On Geoengineering the Albedo Solution to Global Warming and Identifying Key Parameters

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Abstract

A solar geoengineering global warming model is developed with a re-radiation factor and the model is shown to be consistent with the Planck parameter. The re-radiation factor is important in quantifying the relative global warming impact of the albedo effect compared to that of greenhouse gases (GHG). The potential reverse forcing due to a change in the Earth's global albedo compared to GHGs is illustrated. Results of modeling support solar geoengineering solutions with two key parameters from modeling: an albedo-GHG and a Planck-Albedo feedback parameter. Using these, it is concluded that a 1.5% solar geoengineering change in the global albedo could result in a significant resolution to global warming. We also discuss feasibility.

1 Introduction

Solar geoengineering is vital in global warming as results can reverse trends and reduce the probability of a tipping point from occurring. In this paper, a geoengineering model that uses a re-radiation factor, which helps to quantify differences between changes in the global albedo versus greenhouse gas forcing, is developed. The re-radiation parameter is initially obtained in the absence of warming feedbacks with a unique value of 0.612 (or β=0.887). The re-radiation factor is a redefined variable taken from the effective emissivity constant of the planetary system. An application of the model is provided between two different time periods (1950 and 2019). In 2019, the re-radiation parameter takes GHG change and feedback effects into account. Then, the Planck feedback parameter is used to verify model consistency. The model illustrates a reasonable way to view the Earth's energy budget; simplifies estimates without the need for doubling theory, provides a number of useful insights in climatology sensitivity estimates and provides practical solar geoengineering calculation for global warming mitigation [1]. Specifically, a 1.6 albedo-GHG factor along and a Planck-Albedo parameter (having a value of $1\text{W/m}^2/^{\text{o}}\text{K}/\Delta\%\text{albedo}$) is obtained in modeling results. These values greatly simplify solar geoengineering [2, 3] calculations. Using these values, we exemplify a global warming albedo solution and discuss feasibility [1].

2. Data and Method

To introduce the re-radiation engineering model, we will often refer to the Planck parameter and its associated variables that play a key role in development and verifying this model. Therefore, we provide an overview in Appendix A which also includes a unique way to assess its value using an albedo approach (see A.1).

2.1 The Re-radiation Global Warming Model

In geoengineering, we are working with absorption, we define

$$P_{Total} = \sigma T_S^4 = \sigma\left(\frac{T_{TOA}}{\beta}\right)^4 \text{ and } P_\alpha = \sigma T_\alpha^4 = \sigma\left(\beta T_S\right)^4 \tag{1}$$

The definitions of T_{TOA}, T_S and β are provided in Appendix A (Eq. A-1, A-2, A-3). We consider a time when there is ***no feedback issues*** causing warming trends. Then by conservation of energy, the equivalent power re-radiated from GHGs in this model is dependent on P_α

$$P_{GHG} = P_{Total} - P_\alpha = \sigma T_S^4 - \sigma T_\alpha^4 \tag{2}$$

To be consistent with Eq. A-1, $T_\alpha = T_{TOA}$, since typically $T_\alpha \approx 255^{\text{o}}\text{K}$ and $T_s \approx 288^{\text{o}}\text{K}$, then in keeping with a common definition of Beta (see Eq. A-4) for the moment $\beta \approx T_\alpha/T_s \approx T_{TOA}/T_S$.

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This allows us to write the dependence

$$P_{GHG} = \sigma T_S^4 - \sigma T_\alpha^4 = \frac{\sigma T_\alpha^4}{\beta^4} - \sigma T_\alpha^4 = \sigma T_\alpha^4 \left(\frac{1}{\beta^4} - 1 \right) \tag{3}$$

We note that when $\beta^4 = 1$, there are no GHG contributions as required. We now define a re-radiation parameter $f = \beta^4$. We know that some fraction of the blackbody radiation is re-radiated by the GHGs, so f is a re-radiation parameter. That is, the energy, P_{GHG}, must be some fraction of P_α. Consider this is

$$P_{GHG} = f\,P_\alpha = f\,\sigma T_\alpha^4 \tag{4}$$

This we will see is consistent with Eq. 3. Once absorption occurs, initial temperature rise occurs to the Earth, and then part of this energy is reradiated back to Earth by GHGs. It is important in geoengineering to view this as part of the albedo effect. This is a key difference in how we view the total effect from short wavelength absorption by inclusion of re-radiation [2]. Now in order to have consistency in f, we require from Equations 3 and 4

$$P_{GHG} = \sigma T_\alpha^4 \left(\frac{1}{f} - 1 \right) = f\,\sigma T_\alpha^4 \tag{5}$$

This dependence leads us to the solution of the quadratic expression

$$f^2 + f - 1 = 0 \text{ yielding } f_1 = 0.618034 = \beta^4,\ \beta = (0.618034)^{1/4} = 0.88664 \tag{6}$$

This is very close to the common value estimated for β (Appendix A) and this has been obtained through energy balance in the planetary system providing a self-determining assessment. In Section 2.3, we double check this model in another way by balancing energy. Then in Section 3 we will apply the modeling to demonstrate its capability and consistency with the Planck parameter.

2.2 ***Re-radiation Model Applied to Two Different Time Periods***

Global warming can be modeled by looking at two different time periods. We can model the radiation for 1950 consistent with our model in Eq. 2 and 4

- we will assume no feedback issues causing a warming trend in 1950 so that from our model

$$P_{Total\_1950} = P_\alpha + P_{GHG} = P_\alpha + f_1 P_\alpha = P_\alpha (1 + f_1) = 1.618\,P_\alpha \tag{7}$$

where $P_\alpha = S_o\{0.25x(1 - Albedo)\}$ and S_o=1361W/m^2. We can use the value 1.618 (Eq. 6), since we make the assumption of no changes in GHG and feedback issues in 1950. This makes it a reasonable reference number for geoengineering estimates. We can term this as a 1950 albedo-GHG value. Since its value is related to the re-radiation parameter, it changes due to variations in our climate system. However, its 1950 value in our equilibrium model is constrained by the energy balance discussed in Section 2.3 and Eq. 5.

In 2019 due to global warming trends, this model is more complex and harder to separate out terms. However, we choose to model it similarly as results will illustrate justification where

$$P_{Total\,2019} = P_{\alpha'} + P_{GHG'+Feedback} \approx P_{\alpha'} + f_2 P_{\alpha'} \tag{8}$$

Here, $P_{GHG'+Feedback}$ includes GHGs and its increase with feedbacks such as water-vapor concentration, lapse rate effect and other changes including increase in snow-ice albedo variations that are hard to separate out. That is, some of this feedback is related to GHG forcing increases and some is related to albedo change. $P_{\alpha'}$ represents the 2019 point in time with its albedo due to prior changes in UHI absorption, cloud absorption, ice and snow melting, and so forth that can be discerned. The model does not demand rigid accountability in its application (see Sec.3). We note that f, a measure of the emissivity, is ***not*** constant, but must change since the amount of GHGs changes.

To be clear, f is just a fractional parameter related to the emissivity. In 1950 it was a function of the GHGs (with no feedbacks) and is consistent with the estimates for beta. In 2019, it is more complex and according to Eq. 8, must include feedbacks. The value f_2 while close to the beta value in Eq. 6, is no longer identical ($f_1=\beta$) as in 1950 (see Equations 15 and 16). The value f_2 can also be assessed relative to f_1 as described in Section 2.3.2. However, in general, $P_{GHG} \approx P_{GHG'+Feedback}$ (see results in Section 3).

2.3 Energy Balance

Although f_1 has been uniquely defined in Eq. 6, this should also result from balancing the energy in and out of the global system.

2.3.1 Balancing P_{out} and P_{in} in 1950

To balance the energy in 1950 with no global warming imbalance, we can still start with Eq. 7. In equilibrium the radiation that leaves must balance P_α, the absorption energy so that

$$\begin{aligned} Energy_{Out} &= (1-f_1)P_\alpha + (1-f_1)P = (1-f_1)P_\alpha + (1-f_1)\{P_\alpha + f_1 P_\alpha\} \\ &= 2P_\alpha - f_1 P_\alpha - f_1^2 P_\alpha = Energy_{In} = P_\alpha \end{aligned} \tag{9}$$

This is consistent, so that in 1950 Eq. 9 requires the same quadratic solution as Eq. 6. It is apparent that

$$P_\alpha = f_1 P_{Total\_1950} = \beta_1^4 P_{Total\_1950} \tag{10}$$

or

$$P_\alpha = f_1(P_\alpha + f_1 P_\alpha) \;\; or \;\; 1 = f_1(1+f_1) \tag{11}$$

The RHS of Eq. 11 is Eq. 6. This illustrates f_1 from another perspective as the fractional amount of total radiation in equilibrium. As a final check, an application in Section 3, Table 1 results, will illustrate that f_1 provides reasonable results.

2.3.2 Warming Imbalance in 2019

The re-radiation parameters f_1 and f_2, are connected and from Eq. 7 and 8 we have

$$f_2 = f_1 + \left(\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_\alpha}\right) = f_1 + \Delta f \tag{12}$$

In this way f_2 is a function of $f_1=0.618$ and the differences in the global warming residuals that is identified in Eq. 12 as Δf.

3.0 Results and Discussion

Since the re-radiation parameter is fixed for $f_1=0.618$, to obtain $T_{1950}=13.89^{o}C$ ($287.038^{o}K$), the only adjustable parameter left in our model is the Earth's albedo. This value requires an albedo value of 0.3008 (see Table 1) to obtain the correct value T_{1950}. This albedo numbers is reasonable and similar to values cited in the literature [4].

In 2019, the average temperature of the Earth is $T_{2019}=14.84^{o}C$ ($287.99^{o}K$). Here we are not sure of the albedo value since it likely changed due to UHI increase, snow and ice melting and cloud coverage changes. The IPCC value in AR5 [5] is 0.294118 (100/340) is given in AR5 [6]. However, this would represent a 3% change in our model since 1950 which may be an overestimation. In our assessment, we will assume a low middle value of about 1.2% change. Another reason for this choice will become apparent in the resulting analysis. Then, the f_2 parameter is adjusted to *0.6311* to obtain T_{2019}. Table 1 summarizes model results for the specified albedos and observed Earth's surface temperatures. The results yield $P_{Total\ 1950}=384.935\ W/m^2$ and $P_{Total\ 2019}=390.055$ W/m^2.

Table 1 Model results

Year	**T(^{o}K)**	**T_α(^{o}K)**	***f_1, f_2***	**α, α'**	**P_α, $P_{\alpha'}$ (W/m^2)**	**$P_{GHG'+feedback}$ P_{GHG} (W/m^2)**	**P_{Total} (W/m^2)**
2019	287.991	254.83	0.63114	29.719	239.131	150.925	390.056
1950	287.041	254.51	0.6180	30.08	237.903	147.032	384.935
Δ2019-1950	**0.95**	0.328	**1.311%**	0.361 **(1.2%)**	**1.228**	3.893	**5.12**

From Table 1

$$\Delta P_{Total} = P_{2019} - P_{1950} = 5.121 W/m^2 \tag{13}$$

and

$$\Delta T_{Total} = T_{2019} - T_{1950} = 0.95°C \tag{14}$$

as modeled.

3.1 Showing Model Consistency with the Planck Parameter

To show model consistency, the forcing change, 5.121 W/m^2, resulting in a 0.95^{o}K rise, should agree with what is expected when using the Planck feedback parameter.

In order to show model consistency, we will need some exact values for beta using the temperatures in Table 1, these are from the two different time periods (see Eq. A-3)

$$\beta_{1950} = \frac{T_\alpha}{T_S} = \frac{T_{TOA}}{T_S} = \frac{254.51}{287.04} = 0.88667 \ and \ \beta_{1950}^4 = 0.61809 \tag{15}$$

and

$$\beta_{2019} = \frac{T_\alpha}{T_S} = \frac{T_{TOA}}{T_S} = \frac{254.83}{287.99} = 0.88485 \ and \ \beta_{2019}^4 = 0.61304 \tag{16}$$

Although these are very close, we need both values due to the need for high accuracy it needs to be self-consistent with what was found in our model at each time period.

From Equation A-4 in the appendix, we note the Planck parameter from Table 1 can be estimated

$$\lambda_o = -4\frac{\Delta R_{LWR}}{T_S} = -4\left(\frac{237.9W/m^2}{287.04°K}\right)_{1950} = -3.315W/m^2/°K \tag{17}$$

and

$$\lambda_o = -4\frac{\Delta R_{LWR}}{T_S} = -4\left(\frac{239.13W/m^2}{287.99°K}\right)_{2019} = -3.321W/m^2/°K \tag{18}$$

We note these are very close in value showing miner error and reasonable consistency with Planck parameter value, often take as 3.3W/m^2/^{o}K. To finalize our task, we need to use the values found in the model. That is, while there are only small differences between the two time periods for each beta and the two Planck parameters, final warming predictions using a Planck parameter method, requires using values found from the model. This provides needed self-consistency within the model to obtain accuracy for ΔT and reduce compounding error. We then use the generalized form of Eq. 10 (with beta) for the long wavelength estimate in Equation A-4, yielding the warming change in terms of the power and the Planck parameter as

$$\Delta T = T_{1950} - T_{2019} = -4\left\{\left(\frac{\beta^4 P_{Total}}{\lambda_o}\right)_{1950} - \left(\frac{\beta^4 P_{Total}}{\lambda_o}\right)_{2019}\right\} \tag{19}$$

Using Table 1 the temperature warming results obtained with the aid of the Planck parameter and the change in the total energy between the two different time periods is

$$\Delta T = -4\left(\frac{0.6181x384.935W/m^2/°K}{3.315W/m^2/°K} - \frac{0.61304x390.056W/m^2/°K}{3.3215W/m^2/°K}\right) = 0.947°K \tag{20}$$

This equation illustrates consistency of the re-radiation model with the Planck parameter showing surprising accuracy.

3.1 Re-radiation Parameter Discussion

In Table 1, the measure of *Δf=1.45%* fractional increase is mainly due to re-radiation change and associated feedbacks. This is significant. From Eq. 7, 8 and 12 we can illustrate this key characteristic of climate change since

$$\Delta f = f_2 - f_1 = (\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_\alpha}) = (\frac{P_{GHG'+F}}{P_{\alpha'}} - \frac{P_{GHG}}{P_\alpha}) \tag{21}$$

Therefore, *f* is an estimate of climate re-radiation and *Δf* an estimate of climate emissivity change and confounded with feedback effects. It is a measure of GHG forcing increase and the feedback relative to the initial 1950 radiation, and is generally helpful in looking at how our climate is working.

3.2 Comparisons Using the Albedo-GHG Factor

We can look at an important ratio, the power created by the albedo effect compared to GHGs in 1950. The initial radiation is P_α which heats the Earth to 254.51°K, and then according to Eq. 7 and Table 1, the energy increased by P_{GHG} is due to re-radiation fP_α and the ratio is

$$\left\{\frac{P_\alpha + P_{GHG}}{P_{GHG}} = \frac{P_\alpha + f_1 P_\alpha}{f_1 P_\alpha} = \frac{1+f_1}{f_1} = \frac{1.62}{0.62} = 2.62\right\}_{1950} \quad and \quad \left\{\frac{1+f_2}{f_2} = 2.58\right\}_{2019} \tag{22}$$

We note the ratio is reduced in 2019 as P_{GHG} increases along with feedbacks with re-radiation increases. If f could approaches a catastrophic value of unity, this ratio reduces to a minimum of 2.

In this engineering view, we can look at a change in albedo forcing compared with a change in GHGs. The variation in the energy due to an average albedo change and its re-radiation is

$$\Delta P_{\overline{\alpha}} = \Delta P_{\alpha'} + f_2 \Delta P_{\alpha'} = 1.631\ \Delta P_{\alpha'} \tag{23}$$

The average change in GHGs can be written in terms of *Δf*

$$\Delta P_{\overline{GHG'}} = \Delta f\ P_{GHG'} = 1.311\%\left(f_2\, P_{\alpha'}\right) = 0.827\% P_{\alpha'} \tag{24}$$

This resulting ratio is from Table 1

$$\frac{\Delta P_{\overline{\alpha}}}{\Delta P_{\overline{GHG'}}} = \frac{\Delta P_{\alpha'}}{\Delta f}\frac{(1+f_2)}{P_{\alpha'}\, f_2} = \frac{1.228 W/m^2}{0.0131}\frac{1.631}{239.1 W/m^2\ 0.631} = 1.01 \tag{25}$$

Note that this ratio is of course dependent on the 2019 albedo 1.2% change, selected here to obtain unity for illustrative purposes. The ratio, $\Delta P_{\alpha'}/\Delta f_\alpha$, is an interesting aspect of climate change. In 2019, if we have knowledge of values, we can compare the dominant aspect of the warming trend. It also provides us with a measure of solar reversibility

$$\Delta P_{\alpha'} \geq \Delta f\ \frac{P_{\alpha'}\, f_2}{(1+f_2)} 1.02 \approx 1.21 W/m^2 \tag{26}$$

This ratio is dependent on the change in the albedo compared with a GHG change. This does not include the potential for a transient climate response (TCR). It is perhaps not the best way to assess geoengineering estimates. True values of $\Delta\alpha$ and Δf are not easily obtained in 2019. However, it avoids CO_2 doubling estimates, which are also difficult to evaluate. Furthermore, in some instances, a ΔP_α change, create excess GHGs. This has been a concern with cool roofs in the winter. We can estimate similarly as in Eq. 26, weather such a change is beneficial by comparison.

It is important to simplify things further to provide a more productive approach. In reverse solar geoengineering of a global warming solution, it is helpful to have simple reliable values. In this view, the 1.6 albedo-GHG factor (which is reasonably accurate) is an important engineering number. It provides one of the significant values needed in reverse albedo forcing that takes into account the initial absorption change followed by re-radiation. Another important engineering value is described by a Planck-albedo parameter.

3.3 Planck-Albedo Parameter and a Simplified Reverse Forcing Solution

The albedo changes and ΔP_α in Table 1, are: $\%\Delta\alpha = 1.6\%$ and 1.638W/m^2, respectively. We note that we can define a unique Planck-albedo parameter $\lambda_{\%\Delta\alpha} = \Delta P_\alpha / \%\Delta albedo$. To illustrate from Table 1

$$\lambda_{\%\Delta\alpha} = 1.024\ \mathrm{W/m^2/\Delta\%albedo} \tag{27}$$

This parameter can also be expressed per degree (noting the 0.95°K change in Table 1)

$$\lambda_{\%\Delta\alpha\Delta T} \approx 1W/m^2/\Delta\%albedo/°K \tag{28}$$

The helpful parameter [3] is featured here as a modeling tool. We term it the Planck-albedo parameter, since it relates to blackbody (P_α) absorption. A simple numeric example is given in the conclusion to illustrate how it provides helpful estimates along with the albedo-GHG factor. This interesting parameter simplifies from the basic assessments of the two different time periods (see also Eq. A-8) as

$$\lambda_{\%\Delta\alpha} = \frac{(\Delta E_o)_\alpha}{\frac{\alpha_1-\alpha_2}{\alpha_1}100} = \frac{E_o(\alpha_1-\alpha_2)}{\frac{\alpha_1-\alpha_2}{\alpha_1}100} = E_o\alpha_1/100 \approx 1W/m^2/\%\Delta albedo \tag{29}$$

where E_o=340 W/m^2 and when α_1 is 0.294118, the value 1.000W/m^2/Δ%albedo is obtained. We note the value 29.4118% (100/340) is given in AR5 [6]. The parameter's relationship to λ_α is

$$\lambda_\alpha = \lambda_{\%\Delta\alpha\Delta T}\ x\ \%\Delta\alpha \tag{30}$$

and appropriate feedback parameters could including the re-radiation albedo-GHG factor in 2019 [2], for example

$$\lambda_\alpha^\dagger = \lambda_{\%\Delta\alpha\Delta T}\ x\ \%\Delta\alpha\ \ (1+f_2) \tag{31}$$

The albedo-GHG and the Planck-Albedo feedback parameter may be combined in order to provide a simple solar geoengineering solution estimate

$$\Delta P_{Rev\_S} = -\lambda_{\%\Delta\alpha\Delta T}\ \ \%\Delta\alpha\ (1+f_2)\ A = \Delta P_T\,(1+f_2)\ A \tag{32}$$

and from A-14 $\Delta P_{Rev\_LWR} = \beta^4 \Delta P_{Rev\_S}$ the temperature reduction is

$$\Delta T_{Rev} = -\Delta P_{Rev\_LWR}\frac{1}{\lambda_o} \tag{33}$$

Here ΔP_{Rev} is the reverse forcing, A is an estimate of the anticipated GW amplification reduction, and ΔP_T is the reverse forcing from the target area. The equation provides a fairly simple and practical way to estimate ΔP_{Rev}. An example is provided in the conclusion**.** In solar geoengineering, it may be reasonable to not consider a calculation allowance for the climate system to equilibrate [13] since this works both ways.

4.0 Conclusion

In this paper, we provided a re-radiation global warming model. The model shows consistency with the Planck parameter. We noted that the re-radiation parameter increased by about 1.45% due to global warming from 1950 to 2019, illustrating the warming from a different perspective. From the model, a helpful albedo-GHG parameter was quantified having a value of 1.6.

We also found an engineering factor that we termed the Planck-albedo parameter, which is about $\lambda_{\%\Delta\alpha\Delta T} \approx 1W/m^2/\Delta\%albedo/°K$. These findings can be helpful in quickly estimating the effect of an albedo change on global warming and in assessing λ_α. These results support solar geoengineering solutions [3, 7-9].

For example, Feinberg 2020 [2] suggested a goal of 1.5% geoengineering albedo change. Using Equation 30, with a decrease in water-vapor feedback anticipated, we might use a value of A≈2 [10], then

$$\Delta P_{Rev\_S} = -1\text{W/m}^2\text{/\% x 1.5\% x }(1+\text{f}_2)\text{ x 2} = -4.8\text{ Watt/m}^2 \tag{33}$$

One can multiply this by β^4 to compare with IPCC models or to relative to our results in Table 1 with a forcing of 5.12 W/m^2. Equation 31 expressed in terms of reverse temperature warming results is then

$$\Delta T_{Rev} = -0.90°K \tag{34}$$

This would indicate a significant resolution to the current warming trend. As one might suspect, a 1.5% albedo change requires a lot of modified area. Feasibility is discussed in more detail in Feinberg's 2020 [2]. Results of Feinberg [2] indicate the required area of change, if proper hotspots are targeted, is 3.4-17 times smaller than the estimates of the current urbanization area. Other solar geoengineering solutions have been proposed [7-9].

Appendix A

Overview of Planck Feedback Parameter

Estimates on the Planck feedback parameter are varied, typically between $-3.8\mathrm{W/m^2/^oK}$ and $-3.21\mathrm{W/m^2/^oK}$ with some values as large as $-7.1\mathrm{W/m^2/^oK}$ [11]. The IPCC AR4 [12] lists a value of $-3.21\mathrm{W/m^2/^oK}$. Numerous authors have developed different expressions [11]. A typical estimate starts with

$$F_{TOA} = (1-\alpha)\,S_o/4 - \sigma(\beta T_S)^4 = (1-\alpha)\,S_o/4 - R_{LWR} \tag{A-1}$$

where $S_o=1361\mathrm{W/m^2}$, F_{TOA} is the radiation budget at the top of the atmosphere, R_{OLW} is the outgoing long wave radiation (a function of surface temperature and albedo), σ is the Stefan-Boltzmann constant and β is described in this section below and is redefined in terms of a re-radiation parameter in this paper. Then the Planck parameter λ_o can be calculated as

$$\lambda_o = \partial F_{TOA}/\partial T_S = -\partial R_{OLW}/\partial T_S \tag{A-2}$$

This result is

$$\lambda_o = -4\beta^4\sigma T_{S}^3 = -4\beta\sigma T_{TOA}^3 = -\frac{4R_{OLW}}{T_S} \tag{A-3}$$

where β varies in the literature from 0.876 to 0.887 (averaging=0.8815) and $T_s=288^o\mathrm{K}$ [12]. This yields $-3.37\mathrm{W/m^2/^oK}<\lambda_o<-3.21\mathrm{W/m^2/^oK}$. However, from Eq. A-3, β is often taken as the ratio

$$\beta = T_{TOA}/T_{S} = 255°K/288°K = 0.8854 \text{ and } \beta^4 = 0.615 \tag{A-4}$$

A common assessment uses $T_{TOA}=255^o\mathrm{K}$, so that $\lambda_o = -3.33\mathrm{W/m^2/^oK}$. Another expression developed by Schlesinger [6] is dependent on the albedo and surface temperature as

$$\lambda_o = S_o\left(1-\alpha\right)/T_S \tag{A-5}$$

When $S_o=1361$, $0.294118<\alpha<0.3$, and $T_s=288\,^o\mathrm{K}$ then $-3.308\mathrm{W/m^2/^oK} > \lambda_o > -3.3358\mathrm{W/m^2/^oK}$, respectively.

A.1 Estimating the Planck Parameter with an Albedo Method

Consider a global albedo change corresponding to $1^o\mathrm{K}$ rise from solar absorption letting

$$F_{TOA} = 0 = (1-\alpha)E_o - \sigma(T_S)^4 \tag{A-6}$$

where $E_o=S_o/4$. Then a $1^o\mathrm{K}$ change is

$$\Delta T_S = T_2 - T_1 = \left(\frac{E_o}{\sigma}\left(1-\alpha_2\right)\right)^{1/4} - \left(\frac{E_o}{\sigma}\left(1-\alpha_1\right)\right)^{1/4} = 1°K \tag{A-7}$$

Here we will use the AR5 albedo starting value of 0.294118 [6]. We find that the corresponding albedo change is 0.28299 when $E_o=340\mathrm{W/m^2}$. This corresponds to

$$\Delta E_o = E_o\left\{\left(1-\alpha_2\right)-\left(1-\alpha_1\right)\right\} = E_o\left(\alpha_1-\alpha_2\right) = 3.784W/m^2 \tag{A-8}$$

Since this is for a $1^o\mathrm{K}$ rise, then it can also be written as

$$\lambda_{1K}=3.784\mathrm{W/m^2/^oK} \tag{A-9}$$

We note this is related to the surface value, then

$$\lambda_{1K} = -4\sigma T_{S}^3 \tag{A-10}$$

By comparison to above we have

$$\lambda_o=\lambda_{1K}\,\beta = -3.784\mathrm{W/m^2/^oK}=-3.349\mathrm{W/m^2/^oK} \tag{A-11}$$

This is very close to the -3.33 $W/m^2/^oK$ value obtained in the traditional manner.

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