A Solution to the Black Hole Information Paradox

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It's shown that black holes contradict laws of special relativity in some local inertial frames, which violates the equivalence principle. To solve the problem, a new metric for Schwarzschild geometry is derived that doesn't predict black holes and is confirmed by observations.

1 A problem with black holes

The Relativistic Rocket (RR) equations of special relativity (SR) describe both accelerating and decelerating rockets. For example, the equations describe both a rocket that accelerates from Earth to the midpoint between Earth and the star Vega, and that rocket decelerating from that midpoint to arrive at Vega at low speed.

An RR equation for the velocity *v* of a rocket, in the local inertial frame (LIF) in which the rocket blasted off from rest, is

$$v = \frac{at}{\sqrt{1 + (at/c)^2}} \tag{1}$$

where *a* is the rocket's constant proper acceleration, *t* is the time measured in the LIF, and *c* is the speed of light. An RR equation for the time *t* is

$$t = \sqrt{(d/c)^2 + 2d/a} \tag{2}$$

where *d* is the distance covered by the rocket as measured in the LIF.

In the LIF of a freely falling object (FFO), when a rocket having any constant proper deceleration *a* initially approaches the FFO with the velocity *v* given by (1), where the time *t* is given by (2) for any initial distance *d* between the FFO and the rocket, then they reach each other at rest with respect to each other, such as the rocket arriving at Vega.

In the LIF of an FFO, a rocket having a constant proper deceleration initially approaches the FFO with the velocity needed for them to reach each other, as calculated by (1) and (2). But general relativity (GR) disagrees that they can reach each other, when the rocket hovers above a black hole's event horizon and the FFO starts below the event horizon. By contradicting SR within a LIF, GR violates its equivalence principle (EP).

2 What about Rindler horizons?

Rindler horizons don't prevent (1) and (2) from functioning for deceleration as told in section 1. Adapt the barn-pole paradox to see how a rocket can accelerate and decelerate to reach any FFO (e.g. Vega) without its Rindler horizon getting in the way during the deceleration phase: The runner, representing the FFO (e.g. Vega), holds the trailing end of the pole that has any proper length *d*. In the barn frame the runner's speed is such that the pole is completely within the barn when the switch is thrown. Instead of the barn doors closing, a rocket blasts off horizontally from the far door, so that the runner chases the rocket, and a flash of light is emitted from the near door, eventually reaching the rocket. Having passed the light source before it flashed, the runner can reach the rocket before the flash does.

The runner could pass the rocket (e.g. the rocket that decelerates to Vega could overshoot it, so that Vega, representing the runner, passes the rocket) to reach an FFO above it. Now that this is evident, a simpler case of GR's violation of the EP can be shown: In the LIF of an FFO that starts below a black hole's event horizon, the FFO can't reach an FFO that's above the event horizon and moving ever away from the black hole (e.g. escaping). SR allows them to reach each other, as calculated by the usual speed equation $v = d/t$, but GR doesn't. A clerk in the Vega system can use that equation to plan to receive (reach) a freely falling package sent from Earth, without regard for the Rindler horizons of rockets that decelerate to Vega in the package's path.

3 New equations for the motion of a freely falling object

It seems to be generally accepted that the velocity v of an FFO that's dropped in a uniform gravitational field is given by the equation

$$v = at \tag{3}$$

where a is the acceleration and t is the time. NASA gives that equation.

The EP disagrees with (3). Here is a visualization of the EP where a ball is dropped:

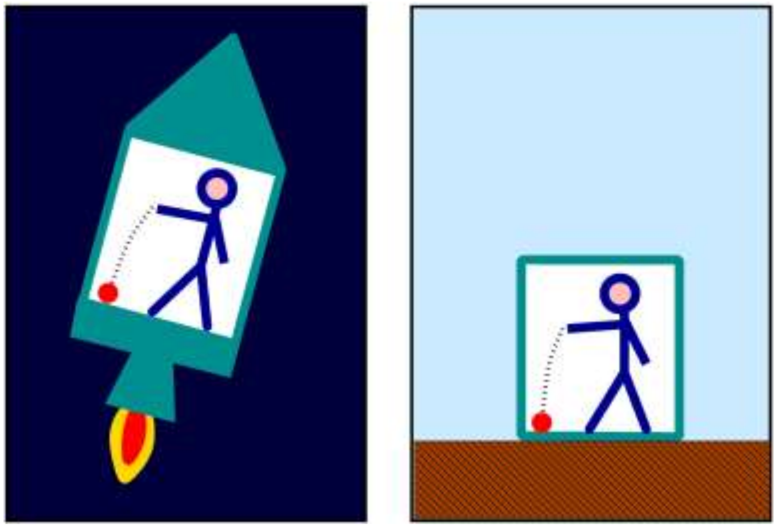

Fig. 1: Ball falling to the floor in an accelerating rocket (left) and on Earth (right). Reprinted by permission of I, Mapos / CC BY-SA.

The EP implies that the laws of SR hold in both scenarios in Fig. 1. The RR equations describe the ball's motion within the rocket. Then the RR equations describe the ball's motion within the box on Earth as well, and so (1), which always returns a velocity < *c*, supplants (3). Note that for Fig. 1 the time *t* in (1) is measured in ball's LIF, and not by the person, who feels the acceleration and is analogous to the rocket.

4 A new equation for escape velocity

Hereafter, geometric units are used, where $c = G$ (the gravitational constant) = 1.

GR's equation for escape velocity v_e is

$$v_e = \sqrt{\frac{2M}{r}} \tag{4}$$

where M is the mass of the massive body in geometric units, and r is the radial coordinate (circumference of a circle centered on the massive body, divided by 2π).

I made a conversion equation that converts (3) to (1), and used it to convert (4) to a new equation for escape velocity that's approximated by (4) and predicts that the escape velocity is < *c* everywhere.

The conversion equation is

$$v_{new} = \frac{v_{old}}{\sqrt{1 + v_{old}^2}}\,. \tag{5}$$

The new equation for escape velocity v_e, derived using (4) and (5), is

$$v_e = \sqrt{\frac{2M}{r + 2M}}\,. \tag{6}$$

5 A new gravitational time dilation factor

Imagine nested spherical shells concentric to a massive body. An observer drops from an arbitrarily large distance, falling freely toward the massive body while measuring, as a fraction x of the observer's own rate of time, the rate of clocks at each shell as they pass right by. Each shell is passed at the escape velocity for that shell. Inputting that speed into the reciprocal of the gamma factor gets the value x for that shell. The escape velocity at an arbitrarily large distance is zero, so $x = 1$ there. The observer is stationary relative to the falling space, so the observer's own rate of time remains the rate of time at an arbitrarily large distance. Then the gravitational time dilation factor, the rate of time at a radial coordinate r, as a fraction of the rate of time at an arbitrarily large distance, is given by the pseudo-equation

gravitational time dilation factor = 1 / gamma factor(escape velocity at *r*). (7)

I verified (7) by deriving GR's gravitational time dilation factor from it, using (4):

$$\frac{t_0}{t_f} = \sqrt{1 - v_e^2} = \sqrt{1 - \sqrt{\frac{2M}{r}}^{\,2}} = \sqrt{1 - \frac{2M}{r}} \tag{8}$$

where t_0 is the proper time between two adjacent events as measured by a clock at the radial coordinate r, and t_f is the time between those events as measured by a clock at an arbitrarily large distance from the massive body.

The new gravitational time dilation factor, derived using (6) and (7), is

$$\frac{t_0}{t_f} = \sqrt{\frac{r}{r + 2M}}\,. \tag{9}$$

6 A new metric for Schwarzschild geometry

The only difference between the metric for flat spacetime in polar coordinates and the Schwarzschild metric is GR's curvature factor, given by (8), that's in the Schwarzschild metric. To derive the new metric for Schwarzschild geometry I used (9) to replace those curvature factors.

The new metric for Schwarzschild geometry is

$$d\sigma^2 = -\frac{r}{r+2M}dt^2 + \frac{r+2M}{r}dr^2 + r^2 d\phi^2 \tag{10}$$

where σ is the proper distance between two adjacent events, t is the time between those events as measured by a clock at an arbitrarily large distance from the massive body, and ϕ is the measure of angle in a plane through the center of the massive body.

7 Experimental confirmation of the new metric

Since (8) better approximates (9) as gravity weakens, hence the Schwarzschild metric better approximates the new metric (10) as gravity weakens, I focused on a test of the Schwarzschild metric for the strongest gravity, the Schwarzschild precession in the orbit of the star S2 around Sgr A*. Both metrics predict 12.1 arcminutes per orbit, in agreement with observations.

For a hypothetical star having S2's orbital eccentricity, when the Schwarzschild metric predicts 12.100 arcminutes per orbit for Schwarzschild precession, then the new metric predicts 12.095 arcminutes per orbit.

For the Schwarzschild precession of Mercury, both metrics predict 42.98 arcseconds per century, in agreement with observations.

The new gravitational time dilation factor (9) goes to zero as r goes to zero. Gravitational time dilation is also known as gravitational redshift. So a massive body can look black when observed from afar.

8 Recommendation

I recommend that the Einstein field equations or their dependencies be updated, so that their solution for Schwarzschild geometry is the new metric (10).