Unknown Templates of the Coding

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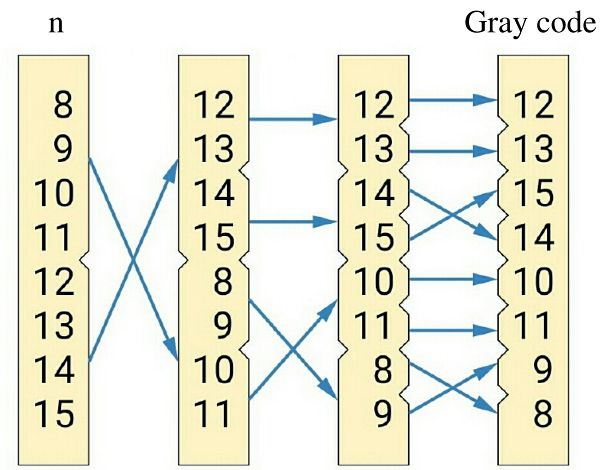
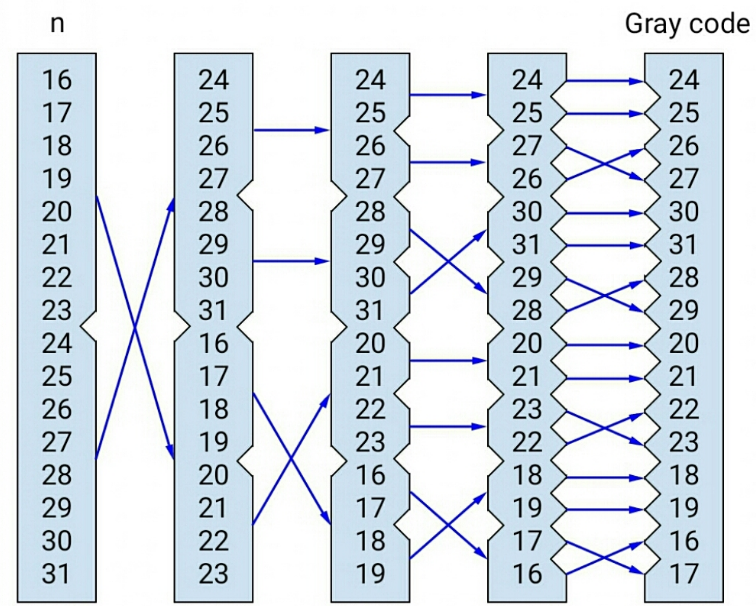
Abstract

The templates described in this article model the encoding systems that convert an integer to a number that has the same number of digits as the first in the binary system. So, each coded group will be a rearrangement of the sequence of the nonnegative integers that have the same number of binary digits. Gray code and its inverse are included in this category.

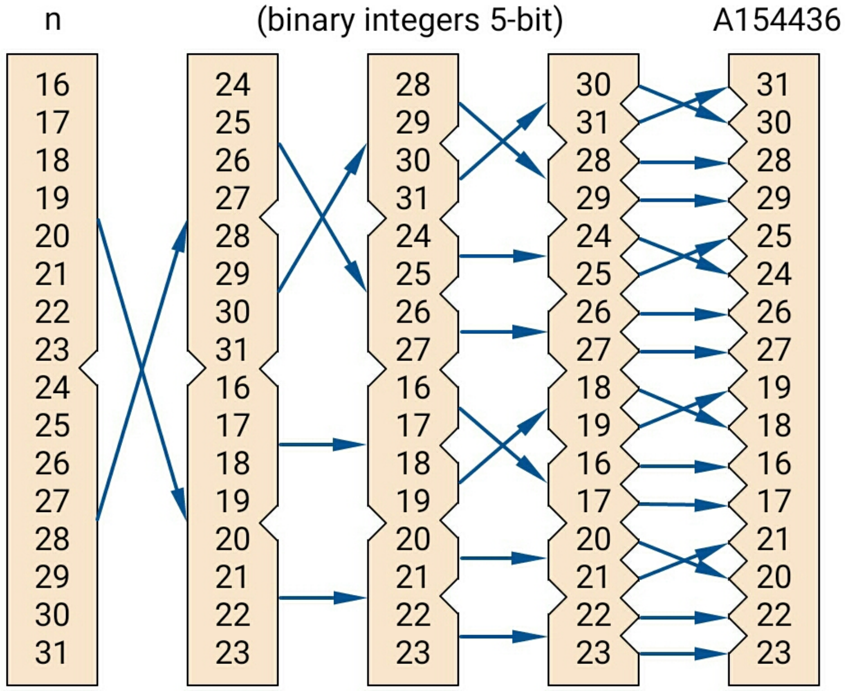
The following diagrams eloquently show the process of converting 4-bit and 5-bit integers to Gray code.



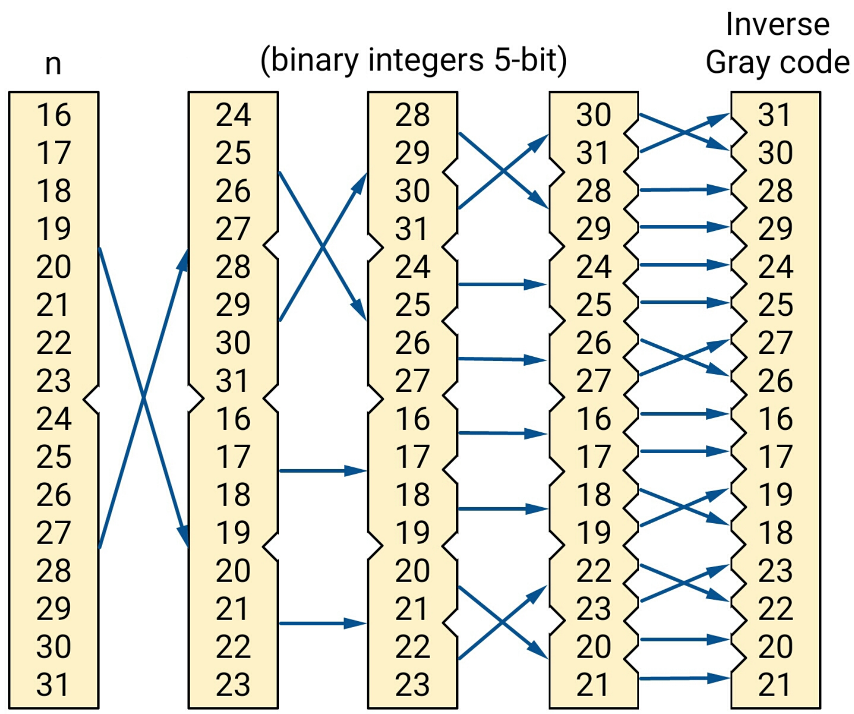


We can distinguish two ways to transfer groups of numbers from column to column, crosswise (×) and parallel (=).

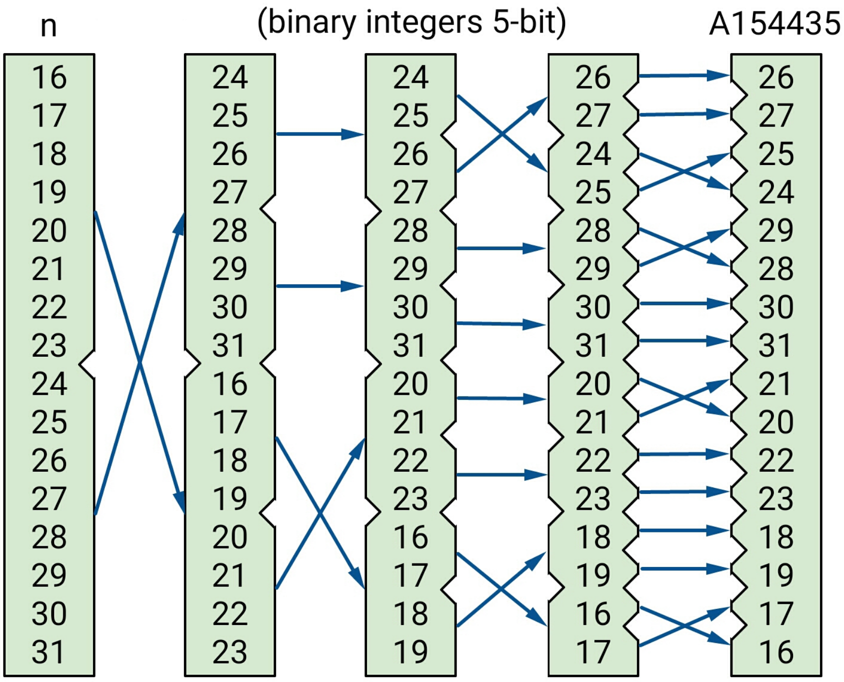
A different arrangement of arrows results in a different oeis sequence (all the sequences mentioned here are described below):



With this seemingly disorganized arrangement of arrows we get the inverse Gray code:



With an equally organized rearrangement of the arrows of the last table, a fourth sequence emerges:



The rules that allow these encodings are defined by invisible sequences of binary numbers, which we will see shortly.

1. Each column after the first contains the same number of pairs of arrows of the type "=" and "×".

2. The number of these pairs is doubled in each subsequent column.

3. The rules for arranging pairs in each column - not always obvious - are determined by the arrangement of the pairs in the previous column.

4. In each diagram, all the vertices or all the bottoms of the columns are occupied by pairs of "×" type arrows.

If we break these rules then absolutely no important sequence will be created in the right column of the diagram. These rules are interpreted by the sequences of the binary numbers that result if we first set "=" = 0 and "×" = 1.

We will focus on the previous diagram with the inverse Gray code.

If we read each successive column from top to bottom, then we will get the binary sequence 1, 10, 1001, 10010110, ... In the decimal system these numbers become 1, 2, 9, 150, 38505, ... and they are terms of the A133468 sequence, which is constructed by the digits of the Thue Morse sequence. If you look closely at the above separated binary terms of this sequence, you will have no difficulty finding the rule of its construction. You can see that this sequence meets all four of the above conditions.

As far as I know there are generalized Thue Morse sequences, and although I have never seen any of them I don't think there is a similar sequence that gathers all these features. But this is a hypothetical question. In addition, I do not know how many sequences have the properties I mention in the abstract. If you know such a sequence, then you may find the template of this construction based on the above rules:

It is easy to first find the rules that connect the second to the third column for three-digit binary numbers. The connection of these columns will be the same when you repeat the same thing for the four-digit numbers. Continue in the same way until you check a sufficient number of columns to make sure no problems are created.

The following sequences were taken from oeis with their definitions.

A003188
Decimal equivalent of Gray code for n.
0, 1, 3, 2, 6, 7, 5, 4, 12, 13, 15, 14, 10, 11, 9, 8, 24, 25, 27, 26, 30, 31, 29, 28, 20, 21, 23, 22, 18, 19, 17, 16, ...

A006068
a(n) is Gray-coded into n (inverse Gray code).
0, 1, 3, 2, 7, 6, 4, 5, 15, 14, 12, 13, 8, 9, 11, 10, 31, 30, 28, 29, 24, 25, 27, 26, 16, 17, 19, 18, 23, 22, 20, 21, ...

A154435
Permutation of nonnegative integers induced by Lamplighter group generating wreath recursion, variant 3: a = s(b,a), b = (a,b), starting from the state a.
0, 1, 3, 2, 6, 7, 5, 4, 13, 12, 14, 15, 10, 11, 9, 8, 26, 27, 25, 24, 29, 28, 30, 31, 21, 20, 22, 23, 18, 19, 17, 16, ...

A154436
Permutation of nonnegative integers induced by Lamplighter group generating wreath recursion, variant 1: a = s(a,b), b = (a,b), starting from the state a.
0, 1, 3, 2, 7, 6, 4, 5, 15, 14, 12, 13, 9, 8, 10, 11, 31, 30, 28, 29, 25, 24, 26, 27, 19, 18, 16, 17, 21, 20, 22, 23, ...

A010060
Thue-Morse sequence: let A_k denote the first 2^k terms; then A_0 = 0 and for k >= 0, A_{k+1} = A_k B_k, where B_k is obtained from A_k by interchanging 0's and 1's.
0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 0, ...

A133468
A080814 complemented, then interpreted as binary and then re-expressed in decimal form (e.g., "1221" = "0110"). Alternately, view as A080814 with "1" mapped to "1" and "2" mapped to "0".
1, 2, 9, 150, 38505, 2523490710, ...

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