One page Proof of Riemann Hypothesis

Dmitri Martila
Institute of Theoretical Physics, Tartu University,
4 Tähe Street, 51010 Tartu, Estonia[*]

(Dated: May 28, 2020)

Abstract

There are tenths of proofs for Riemann Hypothesis and 3 or 5 disproofs of it in arXiv. I am adding to the Status Quo my proof, which uses the achievement of Dr. Zhu.

[*]Electronic address: eestidima@gmail.com

I. PRIOR RESEARCH RESULT

Because the paper of Dr. Zhu [1] is not published in a peer-review journal (for 4 years) and is very complicated, it could contain a fatal mistake. Thus, I do not start with the final result called "The probability of Riemann's hypothesis being true is equal to 1" but rather with the starting information of the papers [1, 2] (one of the papers is peer-reviewed), where is proven, that

$$\lim_{n\to\infty} \inf d(n) = 0\,, \tag{1}$$

where $d(n) = D(n)/n$, and $D(n) = \mathrm{e}^{\gamma}\, n \ln\ln n - \sigma(n)$. Hereby the Riemann Hypothesis holds true, if $\lim_{n\to\infty} \inf D(n) \geq 0$.

II. MY FIRST PROOF

The Eq.(1) means, that $\lim_{n\to\infty} d(n) \geq 0$. However, the limit does not exist, because the number $X = \lim \sigma(n)/n$ can not be determined: the function jumps from one value to another, namely $(\sigma(n) - \sigma(n+j))/n \neq 0$ if $n \to \infty$ for $j < \infty$. Therefore, instead of Eq.(1) it is mathematically correct to write: $d(n) = D(n)/n \geq 0$, when $n \gg 1$. The expression $n \gg 1$ means, that the n is always finite $n < \infty$. But for any finite n the $D(n)/n \geq 0$ implies, that $D(n) \geq 0$.

III. MY SECOND PROOF

The main problem of the available Riemann Hypothesis proofs is the fatal mistakes somethere in the text. If text is complicated enough, the mistake is practically impossible to find. The result of Dr. Zhu in Eq.(1) comes from too many theorems: 1,2,3 in the Ref. [2], so the risk of having mistake is very high. But now I will demonstrate, that it is enough to hope for validity of the Therem 2 in the Ref. [2], i.e. I can prove the Riemann Hypothesis even without his theorems one and three. Recall, that the Riemann Hypothesis has been shown to hold unconditionally for n up to $N = \exp(\exp(26))$ as writen in Refs. [2, 3]. Thus, it is enough to check Riemann Hypothesis for the $n \gg 1$ area. Therefore, we do not need Theorem 3, because it is a trivial fact Dr. Zhu is proving, that if $D(n) \geq 0$ for $n > N \gg 1$ the Riemann Hypothesis is correct. And we do not need Theorem 1, because Theorem 2 already says, that the $\lim_{n\to\infty} \inf d(n) \geq 0$ and so $d(n) = D(n)/n \geq 0$, when $n \gg 1$.

---

[1] Yuyang Zhu, The probability of Riemann's hypothesis being true is equal to 1, arXiv:1609.07555v2 [math.GM] (2018)

[2] P. Solé and Y. Zhu. An Asymptotic Robin Inequality. INTEGERS, Nr.A81, 16 (2016), http://math.colgate.edu/∼integers/q81/q81.pdf

[3] K. Briggs, Abundant numbers and the Riemann hypothesis, Experiment. Math. 15 (2), 251–256 (2006).