On a Possible Logarithmic Connection between Einstein's Constant and the Fine-Structure Constant, in Relation to a Zero-energy Hypothesis

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Short Research Article

ABSTRACT

This paper brings into attention *a possible logarithmic connection between Einstein's constant and the fine-structure constant*, based on a hypothetical electro-gravitational resistivity of vacuum: we also propose *a zero-energy hypothesis* (**ZEH**) which is essentially a *conservation principle applied on zero-energy* that mainly states a general quadratic equation having a pair of conjugate mass solutions for each set of coefficients, thus predicting a new type of mass "symmetry" called here "*mass conjugation*" between elementary particles (**EPs**) which predicts the zero/non-zero rest masses of all known/unknown EPs to be conjugated in boson-fermion pairs; ZEH proposes a general formula for all the rest masses of all EPs from Standard model, also indicating a possible bijective connection between the three types of neutrinos and the massless bosons (photon, gluon and the hypothetical graviton), between the electron/positron and the W boson and predicting two distinct types of neutral massless fermions (modelled as conjugates of the Higgs boson and Z boson respectively) which are plausible candidates for dark energy and dark matter. ZEH also offers a new interpretation of Planck length as the approximate length threshold above which the rest masses of all known elementary particles have real number values (with mass units) instead of complex/imaginary number values (as predicted by the unique quadratic equation proposed by ZEH).

Keywords: Einstein's constant; fine-structure constant; electro-gravitational resistivity of vacuum (EGRV); zero-energy hypothesis (ZEH); conservation principle applied on zero-energy; elementary particles (EPs); mass conjugation; neutral massless fermions; dark energy; dark matter; a new interpretation of Planck length.

1. ON A POSSIBLE LOGARITHMIC CONNECTION BETWEEN EINSTEIN'S CONSTANT AND THE FINE-STRUCTURE CONSTANT

1.1 Introduction

This paper continues the work from other two past articles published by the author [1,2] by arguing for a possible base-2 logarithmic connection between large (gravitational) and small (electromagnetic) dimensionless constants of nature, which can be regarded as an alternative to the notorious Dirac's large number hypothesis.

1.2 Motivating Points

If the very large dimensionless physical constants (**DPCs**) (which are gravity-related in general, like *the inverse of the gravitational coupling constant* for example $\alpha_G{}^{-1} = \hbar c / \left(G m_e{}^2\right) \cong 10^{45}$) are deeply related with the small DPCs (usually close to 1 and related to quantum mechanics, like the *fine*

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structure constant $\alpha_0 \cong 137^{-1}$ for example, which is the value at rest of the running coupling constant of the electromagnetic field), by any (yet unknown) mathematical function, then a logarithmic function (**LF**) would be the simplest (and thus the most natural) candidate solution of connecting these large and small DPCs, as other authors also considered in the past [3,4]. Furthermore, even if it is not the case of such a logarithmical connection, possible LFs (connecting those DPCs) would still have to be ruled out first.

1.3 Observations

1.3.1 First observation

Each of all known electromagnetically-charged elementary particles (**CEP**) in the Standard model has a non-zero rest energy which, in turn, is always associated with non-zero spacetime curvature (gravity) as implied by General relativity. Furthermore, because the electron (with elementary electromagnetic charge $-|e|$, rest mass m_p and rest energy $E_e = m_e c^2$) is the lightest known CEP with the largest known (absolute)charge-to-(rest)energy ratio in nature $\phi_{\max} = |e| / E_e$, thus *electromagnetic charge appears to cannot exist (and thus cannot manifest) without a minimum degree of spacetime curvature determined by* E_e *(which contributes to the energy tensor) and the almost infinitesimal Einstein's constant* $\kappa = 8\pi G / c^4 \left(\cong 2.1\times10^{-43} N^{-1}\right)$ (the coupling constant of Einstein's field equation).

1.3.2 Second observation

There is a simple logarithmic function which appears to relate both κ and $\phi_{\max}$ to the fine-structure constant at rest $\alpha_0 = k_e q_e^2 / \left(\hbar c\right)\left(\cong 137^{-1}\right)$ (with $k_e = 1/\left(4\pi\varepsilon_0\right) \cong 8.99\times10^9 Nm^2 / C^2$ being the *Coulomb's constant in vacuum* measured at low non-relativistic energy scales) [5] which is the *asymptotical minimum* at rest of the electromagnetic running coupling constant $\alpha\left(E\right) = \alpha_0 / \left(1 - \alpha_0 f(\mathrm{E})\right)$ [1] [6]:

$$\alpha_0 \overset{99.92\%}{\cong} \left[\log_2\left(\kappa^{-1} k_e \phi_{\max}{}^2\right)\right]^{-1} \left(\cong 136.93^{-1}\right) \qquad (1)$$

α_0 may be directly related to $\left[\log_2\left(\kappa^{-1} k_e \phi_{\max}{}^2\right)\right]^{-1}$ with the following numbered arguments and explanations:

(**1**) As anticipated in the previous "Motivating points" sub-section of this paper, if the very *large dimensionless physical constants* (**DPCs**) (which are gravity-related in general, like $\kappa^{-1} k_e \phi_{\max} \cong 10^{41}$ for example) are deeply related with the small DPCs (usually close to 1 and related to quantum mechanics, like α_0 for example), by any (yet unknown) mathematical function, then a *logarithmic function* (**LF**) would be the simplest (and thus the most natural) candidate solution of connecting these large and small DPCs, as other authors also considered in the past. Furthermore, even if it is not the case of such a logarithmical connection, possible LFs (connecting those DPCs) would still have to be ruled out first.

(**2**) A direct logarithmic relation between an electromagnetic minimum of α_0 and an "electro-gravitational" maximum of nature $\phi_{\max}$ is quite intuitive;

(**3**) $\kappa^{-1}\left(\cong 4.82\times10^{42} N\right)$ (which is relatively close to the Planck force $F_{Pl} = c^4 / G \cong 1.2\times10^{44} N$) may be interpreted as a *global average "tension" of the spacetime fabric (as also interpreted by other authors* [7]) *which strongly opposes to any spacetime curvature (**SC**) induced by any source of energy (including electromagnetic and/or gravitational energy tensors)*: because of this resistance to

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[1] *the leading log approximation of* $\alpha\left(E\right)$*, which is only valid for large energy scales* $E >> E_e$ *, with* $f(\mathrm{E}) = \ln\left[\left(E / E_e\right)^{2/(3\pi)}\right]$ *, but uncertain validity for energy scales close to Planck energy* $E_{Pl} = \sqrt{\hbar c^5 / G} \cong 10^{19} GeV$

any induced SC (by any rest energy and/or movement of any bosonic or fermionic EP), κ^{-1} is identified with the approximate value at rest of an (energy/length-)scale-dependent *electro-gravitational resistivity of vacuum* (**EGRV**) represented by $R(E)$ with an asymptotic maximum value at rest $R_0 = \frac{2^{1/\alpha_0}}{k_e \phi_{\max}{}^2}\left(\cong 5.19\times 10^{42}\,N\right)$ (at zero-energy scale $E_0 = 0J$ which is physically unattainable, which makes R_0 an asymptotical maximum) estimated to exactly correspond to the *asymptotic minimum* α_0 (which α_0 corresponds to the theoretical-only energy scale $E_0 = 0J$ for which $f(\mathrm{E}_0) = \ln(0)$ and $\alpha_0 f(\mathrm{E}_0) = \alpha_0 \ln(0)$ have no real value and $\alpha(E) = \alpha_0 / \left(1 - \alpha_0 f(\mathrm{E})\right)$ is "renormalizable" to $\alpha(E_0) = \alpha_0 / 1 = \alpha_0$ which obviously represents an asymptotical minimum), so that $\alpha_0 = \left[\log_2\left(R_0 k_e \phi_{\max}{}^2\right)\right]^{-1}$. EGRV (measured by $R(E)$ and R_0 at rest) may be considered a truly fundamental parameter of spacetime with both c and G being actually determined by $R(E)$ and thus being indirect measures of EGRV. Another argument for α_0 measuring EGRV (which α_0 is alternatively defined as the probability of a real electron to emit or absorb a real photon) is that EGRV actually opposes to the photon emission process, in the sense that, for any real EP to emit a real photon, that photon first needs to overcome EGRV.

(**4**) EGRV is very plausibly determined by the short-lived *virtual particle-antiparticle pairs* (**VPAPs**) emerging from the vacuum, which VPAPs interact with both photons and gravitational waves plausibly limiting their speed to a common maximum speed-limit for both speed of gravity and speed of light in vacuum. Charged EPs (composing charged VPAPs) interact much more strongly with photons than neutral EPs (composing neutral VPAPs) so that $R(E)$ may actually depend on (and vary with) the ratio between the volumic concentrations of charged and neutral virtual EPs at various length scales of vacuum.

(**5**) By replacing $k_e \phi_{\max}{}^2$ with its equivalent $\alpha_0 \hbar c / E_e{}^2$, α_0 and R_0 become related by a special type of exponential equation such as:

$$\left(\frac{1}{\alpha_0}\right) 2^{1/\alpha_0} = \frac{R_0 \hbar c}{E_e{}^2} \qquad (2)$$

(**6**) Based on the previous equality, α_0 may also be considered as an indirect measure of EGRV and inversely redefined as the unique positive solution w of the exponential equation $(1/\mathrm{w})\,2^{1/\mathrm{w}} = C$, with:

$$C = \frac{R_0 \hbar c}{E_e{}^2}\left(\overset{>}{\cong} \frac{\kappa^{-1}\hbar c}{E_e{}^2}\right) \qquad (3)$$

(with $R_0 \overset{107.8\%}{\cong} \kappa^{-1}$ being the predicted asymptotic maximum of $R(E)$ for unattainable zero-energy scale $E = E_0 = 0J$)

This equation can be solved by using the *Lambert function* only after converting it to its natural-base (e) variant $\left(\ln(2)/\mathrm{w}\right) e^{\ln(2)/\mathrm{w}} = C \ln(2)$ so that:

$$\alpha_0 = \frac{\ln(2)}{W\left(C \ln(2)\right)} \qquad (4)$$

(**7**) By considering $\hbar$, E_e and c to all be (energy/length-) scale-invariant and based on $\alpha(E) = \alpha_0 / \left(1 - \alpha_0 f(\mathrm{E})\right)$, we remind that Coulomb's constant k_e is not scale-invariant, but is actually variable with the energy/length scale and is currently defined in modern physics as a function of $\alpha(E)$ such as $k_e(E) = \alpha(E)\hbar c / e^2$ (with $k_e = k_e(E_0) = \alpha_0 \hbar c / e^2$) which is equivalent to

$k_e(E)=\dfrac{\alpha_0\hbar c/e^2}{\left(1-\alpha_0 f(\mathrm{E})\right)}$. Starting from the previous definition $R_0=\dfrac{2^{1/\alpha_0}}{k_e\phi_{\max}{}^2}$, $R(E)$ *can be inversely deduced and generalized as* $R(E)=\dfrac{2^{1/\alpha(\mathrm{E})}}{k_e(E)\cdot\phi_{\max}{}^2}=\dfrac{2^{(1-\alpha_0 f(\mathrm{E}))/\alpha_0}}{\dfrac{\alpha_0\hbar c/e^2}{\left(1-\alpha_0 f(\mathrm{E})\right)}\cdot\dfrac{e^2}{E_e{}^2}}$, with $R_0=R(E_0)=\dfrac{2^{1/\alpha_0}}{k_e\phi_{\max}{}^2}=\dfrac{2^{1/\alpha_0}}{\left(\dfrac{\alpha_0\hbar c}{e^2}\right)\dfrac{e^2}{E_e{}^2}}=\boxed{\dfrac{2^{1/\alpha_0}}{\alpha_0\hbar c/E_e{}^2}}$; $R(E)$ can be further simplified to $R(E)=\dfrac{2^{1/\alpha_0-f(\mathrm{E})}}{\dfrac{\alpha_0\hbar c/E_e{}^2}{\left(1-\alpha_0 f(\mathrm{E})\right)}}=\dfrac{\boxed{2^{1/\alpha_0}}/2^{f(\mathrm{E})}}{\dfrac{\boxed{\alpha_0\hbar c/E_e{}^2}}{\left(1-\alpha_0 f(\mathrm{E})\right)}}$: the ratio marked with borders from the previous $R(E)$ formula is in fact R_0 which allows $R(E)$ to be further simplified to $R(E)=\dfrac{R_0\left(1-\alpha_0 f(\mathrm{E})\right)}{2^{f(\mathrm{E})}}$; α_0 was redefined as equal to $\dfrac{\ln(2)}{W(C\ln(2))}$ according to equation (**4**) and is also roughly approximable to $\alpha_0\cong\dfrac{1}{\log_2(C)}$ which allows to further simplify $R(E)$ to a final form definable as:

$$R(E)=\frac{R_0}{2^{\mathrm{f(E)}}}\left[1-\frac{\ln(2)f(\mathrm{E})}{W(C\ln(2))}\right]\cong\frac{R_0}{2^{\mathrm{f(E)}}}\left[1-\frac{f(\mathrm{E})}{\log_2(C)}\right] \quad (5)$$

This generalized $R(E)\left[=f(R_0)\right]$ *allows the generalization of* $C\left(=\dfrac{R_0\hbar c}{E_e{}^2}\right)$ *to* $C(E)=\dfrac{R(E)\hbar c}{E_e{}^2}$ *;* $\alpha(E)$ *can be also redefined as a function of this generalized* $C(E)=f\left[R(E)\right]$ *such as:*

$$\alpha(E)=\frac{\ln(2)}{W(\ln(2)C(\mathrm{E}))}=\frac{\ln(2)}{W\left(\ln(2)R(\mathrm{E})\hbar c/E_e{}^2\right)}\left(\cong\frac{\alpha_0}{\left(1-\alpha_0 f(\mathrm{E})\right)}\right) \quad (6)$$

The variation of the ratio $p(x)=\log_{10}\left(R\left(10^x MeV\right)/R_0\right)$ (illustrating the variation of $R(E)$ with the energy scale $E=10^x MeV$ in respect to R_0, which R_0 is the asymptotical maximum of $R(E)$) is graphed next:

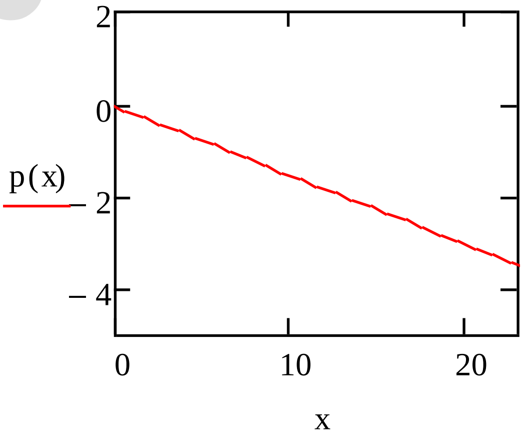


Fig. 1. The graph of $p(x) = \log_{10}\left(R\left(10^x MeV\right)/R_0\right)$, for integer $x \in [0,\ 22]$, with $x \cong 22$ for Planck energy $E_{Pl} = \sqrt{\hbar c^5 / G} \cong 10^{22} MeV$

Table 1. The comparative values of the predicted $\alpha(E) = \dfrac{\ln(2)}{W\left(\ln(2)R(\mathrm{E})\hbar c / E_e{}^2\right)}$ and the leading log approximation of $\alpha(E)\left[= \alpha_0 / \left(1 - \alpha_0 f(\mathrm{E})\right)\right]$ for $E = 10^x MeV$ and various values of $x \in [0,\ 22]$

$x \in [0,\ 22]$ and $E = 10^x MeV$	$\alpha(E) = \dfrac{\ln(2)}{W\left(\ln(2)R(\mathrm{E})\hbar c / E_e{}^2\right)}$	$\alpha(E) = \alpha_0 / \left(1 - \alpha_0 f(\mathrm{E})\right)$
0 (E=1 MeV)	136.9^{-1}	136.9^{-1}
5 (E=10^5 MeV)	134.5^{-1}	134.5^{-1}
10 (E=10^{10} MeV)	132^{-1}	132^{-1}
15 (E=10^{15} MeV)	129.6^{-1}	129.6^{-1}
20 (E=10^{20} MeV)	127.1^{-1}	127.1^{-1}

The prediction of a maximum allowed speed for any wave travelling in our universe and the graviton. Like $R_0\left(= R(E_0) \cong 5.19 \times 10^{42} N\right)$, note that $R(E)$ is measured in Newtons (force-units which can be expressed as Joule/meter thus *linear energy density units*): $R(E)$ is thus the huge *linear energy density of the vacuum* (expressed in J/m units and generated by the evancescent VPAPs which perpetually pop out from the vacuum and produce a huge tension in the spacetime fabric [7] measured by $R(E)$) which opposes to any gravitational or electromagnetic (transverse) wave propagating in the vacuum thus limiting the *wavefront speed* of those propagating physical waves to a maximum speed $v_{\max}$: in Einstein's General relativity (**GR**), both the speed of the electromagnetic waves (photons) in vacuum (c) and speed of gravity (v_g) are stated to be upper-bounded and equal to $v_{\max}$. The largeness of $R(E)$ indicates a very "rigid" spacetime (made so rigid by this huge inner tension, with its rigidness varying direct-proportionally with the length/size scale at which it is measured) which allows to be permeated only by waves with very small amplitudes (like in the case of the electromagnetic and gravitational transverse waves for example). No matter the nature of the physical wave (**PW**) travelling in the vacuum, this PW has to first attain *a minimal momentum* needed for it to produce at least a minimal deformation allowed by the hugely-tensioned spacetime fabric: in the case of electromagnetic waves (**EMWs**), this minimal (quantum angular) momentum is measured by Planck constant h; in the case of the gravitational waves (**GWs**), a minimal (needed) momentum h_g also very plausibly exists (as a sine-qua-non condition to surpass EGRV) and may actually be the quantum angular momentum of the hypothetical graviton (**gr**), which may have an energy scalar similar to the photon such as $E_{gr} = h_g \nu$ (with ν being the frequency of that hypothetical graviton). From the perspective of any straightly-traveling wavefront of any PW, spacetime can be regarded as an immense mesh of interwoven hugely-rigid (and relatively straight) strings (**str**) with (scale-dependent) *inner tension* $T_{str}(E) = R(E)$ and (scale-dependent) *linear massic density* $\rho_{str}(E) = R(E)/c^2$ (measured in kg/m linear density units and the energy-mass conversion factor c [the speed of light in vacuum] being scale independent): the *maximum allowed speed* $v_{\max}$ for any (low-amplitude) transverse wave traveling on such a rigid string is given by the famous Galileo's formula $v_{\max}=\sqrt{\frac{T_{str}(E)}{\rho_{str}(E)}}\left(=\sqrt{c^2}=c\right)$ (which is only valid for low-amplitude transverse waves, like EMWs and GWs actually are). **In conclusion**, $R(E)$ measures the (scale-dependent) rigidness of the spacetime fabric (**STF**) and predicts the existence of a maximum speed $v_{\max}(=c)$ for any PW traveling in this STF, but also the existence of minimal momentum for all EMWs (measured by h) and a distinct minimal momentum for all GWs (measured by the Planck-like gravitational quantum angular momentum h_g of the hypothetical graviton with energy-scalar $E_{gr}=h_g v$).

(**8**) Based on the approximate equality $R_0 \overset{107.8\%}{\cong} \kappa^{-1}\left(\frac{c^4}{8\pi G}\right)$, an asymptotic minimum for big G (corresponding to the unattainable zero-energy scale) can be inversely deduced as $G_0=\frac{c^4}{8\pi R_0}\left(\cong 6.2\times 10^{-11} m^3 kg^{-1} s^{-2}\cong 0.93G\right)$.

Based on $G_0=f(R_0)$ we also propose a generalized quantum gravitational constant $G_q(E)$ (which also varies with energy scale E) as derived from the same $R(E)$, also implying that big G may be actually a function of both the speed of light in vacuum (c) and speed of gravity $v_g(=c)$ (c^4 and $v_g{}^4$ to be more specifically) and EGRV, such as:

$$G_q(E)=\frac{v_{\max}{}^4}{8\pi R(E)}=\frac{c^4}{8\pi R(E)}=\frac{v_g{}^4}{8\pi R(E)} \quad (7)$$

If the gravitational waves (carriers of the gravitational force/field) emitted by any physical body spread simultaneously in the four distinct dimensions of a 4D spacetime (as also stated by Einstein's general relativity), then it is quite intuitively for the gravitational coupling "constant" $G_q(E)$ (which is argued here to be actually a composite "constant") to direct-proportionally depend on $v_g{}^4\left(=c^4\right)$ and inverse-proportionally depend on the (scale-dependent) electro-gravitational resistivity of vacuum (**EGRV**) $R(E)$ (which is an obstacle for those emitted gravitational waves to propagate any change in spacetime curvature and reach all their potential physical "targets").

From the previous relation, one may also note that any subtle variation of v_g and/or $R(E)$ may produce a slight variation of big G numerical value: this fact may actually explain the apparently paradoxal divergence (with deviations up to $\pm 1\%$) of big G experimental values despite the technical advances in the design of the modern experiments.

The variation of the ratio $q(x)=\log_{10}\left(Gq\left(10^x MeV\right)/G\right)$ (illustrating the variation of $G_q(E)$ with the energy scale $E=10^x MeV$ in respect to $G(\cong G_0)$, which G_0 is the asymptotical minimum of $G_q(E)$) is graphed next:

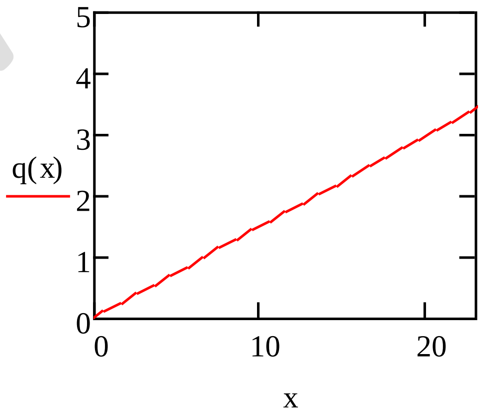


Fig. 2. The graph of $q(x)=\log_{10}\left(Gq\left(10^x MeV\right)/G\right)$**, for integer** $x\in[0,\ 22]$**, with** $x\cong 22$ **for Planck energy** $E_{Pl}=\sqrt{\hbar c^5/G}\cong 10^{22}MeV$

Table 2. The predicted values of $G_q(E) = \dfrac{c^4}{8\pi R(E)}$ for $E = 10^x MeV$ and various values of $x \in [0,\ 22]$

$x \in [0,\ 22]$ and $E = 10^x MeV$	$G_q(E)/G$
0 (E=1 MeV)	1.025
5 ($E=10^5$ MeV)	5.7
10 ($E=10^{10}$ MeV)	31.4
15 ($E=10^{15}$ MeV)	174.2
20 ($E=10^{20}$ MeV)	965.7
22 ($E=10^{22}$ MeV=E_{Pl})	1916

Final note of this 1st paper section. It is important to remember that $R(E)$, $\alpha(E) = \dfrac{\ln(2)}{W\left(\ln(2)R(\mathrm{E})\hbar c / E_e{}^2\right)}$ and $G_q(E) = \dfrac{c^4}{8\pi R(E)}$ were all deducted starting from the leading log approximation (**LLA**) of $\alpha(E)\left[=\alpha_0 / \left(1-\alpha_0 f(\mathrm{E})\right)\right]$, which has uncertain validity for energy scales close to Planck energy $E_{Pl}\left(\cong 10^{19} GeV\right)$ and that is why $R(E)$, $\alpha(E)\left[= f\left(R(\mathrm{E})\right)\right]$ and $G_q(E)$ also have uncertain validity for energy scales close to E_{Pl}: we argue that the zero-energy hypothesis (**ZEH**) proposed next may significantly help in solving this "bothering" uncertainty of LLA (which uncertainty is still a big problem of quantum electrodynamics) and also correct $R(E)$ and $G_q(E)$ values for large energy scales $E \cong E_{Pl}$.

2. A ZERO-ENERGY HYPOTHESIS (ZEH) APPLIED ON VIRTUAL PARTICLE-ANTIPARTICLE PAIRS (VPAPs)

We also propose *a zero-energy hypothesis* (**ZEH**) applied on any virtual particle-antiparticle pair (**VPAP**) popping out from the quantum vacuum at hypothetical length scales comparable to Planck scale. ZEH can be regarded as an extension of the notorious *zero-energy universe hypothesis* first proposed by the theoretical physicist Pascual Jordan [8], assuming minimal curvature (thus almost flat spacetime) at Planck scale. Presuming the gravitational and electrostatic inverse-square laws to be valid down to Planck scales and considering a VPAP composed from two electromagnetically-charged EPs (**CEPs**) each with non-zero rest mass m_{EP} and energy $E_{EP} = m_{EP}c^2$, electromagnetic charge q_{EP} and negative energies of attraction $E_g = -Gm_{EP}{}^2 / r$ [9] and $E_q = -k_e \left|q_{EP}\right|^2 / r$, *ZEH specifically states that*:

$$2E_{EP} + E_g + E_q = 0 \qquad (8)$$

Defining the ratios $\phi_g = G / r$ and $\phi_e = k_e / r$ the previous equation is equivalent to the following simple quadratic equation with unknown $x\left(= m_{EP}\right)$:

$$\phi_g x^2 - \left(2c^2\right)x + \phi_e q_{EP}{}^2 = 0 \qquad (9)$$

The previous equation is easily solvable and has two possible solutions which are both positive reals if $c^4 \ge \phi_g \phi_e q_{EP}{}^2 \ge 0$:

$$m_{EP} = x = \frac{c^2 \pm \sqrt{c^4 - \phi_g \phi_e q_{EP}{}^2}}{\phi_g} \qquad (10)$$

The realness condition $c^4 \ge \phi_g \phi_e q_{EP}{}^2 \ge 0$ implies the existence of a minimum distance between any two EPs (composing the same VPAP) $r_{\min} = q_{EP}\sqrt{Gk_e} / c^2 \cong 10^{-1} l_{Pl}$ (for $q_{EP(\cong e)} \in \left\{e, \pm \tfrac{1}{3}e, \pm \tfrac{2}{3}e\right\}$ and with l_{Pl} being the Planck length): obviously, for distances lower than $r_{\min}$ the previous equation has only imaginary solutions $x = m_{EP}$ for any charged EP; by this fact, ZEH offers a *new interpretation of the Planck length, as being the approximate distance under which charged EPs cannot have rest masses/energies valued with real numbers*; because k_e is actually variable with the energy/length scale and currently defined as a function of $\alpha(E)$ such as

$k_e(E) = \alpha(E)\hbar c / e^2$, $r_{\min}$ can be generalized as $r_{\min}(E) = (q_{EP} / \mathrm{e})\sqrt{G\alpha(E)\hbar c} / c^2$ (and can slightly vary as such). Note that $r_{\min}$ can be additionally corrected to include the strong force (implying color charge) and/or weak force (implying weak charge) between any quark (or gluon, or lepton coupling with the weak field) and its antiparticle (composing the same VPAP): however, these potential corrections are estimated to only slightly modify $r_{\min}(E)$ values so that they're not detailed this paper.

Both generic $x = m_{EP}$ solutions of the previous equation (**10)** indicate that, *because* m_{EP} *has discrete values only,* ϕ_G *(and* E_g *implicitly) and* ϕ_e *(and* E_q *implicitly) should all have discrete values only*. More interestingly, for neutral EPs (**NEPs**) with $q_{EP} = 0$ (which implies $\phi_g \phi_e q_{EP}{}^2 = 0$) and $r \geq r_{\min}(> 0m)$, $x = m_{EP}$ solutions may take *both*:

(**1**) non-zero positive values $m_{EP} = \left(c^2 + \sqrt{c^4}\right) / \phi_g = 2c^2 / \phi_g\,(> 0\,kg)$ (like in the case of all three types of neutrinos, the Z boson and the Higgs boson) AND

(**2**) zero values $m_{EP} = \left(c^2 - \sqrt{c^4}\right) / \phi_g = 0\,kg$ (like in the case of the gluon and the photon which both have zero rest mass $m_{EP}(= 0kg)$ and are assigned only relativistic mass/energy by the Standard model).

Important remark. In other words, formula (**10**) allows NEPs to be divided in two major families (NEPs with non-zero rest mass and NEPs possessing only relativistic mass) which is an indirect proof that m_{EP} is a function of q_{EP} (as requested/imposed by q_{EP}) and *not* vice-versa, as if the q_{EP} quantum also imposes fixed/discrete gradients $\Delta m = m_{EP(2)} - m_{EP(1)}\,(\geq 0\,kg) = f\left(q_{EP}\right)$ between various types of EPs.

ZEH additionally states that the two conjugated elementary mass solutions $m_{EP} = \left(c^2 \pm \sqrt{c^4 - \phi_g \phi_e q_{EP}{}^2}\right) / \phi_g$ (of ZEH's main equation) actually define a boson-fermion pair (with conjugated masses) called here "*conjugated boson-fermion pair*" (**CBFP**). ZEH actually conjectures *a new type of boson-fermion symmetry/"mass-conjugation"* based on ZEH's main *quadratic equation* (with partially unknown coefficients): ZEH mainly predicts *two distinct types of massless neutral fermions* (modelled as conjugates of the Higgs boson and Z boson respectively) with zero charge and zero rest mass (which, implicitly, don't couple electromagnetically and gravitationally and thus may be plausibly the main constituents of *dark matter* and *dark energy*), a bijective mass-conjugation between the three types of neutrinos and the massless bosons (gluon, photon and the hypothetical graviton) and a relation of mass-conjugation between the electron/positron and the $W^{\pm}$ boson (**see next**).

For the beginning, let us start to estimate the values of ϕ_g for the known electromagnetically-neutral EP (**NEP**). For $q_{EP} = 0$, the conjugated solutions expressed by formula (**10**) simplify for any NEP such as $m_{NEP} = \left(c^2 \pm c^2\right) / \phi_g$, resulting $\phi_{g(NEP)} = \left(c^2 \pm c^2\right) / m_{NEP}$.

Focusing on Higgs boson and Z boson and their ZEH-predicted correspondent/conjugated massless fermions. In a first step and defining the unit of measure of $\phi_g\left(= 2c^2 / m_{nEP}\right)$ as $u = m^2 kg^{-1} s^{-2}$, ZEH directly estimates ϕ_g for the Z boson (**Zb**) and Higgs boson (**Hb**) (with both Zb and Hb having non-zero rest energies) such as $\phi_{g(Zb)}\left(= 2c^2 / m_{Zb}\right) \cong 10^{42} u$ and $\phi_{g(Hb)}\left(= 2c^2 / m_{Hb}\right) \cong 8 \times 10^{41} u$. ZEH states that both Zb and Hb have two distinct correspondent/conjugated massless neutral fermions formally called the "Z fermion" (**Zf**) (which shares the same $\phi_{g(Zb)}\left(\cong 10^{42} u\right)$ with Zb) and the "Higgs fermion" (**Hf**) (which shares the same $\phi_{g(\mathrm{Hb})}\left(\cong 8\times10^{41}u\right)$ with Hb) with zero rest masses $m_{Zf}=\left(c^2-c^2\right)/\phi_{g(Zb)}\left(=0\,kg\right)$ and $m_{Hf}=\left(c^2-c^2\right)/\phi_{g(\mathrm{Hb})}\left(=0\,kg\right)$ (thus both moving with the speed of light in vacuum and possessing only relativistic masses instead of rest masses). Based on the previously defined $r_{\min}\left(\cong 10^{-1}l_{Pl}\right)$, we then obtain $G_{Zb(\min)}\left(=\phi_{g(Zb)}r_{\min}\right)\cong G_{Hb(\min)}\left(=\phi_{g(Hb)}r_{\min}\right)\cong 2\times10^{16}G$: based on these huge predicted lower bounds for big G values at Planck scales, ZEH states that E_g *may reach the same magnitude as* E_q $\left(E_g\cong E_q\Leftrightarrow\phi_g {m_{EP}}^2\cong\phi_e {q_{EP}}^2\right)$ *at scales comparable to Planck scale* and that $R(E)$ *(thus* $\alpha(E)$ *and* $G_q(E)$ *) may actually take discrete values only*. Additionally, ZEH helps correcting the previously defined $G_q(E)$ *by assigning it values much larger than the previously tabled* $G_q\left(10^{22}MeV\right)=1916\cdot G$ *at scales comparable to* $r_{\min}$.

Focusing on all three types of neutrinos, photon, gluon and hypothetical graviton. In a second step, ZEH estimates the lower bounds of ϕ_g for all known three neutrinos, as deducted from the currently estimated upper bounds of the non-zero rest energies of all three known types of neutrino: the electron neutrino (**en**) with $E_{en}<1eV$ [10], the muon neutrino (**mn**) with $E_{mn}<0.17MeV$ [11] and the tau neutrino (**tn**) with $m_{tn}<18.2MeV$ [12,13]: $\phi_{g(en)}>_{\cong}10^{53}u$, $\phi_{g(mn)}>_{\cong}6\times10^{47}u$ and $\phi_{g(tn)}>_{\cong}6\times10^{45}u$, with $\phi_{g(en)}$ being assigned a very large big G lower bound $G_{en(\min)}\left(=\phi_{g(\mathrm{en})}r_{\min}\right)\cong 2\times10^{28}G$ thus strengthening the previously introduced (sub-)hypothesis $\phi_g {m_{EP}}^2\cong\phi_e {q_{EP}}^2$ at scales close to Planck scale. **Remark**. It is easy to observe that ZEH generally predicts progressively larger "real" big G values for progressively smaller m_{EP} : an additional explanation for this correlation shall be offered later in this paper. We must also remind that a specific virtual EP (**VEP**) may have a variable mass lower or equal to the mass m_{EP} of the real "version" of the same EP $\left(m_{VEP}\le m_{EP}\right)$ and that is why the "virtual" big G values assigned to the gravitational field acting between a virtual particle and its antiparticle (part of the same VPAP) may be even larger than the previously calculated ones.

ZEH cannot directly estimate the values of $\phi_{g(nEP)}$ for the massless photon (**ph**) $\phi_{g(ph)}$ and the gluon (**gl**) $\phi_{g(gl)}$ due to the division-by-zero error/paradox. However, ZEH additionally states that $\phi_{g(ph)}$ and $\phi_{g(gl)}$ may have very large values coinciding with $\phi_{g(en)}$, $\phi_{g(mn)}$ and $\phi_{g(tn)}$. More specifically, ZEH speculates that $\phi_{g(ph)}>\phi_{g(gl)}$ and that there also exists a massless graviton (**gr**) defined by $\phi_{g(gr)}>\phi_{g(ph)}\left(>\phi_{g(gl)}\right)$ so that: $\boxed{\phi_{g(gr)}=\phi_{g(en)}}$, $\boxed{\phi_{g(ph)}=\phi_{g(mn)}}$ and $\boxed{\phi_{g(gl)}=\phi_{g(tn)}}$.

Focusing on the electron-W boson conjugated pair. In a third step, ZEH additionally states that the W boson and the electron also form a conjugate boson-fermion pair with rest masses $m_e=\left(c^2-\sqrt{c^4-\phi_{g(\mathrm{W}/e)}\phi_{e(\mathrm{W}/e)}{q_e}^2}\right)/\phi_{g(\mathrm{W/e})}$ and $m_W=\left(c^2+\sqrt{c^4-\phi_{g(\mathrm{W}/e)}\phi_{e(\mathrm{W}/e)}{q_e}^2}\right)/\phi_{g(\mathrm{W/e})}$.

The common term $\sqrt{c^4-\phi_{g(W-e)}\phi_{e(W-e)}{q_e}^2}$ of both rest masses (m_e and m_W) disappears when summing $m_e+m_W=2c^2/\phi_{g(\mathrm{W/e})}$, from which their common/shared $\phi_{g(\mathrm{W/e})}$ ratio can be reversely estimated as

$\boxed{\phi_{g(\mathrm{W/e})} = 2c^2 / \left(m_e + m_W\right) \cong 1.25 \times 10^{42} u}$, which is relatively close to $\phi_{g(Zb)} \left(\cong 10^{42} u\right)$ and $\phi_{g(\mathrm{H}b)} \left(\cong 8 \times 10^{41} u\right)$. The other $\phi_{\mathrm{e(W/e)}}$ ratio can be also reversely estimated from both m_W (or m_e) and $\phi_{g(\mathrm{W/e})}$ as $\boxed{\phi_{\mathrm{e(W/e)}} \cong 6.4 \times 10^{24} F^{-1}}$.

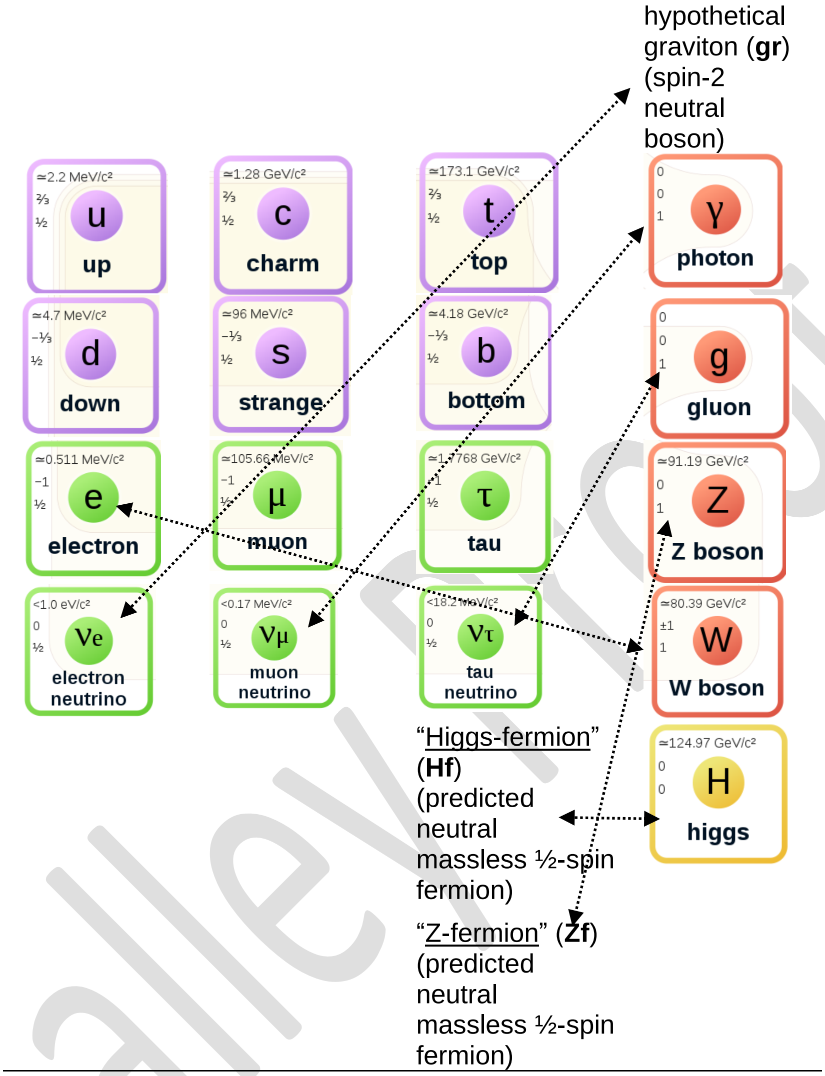
All the proposed pairs of EP mass-conjugates (as stated by ZEH are illustrated in the **next table** (each with their specific assigned ϕ_g and ϕ_e ratios).

Table 3. The pair of conjugated EPs predicted by ZEH

Boson (/correspondent conjugate boson of a known fermion)	**Fermion (/correspondent conjugate fermion of a known boson)**	**Common/ shared ϕ_g ratio of a conjugated boson-fermion pair**	**Common/ shared ϕ_e ratio of a conjugated boson-fermion pair**
Non-quark EPs as treated by ZEH			
hypothetical graviton (**gr**) (spin-2 neutral boson)	electron neutrino (**en**)	$\phi_{g(gr)} = \phi_{g(en)}$ $\left(> 1.1 \times 10^{53} u\right)$	?
photon (**ph**) (spin-1 neutral boson)	muon neutrino (**mn**)	$\phi_{g(ph)} = \phi_{g(mn)}$ $\left(> 6 \times 10^{47} u\right)$	?
gluon (**gl**) (spin-1 neutral boson, with color charge only)	tauon neutrino (**tn**)	$\phi_{g(gl)} = \phi_{g(tn)}$ $\left(> 5.6 \times 10^{45} u\right)$	?
Z boson (**Zb**) (spin-1 neutral boson)	"Z-fermion" (**Zf**) (predicted neutral massless ½-spin fermion)	$\phi_{g(Zb)}$ $\cong 10^{42} u$	?
Higgs boson (**Hb**) (spin-0/scalar neutral boson)	"Higgs-fermion" (**Hf**) (predicted neutral massless ½-spin fermion)	$\phi_{g(Hb)}$ $\cong 8 \times 10^{41} u$	?
W boson (**Wb**) (spin-1 charged boson)	electron (**e**)	$\phi_{g(\mathrm{W/e})} \cong$ $1.25 \times 10^{42} u$	$\phi_{\mathrm{e(W/e)}} \cong$ $6.4 \times 10^{24} F^{-1}$

All the proposed pairs of EP-conjugates (as stated by ZEH) are also illustrated in the **next table**: as it can be seen from this next table, ZEH transforms the already "classical" 2D table of EPs (from the Standard model [**SM**] of particle physics) in a 3D structure/table in which EPs are grouped *not only* in boson and fermion families/subfamilies, BUT they are also grouped and inter-related by an "underneath" relation of boson-fermion mass conjugation, all based on the same simple semi-empirical quadratic equation proposed by ZEH.

Table 4. The pairing of conjugated EPs predicted by ZEH and marked by interconnecting arrows. Source of image extracts: en.wikipedia.org/wiki/File:Standard_Model_of_Elementary_Particles.svg
◄·····► (marks each pair of conjugates stated by ZEH, except quarks)



In a checkpoint conclusion, ZEH has the potential to explain *the non-zero rest masses of 12 known and hypothetical EPs (Zb & Zf, Hb & Zf, gr & en, ph & mn, gl & tn, Wb & electron) plus their antiparticles by only seven discrete ratios*: $\phi_{g(Zb)}\left(=\phi_{g(Zf)}\right)$, $\phi_{g(Hb)}\left(=\phi_{g(Hf)}\right)$, $\phi_{g(gr)}\left(=\phi_{g(en)}\right)$, $\phi_{g(ph)}\left(=\phi_{g(mn)}\right)$, $\phi_{g(gl)}\left(=\phi_{g(tn)}\right)$ and $\phi_{g(\mathrm{W/e})}\left(\&\,\phi_{\mathrm{e(W/e)}}\right)$.

ZEH uses the predicted minimum length/distance $r_{\min}\left(=\left|q_{EP}\right|\sqrt{Gk_e}/c^2\cong 10^{-1}l_{Pl}\right)$ needed for any virtual particle-antiparticle pair (**VPAP**) to pop out from the vacuum at the first place (as stated and predicted by ZEH for all rest masses to be describable by real numbers with mass units) *and* all the ZEH-predicted ϕ_g and ϕ_e ratios (briefly listed in the first table of this paper) to predict (**pr.**) the big G and Coulomb's constant k_e values at scales $r_{\min}\left(\cong 10^{-1} l_{Pl}\right)$ comparable to Planck scale as $\boxed{G_{pr}=\phi_{g(pr)} r_{\min}}$ and $\boxed{k_{e(pr)}=\phi_{e(pr)} r_{\min}}$ (**see the next table**).

Table 5. The predicted big G values $G_{pr}=\phi_{g(pr)} r_{\min}$ and Coulomb's constant values $k_{e(pr)}=\phi_{e(pr)} r_{\min}$ for all pairs of conjugated EPs predicted by ZEH

Pair of conjugated EPs	**Common/ shared ϕ_g and ϕ_e ratios**	G_{pr} $\left(=\phi_{g(pr)} r_{\min}\right)$	$k_{e(pr)}$ $\left(=\phi_{e(pr)} r_{\min}\right)$
Non-quark EPs as treated by ZEH			
hypothetical graviton (**gr**) & electron neutrino (**en**)	$\phi_{g(gr)}=\phi_{g(en)}$ $\left(>1.1\times10^{53} u\right)$?	$>2.1\times10^{27} G$	?
photon (**ph**) - muon neutrino (**mn**)	$\phi_{g(ph)}=\phi_{g(mn)}$ $\left(>6\times10^{47} u\right)$?	$>1.2\times10^{22} G$ (the same for all photons, no matter their frequency)	?
gluon (**gl**) - tauon neutrino (**tn**)	$\phi_{g(gl)}=\phi_{g(tn)}$ $\left(>5.6\times10^{45} u\right)$?	$>1.2\times10^{20} G$	?
Z boson (**Zb**) & "<u>Z-fermion</u>" (**Zf**)	$\phi_{g(Zb)}$ $\cong 10^{42} u$	$\cong 2.1\times10^{16} G$	?
Higgs boson (**Hb**) & "<u>Higgs-fermion</u>" (**Hf**)	$\phi_{g(Hb)}$ $\cong 8\times10^{41} u$	$\cong 1.7\times10^{16} G$	?
W boson (**Wb**) & electron (**e**)	$\phi_{g(\mathrm{W/e})}\cong$ $1.25\times10^{42} u$ $\phi_{\mathrm{e(W/e)}}\cong$ $6.4\times10^{24} F^{-1}$	$\cong 2.6\times10^{16} G$	$\cong 10^{-21} k_e$

Interpretation. From the previous table, one can easily remark that ZEH predicts a big G which may increase (when decreasing the length scale of measurement up to values $G_{pr}\left(=\phi_{g(\mathrm{en})} r_{\min}\right)>2.1\times10^{27} G$ at $r_{\min}\left(\cong 10^{-1} l_{Pl}\right)$ length scales (comparable to Planck scale): concomitantly (and accordingly to the same table) and interestingly, ZEH predicts that Coulomb's constant k_e may drop down to values $k_{e(pr)}\left(=\phi_{e(\mathrm{W/m})} r_{\min}\right)\cong 10^{-21} k_e$ at the same length scales close to $r_{\min}\left(\cong 10^{-1} l_{Pl}\right)$.

Important observation. For the electron rest mass $\left(m_e\right)$ at macroscopic scales X (for which $r\left(>>r_{\min}\right)$ $G_{pr}\cong G$) for example, the $\frac{k_e q_e^{2}}{G m_e^{2}}\left(\cong 4.2\times10^{42}\right)$ dimensionless ratio reaches almost 43 orders of magnitude (in favor of the $k_e q_e^{2}$ numerator): interestingly, at Planck (**Pl**) scales the ZEH-predicted big G may grow by at least 27 orders of magnitude (up to $G_{\mathrm{Pl}}\cong 10^{27} G$) and k_e may drop by at least 21 orders of magnitude (down to $k_{e(Pl)} \cong 10^{-21} k_e$) which may bring the ratio $\frac{k_{e(\mathrm{Pl})} q_e^{\,2}}{G_{\mathrm{Pl}} m_e^{\,2}}$ relatively close to 1; the Coulomb's constant k_e is currently defined as a function of the running coupling constant of the electromagnetic field (**EMF**) $\alpha(E) = \alpha_0 / \left(1 - \alpha_0 f(\mathrm{E})\right)$ so that $k_e(\mathrm{E}) = \alpha(\mathrm{E}) \hbar c / q_e^{\,2}$: the currently known $\alpha(E)$ (which is currently predicted by its leading log approximation [**LLA**] to can only grow when approaching Planck energy/length scales E_{Pl}) is thus alternatively predicted by ZEH to actually slightly grow (as described by LLA) at first (when decreasing the length scale) but then to drop significantly down to $\alpha_{Pl} = \alpha(E_{Pl})$ so that $k_{e(Pl)} = \alpha_{Pl} \hbar c / q_e^{\,2} \left(\cong 10^{-21} k_e\right)$ which is equivalent to $\boxed{\alpha_{Pl} \cong 10^{-21} \alpha_0}$ (which tends to the value of the *gravitational coupling constant* $\alpha_G = \frac{G m_e^{\,2}}{\hbar c} \cong 10^{-45} \cong 10^{-43} \cdot \alpha_0$) and indicates EMF to probably possess *asymptotic freedom* (like the strong nuclear field was already proved to have). Based on the previous observation, ZEH additionally states (and predicts) that the gravitational field (**GF**) progressively grows in strength when approaching the $r_{\min} \left(\cong 10^{-1} l_{Pl}\right)$ length-scale (up to $G_{\mathrm{Pl}} > 10^{27} G$) and the electromagnetic field (**EMF**) slightly grows (as described by LLA) and then drops in strength (when approaching the same $r_{\min}$ length-scale) down to $k_{e(Pl)} < 10^{-21} k_e$ and $\alpha_{Pl} \cong 10^{-21} \alpha_0$ reaching the following equality at $r_{\min}$ scales:

$$G_{Pl} m_e^{\,2} \cong k_{e(Pl)} q_e^{\,2} \left(\cong m_e c^2 r_{\min}\right) \tag{11}$$

The previous equation is essentially *a fundamental principle of electro-gravitational strength balance/symmetry at Planck scales*, a principle which allows (as a sine-qua-non condition added to *Heisenberg's uncertainty principle*) the existence of virtual particle-antiparticle pairs (VPAPs) from the first place.

ZEH uses the same minimum length/distance $\boxed{r_{\min} \left(= \left|q_{EP}\right| \sqrt{G k_e} \,/\, c^2 \cong 10^{-1} l_{Pl}\right)}$ needed for any virtual particle-antiparticle pair (**VPAP**) to pop out from the vacuum at the first place (as stated and predicted by ZEH for all rest masses to be describable by real numbers with mass units) to predict a series of "practical" (**pr.**) radii $\left(r_{pr}\right)$ needed for each known/unknown point-like EP (to pop out as a VPAP in the first place) and a finite maximum allowed massic/energetic density in our universe (**OU**).
For big G values to grow progressively with a decreasing length scale r_{pr} , ZEH proposes/conjectures that *both* the very large (but finite!) maximum $G_{\max} = G_{Pl} \left(> 2.1 \times 10^{27} G\right)$ and very small (but finite!) $r_{\min} \left(\cong 10^{-1} l_{Pl}\right)$ *bijectively* correspond only to the electron neutrino (**en**) (with very small BUT finite rest mass $m_{en} < 1 eV / c^2$) which thus generates *a conjectured maximum (large but finite!) allowed (3D spherical) "practical" massic density in our universe (**OU**) identified with the massic density of en (which has a predicted lower bound significantly smaller than Planck density* $\rho_{Pl} = m_{Pl} / l_{Pl}^{\,3} \cong 10^{96} kg\, m^{-3}$**)**:

$$\rho_{OU(\max)} = \rho_{en} \left(\cong \frac{m_{en}}{4\pi/3\, r_{\min}^{\,3}} \right) > 1.6 \times 10^{71} kg\, m^{-3} \tag{12}$$

Furthermore, ZEH ambitiously (and additionally) conjectures that the pre-Big-Bang singularity (**pBBS**) was *not* infinitely dense (thus wasn't a true gravitational singularity with infinite density!) but had a large-but-finite density ρ_{pBBS} equal to $\rho_{en} \left(> 1.6 \times 10^{71} kg\, m^{-3}\right)$ OR in the $\left[\rho_{en}, \rho_{Pl}\right]$ closed interval, thus being a quasi-singularity with $\boxed{\rho_{pBBS} = \rho_{OU(\max)}}$ or $\rho_{pBBS} \in \left[\rho_{en}, \rho_{Pl}\right]$ with all known/unknown EPs being redefined as remnant "crocks" of this pBBS and sharing approximately the same unique density $\boxed{\rho_{EP} \cong \rho_{pBBS} \left(= \rho_{OU(\max)}\right)}$ (ZEH's unique-density conjecture [**UDC**]).

Based on the previously defined UDC, ZEH also proposes a simple formula for calculating the practical radii $r_{pr(EP)}$ of any known type of known/unknown EP with non-zero rest mass:

$$r_{pr(EP)} \overset{>}{\cong} r_{\min} \sqrt[3]{m_{EP} / m_{en}} \quad (13)$$

For example, the previously formula predicts that the Higgs boson (**Hb**) has a practical radius with a lower bound defined by $r_{pr(Hb)} \cong r_{\min} \sqrt[3]{m_{Hb} / m_{en}} \cong 5 \times 10^3 r_{\min}$, with all the other known/unknown EPs with non-zero rest masses smaller than m_{Hb} having their practical radii approximately in the closed interval $\left[r_{\min}, 5 \times 10^3 r_{\min}\right]$.

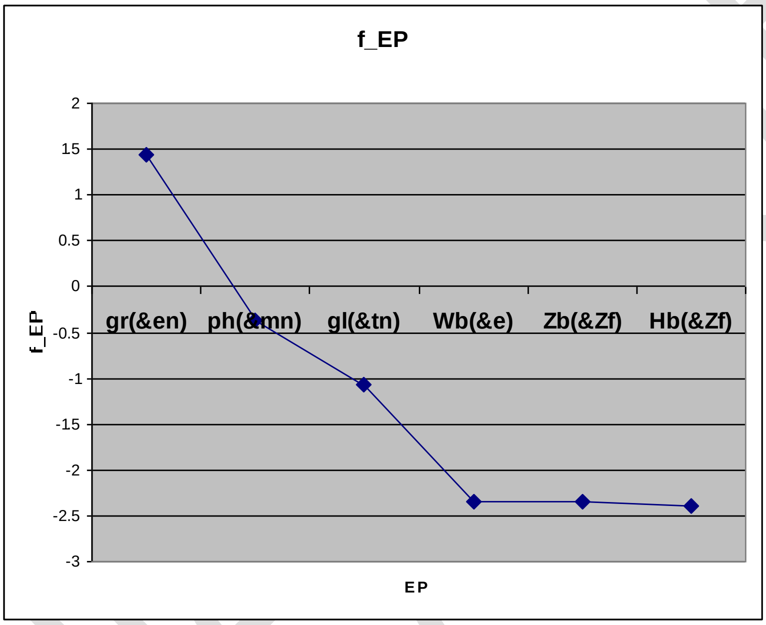


Fig. 3. The variation of f_{EP} with $r_{pr(EP)}$ which illustrates the increase of big G $\left(G_{EP}\right)$ values when the practical radius $r_{pr(EP)}$ (of each EP type in part) decreases, with all previously discussed EPs being arranged in the ascending order of their $r_{pr(EP)}$ values (from left to right). The rhombic blue points from this graph (indirectly) correspond to each G_{EP} value (assigned to each type of EP) and the slopes of the segments between each any two adjacent points (indirectly) correspond to each $\phi_{g(EP)}$ (assigned to the EP that corresponds to the left rhombic point of each segment in part)

ZEH also states that known/unknown EPs with non-zero rest masses larger than m_{en} and practical radii larger than $r_{\min}$ correspond to smaller big G values $G_{EP} < G_{\max} \left(= G_{Pl}\right)$; more specifically, ZEH actually proposes the following generalizations for any neutral or charged EP:

$$G_{EP} m_{EP}{}^2 \cong 2 m_{EP} c^2 r_{\mathrm{pr(EP)}} \quad \left(\textbf{for neutral EPs}\right) \quad (14)$$

$$G_{EP} m_{EP}{}^2 \cong k_{e(\mathrm{EP})} q_{EP}{}^2 \left(\cong m_{EP} c^2 r_{pr(\mathrm{EP})}\right) \quad \left(\textbf{for charged EPs}\right) \quad (15)$$

Based on the previous two equations, the big G values corresponding to each length-scale measured by each practical radii in part (of each type of EP in part) can be reversely deduced as:

$$G_{EP} \cong \frac{2c^2}{m_{EP}} r_{\mathrm{pr(EP)}} \cong \frac{2c^2 r_{\min}}{m_{EP}} \sqrt[3]{\frac{m_{EP}}{m_{en}}} \quad (16)$$
$$(\textbf{for neutral EPs})$$

$$G_{EP} \cong \frac{c^2}{m_{EP}} r_{\mathrm{pr(EP)}} \cong \frac{c^2 r_{\min}}{m_{EP}} \sqrt[3]{\frac{m_{EP}}{m_{en}}} \quad (17)$$
$$(\textbf{for charged EPs})$$

To illustrate the growth of G_{EP} with the decrease in the length scale measured by $r_{\mathrm{pr(EP)}}$ ZEH proposes the double-logarithmic ratio $f_{EP} = \log_{10}\left(\frac{\log_{10}\left(G_{EP}/G\right)}{r_{pr(EP)}/r_{\min}}\right)$ which is graphed next.

These new large G_{EP} values integrate with $G_q(E)$ (and corrects it in the Planck domain), which $G_q(E)$ grows in a smooth manner from macroscopic scales down to scales close to (but larger than) $r_{pr(\mathrm{Hb})} = 5\times10^3 r_{\min}$, but then grows abruptly and in a quantum saltatory manner (by a set of discrete slopes $\phi_{g(EP)} = G_{EP}/r_{pr(EP)}$) in the approximate $\left[r_{\min}, r_{\mathrm{pr(Hb)}}\right]$ interval, probably because a granular structure of spacetime in that closed interval $\left[r_{\min}, r_{\mathrm{pr(Hb)}}\right]$.

In a checkpoint conclusion, the zero/non-zero discrete values of rest masses of known and unknown EPs $\left(m_{EP} = \frac{c^2 \pm \sqrt{c^4 - \phi_g \phi_e q_{EP}{}^2}}{\phi_g}\right)$ are all stated by ZEH to be actually generated by the quantized electromagnetic charge and by the discrete values of $R(E)$ and $G_q(E)$ implicitly in the sub-domain of length scales $\left[r_{\min}, r_{\mathrm{pr(Hb)}}\right]$, a quantization/discreteness probably determined by a granular/quantum structure of spacetime in that sub-domain of length scales (allowing only discrete practical radii r_{pr}).

Focusing on the three generations of quarks and both the muon and the tauon. ZEH may deal with the known quarks and the muon plus tauon (which are considered two distinct excited states of the electron) in multiple ways, all speculative however:

(**1**) Each quark in part may have its own correspondent boson mass-conjugate (named here "*quark-boson*", because its has the same fractional charge as its mass-conjugate quark); however, ZEH-3a doesn't allow to directly estimate the ϕ_g and ϕ_{e} ratios for each (quark-) boson-quark pair, because the true existence of these theoretical quark-bosons (and their rest masses) is uncertain: the possible existence of *quark bosons* (with fractional charge) obviously implies the possible existence of *additional "exotic" fundamental physical forces/fields still unknown in the present*. If the ZEH's mass-conjugation principle wouldn't apply to quarks (which is probably not the case), the discrete values of ϕ_g for all the other (charged) EPs with $q_{EP} = f \cdot e$ ($m_{EP} = x$ and $f \in \{\pm1, \pm\tfrac{2}{3}, \pm\tfrac{1}{3}\}$) could have been easily determined by using the previously introduced additional statement of ZEH $\left(\phi_g m_{EP}{}^2 \cong \phi_e q_{EP}{}^2\right)$ which simplifies the initial equation (**9**) which becomes $\phi_g x^2 - \left(2c^2\right)x + f^2\phi_g x^2 = 0$ (which, by dividing both left and right terms with $x(\neq 0)$ becomes equivalent to $\left(1+f^2\right)\phi_g x - \left(2c^2\right) = 0$ and $\left(1+f^2\right)\phi_g x = 2c^2$) and allows the estimation of $\phi_g \cong \frac{2c^2}{\left(1+f^2\right)m_{EP}}$ as approximately $\phi_g \cong 2c^2/\left(2m_{EP}\right) \cong c^2/m_{EP}$ for all known $|e|$-charged leptons (with $f = \pm1$), with slight variations in the case of quarks (depending on the exact fractional charge of those quarks):

$\phi_g \cong 2c^2 / \left(\tfrac{13}{9} m_{EP} \right)$ (in the case of $\tfrac{2}{3}|e|$ -quarks) and $\phi_g \cong 2c^2 / \left(\tfrac{10}{9} m_{EP} \right) \cong c^2 / \left(\tfrac{5}{9} m_{EP} \right)$ in the case of $\tfrac{1}{3}|e|$-quarks).

(**2**) Quarks with the same fractional charge however (which are aligned horizontally in the particles table of the Standard model), may be actually conjugated in fermion-fermion (quark-quark) pairs like *up-charm pair (of conjugates) quarks* (**uq-cq**), *down-strange pair of quarks* **(dq-sq)**, *top-"X_top" pair of quarks* **(tq-Xtq)** and *bottom-"X_bottom" pair of quarks* **(bq-Xbq) pair**, with the here-named "X_top" quark (with electromagnetic charge [**emc**] ±2/3e) and "X_bottom" quark (with emc ±1/3e) actually forming a **4th generation of quarks** (which is still a subject of active research at the LHC today and was first predicted by Sheldon Lee Glashow and James Bjorken which allowed for a better description of the weak interaction and implied a mass formula that correctly reproduced the masses of the known mesons); in this case, ZEH has the advantage to can directly estimate these common/shared ratios: $\phi_{\mathrm{g(u/cq)}}$ & $\phi_{e\mathrm{(u/cq)}}$ (shared by uq-cq) and $\phi_{\mathrm{g(d/sq)}}$ & $\phi_{e(d\mathrm{/sq})}$ (shared by dq-sq); for the uq-cq we have $m_{uq} + m_{cq} = 2c^2 / \phi_{g\mathrm{(u/cq)}}$, from which their common/shared $\phi_{g(u\mathrm{/cq})}$ ratio can be reversely estimated as $\boxed{\phi_{g\mathrm{(u/cq)}} = 2c^2 / \left(m_{uq} + m_{cq} \right) \cong 7.8 \times 10^{43} u}$: the other $\phi_{\mathrm{e(u/cq)}}$ ratio can be also reversely estimated from both m_{cq} (or m_{uq}) and $\phi_{g\mathrm{(u/cq)}}$ as $\boxed{\phi_{\mathrm{e(u/cq)}} \cong 6.4 \times 10^{25} F^{-1}}$, with assigned $\boxed{k_{e\mathrm{(u/cq)}} = \phi_{e\mathrm{(u/cq)}} r_{\min} \cong 10^{-20} k_e}$; for dq-sq we have $m_d + m_s = 2c^2 / \phi_{g\mathrm{(d/sq)}}$, from which their common/shared $\phi_{g\mathrm{(d/sq)}}$ ratio can be reversely estimated as $\boxed{\phi_{g\mathrm{(d/sq)}} = 2c^2 / \left(m_{dq} + m_{sq} \right) \cong 10^{45} u}$: the other $\phi_{\mathrm{e(d/sq)}}$ ratio can be also reversely estimated from both m_{dq} (or m_{sq}) and $\phi_{g\mathrm{(d/sq)}}$ as $\boxed{\phi_{\mathrm{e(d/sq)}} \cong 5.1 \times 10^{26} F^{-1}}$, with assigned $\boxed{k_{e\mathrm{(d/sq)}} = \phi_{e\mathrm{(d/sq)}} r_{\min} \cong 7.9 \times 10^{-20} k_e}$.

(3) Another possibility is that only the first two generations of quarks may be actually reciprocally conjugated in fermion-fermion (quark-quark) pairs like uq-cq and dq-sq; the 3rd generation of quarks may actually be conjugated with two unknown "quark bosons" (with fractional charge) called here "*top-boson*" (**Tb**) (with emc ±2/3e and conjugating to the top-quark) and "*bottom-boson*" (**Bb**) (with emc ±1/3e and conjugating to the bottom-quark).

(**4**) In the case of the muon (**m**) and tauon (**t**) (which are currently considered two distinct excited states of the electron) ZEH offers two possibilities of mass-conjugation: (**a**) the muon and the tauon may be conjugated with two predicted hypothetical bosons (which are analogously considered two distinct excited (super-heavy) states of the W boson) called here the "W muonic boson" (**Wm**) and the "W tauonic boson" (**Wt**) respectively, which Wm and Wt are probably much heavier than the W boson and the Higgs boson, thus indirectly suggesting the existence of the 4th generation of quarks (which may produced by the decay of these super-heavy Wm and Wt); (**b**) the 2nd possibility is that the muon (**m**) and tauon (**t**) could be actually *reciprocal conjugates* (thus not necessarily conjugated with other two [previously predicted] bosons [heavier than the W boson]: **Wm** and **Wt**), so that $m_m + m_t = 2c^2 / \phi_{g(m/t)}$, from which their common/shared $\phi_{g\mathrm{(m/t)}}$ ratio can be reversely estimated as $\boxed{\phi_{g\mathrm{(m/t)}} = 2c^2 / \left(m_m + m_t \right) \cong 5.36 \times 10^{43} u}$, which is approximately 15-20 times larger than $\phi_{g(Zb)} \left(\cong 10^{42} u \right)$ and $\phi_{g(\mathrm{H}b)} \left(\cong 8 \times 10^{41} u \right)$. The other $\phi_{\mathrm{e}(m/t)}$ ratio can be also reversely estimated from both m_t (or m_m) and $\phi_{g\mathrm{(m/t)}}$ as $\boxed{\phi_{\mathrm{e(m/t)}} \cong 1.2 \times 10^{27} F^{-1}}$ with assigned $\boxed{k_{e\mathrm{(m/t)}} = \phi_{e\mathrm{(m/t)}} r_{\min} \cong 1.9 \times 10^{-19} k_e}$.

(**5**) Analogously to the case of the muon (**m**) and tauon (**t**) ZEH also offers two additional possibilities of mass-conjugation in the case of the three known generations of quarks: (**a**) the 1st generation quarks (uq & dq) may be actually conjugated with two distinct *quark bosons* with fractional charge (the here called "*up-boson*" [**Ub**] with emc ±2/3e [conjugated to uq] and the "**down-boson**" [**Db**] with emc ±1/3e [conjugated to dq]) *and* the other two quark generations (cq & tq [which are considered two distinct excited states of the same uq] plus sq & bq [which are considered two distinct excited states of the same dq]) may be actually reciprocal conjugates on horizontal so that $m_{cq} + m_{tq} = 2c^2 / \phi_{g(c/tq)}$ (and $m_{sq} + m_{bq} = 2c^2 / \phi_{g(s/bq)}$ respectively), from which their common/shared $\phi_{g(\text{c/tq})}$ ratio (and $\phi_{g(\text{s/bq})}$ ratio respectively) can be reversely estimated as $\boxed{\phi_{g(\text{c/tq})} = 2c^2 / \left(m_{cq} + m_{tq}\right) \cong 5.7\times10^{41} u}$ (and $\boxed{\phi_{g(\text{s/bq})} = 2c^2 / \left(m_{sq} + m_{bq}\right) \cong 2.3\times10^{43} u}$ respectively): the other $\phi_{\text{e(c/}tq)}$ ratio can be also reversely estimated from both m_{tq} (or m_{cq}) and $\phi_{g(\text{c/tq})}$ as $\boxed{\phi_{\text{e(c/tq)}} \cong 3.6\times10^{28} F^{-1}}$ with assigned $\boxed{k_{\text{e(c/tq)}} = \phi_{e(\text{c/tq})} r_{\min} \cong 5.5\times10^{-18} k_e}$; the other $\phi_{\text{e(s/b}q)}$ ratio can be also reversely estimated from both m_{bq} (or m_{sq}) and $\phi_{g(s/\text{bq})}$ as $\boxed{\phi_{\text{e(s/bq)}} \cong 1.05\times10^{28} F^{-1}}$ with assigned $\boxed{k_{e(\text{s/bq})} = \phi_{e(\text{s/bq})} r_{\min} \cong 1.6\times10^{-18} k_e}$.

In a checkpoint conclusion, what distinguishes ZEH is actually the contrast between its simplicity and the richness/diversity of explanations, correlations and predictions it offers. The author of this paper resonates to the famous Dirac's vision on the importance of mathematical beauty in physical equations: "*The research worker, in his efforts to express the fundamental laws of Nature in mathematical form, should strive mainly for mathematical beauty […]It often happens that the requirements and beauty are the same, but where they clash the latter must take precedence.*"

3. DISCUSSION

The energy/length scale-dependent *electro-gravitational resistivity of vacuum* $R(E)$ (introduced in the 1st section of this paper) is a powerful concept which predicts the existence of the graviton (as a spin-2 boson with an assigned Planck-like gravitational constant $h_g << h$ measuring its quantum angular momentum), retrodicts a minimum quantum angular momentum h for electromagnetic waves (thus the quantum nature of photons), predicts a maximum speed $v_{\max}\left(=c=v_g\right)$ for any physical wave allowed in our universe and predicts both a variable $G_q(E)(>>G)$ and $\alpha(E)$ bringing General relativity and quantum field theory more closer to one another by offering a solution to the *hierarchy problem* in physics.
The zero-energy hypothesis (**ZEH**) proposed in this paper is another powerful concept which is essentially a *conservation principle applied on zero-energy* that mainly states a *general quadratic equation* having a pair of conjugate mass solutions for each set of coefficients, thus predicting a new type of mass "symmetry" called here "*mass conjugation*" between elementary particles which predicts the zero/non-zero rest masses of all known/unknown EPs to be conjugated in boson-fermion pairs; more specifically, ZEH proposes a general formula for all the rest masses of all elementary particles from Standard model, also indicating a possible bijective connection between the three types of neutrinos and the massless bosons (photon, gluon and the hypothetical graviton), between the electron/positron and the W boson and predicting two distinct types of neutral massless fermions (modelled as conjugates of the Higgs boson and Z boson respectively) which are plausible candidates for dark energy and dark matter. ZEH also offers a new interpretation of Planck length as the approximate length threshold above which the rest masses of all known elementary particles have real number values (with mass units) instead of complex/imaginary number values (as predicted by the unique quadratic equation

proposed by ZEH). ZEH also helps correcting $R(E)$ as based on the currently known the leading log approximation (**LLA**) of $\alpha(E)\left[=\alpha_0/\left(1-\alpha_0 f(\mathrm{E})\right)\right]$ and predicts the behaviour of the electromagnetic field and gravitational field at Planck length/energy scales; ZEH also predicts a granular/quantum structure of spacetime near the Planck scale and the existence of a *pre-Big Bang quasi-singularity* with large but finite density.

4. FINAL CONCLUSIONS

The combination between the concept of *electrogravitational resistivity of vacuum* and *the zero-energy hypothesis* may help solving the *hierarchy problem*, the infinite-density *singularity problem* (of General relativity) and crystallizes new directions in theoretical physics beyond the Standard model, including the prediction of two massless fermions (that represent good candidates for dark energy and dark matter), a granular/quantum structure of spacetime near the Planck scale and the existence of a pre-Big Bang quasi-singularity with large but finite density.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Andrei-Lucian Drăgoi (July 2017). “On a Plausible Triple Electro-gravito-informational Significance of the Fine Structure Constant”, Physical Science International Journal (PSIJ), ISSN: 2348-0130, Vol. 15, Issue 3. DOI: 10.9734/PSIJ/2017/34613 (URL-CrossRef.org). URL0 (original source); URL1 (Research Gate source)
2. Andrei-Lucian Drăgoi (May 2018). “(Toy-model) A Simple “Digital” Vacuum Composed of Space Voxels with Quantized Energetic States”, Physical Science International Journal (PSIJ), ISSN: 2348-0130, Vol.: 18, Issue.: 1. DOI: 10.9734/PSIJ/2018/41391. URL0 (original source); URL1 (Research Gate source);
3. E. Teller (1948). “On the Change of Physical Constants”. Physical Review (April 1948) Vol. 73, Issue 7, page 801. URL1 (https://journals.aps.org/pr/abstract/10.1103/PhysRev.73.801), URL2 (https://journals.aps.org/pr/issues/73/7). See also Barrow J.D. (1981) (which also cites Teller’s paper). “The lore of large numbers: some historical background to the anthropic principle”, Q. Jl. R. Astr. Soc. 22 (1981) 388-420. This extract can also be found at page 397: http://articles.adsabs.harvard.edu//full/1981QJRAS..22..388B/0000397.000.html
4. Sirag S-P. (1980, 1983). “Physical constants as cosmological constraints” (Received on November 22, 1980), International Journal of Theoretical Physics, 1983, vol. 22, no. 12, pages 1067–1089. DOI: 10.1007/BF02080315. Bibcode: 1983IJTP...22.1067S. URL1 (https://link.springer.com/article/10.1007/BF02080315) and URL2 (https://ui.adsabs.harvard.edu/abs/1983IJTP...22.1067S/abstract)
5. Bouchendira, Rym; Cladé, Pierre; Guellati-Khélifa, Saïda; Nez, François; Biraben, François (2011). "New determination of the fine-structure constant and test of the quantum electrodynamics" (PDF). Physical Review Letters (Submitted manuscript). 106 (8): 080801. arXiv:1012.3627. Bibcode: 2011PhRvL.106h0801B. doi: 10.1103/PhysRevLett.106.080801. PMID 21405559.
6. Aitchison IJR and Hey AJG (2003). “Gauge theories in particle physics: A practical introduction, Fourth Edition - 2 Volumes set 4th Edition” (book). 2nd volume. Chapter 15.2.3 (The renormalization group equation and large −q2 behavior in QED). 2009;123 (equation 15.45, from pdf page no. 136). URL: www.taylorfrancis.com/books/9780367805746
7. Estakhr, Ahmad Reza (2016). “Planck Force is Tension of Spacetime (General Relativity & Estakhr's expression of

Einstein Field Equation)". 2016 Annual Meeting of the Far West Section of the American Physical Society (APS), Volume 61, Number 17, Abstract ID: BAPS.2016.FWS.G1.31, Abstract: G1.00031, Friday–Saturday, October 28–29, 2016; Davis, California, Session G1: Poster Session 4:00 PM, Friday, October 28, 2016, Conference Center Room: Lobby. Retrieved on 20.10.2019 from URL http://meetings.aps.org/link/BAPS.2016.FWS.G1.31 which redirects to http://meetings.aps.org/Meeting/FWS16/Event/286265

8. Edward P. Tryon (1973). "Is the Universe a Vacuum Fluctuation?", Nature, vol. 246, p.396–397, 1973.
9. Alan H. Guth (1997). "The Inflationary Universe. The Quest for a New Theory of Cosmic Origins. With a Foreword by Alan Lightman. (1st Edition)" (Hardcover, 384 Pages, Published 1997 by Jonathan Cape Ltd). ISBN 0-224-04448-6. Appendix A
10. Battye, Richard A.; Moss, Adam (2014). "Evidence for Massive Neutrinos from Cosmic Microwave Background and Lensing Observations". Physical Review Letters. 112 (5): 051303. arXiv:1308.5870. Bibcode:2014PhRvL.112e1303B. doi:10.1103/PhysRevLett.112.051303. PMID 24580586.
11. K. Assamagan, Ch. Brönnimann, M. Daum, H. Forrer, R. Frosch, P. Gheno, R. Horisberger, M. Janousch, P. -R. Kettle, Th. Spirig, and C. Wigger (1996). "Upper limit of the muon-neutrino mass and charged-pion mass from momentum analysis of a surface muon beam". Phys. Rev. D 53, 6065 – Published 1 June 1996. DOI: 10.1103/PhysRevD.53.6065. URL: https://journals.aps.org/prd/abstract/10.1103/PhysRevD.53.6065
12. Barate, R.; Buskulic, D.; Decamp, D. et. al (1998). "An Upper limit on the tau-neutrino mass from three-prong and five-prong tau decays". Europ Phys J C 2 (1998): 395-406. doi:10.1007/s100520050149. URLs: (1) https://epubs.stfc.ac.uk/work/26893; (2) www.researchgate.net/publication/30403534
13. Patrignani, C. et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016). See: Introduction to the neutrino properties listings, revised in August 2013 by P. Vogel (Caltech) and A. Piepke (University of Alabama). URL: http://pdg.lbl.gov/2016/listings/rpp2016-list-neutrino-prop.pdf