A solution to the black hole information paradox

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I show that, contrary to generally accepted thinking about Rindler horizons, special relativity allows a free-falling object to reach a rocket having any constant proper acceleration, starting from any distance below the rocket as measured in the free-falling object's frame. I show that general relativity thereby violates its equivalence principle re black holes, and explain how these facts hold true despite Rindler horizons. To fix the violation and solve the black hole information paradox, a new metric for Schwarzschild geometry is derived, that doesn't predict black holes and is confirmed by observations.

1 A problem with black holes

The Relativistic Rocket (RR) equations of special relativity (SR) apply to both accelerating and decelerating rockets. For instance, they describe either half of a trip between Earth and the star Vega, where the rocket accelerates and decelerates at 1 g (1 Earth gravity) to arrive at Vega at low speed.

An RR equation for the velocity v of a rocket, in a local inertial frame (LIF) in which the rocket blasted off from rest (e.g. the Earth-Vega frame), is

$$v = \frac{at}{\sqrt{1 + (at/c)^2}} \tag{1}$$

where a is the rocket's constant proper acceleration > 0, t is the time in the LIF, and c is the speed of light.

An RR equation for the time t is

$$t = \sqrt{(d/c)^2 + 2d/a} \tag{2}$$

where d is the distance covered by the rocket in the LIF.

The RR site notes that we can "run the film backwards" to reason that the above equations must still apply to a decelerating rocket, which means they work in reverse, like this: In the LIF of a free-falling object, when a rocket decelerates toward the object with the initial velocity v given by (1), where the time t is given by (2) for the initial distance d between them, then they reach each other at relative rest, such as the rocket arriving at Vega. (They come to *momentary* relative rest when the rocket's engines remain running.) Thus SR allows a free-falling object to reach a rocket having any acceleration a, starting from any distance d below the rocket as measured in the free-falling object's LIF. For example, a rocket can in principle decelerate from the Earth-Vega midpoint to Vega at 10^{15} g, or any higher acceleration a.

Therefore SR allows a free-falling ball that's just below the event horizon of a static supermassive black hole, and below a rocket hovering above the event horizon and well within the ball's LIF, to reach the rocket. Which is to say they can reach each other in principle. General relativity (GR) predicts they can't reach each other even in principle, in violation of its equivalence principle (EP) that says SR's laws must hold in any and every LIF.

SR's (1) and (2) predict that the ball can reach the hovering rocket in principle, for any inputs for the rocket's acceleration a and the initial distance d between them in the ball's LIF, just as they predict that Vega can reach a rocket that decelerates at any acceleration a from the Earth-Vega midpoint to Vega. There are no special or preferred inertial frames in SR, so the rocket's initial velocity v in the ball's LIF can in principle be any value in the interval $-c < v < c$, or else the EP is violated. Since the EP requires that the ball be able to reach the hovering rocket in principle, black holes can't validly exist in Schwarzschild geometry along with the EP.

2 What about Rindler horizons?

SR allows the ball to reach the hovering rocket despite any Rindler horizon. Let the rocket that travels from Earth to Vega accelerate and decelerate at $a = 1.03$ ly/yr$^2 \approx 1\ g$. At the start of its braking phase its Rindler horizon is $c^2/a = 0.97$ ly below it in its frame, in Vega's direction. The rocket-Vega distance $d = 12.5$ ly in the Earth-Vega frame (half of the Earth-Vega distance in that frame) is length contracted in its frame to $d/\gamma = 0.90$ ly (where γ is the gamma factor), such that Vega is above its Rindler horizon and able to reach it. SR likewise predicts that the ball is initially above the hovering rocket's Rindler horizon when the ball and the rocket approach each other fast enough initially. For instance, SR predicts that

when the hovering rocket's acceleration $a =$	1.03×10^{15} ly/yr^2 (~$10^{15}\ g$),
and the ball's initial distance d below the rocket in the ball's LIF $=$	1 light-microsecond (~300 m),
and, as calculated using SR's (1) and (2), the rocket's initial velocity v toward the ball (i.e. the rocket decelerates toward the ball) in the ball's LIF $=$	$0.99956c$,
then, as calculated using (2), they reach each other at relative rest in time t in the ball's LIF, $=$	1.03 μs,
and the rocket's Rindler horizon is c^2/a below the rocket in its frame, $=$	9.19 m,
and the ball's initial distance d/γ below the rocket in the rocket's frame $=$	8.91 m,

so, since the initial distance $d/\gamma < c^2/a$, the ball is initially above the rocket's Rindler horizon.

When the rocket's initial velocity v toward the ball is given using SR's (1) and (2), then, for any inputs for the rocket's acceleration a and the initial distance d between them, they reach each other at relative rest, and the initial distance $d/\gamma < c^2/a$ (no matter how small c^2/a is, even less than a nanometer), so the ball is initially above the rocket's Rindler horizon.

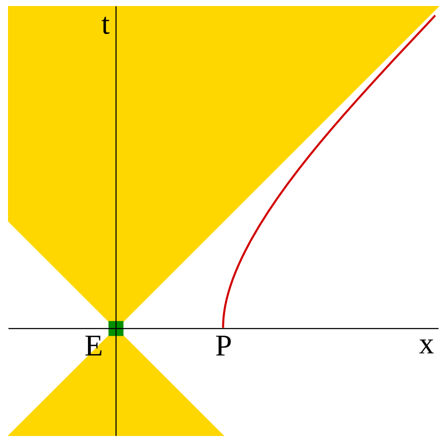

Figure 1: Spacetime diagram showing a uniformly accelerated particle, P, and an event E. The event's future light cone never intersects the particle's world line. By Christopher Thomas / tiZom / CC BY-SA.

In Fig. 1, let P be the hovering rocket. The diagram is for a LIF that momentarily comoves with the rocket at $t = 0$; the rocket blasts off from rest in this frame. Let the event E occur at the rocket's Rindler horizon. As explained above, SR allows the ball to be initially above the rocket's Rindler horizon, between E and P in this diagram/LIF at $t = 0$, even when the E-P distance ($= c^2/a$) is less than a nanometer. So the ball can reach the rocket in principle.

To better see this, adapt the barn-pole paradox: The runner, representing a free-falling object (e.g., the ball, or Vega), holds the trailing end of the pole that has any proper length d. In the barn frame the runner's velocity v is such that the pole (which is length contracted to d/γ) is completely within the barn when the switch is thrown. Instead of the barn doors closing, a rocket blasts off horizontally from the far door (with an acceleration a), so that the runner chases the rocket, and a flash of light emits from the near door, eventually reaching the rocket (so in the barn frame at blastoff the light source is above the rocket's Rindler horizon, which is c^2/a below the rocket). Having passed the light source before it flashed (so $d/\gamma < c^2/a$), the runner can in principle reach the rocket before the flash does. In the runner's frame at $t = 0$ the rocket blasts off distance d ahead of the runner, and decelerates toward the runner with an initial velocity v. They reach each other at relative rest when that initial velocity is given using SR's (1) and (2). See also the ladder paradox.

SR allows the ball to reach another free-falling object at a point just above the hovering rocket. In the ball's LIF the time t would be given by $t = d/v$ (where d is the initial distance between the ball and the other object, and v is the other object's velocity), just as that equation applies in the Vega system to receive/reach a free-falling package sent from Earth, irrespective of the Rindler horizons of rockets that decelerate toward Vega along the package's route.

All the above shows that the EP requires the escape velocity to be $< c$ everywhere in Schwarzschild geometry. A new metric for Schwarzschild geometry is derived below, that doesn't predict black holes.

3 New equations for free-fall motion

NASA says the velocity v of a free-falling object that was dropped in a uniform gravitational field with no air resistance is given by the equation

$$v = at \tag{3}$$

where a is the acceleration and t is the time.

The EP shows that (3) is invalid:

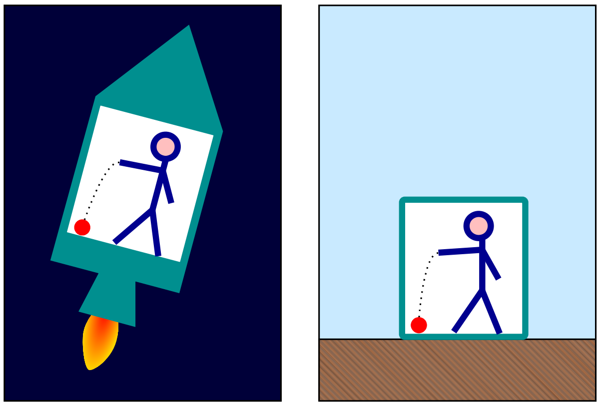

Figure 2: Ball falling to the floor in an accelerating rocket (left) and on Earth (right). By Pbroks13/Mapos / CC BY-SA.

The EP implies that SR's laws hold in both scenarios in Fig. 2. The RR equations describe the ball's motion within the rocket. Then the RR equations describe the ball's motion within the box on Earth as well, and so the velocity (1), which always returns a value $< c$, supplants NASA's (3). The time t in (1) is measured in the ball's LIF.

4 A new equation for escape velocity

Hereafter, geometric units are used, where $c = G$ (the gravitational constant) $= 1$.

GR's equation for escape velocity v_e is

$$v_e = \sqrt{\frac{2M}{r}} \tag{4}$$

where M is the mass of the massive body in geometric units, and r is the radial coordinate (circumference of a circle centered on the massive body, divided by 2π).

I made a conversion equation that converts the old free-fall velocity (3) to the new free-fall velocity (1), and used it to convert GR's escape velocity (4) to a new equation for escape velocity that's approximated by (4) and predicts that the escape velocity is $< c$ everywhere.

The conversion equation is

$$v_{new} = \frac{v_{old}}{\sqrt{1 + v_{old}^2}} \,. \tag{5}$$

The new equation for escape velocity v_e, derived using GR's escape velocity (4) and the conversion equation (5), is

$$v_e = \sqrt{\frac{2M}{r + 2M}} \,. \tag{6}$$

5 A new gravitational time dilation factor

Imagine nested spherical shells concentric to a massive body. An observer drops from an arbitrarily large distance, falling freely toward the massive body while measuring, as a fraction x of the observer's own rate of time, the rate of clocks at each shell as they pass right by. Each shell passes at the escape velocity there. Inputting that velocity into the reciprocal of the gamma factor gets the value x for that shell. The escape velocity at an arbitrarily large distance is zero, so $x = 1$ there. The observer remains stationary relative to the falling space, so the observer's own rate of time remains the rate of time at an arbitrarily large distance. Then the gravitational time dilation factor, the rate of time at a radial coordinate r, as a fraction of the rate of time at an arbitrarily large distance, is given by the pseudo-equation

$$\text{gravitational time dilation factor} = 1 \;/\; \text{gamma factor(escape velocity at } r\text{)}. \tag{7}$$

I verified (7) by deriving GR's gravitational time dilation factor from it, using GR's escape velocity (4):

$$\frac{t_0}{t_f} = 1 \Big/ \frac{1}{\sqrt{1 - v_e^2}} = \sqrt{1 - v_e^2} = \sqrt{1 - \left(\sqrt{\frac{2M}{r}}\right)^2} = \sqrt{1 - \frac{2M}{r}} \tag{8}$$

where t_0 is the proper time between two adjacent events as measured by a clock at the radial coordinate r, and t_f is the time between those events as measured by a clock at an arbitrarily large distance from the massive body.

The new gravitational time dilation factor, derived using the pseudo-equation (7) and the new escape velocity (6), is

$$\frac{t_0}{t_f} = \sqrt{\frac{r}{r + 2M}} \,. \tag{9}$$

6 A new metric for Schwarzschild geometry

The only difference between the metric for flat spacetime in polar coordinates and the Schwarzschild metric is GR's curvature factor $(1 - 2M/r)$ that's in the Schwarzschild metric and given by the old gravitational time dilation factor (8). To derive the new metric for

Schwarzschild geometry I used the new gravitational time dilation factor (9) to replace those curvature factors.

The new metric for Schwarzschild geometry is

$$d\sigma^2 = -\frac{r}{r+2M}dt^2 + \frac{r+2M}{r}dr^2 + r^2 d\phi^2 \tag{10}$$

where σ is the proper distance between two adjacent events, t is the time between those events as measured by a clock at an arbitrarily large distance from the massive body, and ϕ is the measure of angle in a plane through the center of the massive body.

7 Experimental confirmation of the new metric

The only change made to the Schwarzschild metric was to the escape velocity that's built into it. Since the old escape velocity (4) better approximates the new escape velocity (6) as gravity weakens, the Schwarzschild metric better approximates the new metric (10) as gravity weakens.

For the Schwarzschild precession in the orbit of the star S2 around Sgr A*, both metrics predict $12.1'$ per orbit, in agreement with observations. When the Schwarzschild metric predicts $12.100'$ per orbit for S2's Schwarzschild precession, the new metric predicts $12.095'$ per orbit.

For the Schwarzschild precession of Mercury, both metrics predict $42.98''$ per Julian century, in agreement with observations. When the Schwarzschild metric predicts $42.9799''$ per Julian century for Mercury's Schwarzschild precession, so does the new metric.

The new gravitational time dilation factor (9) goes to zero as r goes to zero. Gravitational redshift indicates gravitational time dilation, so a star can look black when viewed from afar.

8 Recommendation

I recommend that the Einstein field equations or their dependencies be updated, so that their solution for Schwarzschild geometry is the new metric (10).