some special extended formulas as follows.

$$e=\frac{5760}{2447}\sum_{k=5}^{\infty}\frac{\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}=\frac{2^{7}\cdot 3^{2}\cdot 5}{2\cdot 9\cdot 136-1}\sum_{k=5}^{\infty}\frac{\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}$$

$$\frac{1}{e}=\frac{5760}{337}\sum_{k=5}^{\infty}\frac{(-1)^{k-5}\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}=\frac{2^{7}\cdot 3^{2}\cdot 5}{3\cdot 112+1}\sum_{k=5}^{\infty}\frac{(-1)^{k-5}\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}$$

$$e=\frac{2304}{959}\sum_{k=6}^{\infty}\frac{\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}=\frac{2^{8}\cdot 3^{2}}{7\cdot 137}\sum_{k=6}^{\infty}\frac{\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}$$

$$\frac{1}{e}=\frac{3840}{137}\sum_{k=6}^{\infty}\frac{(-1)^{k-6}\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}=\frac{2^{8}\cdot 3\cdot 5}{137}\sum_{k=6}^{\infty}\frac{(-1)^{k-6}\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}$$

4. Relationships between the new formulas of e and nuclides

$$e=\frac{8\cdot 3}{11}\sum_{k=3}^{\infty}\frac{\sum_{l=2}^{k-1}l\sum_{m=1}^{l-1}m}{k!}\qquad \frac{1}{e}=\frac{8\cdot 3}{5}\sum_{k=3}^{\infty}\frac{(-1)^{k-3}\sum_{l=2}^{k-1}l\sum_{m=1}^{l-1}m}{k!}$$

$${}^{7}_{3}Li_{4}\quad {}^{9}_{4}Be_{5}\quad {}^{10,11}_{5}B_{5,6}\quad {}^{12,13}_{6}C_{6,7}\quad {}^{14,15}_{7}N_{7,8}\quad {}^{20,21,22}_{10}Ne_{10,11,12}\quad {}^{23}_{11}Na_{12}\quad {}^{24,25,26}_{12}Mg_{12,13,14}\quad {}^{48}_{22}Ti_{26}$$

$$e=\frac{16}{7}\sum_{k=4}^{\infty}\frac{\sum_{l_1=3}^{k-1} l_1\sum_{l_2=2}^{l_1-1} l_2\sum_{m=1}^{l_2-1} m}{k!}$$

$$\frac{1}{e}=\frac{16\cdot 3}{5}\sum_{k=4}^{\infty}\frac{(-1)^{k-4}\sum_{l_1=3}^{k-1} l_1\sum_{l_2=2}^{l_1-1} l_2\sum_{m=1}^{l_2-1} m}{k!}$$

$$\frac{1}{e^4}=-12\sum_{k=4}^{\infty}\frac{(-4)^{k-4}\sum_{l_1=3}^{k-1} l_1\sum_{l_2=2}^{l_1-1} l_2\sum_{m=1}^{l_2-1} m}{k!}$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{32,34,36}_{16}S_{16,18,20}$ ${}^{40,42,44,48}_{20}Ca_{20,22,24,28}$ ${}^{45}_{21}Sc_{24}$ ${}^{50,52,54}_{24}Cr_{26,28,30}$

${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{58,60,64}_{28}Ni_{30,32,36}$ ${}^{64,36,70}_{30}Zn_{34,36,40}$ ${}^{66,68,72}_{32}Ge_{34,36,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{80,84}_{36}Kr_{44,48}$

${}^{112}_{48}Cd_{64}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{200}_{80}Hg_{120}$ ${}^{285}_{112}Cn^{*}_{173}$

$$e=\frac{5760}{2447}\sum_{k=5}^{\infty}\frac{\sum_{l_1=4}^{k-1} l_1\sum_{l_2=3}^{l_1-1} l_2\sum_{l_3=2}^{l_2-1} l_3\sum_{m=1}^{l_3-1} m}{k!}=\frac{2^7\cdot 3^2\cdot 5}{2\cdot 9\cdot 136-1}\sum_{k=5}^{\infty}\frac{\sum_{l_1=4}^{k-1} l_1\sum_{l_2=3}^{l_1-1} l_2\sum_{l_3=2}^{l_2-1} l_3\sum_{m=1}^{l_3-1} m}{k!}$$

$$\frac{1}{e}=\frac{5760}{337}\sum_{k=5}^{\infty}\frac{(-1)^{k-5}\sum_{l_1=4}^{k-1} l_1\sum_{l_2=3}^{l_1-1} l_2\sum_{l_3=2}^{l_2-1} l_3\sum_{m=1}^{l_3-1} m}{k!}=\frac{2^7\cdot 3^2\cdot 5}{3\cdot 112+1}\sum_{k=5}^{\infty}\frac{(-1)^{k-5}\sum_{l_1=4}^{k-1} l_1\sum_{l_2=3}^{l_1-1} l_2\sum_{l_3=2}^{l_2-1} l_3\sum_{m=1}^{l_3-1} m}{k!}$$

${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{64,68,70}_{30}Zn_{34,38,40}$ ${}^{74,80,82}_{34}Se_{40,46,48}$

${}^{112,114,118,119,120,122}_{50}Sn_{62,64,68,69,70,72}$ ${}^{168}_{68}Er_{100}$ ${}^{112}_{48}Cd_{64}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{285}_{112}Cn^{*}_{173}$

$$e=\frac{2304}{959}\sum_{k=6}^{\infty}\frac{\sum_{l_1=5}^{k-1} l_1\sum_{l_2=4}^{l_1-1} l_2\sum_{l_3=3}^{l_2-1} l_3\sum_{l_4=2}^{l_3-1} l_4\sum_{m=1}^{l_4-1} m}{k!}$$

$$=\frac{2^8\cdot 3^2}{7\cdot 137}\sum_{k=6}^{\infty}\frac{\sum_{l_1=5}^{k-1} l_1\sum_{l_2=4}^{l_1-1} l_2\sum_{l_3=3}^{l_2-1} l_3\sum_{l_4=2}^{l_3-1} l_4\sum_{m=1}^{l_4-1} m}{k!}$$

$$\frac{1}{e}=\frac{3840}{137}\sum_{k=6}^{\infty}\frac{(-1)^{k-6}\sum_{l_1=5}^{k-1} l_1\sum_{l_2=4}^{l_1-1} l_2\sum_{l_3=3}^{l_2-1} l_3\sum_{l_4=2}^{l_3-1} l_4\sum_{m=1}^{l_4-1} m}{k!}$$

$$=\frac{2^8\cdot 3\cdot 5}{137}\sum_{k=6}^{\infty}\frac{(-1)^{k-6}\sum_{l_1=5}^{k-1} l_1\sum_{l_2=4}^{l_1-1} l_2\sum_{l_3=3}^{l_2-1} l_3\sum_{l_4=2}^{l_3-1} l_4\sum_{m=1}^{l_4-1} m}{k!}$$

${}^{83,84}_{36}Kr_{47,48}$ ${}^{112}_{48}Cd_{64}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{138}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$

${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

5. Relationships between formulas of 2π and nuclides

$$\pi = \frac{22}{7} - \int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx \qquad 2\pi = \frac{4\cdot 11}{7} - 2\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$$

${}^{14,15}_{7}N_{7,8}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{46,47,48,49,50}_{22}Sc_{24,25,26,27,28}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{77}_{34}Se_{43}$

${}^{88}_{38}Sr_{50}$ ${}^{96,98,99,100,102,104}_{44}Ru_{52,54,55,56,58,60}$ ${}^{150}_{62}Sm_{88}$ ${}^{226}_{88}Ra^{*}_{138}$

$$\pi = \frac{355}{113} - \frac{1}{3164}\int_0^1 \frac{x^8(1-x)^8(25+816x^2)}{1+x^2}dx$$

$$2\pi = \frac{2\cdot 5\cdot 71}{113} - \frac{1}{2\cdot 7\cdot 113}\int_0^1 \frac{x^8(1-x)^8(25+16\cdot 3\cdot 17x^2)}{1+x^2}dx$$

${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{31}_{15}P_{16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{55}_{25}Mn_{30}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{112,113}_{48}Cd_{64,65}$

${}^{77,78,80,82}_{34}Se_{43,44,46,48}$ ${}^{118,119}_{50}Sn_{68,69}$ ${}^{121,122}_{51}Sb_{70,72}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{175,11\cdot 16}_{71}Lu_{104,105}$

${}^{170,171,172,173,176}_{70}Yb_{100,101,102,103,106}$ ${}^{189}_{76}Os_{113}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{259}_{102}Fm^{*}_{157}$ ${}^{4\cdot 71,2\cdot 11\cdot 13}_{113}Nh^{ie}_{171,173}$ ${}^{2\cdot 11\cdot 17}_{148}Ch^{ie}_{226}$

6. Relationships between γ, e, γ_c, γ_g and γ_{cg} and nuclides

Euler-Mascheroni constant $\gamma = \lim_{N\to\infty}\left(\sum_{n=1}^{N}\frac{1}{n} - \ln n\right) = 0.5772156649\cdots$

$$\gamma = 0.5772156649\cdots \approx \frac{3\cdot 5}{2\cdot 13} \approx \frac{3\cdot 41}{71}$$

${}^{11}_{5}B_{6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{31}_{15}P_{16}$ ${}^{39}_{19}K_{20}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$

${}^{64,66}_{30}Zn_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{89}_{39}Y_{50}$ ${}^{93}_{41}Nb_{52}$ ${}^{121,3\cdot 41}_{51}Sb_{70,72}$ ${}^{3\cdot 41}_{52}Te_{71}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{18\cdot 17,4\cdot 77}_{123}Ch^{ie}_{183,185}$

$$e = 2.718281828\cdots \approx \frac{3\cdot 29}{32} \approx \frac{193}{71}$$

${}^{48}_{22}Ti_{26}$ ${}^{53}_{24}Cr_{29}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{87,84}_{36}Kr_{47,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{106,112,113,116}_{48}Cd_{58,64,65,68}$

${}^{113,115}_{49}In_{64,66}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{4\cdot 71,286}_{113}Nh^{*}_{171,173}$ ${}^{11\cdot 29,320}_{127}Ch^{ie}_{3\cdot 64,193}$ ${}^{434}_{173}Ch^{ie}_{9\cdot 29}$

$$e^2 \approx \frac{37}{5} \approx \frac{7\cdot 19}{2\cdot 9}$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{38}_{18}Ar_{20}$ ${}^{67,68}_{30}Zn_{37,38}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{84,87}_{38}Sr_{46,49}$

${}^{7\cdot 19}_{55}Cs_{78}$ ${}^{9\cdot 19}_{70}Yb_{101}$ ${}^{185,187}_{75}Tm_{110,112}$ ${}^{284,286}_{113}Nh^{ie}_{9\cdot 19,173}$ ${}^{4\cdot 77}_{123}Ch^{ie}_{185}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{3\cdot 112}_{133}Ch^{ie}_{7\cdot 29}$ ${}^{429,430}_{171}Ch^{ie}_{258,7\cdot 37}$

$$e^{\gamma} = 1.7810724\cdots \approx \frac{3\cdot 19}{32} \approx \frac{7\cdot 43}{167}$$

${}^{7}_{3}Li_{4}$ ${}^{48}_{22}Ti_{26}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{70}_{32}Ge_{38}$ ${}^{77}_{34}Se_{43}$ ${}^{84}_{36}Kr_{48}$ ${}^{86,87}_{38}Sr_{48,49}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{112,113,114}_{48}Cd_{64,65,66}$

${}^{167,168}_{68}Er_{99,100}$ ${}^{171}_{70}Yb_{101}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{284,22\cdot 13}_{113}Nh^{ie}_{9\cdot 19,173}$ ${}^{33\cdot 13,430}_{171}Ch^{ie}_{258,259}$ ${}^{22\cdot 19}_{167}Ch^{ie}_{251}$

$$e^{\gamma_c} = e^{0.0810614668\cdots} = 1.08443755\cdots \approx \frac{7\cdot 11}{71} \approx \frac{5\cdot 131}{4\cdot 151}$$

${}^{69,71}_{31}Ga_{38,40}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{77}_{34}Se_{43}$ ${}^{131}_{54}Xe_{77}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{247}_{96}Cm^{*}_{151}$

$$e^{2\gamma_c} = e^{2\times 0.0810614668\cdots} = 1.17600480\cdots \approx \frac{4\cdot 5}{17} \approx \frac{3\cdot 49}{125} \approx \frac{167}{2\cdot 71}$$

$^{35,37}_{17}Cl_{18,20}$ $^{49}_{22}Ti_{27}$ $^{68}_{30}Ga_{38}$ $^{69,71}_{31}Ga_{38,40}$ $^{85,87}_{37}Rb_{48,50}$ $^{87}_{38}Sr_{49}$ $^{113,115}_{49}In_{64,66}$ $^{118}_{50}Sn_{68}$ $^{121,123}_{51}Sb_{70,72}$

$^{125}_{52}Te_{73}$ $^{147}_{62}Sm_{85}$ $^{167,168}_{68}Er_{99,100}$ $^{175,176}_{71}Lu_{104,105}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}At^{*}_{125}$ $^{312}_{125}Ch^{ie}_{11\cdot 17}$ $^{12\cdot 31}_{147}Ch^{ie}_{225}$ $^{418}_{167}Ch^{ie}_{251}$

$$2(1-\gamma_c) = 2(1-0.0810614668\cdots) \approx \frac{4\cdot 17}{37} \qquad \text{Note: } 2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2$$

$^{35,37}_{17}Cl_{18,20}$ $^{64,67,68}_{30}Zn_{34,37,38}$ $^{74}_{34}Se_{40}$ $^{85,87}_{37}Rb_{38,40}$ $^{118}_{50}Sn_{68}$ $^{145}_{60}Nd_{85}$ $^{168}_{68}Er_{100}$ $^{185,187}_{75}Tm_{110,112}$

$^{210}_{85}At^{*}_{125}$ $^{4\cdot 77}_{123}Ch^{ie}_{185}$ $^{312}_{125}Ch^{ie}_{187}$

$$2(1-\gamma_g) = 2(1-0.7742086474\cdots) \approx \frac{2\cdot 7}{31} \qquad \text{Note: } \frac{\pi}{2} = \left(\frac{e}{e^{\gamma_g}}\right)^2$$

$^{14,15}_{7}N_{7,8}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{31}_{15}P_{16}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{62}_{28}Ni_{34}$ $^{69,71}_{31}Ga_{38,40}$ $^{147,148,150,152}_{62}Sm_{85,86,88,90}$

$$e^{2\gamma_{cg}} \approx \frac{17}{3\cdot 5} \approx \frac{127}{112} \approx \frac{144}{127}, \quad 2\gamma_{cg} = 2\times 0.0628164798\cdots \approx \frac{23}{3\cdot 61} \approx \frac{8\cdot 3}{191} \approx \frac{25}{199} \qquad \frac{\pi}{2} = \left(\frac{e^{\gamma/2}}{e^{\gamma_{cg}}}\right)^2$$

$^{50,51}_{23}V_{27,28}$ $^{53}_{24}Cr_{29}$ $^{64}_{30}Zn_{34}$ $^{83,84}_{36}Kr_{47,48}$ $^{112}_{48}Cd_{64}$ $^{127}_{53}I_{74}$ $^{144}_{60}Nd_{84}$ $^{3\cdot 61,186}_{74}W_{109,112}$ $^{185,187}_{75}Tm_{110,112}$ $^{199}_{80}Hg_{7\cdot 17}$

$^{191,193}_{77}Ir_{114,116}$ $^{209,210}_{83}Bi^{*}_{126,127}$ $^{237}_{93}Np^{*}_{144}$ $^{285}_{112}Cn^{*}_{173}$ $^{18\cdot 17}_{123}Ch^{ie}_{3\cdot 31}$ $^{6\cdot 53,11\cdot 29,320}_{127}Ch^{ie}_{191,192,193}$ $^{328}_{129}Ch^{ie}_{199}$ $^{6\cdot 61}_{144}Ch^{ie}_{222}$

7. Integrated Relationships between 2π, γ, e, γ_c, γ_g and γ_{cg} and nuclides

As we had demonstrated that 2π is directly and indirectly related to nuclides in the previous paper[1], it should be reasonable for some constants comparable to 2π such as γ, e, e^{γ}, $2(1-\gamma_c)$, $2(1-\gamma_g)$ and $2\gamma_{cg}$ to directly relate to nuclides. The above examples exhibit not only the relationships between the above constants and nuclides respectively, but also the integrated relationships between them and nuclides. For example, it seems they cooperatively define all the 4 stable isotopes of Fe as follows.

$$\gamma \approx \frac{3\cdot 5}{2\cdot 13} \quad e \approx \frac{3\cdot 29}{32} \quad e^{\gamma} \approx \frac{3\cdot 19}{32} \quad 2(1-\gamma_g) \approx \frac{2\cdot 7}{31}$$

$$^{54,56,57,58}_{26}Fe_{28,30,31,32}$$

The following is another example of this kind of integrated relationships.

$$2\pi = \frac{4\cdot 11}{7} - 2\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$$

$$2\pi = \frac{2\cdot 5\cdot 71}{113} - \frac{1}{2\cdot 7\cdot 113}\int_0^1 \frac{x^8(1-x)^8(25+16\cdot 3\cdot 17x^2)}{1+x^2}dx$$

$$2\pi \approx \frac{4\cdot 157}{100}, \quad \gamma \approx \frac{3\cdot 5}{2\cdot 13} \approx \frac{3\cdot 41}{71}, \quad e \approx \frac{3\cdot 29}{32} \approx \frac{193}{71}, \quad e^2 \approx \frac{37}{5} \approx \frac{7\cdot 19}{2\cdot 9}, \quad e^{\gamma} \approx \frac{3\cdot 19}{32} \approx \frac{7\cdot 43}{167}$$

$$e^{\gamma_c} \approx \frac{7\cdot 11}{71} \approx \frac{5\cdot 131}{4\cdot 151}, \quad e^{2\gamma_c} \approx \frac{4\cdot 5}{17} \approx \frac{3\cdot 49}{125} \approx \frac{167}{2\cdot 71}, \quad e^{2\gamma_{cg}} \approx \frac{17}{3\cdot 5} \approx \frac{127}{112} \approx \frac{127}{16\cdot 9}$$

$$2(1-\gamma_c) \approx \frac{4\cdot 17}{37}, \quad 2(1-\gamma_g) \approx \frac{2\cdot 7}{31}, \quad 2\gamma_{cg} \approx \frac{23}{3\cdot 61} \approx \frac{8\cdot 3}{191} \approx \frac{25}{199}$$

$${}^{23}_{11}Na_{12} \quad {}^{35,37}_{17}Cl_{18,20} \quad {}^{48,49}_{22}Ti_{26,27} \quad {}^{50,51}_{23}V_{27,28} \quad {}^{53}_{24}Cr_{29} \quad {}^{61}_{28}Ni_{33} \quad {}^{64}_{30}Zn_{34} \quad {}^{69,71}_{31}Ga_{38,40} \quad {}^{77}_{34}Se_{43} \quad {}^{85,87}_{37}Rb_{48,50}$$

$${}^{87}_{38}Sr_{49} \quad {}^{93}_{41}Nb_{52} \quad {}^{95}_{42}Mo_{53} \quad {}^{98,99}_{43}Tc^{*}_{55,56} \quad {}^{100}_{44}Ru_{56} \quad {}^{112-114}_{48}Cd_{64-66} \quad {}^{113,115}_{49}In_{64,66} \quad {}^{121,123}_{51}Sb_{70,72} \quad {}^{3\cdot 41,125}_{52}Te_{71,73}$$

$${}^{127}_{53}I_{74} \quad {}^{131}_{54}Xe_{77} \quad {}^{136-138}_{56}Ba_{80-82} \quad {}^{144}_{60}Nd_{84} \quad {}^{5\cdot 29,146,3\cdot 49}_{61}Pm^{*}_{84,85,86} \quad {}^{144,147-150,152,154}_{62}Sm_{82,85-88,90,92} \quad {}^{157}_{64}Gd_{93}$$

$${}^{162}_{66}Dy_{96} \quad {}^{166,167,168,170}_{68}Er_{98,99,100,102} \quad {}^{169}_{69}Tm_{100} \quad {}^{168,170-174,176}_{70}Yb_{98,100-104,106} \quad {}^{175,176}_{71}Lu_{104,105} \quad {}^{3\cdot 61,6\cdot 31}_{74}W_{109,112}$$

$${}^{185,187}_{75}Tm_{110,112} \quad {}^{188,189}_{76}Os_{112,113} \quad {}^{191,193}_{77}Ir_{114,116} \quad {}^{199}_{80}Hg_{7\cdot 17} \quad {}^{209,210}_{83}Bi^{*}_{126,127} \quad {}^{209}_{84}Po^{*}_{125} \quad {}^{210}_{85}At^{*}_{125} \quad {}^{6\cdot 37}_{86}Rn^{*}_{136}$$

$${}^{223}_{87}Fr^{*}_{136} \quad {}^{2\cdot 116}_{88}Ra^{*}_{138} \quad {}^{237}_{93}Np^{*}_{144} \quad {}^{243}_{95}Am^{*}_{148} \quad {}^{13\cdot 19}_{96}Cm^{*}_{151} \quad {}^{257}_{100}Fm^{*}_{157} \quad {}^{6\cdot 43}_{101}Md^{*}_{157} \quad {}^{7\cdot 37}_{102}No^{*}_{157} \quad {}^{9\cdot 29,262}_{103}Lr^{*}_{158,159}$$

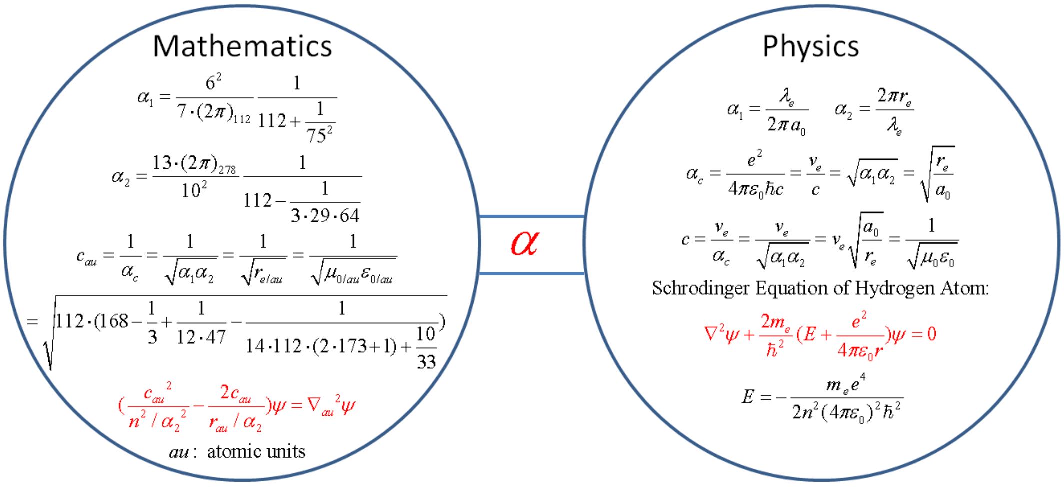
$${}^{281}_{110}Ds^{*}_{171} \quad {}^{15\cdot 19}_{112}Cn^{*}_{173} \quad {}^{4\cdot 71,22\cdot 13}_{113}Nh^{ie}_{9\cdot 19,173} \quad {}^{18\cdot 17,4\cdot 77}_{123}Ch^{ie}_{3\cdot 61,185} \quad {}^{312}_{125}Ch^{ie}_{187} \quad {}^{6\cdot 53,11\cdot 29,320}_{127}Ch^{ie}_{191,3\cdot 64,193} \quad {}^{328}_{129}Ch^{ie}_{199}$$

$${}^{4\cdot 83}_{131}Ch^{ie}_{201} \quad {}^{8\cdot 43,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210} \quad {}^{6\cdot 61}_{144}Ch^{ie}_{6\cdot 37} \quad {}^{12\cdot 31}_{147}Ch^{ie}_{225} \quad {}^{22\cdot 17}_{148}Ch^{ie}_{226} \quad {}^{380}_{151}Ch^{ie}_{229} \quad {}^{400}_{157}Ch^{ie}_{243} \quad {}^{22\cdot 19}_{167}Ch^{ie}_{251}$$

$${}^{33\cdot 13,430}_{171}Ch^{ie}_{6\cdot 43,7\cdot 37} \quad {}^{430,432}_{172}Ch^{ie}_{6\cdot 43,260} \quad {}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$$

These relationships are marvelous, they should not be just coincidences, and they should be science. This mechanism is analogous to that between DNA and protein.

8. Unification of Mathematics and Physics



Unification of Mathematics and Physics through α

Dr. Gang Chen, 2020/7/20-21

Fig. 1

Mathematics and science are usually regarded to be independent to each other, although mathematics is used as tool or language of science. It seems science has mathematic properties, so on the other hand, does mathematics has science features? We found the fine-structure constant α could act as a bridge between mathematics and physics (**Fig. 1**). For example, Schrödinger equation of hydrogen atom could be simplified to a reasonable and pure mathematic equation through α (red color parts).

9. Formula of the Speed of Light in Atomic Unites

In the previous paper[1], many formulas of the speed of light in atomic unites c_{au} had been deduced. Among them the most important should be the following formula.

$$
\begin{aligned}
c_{au}{}^2 &= \frac{1}{\alpha_c{}^2} = \frac{1}{\alpha_1\alpha_2} = \frac{2\pi a_0}{\lambda_e}\frac{\lambda_e}{2\pi r_e} = \frac{a_0}{r_e} = (\frac{c}{v_e})^2 \\
&= 137.035999037435 \times 137.035999111818 = 137.035999074627^2 \\
&= 112 \times (168 - \frac{1}{3} + \frac{1}{12\cdot 47} - \frac{1}{6\cdot 29\cdot 53\cdot 59 - 79/47})
\end{aligned}
$$

Here, we try to revise it to the following more reasonable form.

$$
\begin{aligned}
c_{au}{}^2 &= \frac{1}{\alpha_c{}^2} = \frac{1}{\alpha_1\alpha_2} = \frac{2\pi a_0}{\lambda_e}\frac{\lambda_e}{2\pi r_e} = \frac{a_0}{r_e} = (\frac{c}{v_e})^2 \\
&= 137.035999037435 \times 137.035999111818 = 137.035999074627^2 \\
&= 112 \times (168 - \frac{1}{3} + \frac{1}{12\cdot 47} - \frac{1}{14\cdot 112\cdot (2\cdot 173+1) + \frac{2\cdot 5}{3\cdot 11}}) \\
&= 112 \times 167.668437878402 = 18778.865042381
\end{aligned}
$$

$^{7}_{3}Li_{4}$ $^{10,11}_{5}B_{5,6}$ $^{12,13}_{6}C_{6,7}$ $^{14,15}_{7}N_{7,8}$ $^{20,21,22}_{10}Ne_{10,11,12}$ $^{23}_{11}Na_{12}$ $^{24,25,26}_{12}Mg_{12,13,14}$ $^{28,29,30}_{14}Si_{14,15,16}$

$^{31}_{15}P_{16}$ $^{32,33}_{16}S_{16,17}$ $^{47,49}_{22}Ti_{25,27}$ $^{55}_{25}Mn_{30}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{63,65}_{29}Cu_{34,36}$ $^{58,60,61,64}_{28}Ni_{30,32,33,36}$

$^{64,66,68}_{30}Zn_{34,36,38}$ $^{69,71}_{31}Ga_{38,40}$ $^{70,72}_{32}Ge_{38,40}$ $^{75}_{33}As_{42}$ $^{83,84}_{36}Kr_{47,48}$ $^{98,99}_{43}Tc^{*}_{55,56}$ $^{98,99,100}_{44}Ru_{54,55,56}$

$^{107,109}_{47}Ag_{60,62}$ $^{110,112-114}_{48}Cd_{62,64-66}$ $^{113,115}_{49}In_{64,66}$ $^{131}_{54}Xe_{77}$ $^{133}_{55}Cs_{78}$ $^{137}_{56}Ba_{81}$ $^{158}_{64}Gd_{94}$ $^{168}_{68}Er_{100}$

$^{169}_{69}Tm_{100}$ $^{173}_{70}Yb_{103}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{4\cdot 47,189}_{76}Os_{112,113}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{244}_{94}Pa^{*}_{150}$ $^{285}_{112}Cn^{*}_{173}$

$^{286}_{113}Nh^{ie}_{173}$ $^{288,289}_{115}Mc^{ie}_{173,174}$ $^{312}_{125}Ch^{ie}_{187}$ $^{314}_{126}Ch^{ie}_{4\cdot 47}$ $^{2\cdot 173}_{137}Fy^{ie}_{209}$ $^{418,420}_{168}Ch^{ie}_{250,252}$ $^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,2\cdot 131}$

10. Fibonacci Sequence *He* and its relationships with nuclides

We noticed 0.618×(112+168)=0.618×280≈173, so we construct the following Fibonacci sequence and present its relationships with nuclides.

Fibonacci Sequence *He*: 2 7 9 16 25 41 66 107 173 280

$^{1,2}_{1}H_{0,1}$ $^{4}_{2}He_{2}$ $^{7}_{3}Li_{4}$ $^{9}_{4}Be_{5}$ $^{14,15}_{7}N_{7,8}$ $^{16,17,18}_{8}O_{8,9,10}$ $^{19}_{9}F_{10}$ $^{25}_{12}Mg_{13}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{31}_{15}P_{16}$

$^{32}_{16}S_{16}$ $^{41}_{19}K_{22}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{55}_{25}Mn_{30}$ $^{66}_{30}Zn_{36}$ $^{73}_{32}Ge_{41}$ $^{93}_{41}Nb_{52}$ $^{106,108}_{46}Pd_{60,62}$

$^{107,109}_{47}Ag_{60,62}$ $^{106,108,112,113,114,116}_{48}Cd_{58,60,64,65,66,68}$ $^{113,115}_{49}In_{64,66}$ $^{116}_{50}Sn_{66}$ $^{163}_{66}Dy_{97}$ $^{173}_{70}Yb_{103}$

$^{179}_{72}Hf_{107}$ $^{180}_{73}Ta_{107}$ $^{262}_{103}Lr^{*}_{159}$ $^{270}_{107}Bh^{*}_{163}$ $^{285}_{112}Cn^{*}_{173}$ $^{286}_{113}Nh^{ie}_{173}$ $^{288}_{115}Nh^{ie}_{173}$ $^{434,435}_{173}Ch^{ie}_{261,262}$

11. The Meanings of the Numerical Values of the Fine-structure Constant

$$\alpha_1=\frac{1}{137.035999037435}=\frac{1}{137+\frac{1}{27}-\frac{1}{9\cdot 107}+\frac{1}{4\cdot 79\cdot(2\cdot 23\cdot 163+1)}}$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,49}_{22}Ti_{24,25,27}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{64,66}_{30}Zn_{34,36}$ ${}^{79,80}_{35}Br_{44,46}$ ${}^{92}_{40}Zr_{52}$

${}^{105,106,108}_{46}Pd_{59,60,62}$ ${}^{107,109}_{47}In_{60,62}$ ${}^{5\cdot 27,136,137,6\cdot 23}_{56}Ba_{79,80,81,82}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{156,157,158}_{64}Gd_{92,93,94}$ ${}^{162,163}_{66}Dy_{96,97}$

${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{179,180}_{72}Hf_{107,108}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{197}_{79}Au_{2\cdot 59}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{5\cdot 47,238}_{92}U^{*}_{143,146}$

${}^{247}_{97}Bk^{*}_{150}$ ${}^{243}_{98}Am^{*}_{145}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,11\cdot 19,210}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{410}_{163}Ch^{ie}_{247}$

$$\alpha_2=\frac{1}{137.035999111818}=\frac{1}{137+\frac{1}{27}-\frac{1}{9\cdot 107}+\frac{1}{5\cdot 17\cdot 137\cdot 173}}$$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{49}_{22}Ti_{27}$ ${}^{50,3\cdot 17}_{23}V_{27,28}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{107,109}_{47}In_{60,62}$ ${}^{131}_{54}Xe_{77}$ ${}^{173}_{70}Yb_{103}$

${}^{5\cdot 27,8\cdot 17,137,138}_{56}Ba_{79,80,81,82}$ ${}^{179,180}_{72}Hf_{107,108}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{2\cdot 131}_{103}Lr^{*}_{159}$

${}^{270}_{107}Bh^{*}_{163}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{284}_{113}Nh^{ie}_{173}$ ${}^{288,17^2}_{115}Mc^{ie}_{173,174}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 19,2\cdot 131}$

12. Construct Formulas of the Fine-structure Constant with 137 instead of 112 or 173

$$\alpha_{1-7(137)}=\frac{4\cdot 11}{7\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot 126-1}{2\cdot 125})^{3\cdot 167}}}\;\frac{1}{137+\frac{1}{37\cdot 173-\frac{5}{32}}}$$

$$=1/137.035999037435$$

${}^{14,15}_{7}N_{7,8}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Na_{18,20}$ ${}^{47,48,49,50}_{22}Ti_{25,26,27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{58,60}_{28}Ni_{30,32}$ ${}^{83,84}_{36}Kr_{47,48}$

${}^{70,72,73,74,76}_{32}Ge_{38,40,41,42,44}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{125,126}_{52}Te_{73,74}$

${}^{136,137}_{56}Ba_{80,81}$ ${}^{167,168}_{68}Er_{99,100}$ ${}^{187,188}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{6\cdot 37}_{86}Rn^{*}_{136}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{286}_{113}Nh^{ie}_{173}$

${}^{288,289}_{115}Mc^{ie}_{173,174}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{314}_{126}Ch^{ie}_{188}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$ ${}^{2\cdot 209}_{167}Ch^{ie}_{251}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-39(137)}=\frac{5\cdot 7^2}{39\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{4\cdot 3\cdot 13^2-1}{2\cdot(4\cdot 11\cdot 23+1)})^{3\cdot 7\cdot 193}}}\;\frac{1}{137+\frac{1}{13\cdot 29\cdot(4\cdot 3\cdot 5\cdot 47-1)}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{89}_{39}Y_{50}$

${}^{94,98}_{42}Mo_{52,56}$ ${}^{99,100,102,104}_{44}Ru_{55,56,58,60}$ ${}^{104,106,110}_{46}Pd_{58,60,64}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{136,137}_{56}Ba_{80,81}$

${}^{140,142}_{58}Ce_{82,84}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{160,164}_{66}Dy_{94,98}$ ${}^{169}_{69}Tm_{100}$ ${}^{4\cdot 47,189,192}_{76}Os_{112,113,116}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{227}_{89}Ac^{*}_{138}$

${}^{5\cdot 47,238}_{92}U^{*}_{143,146}$ ${}^{244}_{94}Pu^{*}_{150}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{286}_{113}Nh^{ie}_{173}$ ${}^{288,289}_{115}Mc^{ie}_{173,174}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$ ${}^{314}_{126}Ch^{ie}_{4\cdot 47}$

${}^{318,11\cdot 29,320}_{127}Ch^{ie}_{191,192,193}$ ${}^{4\cdot 89,21\cdot 17}_{141}Ch^{ie}_{215,216}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-46(137)}=\frac{17^2}{2\cdot 23\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{3\cdot 13\cdot 25}{2\cdot(2\cdot 3^5+1)})^{1949}}}\;\frac{1}{137+\frac{1}{2\cdot 5\cdot 13\cdot(4\cdot 5\cdot 163-1)-\frac{7}{8}}}$$

$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Na_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{89}_{39}Y_{50}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{163}_{66}Dy_{97}$ ${}^{227}_{89}Ac^{*}_{138}$
${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{288,17^2}_{115}Mc^{ie}_{173,174}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,209,210}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$

$$\alpha_{1-71(137)}$$

$$=\frac{2\cdot(2\cdot 112-1)}{71\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{6\cdot(6\cdot 11\cdot 17+1)-1}{16\cdot(4\cdot 3\cdot 5\cdot 7+1)})^{27\cdot(6\cdot 83+1)}}}\;\frac{1}{137+\frac{1}{256\cdot 11\cdot(8\cdot 27\cdot 7-1)}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Na_{18,20}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Cr_{32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{105}_{46}Pd_{59}$
${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{128}_{54}Xe_{74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{143,144}_{60}Nd_{83,84}$ ${}^{171,173}_{70}Yb_{101,103}$
${}^{175,11\cdot 16}_{71}Lu_{104,105}$ ${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{4\cdot 71,286}_{113}Nh^{ie}_{171,173}$ ${}^{288,289}_{115}Mc^{ie}_{173,174}$
${}^{2\cdot 163}_{128}Ch^{ie}_{3\cdot 66}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-103(137)}=\frac{8\cdot 81-1}{103\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot 29\cdot(6\cdot 7\cdot 23+1)}{15\cdot(6\cdot 7\cdot 89+1)})^{23\cdot(4\cdot 23\cdot 53+1)}}}\;\frac{1}{137+\frac{2\cdot 41\cdot 163+1}{125\cdot 10^{11}}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{78,81}_{35}Br_{44,46}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{89}_{39}Y_{50}$ ${}^{103}_{45}Rh_{58}$
${}^{92,95,96,97,98,100}_{42}Mo_{50,53,54,55,56,58}$ ${}^{104,105,106,108}_{46}Pd_{58,59,60,62}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112,113,116}_{48}Cd_{64,65,68}$ ${}^{127}_{53}I_{74}$
${}^{129,132,134,136}_{54}Xe_{75,78,80,82}$ ${}^{137,138}_{56}Ba_{81,82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{163}_{66}Dy_{97}$ ${}^{169}_{69}Tm_{100}$
${}^{173,174,176}_{70}Yb_{103,104,106}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{7\cdot 29,5\cdot 41}_{81}Tl_{122,124}$ ${}^{206,9\cdot 23,208}_{82}Pb_{124,125,126}$ ${}^{209}_{83}Bi^{*}_{126}$
${}^{209}_{84}Bi^{*}_{125}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{268,271}_{106}Sg^{*}_{163,165}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot 173,12\cdot 29}_{137,138}Fy^{ie}_{209,210}$ ${}^{410}_{163}Ch^{ie}_{261,262}$

$$\alpha_{1-110(137)}=\frac{4\cdot 173-1}{110\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot(168\cdot 11-1)}{3\cdot(112\cdot 11-1)})^{83\cdot 89}}}\;\frac{1}{137+\frac{1}{3\cdot 23\cdot 71\cdot(270-1)}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{99,100}_{44}Ru_{55,56}$
${}^{105,106,108,110}_{46}Pd_{59,60,62,64}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{110,112,113,114}_{48}Cd_{62,64,65,66}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{131}_{54}Xe_{77}$
${}^{116,118,119,120,122}_{50}Sn_{66,68,69,70,72}$ ${}^{133}_{55}Cs_{78}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{143}_{60}Nd_{83}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{163}_{66}Dy_{97}$ ${}^{168}_{68}Er_{100}$
${}^{169}_{69}Tm_{100}$ ${}^{171,172,173,174,176}_{70}Yb_{101,102,103,104,106}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{191,193}_{77}Ir_{114,116}$
${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{281}_{110}Ds^{*}_{171}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{4\cdot 71,22\cdot 13}_{113}Nh^{ie}_{171,173}$
${}^{288,289}_{115}Nh^{ie}_{173,174}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{418,420}_{168}Ch^{ie}_{250,252}$ ${}^{434,435}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-7(137)}=\frac{7\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\frac{e^{2}}{(\frac{4}{3})^{7}}\cdots\frac{e^{2}}{(\frac{6\cdot 199}{8\cdot 149+1})^{7\cdot 11\cdot 31}}}{4\cdot 11}\frac{1}{137-\frac{1}{17\cdot(4\cdot 5\cdot 7\cdot 31+1)-\frac{13}{3\cdot 19}}}$$

$$=\frac{7\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\frac{e^{2}}{(\frac{4}{3})^{7}}\cdots\frac{e^{2}}{(\frac{6\cdot 199}{8\cdot 149+1})^{7\cdot 11\cdot 31}}}{4\cdot 11}\frac{1}{137-\frac{1}{17\cdot(2\cdot 13\cdot 167-1)-\frac{13}{3\cdot 19}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{48}_{22}Ti_{26}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$

${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{76,77,78,80}_{34}Se_{42,43,44,46}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{89}_{39}Y_{50}$

${}^{99,100,101,102,104}_{44}Ru_{55,56,57,58,60}$ ${}^{110,112,113,114,116}_{48}Cd_{62,64,65,66,68}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$

${}^{122,124,128,130}_{52}Te_{70,72,76,78}$ ${}^{130,131,132}_{54}Xe_{76,77,78}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{149}_{62}Sm_{87}$ ${}^{167,168}_{68}Er_{99,100}$

${}^{171,172,173,174,176}_{70}Yb_{101,102,103,104,106}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{186,187,188,189}_{76}Os_{110,111,112,113,114}$ ${}^{191,193}_{77}Ir_{114,116}$

${}^{192,195,196}_{78}Pt_{114,117,118}$ ${}^{199,204}_{80}Hg_{7\cdot 17,124}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Bi^{*}_{125}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{284,22\cdot 13}_{113}Nh^{ie}_{9\cdot 19,173}$

${}^{310}_{124}Ch^{ie}_{6\cdot 31}$ ${}^{326,328}_{129}Ch^{ie}_{197,199}$ ${}^{344,2\cdot 173}_{136,137}Fy^{ie}_{208,209}$ ${}^{376}_{149}Ch^{ie}_{227}$ ${}^{22\cdot 19}_{167}Ch^{ie}_{251}$ ${}^{14\cdot 31,435}_{173}Ch^{ie}_{9\cdot 19,262}$

13. The Meanings of the Numerical Value of the Speed of Light in Atomic Unites

$$c_{au}=137.035999074627=137+\frac{1}{27}-\frac{1}{9\cdot 107}+\frac{1}{8\cdot 137\cdot(4\cdot 7\cdot 71-1)}$$

$$c_{au}=137.035999074627=137+\frac{1}{27}-\frac{1}{9\cdot 107}+\frac{1}{3\cdot 17\cdot(4\cdot 7\cdot 25\cdot 61+1)}$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$

${}^{31}_{15}P_{16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{45}_{21}Sc_{24}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$

${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,66,68}_{30}Zn_{34,36,38}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{70,72,74}_{32}Ge_{38,40,42}$ ${}^{79,81}_{35}Br_{44,46}$

${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{96,97,98}_{42}Mo_{54,55,56}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{103}_{45}Rh_{58}$ ${}^{105,106,108}_{46}Pd_{59,60,62}$

${}^{96,98,99,100,102,104}_{44}Ru_{52,54,55,56,58,60}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{118,119,122}_{50}Sn_{68,69,72}$ ${}^{131}_{54}Xe_{77}$

${}^{121,123}_{51}Sb_{70,72}$ ${}^{5\cdot 27,8\cdot 17,137,138}_{56}Ba_{79,80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{145,146,147}_{61}Pm^{*}_{84,85,86}$ ${}^{157}_{64}Gd_{93}$ ${}^{162,163}_{66}Dy_{96,97}$

${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{170,171,172,173.174}_{70}Yb_{100,101,102,103,104}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{3\cdot 59,179,180}_{72}Hf_{105,107,108}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$

${}^{188,189}_{76}Os_{112,113}$ ${}^{203,205}_{81}Tl_{122,124}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$

${}^{232}_{90}Th^{*}_{142}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{258}_{101}Md^{*}_{157}$ ${}^{259}_{102}No^{*}_{157}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269,271}_{106}Sg^{*}_{163,165}$ ${}^{270}_{107}Bh^{*}_{163}$

${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{4\cdot 71,286}_{113}Nh^{ie}_{171,173}$ ${}^{288,289}_{115}Mc^{ie}_{173,174}$ ${}^{304}_{122}Ch^{ie}_{182}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{2\cdot 179,360}_{142}Ch^{ie}_{216,218}$ ${}^{14\cdot 27}_{150}Ch^{ie}_{228}$

${}^{400}_{157}Ch^{ie}_{243}$ ${}^{410}_{163}Ch^{ie}_{247}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$ ${}^{33\cdot 13,430}_{171}Ch^{ie}_{6\cdot 43,7\cdot 37}$ ${}^{430,16\cdot 27}_{172}Ch^{ie}_{6\cdot 43,260}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

14. Other Formulas of the Fine-structure Constant with 137

$$\alpha_{1-7(137)\text{-}Wallis} = \frac{11}{7\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{750}{751}\frac{16\cdot 47}{2\cdot 3\cdot 125+1})}\frac{1}{137+\frac{1}{13\cdot 37\cdot 59-\frac{4\cdot 11}{3\cdot 25}}}$$

$=1/137.035999037435$

${}^{7}_{3}Li_{4}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47}_{22}Ti_{25}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$ ${}^{64,66,67,68}_{30}Zn_{34,36,37,38}$
${}^{75}_{33}As_{42}$ ${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$
${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{187,188}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{6\cdot 37}_{86}Rn^{*}_{136}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$

$$\alpha_{1-7(137)-GL} = \frac{11}{2\cdot 7\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot(2\cdot 7\cdot 17+1)+1})}\frac{1}{137+\frac{1}{4\cdot 11\cdot(4\cdot 7^{2}+1)+\frac{19}{27}}}$$

$$= \frac{11}{2\cdot 7\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot(2\cdot 7\cdot 17+1)+1})}\frac{1}{137+\frac{1}{4\cdot 11\cdot(2\cdot 9\cdot 11-1)+\frac{19}{27}}}$$

$=1/137.035999037435$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{49,50}_{22}Ti_{27,28}$ ${}^{50,51}_{23}V_{27,28}$
${}^{59}_{27}Co_{32}$ ${}^{76,77,78,80,82}_{34}Se_{42,43,44,46,48}$ ${}^{98,99,100,101,102}_{44}Ru_{54,55,56,57,58}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$
${}^{162,164,166,167,168,170}_{68}Er_{94,96,98,99,100,102}$ ${}^{344,2\cdot 173}_{136,137}Fy^{ie}_{208,11\cdot 19}$

$$\alpha_{1-7(137)-NC} = \frac{4\cdot 11}{7\cdot(2\pi)_{NC-5}}\frac{1}{137+\frac{1}{23}-\frac{1}{2\cdot(4\cdot 3\cdot 5\cdot 7+1)}+\frac{1}{2\cdot 3\cdot 7\cdot(2\cdot 5\cdot 53\cdot 83+1)}}$$

$$= \frac{4\cdot 11}{7\cdot(2\pi)_{NC-5}}\frac{1}{137+\frac{1}{23}-\frac{1}{2\cdot(4\cdot 3\cdot 5\cdot 7+1)}+\frac{1}{2\cdot 3\cdot 7\cdot(8\cdot 9\cdot 13\cdot 47+1)}}$$

$=1/137.035999037435$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{53}_{24}Cr_{29}$
${}^{95}_{42}Mo_{53}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{127}_{53}I_{74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$
${}^{210}_{85}At^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{6\cdot 53,11\cdot 29,320}_{127}Ch^{ie}_{191,192,13}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,,209,210}$

$$\alpha_{1-39(137)-Wallis} = \frac{5\cdot 7^{2}}{4\cdot 39\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{6080}{6081}\frac{2\cdot(2\cdot 9\cdot 13^{2}-1)}{2\cdot 32\cdot 5\cdot 19+1})}\frac{1}{137+\frac{1}{2\cdot 3\cdot 5^{2}\cdot 7\cdot 11-\frac{1}{10}}}$$

$=1/137.035999037435$

${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$
${}^{31}_{15}P_{16}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{40,41,42,43,44,46,48}_{20}Ca_{20,22,23,24,26,28}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$
${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{58,60,61,64}_{28}Ni_{30,32,33,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,66,68,70}_{30}Zn_{34,36,38,40}$ ${}^{69,71}_{31}Ga_{38,40}$

${}^{79,81}_{35}Br_{44,46}$ ${}^{87,88}_{38}Sr_{49,50}$ ${}^{89}_{39}Y_{50}$ ${}^{90,91,92}_{40}Zr_{50,51,52}$ ${}^{92,94,95,96,97,98,100}_{42}Mo_{50,52,53,54,55,56,58}$ ${}^{103}_{45}Rh_{58}$

${}^{96,98,99,100,101,102,104}_{44}Ru_{52,54,55,56,57,58,60}$ ${}^{112,114,115,116,117,118,119,120,122,124}_{50}Sn_{62,64,65,66,67,68,69,70,72,74}$

${}^{113,115}_{49}In_{64,66}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{173}_{70}Yb_{103}$ ${}^{191,193}_{77}Ir_{104,105}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{286}_{113}Nh^{ie}_{173}$ ${}^{344,2\cdot173,348}_{137}Fy^{ie}_{208,209,210}$

$$\alpha_{1-39(137)-GL}=\frac{5\cdot7^2}{8\cdot39\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot9\cdot5\cdot43+1})}\frac{1}{137+\frac{1}{8\cdot5\cdot7\cdot(2\cdot5\cdot23\cdot27-1)}}$$

$$=\frac{5\cdot7^2}{8\cdot39\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot9\cdot5\cdot43+1})}\frac{1}{137+\frac{1}{8\cdot5\cdot7\cdot(64\cdot97+1)}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{40,42,43,44,46,50}_{20}Ca_{20,22,23,24,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{77}_{34}Se_{43}$

${}^{79,81}_{35}Br_{44,46}$ ${}^{90,91,92,94,96}_{40}Zr_{50,51,52,54,56}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{163}_{66}Dy_{97}$ ${}^{173}_{70}Yb_{103}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{13\cdot19}_{97}Bk^{*}_{150}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{8\cdot43,2\cdot173,348}_{137}Fy^{ie}_{208,209,210}$ ${}^{410}_{163}Ch^{ie}_{13\cdot19}$

$$\alpha_{1-39(137)-NC}=\frac{5\cdot7^2}{39\cdot(2\pi)_9}\frac{1}{137+\frac{109\cdot(12\cdot29+1)}{4\cdot10^{11}}}=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{183}_{74}W_{109}$ ${}^{278}_{109}Mt^{*}_{169}$ ${}^{344,346,12\cdot29}_{137}Fy^{ie}_{208,209,210}$

$\alpha_{1-46(137)-Wallis}$

$$=\frac{17^2}{8\cdot23\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{2924}{9\cdot25\cdot13}\frac{2\cdot7\cdot11\cdot19}{2\cdot2\cdot17\cdot43+1})}\frac{1}{137+\frac{1}{4\cdot3\cdot11\cdot17\cdot43-\frac{13}{17}}}$$

$$=1/137.035999037435$$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Na_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{43}_{20}Ca_{23}$ ${}^{46,47,48,48,50}_{22}Ti_{24,25,26,27,28}$

${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{77}_{34}Se_{43}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ ${}^{86,87,88}_{38}Sr_{48,49,50}$

${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$

${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{17^2}_{115}Mc^{ie}_{174}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-46(137)-GL}=\frac{17^2}{16\cdot23\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot49\cdot19+1})}\frac{1}{137+\frac{1}{16\cdot3\cdot17\cdot79-\frac{3\cdot13}{4\cdot11}}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Na_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,48,49,50}_{22}Ti_{24,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{85,87}_{37}Rb_{48,50}$

${}^{84,86,87,88}_{38}Sr_{46,48,49,50}$ ${}^{89}_{39}Y_{50}$ ${}^{96,98,99,100,101,102,104}_{44}Ru_{52,54,55,56,57,58,60}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$

${}^{125,126,128,130}_{52}Te_{73,74,76,78}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{197}_{79}Au_{118}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$

${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{17^2}_{115}Mc^{ie}_{174}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-46(137)-NC}=\frac{17^2}{2\cdot23\cdot(2\pi)_7}\frac{1}{137+\frac{1}{5\cdot79}-\frac{1}{3\cdot19\cdot(4\cdot3\cdot7\cdot47-1)+\frac{4}{13}}}$$

$=1/137.035999037435$

${}^{7}_{3}Li_{4}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Na_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,71}_{23}V_{27,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$

${}^{79,81}_{35}Br_{44,46}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{101}_{44}Ru_{57}$ ${}^{125,126,128,130}_{52}Te_{73,74,76,78}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{138,139}_{57}La_{81,82}$

${}^{197}_{79}Au_{118}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{17^2}_{115}Mc^{ie}_{174}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$\alpha_{1-71(137)-Wallis}$

$$=\frac{2\cdot112-1}{2\cdot71\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{20208}{20209}\frac{2\cdot5\cdot43\cdot47}{16\cdot3\cdot(4\cdot3\cdot5\cdot7+1)+1})}\frac{1}{137+\frac{2\cdot9\cdot7\cdot13\cdot109}{25\cdot10^{11}}}$$

$=1/137.035999037435$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{69,71}_{31}Ga_{38,40}$

${}^{83,84}_{36}Kr_{47,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{94}_{42}Mo_{52}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112}_{48}Cd_{64}$ ${}^{159}_{65}Tb_{94}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{4\cdot47}_{76}Os_{112}$

${}^{183}_{74}W_{109}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{244}_{94}Pu^{*}_{150}$ ${}^{278}_{109}Mt^{*}_{169}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$\alpha_{1-71(137)-GL}$

$$=\frac{2\cdot112-1}{4\cdot71\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot7\cdot(2\cdot27\cdot17+1)+1})}\frac{1}{137+\frac{1}{2\cdot5\cdot7\cdot31\cdot(2\cdot3\cdot7\cdot31-1)}}$$

$=1/137.035999037435$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{58,60,61,62}_{28}Ni_{30,32,33,34}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{107,109}_{47}Ag_{60,62}$

${}^{96}_{42}Mo_{54}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{173}_{70}Yb_{103}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-71(137)-NC}=\frac{2\cdot(2\cdot112-1)}{71\cdot(2\pi)_{NC-15}}\frac{1}{137+\frac{1}{2\cdot27\cdot25+\frac{3\cdot11}{4\cdot131}}}=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{31}_{15}P_{16}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{75}_{33}As_{42}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{120,122}_{50}Sn_{70,72}$

${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{185,187}_{75}Re_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$

${}^{4\cdot83}_{131}Ch^{ie}_{201}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{14\cdot31,435}_{173}Ch^{ie}_{261,2\cdot131}$

$\alpha_{1-103(137)-Wallis}$

$$=\frac{8\cdot81-1}{4\cdot103\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{168234}{168235}\frac{4\cdot137\cdot(4\cdot7\cdot11-1)}{2\cdot3\cdot11\cdot(4\cdot49\cdot13+1)+1})}\frac{1}{137+\frac{113\cdot157}{5\cdot10^{12}}}$$

$$=\frac{8\cdot81-1}{4\cdot103\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{168234}{168235}\frac{4\cdot137\cdot(4\cdot7\cdot11-1)}{2\cdot3\cdot11\cdot(2\cdot3\cdot25\cdot17-1)+1})}\frac{1}{137+\frac{7\cdot37\cdot137}{10^{13}}}$$

$=1/137.035999037435$

$^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$
$^{67}_{30}Zn_{37}$ $^{75}_{33}As_{42}$ $^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$ $^{85,87}_{37}Rb_{48,50}$ $^{96,98,99,100,101,102,104}_{44}Ru_{52,54,55,56,57,58,60}$
$^{103}_{45}Ru_{58}$ $^{106,108,110,111,112,113,114,116}_{48}Cd_{58,60,62,63,64,65,66,68}$ $^{113,115}_{49}In_{64,66}$ $^{116,118,119,120,124}_{50}Sn_{66,68,69,70,74}$
$^{121,123}_{51}Sb_{70,72}$ $^{131}_{54}Xe_{77}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{157}_{64}Gd_{93}$ $^{162,163}_{66}Dy_{96,97}$ $^{168}_{68}Er_{100}$ $^{173}_{70}Yb_{103}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$
$^{257}_{100}Fm^{*}_{157}$ $^{262}_{103}Lr^{*}_{159}$ $^{285}_{112}Cn^{*}_{173}$ $^{286}_{113}Nh^{ie}_{173}$ $^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{400}_{157}Ch^{ie}_{243}$ $^{434,435}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-103(137)-GL}=\frac{8\cdot81-1}{8\cdot103\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot(2\cdot9\cdot25\cdot7\cdot17+1)+1})}\frac{1}{137+\frac{19\cdot173}{5\cdot10^{11}}}$$
$$=1/137.035999037435$$

$^{10,11}_{5}B_{5,6}$ $^{14,15}_{7}N_{7,8}$ $^{16,17,18}_{8}O_{8,9,10}$ $^{19}_{9}F_{10}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{39,40,41}_{19}K_{20,21,22}$ $^{45}_{21}Sc_{24}$ $^{55}_{25}Mn_{30}$
$^{79,81}_{35}Br_{44,46}$ $^{103}_{45}Ru_{58}$ $^{118,7\cdot17}_{50}Sn_{68,69}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{173}_{70}Yb_{103}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{7\cdot29,205}_{81}Tl_{122,124}$
$^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}At^{*}_{125}$ $^{262}_{103}Lr^{*}_{159}$ $^{285}_{112}Cn^{*}_{173}$ $^{2\cdot157}_{126}Ch^{ie}_{188}$ $^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{434,435}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-103(137)-NC}=\frac{8\cdot81-1}{103\cdot(2\pi)_{NC-29}}\frac{1}{137+\frac{1}{32\cdot3\cdot7\cdot13\cdot29+\frac{2}{15}}}=1/137.035999037435$$

$^{14,15}_{7}N_{7,8}$ $^{27}_{13}Al_{14}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{31}_{15}P_{16}$ $^{45}_{21}Sc_{24}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{63,65}_{29}Cu_{34,36}$ $^{64,70}_{30}Zn_{34,40}$
$^{79,81}_{35}Br_{44,46}$ $^{103}_{45}Ru_{58}$ $^{112,113}_{48}Cd_{64,65}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{173}_{70}Yb_{103}$ $^{7\cdot29,205}_{81}Tl_{122,124}$ $^{262}_{103}Lr^{*}_{159}$
$^{285}_{112}Cn^{*}_{173}$ $^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{434,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-110(137)-Wallis}$$
$$=\frac{4\cdot173-1}{4\cdot110\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{11080}{11081}\frac{2\cdot3\cdot(168\cdot11-1)}{2\cdot8\cdot5\cdot(12\cdot23+1)+1})}\frac{1}{137+\frac{1}{4\cdot27\cdot29\cdot(420+1)}}$$
$$=1/137.035999037435$$

$^{19}_{9}F_{10}$ $^{20,21,22}_{10}Ne_{10,11,12}$ $^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{36,38,40}_{18}Ar_{18,20,22}$ $^{40,42,44,46,48}_{20}Ca_{20,22,23,24,26,28}$
$^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{50,51}_{23}V_{27,28}$ $^{50,52,53,54}_{24}Cr_{26,28,29,30}$ $^{59}_{27}Co_{32}$ $^{63,65}_{29}Cu_{34,36}$ $^{80,82,83,84,86}_{36}Kr_{44,46,47,48}$
$^{90,92,94}_{40}Zr_{50,52,54}$ $^{98,99,100,102,104}_{44}Ru_{54,55,56,58,60}$ $^{103}_{45}Rh_{58}$ $^{102,104,105,106,108,110}_{46}Pd_{56,58,59,60,62,64}$ $^{131}_{54}Xe_{77}$ $^{133}_{55}Cs_{78}$
$^{106,108,110,111,112,113,114,116}_{48}Cd_{58,60,62,63,64,65,66,68}$ $^{112,115,116,118,120,122}_{50}Sn_{62,65,66,68,70,72}$ $^{136,137,138}_{56}Ba_{80,81,82}$
$^{136,138,140,142}_{58}Ce_{78,80,82,84}$ $^{144,145,148}_{60}Ce_{84,85,88}$ $^{168}_{68}Er_{100}$ $^{173}_{70}Yb_{103}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{191,193}_{77}Ir_{114,116}$
$^{200}_{80}Hg_{120}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}At^{*}_{125}$ $^{223}_{87}Fr^{*}_{136}$ $^{226}_{88}Ra^{*}_{138}$ $^{262}_{103}Lr^{*}_{159}$ $^{270}_{108}Hs^{*}_{162}$ $^{293}_{116}Lv^{*}_{177}$ $^{285}_{112}Cn^{*}_{173}$
$^{22\cdot13}_{113}Nh^{ie}_{173}$ $^{288,289}_{115}Mc^{ie}_{173,174}$ $^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{418,420}_{168}Ch^{ie}_{250,252}$ $^{434,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-110(137)-GL}$$
$$=\frac{4\cdot173-1}{8\cdot110\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot(8\cdot9\cdot49-1)+1})}\frac{1}{137+\frac{1}{3\cdot7\cdot11\cdot(8\cdot3\cdot5\cdot41-1)}}$$
$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{70,72,73,74,76}_{32}Ge_{38,40,41,42,44}$ ${}^{75}_{33}As_{42}$ ${}^{93}_{41}Nb_{52}$

${}^{82,84}_{36}Kr_{46,48}$ ${}^{96,98,99,100,101,102,103,104}_{44}Ru_{52,54,55,56,57,58,60}$ ${}^{110,112,113,114,116}_{48}Cd_{62,64,65,66,68}$ ${}^{113,115}_{49}In_{64,66}$

${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{173}_{70}Yb_{103}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$

${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{434,435}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-110(137)-NC}=\frac{4\cdot173-1}{110\cdot(2\pi)_{NC-13}}\frac{1}{137+\frac{1}{3\cdot149}-\frac{1}{29\cdot(2\cdot3\cdot13\cdot137+1)+\frac{4}{7}}}$$

$$=\frac{4\cdot173-1}{110\cdot(2\pi)_{NC-13}}\frac{1}{137+\frac{1}{3\cdot149}-\frac{1}{29\cdot(64\cdot167-1)+\frac{4}{7}}}=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{89}_{39}Y_{50}$ ${}^{99,100,101,102,104}_{44}Ru_{55,56,58,60}$ ${}^{149}_{62}Sm_{87}$

${}^{136,137,138}_{56}Ba_{8,81,82}$ ${}^{167,168}_{68}Er_{99,100}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{376}_{149}Ch^{ie}_{227}$ ${}^{22\cdot19}_{167}Ch^{ie}_{251}$

${}^{434,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-7(137)-wallis}=\frac{7\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{3580}{3579}\frac{4\cdot5\cdot179}{2\cdot(4\cdot3\cdot149+1)+1})}{11}\frac{1}{137-\frac{1}{32\cdot9\cdot7\cdot173+1}}$$

$$=1/137.035999111818$$

${}^{7}_{3}Li_{4}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{149}_{62}Sm_{87}$ ${}^{173}_{70}Yb_{103}$ ${}^{179}_{72}Hf_{107}$

${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{2\cdot149}_{119}Ch^{ie}_{179}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{376}_{149}Ch^{ie}_{227}$ ${}^{434,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-7(137)-GL}=\frac{2\cdot7\cdot(1-\frac{1}{3}+\frac{1}{5}-\cdots+\frac{1}{2\cdot2\cdot17\cdot67+1})}{11}\frac{1}{137-\frac{1}{4\cdot7\cdot19\cdot(2\cdot9\cdot31-1)-\frac{10}{37}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$

${}^{64,66,67,68}_{30}Zn_{34,36,37,38}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{74,76,77,78}_{34}Se_{40,42,43,44}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{84,86,87,88}_{38}Sr_{46,48,49,50}$ ${}^{117,118}_{50}Sn_{67,68}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{165}_{67}Ho_{98}$ ${}^{173}_{70}Yb_{103}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{434,435}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-7(137)-NC}=\frac{7\cdot(2\pi)_{NC-9}}{4\cdot11}\frac{1}{137-\frac{1}{7\cdot17}+\frac{1}{3\cdot19\cdot(8\cdot61-1)-\frac{61}{100}}}=1/137.035999111818$$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{61,62}_{28}Ni_{33,34}$ ${}^{77}_{34}Se_{43}$ ${}^{118,7\cdot17}_{50}Sn_{68,69}$

${}^{99,100,101,102,104}_{44}Ru_{55,56,57,58,60}$ ${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{145,146,147}_{61}Pm^{*}_{84,85,86}$ ${}^{168}_{68}Er_{100}$

${}^{173}_{70}Yb_{103}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{199,200}_{80}Hg_{119,120}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{434,435}_{173}Ch^{ie}_{261,262}$

15. Formulas of the Fine-structure Constant with 83

As the 83[th] element ${}_{83}$Bi should be the end of stable elements and the start of radioactive elements (except ${}_{43}$Tc and ${}_{61}$Pm) in the periodic table of elements, some formulas of the fine-structure constant could be constructed with the factor 83 instead of 112, 173 and 137 as follows.

$$\alpha_{1-31(83)} = \frac{2\cdot 59}{31\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{9\cdot 83}{2\cdot(2\cdot 11\cdot 17-1)})^{1493}}}\frac{1}{83+\frac{1}{3\cdot 29\cdot(6\cdot 83-1)-\frac{19}{2\cdot 37}}}$$

$$= \frac{2\cdot 59}{31\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{9\cdot 83}{2\cdot(12\cdot 31+1)+1})^{1493}}}\frac{1}{83+\frac{1}{3\cdot 29\cdot(16\cdot 31+1)-\frac{19}{2\cdot 37}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{59}_{27}Co_{32}$
${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Co_{34,36}$ ${}^{64,66,67,68}_{30}Zn_{34,36,37,38}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$
${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{84.86,87,88}_{38}Sr_{46,48,49,50}$ ${}^{99,100,102}_{44}Ru_{55,56,58}$ ${}^{105,107}_{46}Pd_{59,62}$ ${}^{107,109}_{47}Ag_{60,62}$
${}^{110,111,112,113,114,116}_{48}Cd_{62,63,64,65,66,68}$ ${}^{2\cdot 59}_{50}Sn_{68}$ ${}^{126}_{52}Te_{74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$
${}^{143}_{60}Nd_{83}$ ${}^{149}_{62}Sm_{87}$ ${}^{166,167,168,170}_{68}Er_{98,99,100,102}$ ${}^{173}_{70}Yb_{103}$ ${}^{184,6\cdot 31}_{74}W_{110,112}$ ${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$
${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{286}_{113}Nh^{ie}_{173}$ ${}^{294}_{118}Og^{ie}_{176}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137.138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-31(83)-Wallis} = \frac{59}{2\cdot 31\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{2238}{2239}\frac{4\cdot 5\cdot 112}{2\cdot 3\cdot(2\cdot 11\cdot 17-1)+1})}\frac{1}{83+\frac{1}{2\cdot 83\cdot(3\cdot 128-1)-\frac{5}{11}}}$$

$$=1/137.035999037435$$

${}^{10,11}_{5}B_{5,6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{112}_{48}Cd_{64}$ ${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137.138}Fy^{ie}_{208,209,210}$

$$\alpha_{1\text{-}31(83)\text{-}GL} = \frac{59}{4\cdot 31\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 16\cdot 89+1})}\frac{1}{83+\frac{1}{13\cdot 17\cdot(14\cdot 73-1)-\frac{13}{4\cdot 7}}}$$

$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{58,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{73}_{32}Ge_{41}$ ${}^{89}_{39}Y_{50}$ ${}^{125}_{52}Te_{73}$
${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{24\cdot 13}_{125}Ch^{ie}_{11\cdot 17}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137.138}Fy^{ie}_{16\cdot 13,209,210}$

$$\alpha_{1\text{-}31(83)\text{-}NC}=\frac{2\cdot 59}{31\cdot(2\pi)_{NC-7}}\frac{1}{83+\frac{1}{13^2}-\frac{1}{2\cdot(4\cdot 81\cdot 47-1)-\frac{47}{61}}}=1/137.035999037435$$

$^{14,15}_{7}N_{7,8}$ $^{27}_{13}Al_{14}$ $^{59}_{27}Co_{32}$ $^{61}_{28}Ni_{33}$ $^{69,71}_{31}Ga_{38,40}$ $^{79,81}_{35}Br_{44,46}$ $^{83}_{36}Kr_{47}$ $^{107,109}_{47}Ag_{60,62}$ $^{136,137,138}_{56}Ba_{80,81,82}$

$^{145}_{61}Pm^{*}_{84}$ $^{169}_{69}Tm_{100}$ $^{173}_{70}Yb_{103}$ $^{203,205}_{81}Tl_{2\cdot 61,124}$ $^{285}_{112}Cn^{*}_{173}$ $^{344,2\cdot 173,12\cdot 29}_{136,137.138}Fy^{ie}_{16\cdot 13,209,210}$ $^{426}_{169}Ch^{ie}_{257}$

$$\alpha_{2-8(83)}=\frac{8\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot(4\cdot 25\cdot 7+1)}{3\cdot(4\cdot 9\cdot 13-1)})^{2803}}}{83}\frac{1}{83-\frac{1}{2\cdot 31\cdot(2\cdot 3\cdot 5\cdot 7\cdot 13+1)-\frac{1}{10}}}$$

$=1/137.035999111818$

$^{7}_{3}Li_{4}$ $^{9}_{4}Be_{5}$ $^{10,11}_{5}B_{5,6}$ $^{12,13}_{6}C_{6,7}$ $^{14,15}_{7}N_{7,8}$ $^{16,17,18}_{8}O_{8,9,10}$ $^{19}_{9}F_{10}$ $^{20,21,22}_{10}Ne_{10,11,12}$ $^{27}_{13}Al_{14}$ $^{28,29,30}_{14}Si_{14,15,16}$

$^{31}_{15}P_{16}$ $^{55}_{25}Mn_{30}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{58,60,62,64}_{28}Ni_{30,32,34,36}$ $^{63,65}_{29}Cu_{34,36}$ $^{69,71}_{31}Ga_{38,40}$ $^{79,81}_{35}Br_{44,46}$ $^{89}_{39}Y_{50}$

$^{83,84,86}_{36}Kr_{47,48,50}$ $^{90,7\cdot 13,92,94}_{40}Zr_{50,51,4\cdot 13,54}$ $^{107,109}_{47}Ag_{60,62}$ $^{110,112,113}_{48}Cd_{62,64,65}$ $^{115,9\cdot 13,120,124}_{50}Sn_{65,67,70,74}$

$^{124,125,126,130}_{52}Te_{72,73,74,78}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{157}_{64}Gd_{93}$ $^{159}_{65}Tb_{94}$ $^{173}_{70}Yb_{103}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{200}_{80}Hg_{120}$ $^{209}_{83}Bi^{*}_{126}$

$^{209}_{84}Po^{*}_{125}$ $^{210}_{85}At^{*}_{125}$ $^{257}_{100}Fm^{*}_{157}$ $^{285}_{112}Cn^{*}_{173}$ $^{286}_{113}Nh^{ie}_{173}$ $^{300}_{120}Ch^{*}_{180}$ $^{330}_{130}Ch^{*}_{200}$ $^{344,2\cdot 173,348}_{136,137.138}Fy^{ie}_{16\cdot 13,209,210}$

$$\alpha_{2-8(83)-Wallis}=\frac{32\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{4202}{4203}\frac{4\cdot(2\cdot 3\cdot 25\cdot 7+1)}{2\cdot 11\cdot(2\cdot 5\cdot 19+1)+1})}{83}\frac{1}{83-\frac{1}{4\cdot 7\cdot 89\cdot(4\cdot 83-1)}}$$

$=1/137.035999111818$

$^{19}_{9}F_{10}$ $^{20,21,22}_{10}Ne_{10,11,12}$ $^{23}_{11}Na_{12}$ $^{39,40,41}_{19}K_{20,21,22}$ $^{47}_{22}Ti_{25}$ $^{83,84,86}_{36}Kr_{47,48,50}$ $^{89}_{39}Y_{50}$ $^{143,144}_{60}Nd_{83,84}$ $^{227}_{89}Ac^{*}_{138}$

$$\alpha_{2-8(83)-GL}=\frac{64\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 4\cdot 3\cdot 223+1})}{83}\frac{1}{83-\frac{1}{4\cdot 9\cdot 13\cdot(4\cdot 127+1)-\frac{7}{11}}}$$

$=1/137.035999111818$

$^{14,15}_{7}N_{7,8}$ $^{19}_{9}F_{10}$ $^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{53}_{24}Cr_{29}$ $^{83}_{36}Kr_{47}$ $^{127}_{53}I_{74}$ $^{191,193}_{77}Ir_{114,116}$ $^{209}_{83}Bi^{*}_{126}$ $^{223}_{87}Fr^{*}_{136}$ $^{6\cdot 53}_{127}Ch^{ie}_{191}$

$$\alpha_{2-8(83)-NC}=\frac{8\cdot(2\pi)_{NC-9}}{83}\frac{1}{83-\frac{1}{2\cdot 149}+\frac{1}{2\cdot 3\cdot 79\cdot(2\cdot 3\cdot 47-1)-\frac{7}{8}}}$$

$=1/137.035999111818$

$^{14,15}_{7}N_{7,8}$ $^{16,17,18}_{8}O_{8,9,10}$ $^{19}_{9}F_{10}$ $^{79,81}_{35}Br_{44,46}$ $^{83}_{36}Kr_{47}$ $^{135,136,137,138}_{56}Ba_{79,80,81,82}$ $^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ $^{149}_{62}Sm_{87}$

$^{2\cdot 79}_{64}Gd_{2\cdot 47}$ $^{197}_{79}Au_{118}$ $^{285}_{112}Cn^{*}_{173}$ $^{2\cdot 149}_{119}Ch^{ie}_{179}$ $^{344,2\cdot 173,348}_{136,137.138}Fy^{ie}_{16\cdot 13,209,210}$ $^{4\cdot 89}_{141}Ch^{ie}_{215}$ $^{8\cdot 47}_{149}Ch^{ie}_{227}$

16. Formulas of the Fine-structure Constant with 83^2

$$\alpha_{1-50/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37}-\frac{4\cdot 3\cdot 5\cdot 41-1}{125\cdot 10^{10}}}{83^2}=1/137.035999037435$$

Note: $\frac{1}{3}-\frac{1}{16}=\frac{13}{16\cdot 3}$

${}^{55}_{25}Mn_{30}$ ${}^{67}_{30}Zn_{37}$ ${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$ ${}^{85,87}_{37}Nb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{93}_{41}Nb_{52}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{112,113}_{48}Cd_{64,65}$
${}^{117,122,124}_{50}Sn_{67,72,74}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{143}_{60}Nd_{83}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{207,16\cdot13}_{82}Pb_{125,126}$ ${}^{16\cdot13,209}_{83}Bi^{*}_{125,126}$ ${}^{209}_{84}Po^{*}_{125}$
${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot143}_{113}Nh^{ie}_{173}$ ${}^{24\cdot13}_{125}Ch^{ie}_{187}$ ${}^{8\cdot43,2\cdot173,348}_{136,137,138}Fy^{ie}_{16\cdot13,209,210}$

$$\alpha_{1-50/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37+\frac{59\cdot(4\cdot 53-1)}{25\cdot 10^{5}}}}{83^2}=1/137.035999037435$$

${}^{53}_{24}Cr_{29}$ ${}^{59}_{27}Co_{32}$ ${}^{95}_{42}Mo_{53}$ ${}^{105,106}_{46}Pd_{59,60}$ ${}^{2\cdot59,124}_{50}Sn_{68,74}$ ${}^{127}_{53}I_{74}$ ${}^{141}_{59}\mathrm{Pr}_{82}$

$$\alpha_{1-50/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37+\frac{1}{200+\frac{2\cdot 7}{17}}}}{83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37+\frac{17}{2\cdot 3\cdot(8\cdot 71+1)}}}{83^2}$$
$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{77}_{34}Se_{43}$ ${}^{118,7\cdot17,120,122,124}_{50}Sn_{68,69,70,72,74}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{143}_{60}Nd_{83}$
${}^{175,176}_{71}Lu_{104,105}$ ${}^{200}_{80}Hg_{120}$ ${}^{4\cdot71,2\cdot143}_{113}Nh^{ie}_{171,173}$

$$\alpha_{2-50/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37}-\frac{27\cdot 11\cdot(2\cdot 9\cdot 11-1)}{2\cdot 10^{12}}}{83^2}=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$ ${}^{85,87}_{37}Nb_{48,50}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$
${}^{117,124}_{50}Sn_{67,74}$ ${}^{143}_{60}Nd_{83}$ ${}^{16\cdot13,209}_{83}Bi^{*}_{125,126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot143}_{113}Nh^{ie}_{173}$ ${}^{8\cdot43,2\cdot173,348}_{136,137,138}Fy^{ie}_{16\cdot13,209,210}$

$$\alpha_{2-50/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37+\frac{43\cdot 79\cdot 109}{5\cdot 10^{6}}}}{83^2}=1/137.035999111818$$

${}^{79,81}_{35}Br_{44,46}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{118}_{50}Sn_{68}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{183}_{74}W_{109}$ ${}^{197}_{79}Au_{118}$ ${}^{278}_{109}Mt^{*}_{169}$

$$\alpha_{2-50/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37+\frac{2}{27+\frac{7}{1000}}}}{83^2}=\frac{50+\frac{13}{16\cdot 3}+\frac{1}{43\cdot 37+\frac{16\cdot 125}{113\cdot(2\cdot 7\cdot 17+1)}}}{83^2}$$
$$=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{85,87}_{37}Nb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{97,99}_{43}Tc^{*}_{54,56}$ ${}^{112,113}_{48}Cd_{64,65}$
${}^{143}_{60}Nd_{83}$ ${}^{188,27\cdot7}_{76}Os_{112,113}$ ${}^{16\cdot13,209}_{83}Bi^{*}_{125,126}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot143}_{113}Nh^{ie}_{173}$

17. Formulas of the Fine-structure Constant with 112×173

$$\alpha_{1-141/112/173}=\frac{3\cdot 47+\frac{1}{2}-\frac{1}{9}+\frac{1}{8\cdot 27}-\frac{1}{2\cdot 31\cdot(2\cdot 5\cdot 107-1)-\frac{5}{17}}}{112\cdot 173}=1/137.035999037435$$

$${}^{10,11}_{5}B_{5,6}\quad {}^{14,15}_{7}N_{7,8}\quad {}^{16,17,18}_{8}O_{8,9,10}\quad {}^{19}_{9}F_{10}\quad {}^{27}_{13}Al_{14}\quad {}^{28,29,30}_{14}Si_{14,15,16}\quad {}^{31}_{15}P_{16}\quad {}^{35,37}_{17}Cl_{18,20}\quad {}^{36,38,40}_{18}Ar_{18,20,22}$$

$${}^{50,51}_{23}Cl_{27,28}\quad {}^{54,56,57,58}_{26}Fe_{28,30,31,32}\quad {}^{59}_{27}Co_{32}\quad {}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}\quad {}^{69,71}_{31}Ga_{38,40}\quad {}^{82,83,84}_{36}Kr_{46,47,48}$$

$${}^{96}_{42}Mo_{54}\quad {}^{105,108}_{46}Pd_{59,62}\quad {}^{107,109}_{47}Ag_{60,62}\quad {}^{112,113}_{48}Cd_{64,65}\quad {}^{131}_{54}Xe_{77}\quad {}^{135,136,137,138}_{56}Ba_{79,80,81,82}\quad {}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$$

$${}^{147}_{62}Sm_{85}\quad {}^{173}_{70}Yb_{103}\quad {}^{179,180}_{72}Hf_{107,108}\quad {}^{185,187}_{75}\mathrm{Re}_{110,112}\quad {}^{4\cdot 47,27\cdot 7}_{76}Os_{112,113}\quad {}^{209}_{83}Bi^{*}_{126}\quad {}^{209}_{84}Po^{*}_{125}\quad {}^{210}_{85}At^{*}_{125}$$

$${}^{2\cdot 131}_{103}Lr^{*}_{159}\quad {}^{270}_{107}Bh^{*}_{163}\quad {}^{15\cdot 19}_{112}Cn^{*}_{173}\quad {}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,11\cdot 19,210}\quad {}^{4\cdot 71,22\cdot 13}_{113}Nh^{ie}_{9\cdot 19,173}\quad {}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,2\cdot 131}$$

$$\alpha_{1-141/112/173}=\frac{3\cdot 47+\frac{1}{2}-\frac{1}{9}+\frac{1}{8\cdot 27}-\frac{1}{9\cdot 17\cdot(16\cdot 27-1)-\frac{2\cdot 17}{47}}}{112\cdot 173}=1/137.035999111818$$

$${}^{32,33,34,36}_{16}S_{16,17,18,20}\quad {}^{35,37}_{17}Cl_{18,20}\quad {}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}\quad {}^{50,51}_{23}Cl_{27,28}\quad {}^{60,62,64}_{28}Ni_{32,34,36}\quad {}^{83,84}_{36}Kr_{47,48}$$

$${}^{91,94}_{40}Zr_{51,54}\quad {}^{94,96,98}_{42}Mo_{52,54,56}\quad {}^{107,109}_{47}Ag_{60,62}\quad {}^{121,123}_{51}Sb_{70,72}\quad {}^{151,153}_{63}Eu_{88,90}\quad {}^{152,154,158,160}_{64}Gd_{88,90,94,96}$$

$${}^{159}_{65}Tb_{94}\quad {}^{160}_{66}Dy_{94}\quad {}^{244}_{94}Pu^{*}_{150}\quad {}^{251}_{98}Cf^{*}_{9\cdot 17}\quad {}^{36\cdot 7}_{99}Es^{*}_{9\cdot 17}\quad {}^{128\cdot 3}_{153}Ch^{ie}_{3\cdot 77}$$

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Appendix I: Research History

1	1-10	2020/7/8-23
2	11-12	2020/7/11-12
3	12-14	2018/8/21-9/6
3-1	13	2018/8/21-25
3-2	13	2018/9/2-6
3-3	14	2018/9/2-6
3-4	14	2018/9/5-6
4	14-15	2020/6/28-30
5	16	2020/6/30-7/1
6	16-17	2020/6/30-7/1,7/5
7	17-18	2020/7/3-5
8	18-19	2020/7/20-21
9	19	2020/7/21-25
10	19-20	2020/7/27-28
11	20	2020/7/28
12	20-22	2020/7/28-30
13	22	2020/8/2
14	23-27	2020/8/4-8
15	28-29	2020/8/11-13
16	30	2020/8/13-14
17	31	2020/8/14
Preparing this paper	1-32	2020/7/8-2020/8/15

Notes: Dates were recorded according to Beijing Time; *ie* means ideal extended elements; *GL* means Gregory-Leibniz formula.